ELSEVIER

17 June 2002

Physics Letters A 298 (2002) 295–300

PHYSICS LETTERS A

www.elsevier.com/locate/pla

Inverse quantum Zeno effect in quantum oscillations

Alexander D. Panov

Skobeltsyn Institute of Nuclear Physics, Moscow State University, Moscow 119899, Russia

Received 12 March 2002; received in revised form 19 April 2002; accepted 26 April 2002

Communicated by V.M. Agranovich

Abstract

It is shown that inverse quantum Zeno effect (IZE) could exist in a three-level system with Rabi oscillations between discrete atomic states. An experiment to observe IZE in such a system is proposed. © 2002 Elsevier Science B.V. All rights reserved.

PACS: 03.65.Bz

Keywords: Quantum measurement; Inverse quantum Zeno effect; Rabi oscillations

1. Introduction

It is known that frequently repeated discrete quantum measurements can hinder quantum transitions. This phenomenon is known as quantum Zeno effect (QZE) [1,2]. This effect was observed experimentally in systems with forced Rabi oscillations between discrete atomic levels [3] and in spontaneously decaying systems [4]. It was shown also that there are regimes when repeated discrete measurements can accelerate spontaneous decay [5–7], and this phenomenon was found experimentally as well [4]. This effect is known as anti-Zeno effect or inverse Zeno effect (IZE).

In the present Letter the definitions for QZE and IZE are admitted in the agreement with that introduced by Facchi and Pascazio [8] with one modification