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27 September 1999

PHYSICS LETTERS A

Physics Letters A 260 (1999) 441–446

www.elsevier.nl/locate/physleta

General equation for Zeno-like effects in spontaneous exponential decay

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Received 20 July 1999; accepted 30 July 1999

Communicated by V.M. Agranovich

Abstract

Different perturbation mechanisms of the spontaneous decay constant are shown: the inelastic interaction of emitted particles with particle detector, decay onto an unstable level, Rabi transition from the final state of decay (electromagnetic field domination) and some others are really the special kinds of one general effect – the perturbation of the decay constant by dissipation of the final state of decay. Such phenomena are considered to be Zeno-like effects and the general formula for the perturbed decay constant is deduced. © 1999 Published by Elsevier Science B.V. All rights reserved.

PACS: 03.65.Bz

Keywords: Measurement theory; Zeno effect; Dynamical description; Spontaneous decay

1. Introduction

The term 'quantum Zeno paradox' was introduced in [1,2]. It was argued there that an unstable particle which is continuously observed to see whether it decays will never be found to decay. Analogous ideas were also discussed in some earlier works [3,4] (for a review see [5]). Continuous observations (or measurements) of a system were described phenomenologically in [1–4], as a sequence of very frequent instantaneous collapses of the system's wave function. Later the idea of a quantum Zeno paradox had been developed in two main directions. First, the idea was applied to forced transitions of the Rabi type between discrete levels [6] and was experimentally proved in this form [7]. As a result the quantum

Zeno paradox is considered now as a real event (quantum Zeno effect, QZE), not as a paradox. Second, it was recognized that a phenomenological description of the QZE (using a projection postulate and wave function collapses) is not necessary. Dynamical considerations of the QZE were presented [5,8–15] and it was shown that the main features of the QZE found earlier may be reproduced. It was also argued that a dynamical consideration of the QZE could show some essentially new features of this phenomenon [13].

The situation of continuous observation of spontaneous exponential decay is especially interesting. It was argued that it is impossible to observe QZE in spontaneous decay [16,8,6]. But using a dynamical approach to QZE, it was speculated [13–15] that perturbations in the decay rate could in principle occur and, moreover, they could be strong [13]. In each of the three papers [13–15] mechanisms of the

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perturbation of the decay rate seems to be quite different. In [13] the inelastic interaction of emitted particles with a particle detector was analyzed. In [14] the forced electromagnetic transition from the final discrete state of the decay to another (third) discrete state was studied. And in [15] the spontaneous decay of the final discrete state to another discrete state was considered. Three different formulae describing QZE were deduced in [13–15] for these three different mechanisms. But there are common points in all these processes. A measurement mechanism has no direct influence on the initial state of the system (undecayed particle) in all cases. But as soon as the system comes to the final state of the decay, it starts to interact with some outer systems (devices or fields) that rapidly leads to the destruction of the final state of the decay. The final state of decay means the united state of decaying system with outer systems. Consequently, both changing the outer systems states and changing the decaying system states after decay is a destruction of the final state. In the present paper we show that all three perturbation mechanisms of the decay rate described in [13–15] are special kinds of a general mechanism, connected with a destruction of the final decay state, and we deduce the general formula for it. Then the formulae of [13–15] turn out to be the special cases of our new general formula.

2. General formula for Zeno-like effects

Let's consider some quantum system S , being in the initial state $|\Psi_0\rangle$ at the time $t=0$. Undisturbed Hamiltonian of the system S is H_0 , $H_0|\Psi_0\rangle = \mathcal{E}_0|\Psi_0\rangle$ where \mathcal{E}_0 is the initial eigenenergy. Under influence of perturbation V , the system transits from the initial discrete state $|\Psi_0\rangle$ to continuum of states $\{|\xi\rangle\}$ which is orthogonal to $|\Psi_0\rangle$. We consider that
$$V = \sum_{\xi} (|\xi\rangle\langle\Psi_0|v(\xi) + |\Psi_0\rangle\langle\xi|v^*(\xi)),$$

where $v(\xi) = \langle\xi|V|\Psi_0\rangle$ is the matrix element for the transition. We can suppose without restriction of generality that $\langle\Psi_0|V|\Psi_0\rangle = 0$. We suppose also that the perturbation V is time-independent and small, thus the transition $|\Psi_0\rangle \rightarrow \{|\xi\rangle\}$ may be approximated by a spontaneous exponential decay for sufficiently large times.

Let's suppose that in addition to the small perturbation V there exist another interaction Hamiltonian $W(t)$, which is not small and has dependence on time. Thus, the full Hamiltonian of the system S is $H = H_0 + V + W(t)$. The interaction $W(t)$ has the feature

$$W(t)|\Psi_0\rangle = 0,$$

i.e. it does not influence the initial state of the system. But $W(t)$ may cause a transition from subspace $\{|\xi\rangle\}$ to another subspace $\{|\eta\rangle\}$ which is orthogonal to both $|\Psi_0\rangle$ and $\{|\xi\rangle\}$. Let Γ be the decay constant of the state $|\Psi_0\rangle$. So, what is perturbed value of Γ if we take into account the interaction $W(t)$?

We find the no-decay amplitude

$$F(t) = \langle\Psi_0|\Psi(t)\rangle e^{i\mathcal{E}_0 t} \quad (\hbar = 1). \quad (1)$$

To find it we solve the Shrödinger equation

$$\begin{aligned} |\dot{\Psi}(t)\rangle &= -i(H_0 + V + W(t))|\Psi(t)\rangle; \\ |\Psi(0)\rangle &= |\Psi_0\rangle. \end{aligned} \quad (2)$$

We cannot apply the perturbation theory for $W(t)$, because this interaction is not small, but we can do this with respect to V . First let's consider the Shrödinger equation for the Hamiltonian without interaction V :

$$|\dot{\Psi}(t)\rangle = -i(H_0 + W(t))|\Psi(t)\rangle \quad (3)$$

and let the solution of Eq. (3) be

$$|\Psi(t)\rangle = U(t, t_1)|\Psi(t_1)\rangle.$$

Let's introduce the interaction picture as

$$|\Psi_1(t)\rangle = U^+(t, 0)|\Psi(t)\rangle.$$

Then Eq. (2) may be rewritten as

$$|\dot{\Psi}_1(t)\rangle = -iV_1(t)|\Psi_1(t)\rangle, \quad |\Psi_1(0)\rangle = |\Psi_0\rangle, \quad (4)$$

where $V_1(t)$ is the potential V in the interaction picture:

$$V_1(t) = U^+(t, 0)VU(t, 0).$$

In the second order of perturbation theory we easily find an equation for derivation of no-decay amplitude from Eq. (4):

$$\frac{dF}{dt} = -\int_0^t dt_1 \langle\Psi_0|V_1(t)V_1(t_1)|\Psi_0\rangle. \quad (5)$$

Let ω_ξ be the eigenenergy of the state $|\xi\rangle$: $H_0|\xi\rangle = \omega_\xi|\xi\rangle$. It is not difficult to show that the matrix element under the integral in Eq. (5) may be rewritten as

$$\langle\Psi_0|V_1(t)V_1(t_1)|\Psi_0\rangle = \sum_{\xi} e^{i(\mathcal{E}_0 - \omega_\xi)(t-t_1)} |v(\xi)|^2 D(t, t_1), \quad (6)$$

where we have introduced the *dissipation function*:

$$D(t, t_1) = \frac{\langle\Psi_0|VU(t, t_1)V|\Psi_0\rangle}{\langle\Psi_0|V\exp[-iH_0(t-t_1)]V|\Psi_0\rangle}. \quad (7)$$

The dissipation function describes the dissipation of final decay states caused by the interaction $W(t)$. It is easily to see that if $W(t) = 0$ then $D(t, t_1) \equiv 1$. Let the index ξ of the state be the set of the eigenenergy ω of the state and some other quantum numbers α : $|\xi\rangle = |\omega, \alpha\rangle$. Let's introduce the function $M(\omega)$ as follows:

$$M(\omega) = \sum_{\alpha} |v(\omega, \alpha)|^2$$

and then change \sum_{ω} to $\int d\omega$. Then Eq. (6) is

$$\langle\Psi_0|V_1(t)V_1(t_1)|\Psi_0\rangle = \int d\omega M(\omega) e^{i(\mathcal{E}_0 - \omega)(t-t_1)} D(t, t_1). \quad (8)$$

For sufficiently large (but not very large) times $F(t) = \exp(-\gamma t) \approx 1 - \gamma t$, consequently $\gamma = -dF/dt$. To obtain the decay constant of the state $|\Psi_0\rangle$: $\Gamma = 2\text{Re}\gamma$ we substitute Eq. (8) in Eq. (5) and formally tend t to infinity supposing that this limit exists. We deduce

$$\Gamma = 2\pi \int d\omega M(\omega) \Delta(\omega - \mathcal{E}_0), \quad (9)$$

where the function $\Delta(\epsilon)$ is defined as

$$\Delta(\epsilon) = \frac{1}{\pi} \text{Re} \lim_{t \rightarrow \infty} \int_0^t dt_1 e^{-i\epsilon(t-t_1)} D(t, t_1). \quad (10)$$

If $W(t) = 0$, it is easily to see that $\Delta(\epsilon) = \delta(\epsilon)$, where $\delta(\epsilon)$ is usual Dirac's delta-function. Then Eq. (9) is transformed to $\Gamma_0 = 2\pi M(\mathcal{E}_0)$, i.e. to usual Fermi's Golden Rule, as one could expect. Thus, Eq. (9) is a generalization of usual Golden Rule for case of unstable final states of decay. The main difference between usual Golden Rule and Eq. (9) is that in the

first case Γ_0 is expressed through the single value of function $M(\omega)$, but in the second case Γ is expressed through convolution of $M(\omega)$ with spreaded function $\Delta(\omega - \mathcal{E}_0)$. Now we use the Eq. (9) for studying of some particular systems.

3. Detection of emitted particles

Let's suppose that some system X (for example, an atom) transits spontaneously from the initial excited state $|x_0\rangle$ to ground state $|x_1\rangle$ emitting some particle Y (a photon or an electron). We consider this particle as a separate quantum system, which is initially in the ground (vacuum) state $|y_0\rangle$ and then transits to continuum $|\omega^Y, \alpha^Y\rangle$, where ω^Y is energy of state and α^Y represents all other quantum numbers. Particle Y inelastically scatters on a third system Z (a surrounding media or a particle detector) due to time-independent interaction W . As a result, system Z transits from the initial ground state $|z_0\rangle$ to continuum $|\zeta\rangle$, and this process may be considered as registration of decay. We suppose that interaction V does not act on system Z and that interaction W does not act on system X . It is a special case of situation described in Section 2 and we can write:

$$S = X \otimes Y \otimes Z, \quad (11)$$

$$H_0 = H_0^X \otimes I_Y \otimes I_Z + I_X \otimes H_0^Y \otimes I_Z + I_X \otimes I_Y \otimes H_0^Z, \quad (12)$$

$$|\Psi_0\rangle = |x_0\rangle \otimes |y_0\rangle \otimes |z_0\rangle \equiv |x_0 y_0 z_0\rangle, \quad (13)$$

$$\mathcal{E}_0 = \omega_0^X + \omega_0^Y + \omega_0^Z, \quad (14)$$

$$V = V_{XY} \otimes I_Z; \quad W = I_X \otimes W_{YZ},$$

$$\{|\xi\rangle\} = \{|x_1\rangle|\omega^Y, \alpha^Y\rangle|z_0\rangle\};$$

$$\{|\eta\rangle\} = \{|x_1\rangle|\omega^Y, \alpha^Y\rangle|\zeta\rangle\},$$

$$\omega = \omega_1^X + \omega^Y + \omega_0^Z \quad (15)$$

$$U(t, t_1) = \exp[-i(H_0 + W)(t - t_1)], \quad (16)$$

where notations are obvious. Using the notations

$$H_0^{YZ} = H_0^Y \otimes I_Z + I_Y \otimes H_0^Z,$$

$$|\tilde{y}\rangle = \int_{\epsilon} d\omega^Y \sum_{\alpha^Y} v(\omega^Y, \alpha^Y) |\omega^Y, \alpha^Y\rangle,$$

one can obtain the expression for the dissipation function from Eq. (7):

$$D(t, t_1) \equiv D_s(t - t_1) = \frac{\langle \tilde{y}_{z_0} | e^{-i(H_0^{YZ} + W_{YZ})(t-t_1)} | \tilde{y}_{z_0} \rangle}{\langle \tilde{y}_{z_0} | e^{-iH_0^{YZ}(t-t_1)} | \tilde{y}_{z_0} \rangle} \quad (17)$$

and the expression for the function $\Delta_s(\epsilon)$ from Eq. (10):

$$\Delta_s(\epsilon) = \frac{1}{\pi} \text{Re} \int_0^\infty D_s(\tau) e^{-i\epsilon\tau} d\tau = \frac{1}{2\pi} \int_{-\infty}^{+\infty} D_s(\tau) e^{-i\epsilon\tau} d\tau. \quad (18)$$

The subscript s is an abbreviation of ‘scattering’. As one can see from Eq. (15), $M(\omega)$ in Eq. (9) depends on ω^Y only. Thus, we can rewrite Eq. (9) as

$$\Gamma = 2\pi \int d\omega^Y M(\omega^Y) \Delta_s(\omega^Y - \omega_f^Y), \quad (19)$$

where $\omega_f^Y = \omega_0^Y + \omega_0^X - \omega_1^X = \omega_0^Y + \omega_{01}$ is the expected value of the final energy of particle Y in accordance with the energy conservation law. The formulae (19), (18), (17) show the perturbed value of decay rate for considered problem and coincide with the formulae (26), (27) and (18) of [13] for the same case. Further analysis of this formulae can be found in [13].

4. Decay onto unstable level

Let's consider three-level system X which makes a cascade transition from the initial state $|x_0\rangle$ to the state $|x_1\rangle$ and then to the state $|x_2\rangle$ with emission of two particles Y and Z , respectively. So, what is the influence of instability of state $|x_1\rangle$ on decay constant of the state $|x_0\rangle$? Again, it is a particular case of general situation described in Section 2. We consider particles Y and Z as separate systems which are in the initial vacuum states $|y_0\rangle$ and $|z_0\rangle$ at the moment $t = 0$ and then they transit to continuum $\{|\omega^Y, \alpha^Y\rangle\}$ and $\{|\omega^Z, \alpha^Z\rangle\}$, respectively. The transition $|x_0\rangle \rightarrow |x_1\rangle$ is caused by interaction $V = V_{XY} \otimes I_Z$, and transition $|x_1\rangle \rightarrow |x_2\rangle$ is caused by interaction $W = W_{XZ} \otimes I_Y$, where

$$V_{XY} = \int d\omega^Y \sum_{\alpha^Y} |x_1\rangle |\omega^Y, \alpha^Y\rangle \langle x_0| \langle y_0| v(\omega^Y, \alpha^Y) + \text{E.C.} \quad (20)$$

$$W_{XZ} = \int d\omega^Z \sum_{\alpha^Z} |x_2\rangle |\omega^Z, \alpha^Z\rangle \langle x_1| \langle z_0| w(\omega^Z, \alpha^Z) + \text{E.C.} \quad (21)$$

It is seen from Eqs. (20) and (21) that V causes transition $|x_0\rangle \rightarrow |x_1\rangle$ only and W causes transition $|x_1\rangle \rightarrow |x_2\rangle$ only. This process is characterized by relations

$$\{|\xi\rangle\} = \{|x_1\rangle |\omega^Y, \alpha^Y\rangle |z_0\rangle\};$$

$$\{|\eta\rangle\} = \{|x_2\rangle |\omega^Y, \alpha^Y\rangle |\omega^Z, \alpha^Z\rangle\}$$

and by a number of relations, coinciding with Eqs. (11) and (12), (13), (14), (15), (16).

It is not difficult to show that the dissipation function Eq. (7) now has the form

$$D(t, t_1) \equiv D_u(t - t_1) = \langle x_1 z_0 | e^{-i(H_0^{XZ} + W_{XZ})(t-t_1)} | x_1 z_0 \rangle e^{i\mathcal{E}_0^{XZ}(t-t_1)}, \quad (22)$$

where $H_0^{XZ} = H_0^X \otimes I_Z + I_X \otimes H_0^Z$ and $\mathcal{E}_0^{XZ} = \omega_1^X + \omega_0^Z$. The subscript ‘u’ is an abbreviation of ‘unstable’. We see from Eq. (22) that the dissipation function now has a simple physical sense. This is nothing more than a no-decay amplitude of the state $|x_1\rangle$ in relation to decay under the influence of interaction W . Thus, we can use the approximation

$$D_u(\tau) = e^{-\lambda\tau}, \quad (23)$$

where λ is a complex decay constant of level $|x_1\rangle$. Let $\lambda = \lambda_r - i\lambda_i$, where λ_r and λ_i are real numbers. Then we obtain from Eq. (23) and Eq. (10)

$$\Delta_u(\epsilon) = \frac{1}{\pi} \frac{\lambda_r}{\lambda_r^2 + (\epsilon - \lambda_i)^2} \quad (24)$$

and we obtain from Eq. (9) and Eq. (24)

$$\Gamma = 2\pi \int d\omega^Y M(\omega^Y) \frac{1}{\pi} \frac{\lambda_r}{\lambda_r^2 + [\omega^Y - (\omega_f^Y + \lambda_i)]^2}, \quad (25)$$

with the same notations as in Eq. (19). For the special case $\lambda_1 = 0$, $\lambda_r \ll \omega_f^Y$, $M(\omega^Y) = \text{const}$ for $\omega^Y > 0$ we obtain the formula, similar to Eq. (20) in [15].

5. Rabi transition from final state of decay

Now we analyze the last particular case of general problem of Section 2. The situation is similar to that described in Section 4, but the instability of the state $|x_1\rangle$ is caused by a forced resonance Rabi transition to another state $|x_2\rangle$. We describe Rabi transition semiclassically by the time-dependent interaction

$$W_X(t) = \Omega(|x_1\rangle\langle x_2| + |x_2\rangle\langle x_1|)\cos\omega_{21}t,$$

where $\omega_{21} = \omega_2^X - \omega_1^X$ and Ω is the Rabi frequency. Spontaneous transition $|x_0\rangle \rightarrow |x_1\rangle$ is described in same manner as in the previous sections. We have

$$S = X \otimes Y$$

$$H = H_0^X \otimes I_Y + I_X \otimes H_0^Y + V_{XY} + W_X(t) \otimes I_Y$$

$$|\Psi_0\rangle = |x_0 y_0\rangle; \quad \mathcal{E}_0 = \omega_0^X + \omega_0^Y$$

$$\{|\xi\rangle\} = \{|x_1\rangle|\omega^Y, \alpha^Y\rangle\}; \quad \{|\eta\rangle\} = \{|x_2\rangle|\omega^Y, \alpha^Y\rangle\}$$

$$\omega = \omega_1^X + \omega^Y.$$

Based on Eq. (7), it's not difficult to show that the dissipation function is

$$D(t, t_1) = \langle x_1 | x(t) \rangle e^{i\omega_1^X(t-t_1)}, \quad (26)$$

where $|x(t)\rangle$ is the solution of equation

$$|\dot{x}(t)\rangle = -i[H_0^X + W_X(t)]|x(t)\rangle,$$

$$|x(t_1)\rangle = |x_1\rangle. \quad (27)$$

Using the rotating wave approximation we find from Eq. (27)

$$\langle x_1 | x(t) \rangle = \cos\frac{\Omega}{2}(t-t_1)e^{-i\omega_1^X(t-t_1)}. \quad (28)$$

Substituting the scalar product from Eq. (28) in Eq. (26) we obtain

$$D(t, t_1) \equiv D_R(t-t_1) = \cos\frac{\Omega}{2}(t-t_1). \quad (29)$$

The subscript *R* is an abbreviation of 'Rabi'. Using Eq. (29) and Eq. (10) we find

$$\Delta_R(\epsilon) = \frac{1}{2} \left[\delta\left(\epsilon - \frac{\Omega}{2}\right) + \delta\left(\epsilon + \frac{\Omega}{2}\right) \right]$$

and then we obtain from Eq. (9)

$$\Gamma = \pi \left[M\left(\omega_f^Y - \frac{\Omega}{2}\right) + M\left(\omega_f^Y + \frac{\Omega}{2}\right) \right]. \quad (30)$$

The resultant Eq. (30) coincides with conclusion of [14] (see [14], Eq. (2.31)). But the method by which this conclusion has been obtained in [14] significantly differs from our method. The forced transition from the level $|x_1\rangle$ (in the terms of present paper) to the level $|x_2\rangle$ was described by means of full quantum method, using quantized electromagnetic field instead of classical potential, rather than semiclassically. Instead of value $\Omega/2$ in our Eq. (30), the quantity *B* arises in Eq. (2.31) [14]:

$$B = |\Phi_0| \sqrt{N_0},$$

where Φ_0 is the transition matrix element and N_0 is the number of field quanta in resonance with $|x_1\rangle \rightarrow |x_2\rangle$ transition. But it is not difficult to show that value *B* is precisely the half of Rabi frequency. Thus, full quantum and semiclassical methods produce the same results. One can note that full quantum description of Rabi transition can be treated in the frame of general formalisms of Section 2 as well.

6. Discussion

It is easily seen from the formulae (19), (25), (30) that, in the formal limit of very fast dissipation of the final state of decay caused by interaction with environment, the spontaneous exponential decay is frozen. Really, the function $M(\omega)$ has only finite width. Very fast dissipation of the final state means that function $\Delta(\epsilon)$ becomes very wide. But $\int \Delta(\epsilon) d\epsilon = 1$ in all cases. Thus, the integral in the right hand side of Eqs. (19) and (25), (30) tends to zero as the width of function $\Delta(\epsilon)$ tends to infinity, consequently $\Gamma \rightarrow 0$. This is an expression of Zeno paradox in dynamical consideration (without using projection postulate). But dissipation of final states of decay does not necessarily cause decrease of decay

constant. When dissipation is not very strong, the behavior of decay constant may be rather complex, its behavior depends on fine features of the function $M(\omega)$. For example, if we consider realistic case $\Omega \ll \omega_{01}$ for Rabi transition from the final state of usual electromagnetic transition, the relation of disturbed to no-disturbed decay constant is

$$\frac{\Gamma}{\Gamma_0} = 1 + \frac{3}{4} \frac{\Omega^2}{\omega_{01}^2},$$

i.e. $\Gamma > \Gamma_0$. And only in the case of very fast dissipation of final state, decay constant starts to decrease.

Let's note that the decay constant perturbation by Rabi transition from final decay state (Section 5) seems to be not an usual QZE, because there are no irreversible changes in the environment following such transition, consequently there is no event of measurement. This is clearly seen from the semiclassical picture of this phenomenon. But such phenomenon is closely related to QZE and we can consider it as a Zeno-like effect. Consequently our main formula (9) describes not only Zeno effect itself, but a wide class of Zeno-like effects. There are also some other phenomena which can be described in the frame of general theory of Section 2, but have been excluded from our consideration: spontaneous oscillation in the final states of decay or combinations of different mechanisms considered above are examples of such phenomena.

Acknowledgements

The author acknowledges the fruitful discussions with M.B. Mensky. The work was supported in part by the Russian Foundation of Basic Research, grant 98-01-00161.

References

- [1] B. Misra, E.C.G. Sudarshan, *J. Math. Phys.* 18 (1977) 756.
- [2] C.B. Chiu, E.C.G. Sudarshan, B. Misra, *Phys. Rev. D* 16 (1977) 520.
- [3] A. Beskow, J. Nilsson, *Arkiv Fys.* 34 (1974) 561.
- [4] A. Degasperis, L. Fonda, G.C. Ghirardi, *Nuovo Cimento A* 21 (1974) 471.
- [5] D. Home, M.A.B. Whitaker, *Ann. Phys. (NY)* 258 (1997) 237.
- [6] R.J. Cook, *Phys. Scr.* 21 (1988) 49.
- [7] W.M. Itano, D.J. Heinzen, J.J. Bollinger, D.J. Wineland, *Phys. Rev. A* 41 (1990) 2295.
- [8] K. Kraus, *Found. Phys.* 11 (1981) 547.
- [9] A. Sudbery, *Ann. Phys. (NY)* 157 (1984) 512.
- [10] E. Joos, *Phys. Rev. D* 29 (1984) 1626.
- [11] S.M. Chumakov, K.-E. Hellwig, A.L. Rivera, *Phys. Lett. A* 197 (1995) 73.
- [12] S. Pascazio, M. Namiki, *Phys. Rev. A* 50 (1994) 4582.
- [13] A.D. Panov, *Ann. Phys. (NY)* 249 (1996) 1.
- [14] E. Mihokova, S. Pascazio, L.S. Schulman, *Phys. Rev. A* 56 (1997) 25.
- [15] M.B. Mensky, *Phys. Lett. A* 257 (1999) 227.
- [16] G.C. Ghirardi, C. Omero, T. Weber, A. Rimini, *Nuovo Cimento A* 52 (1979) 421.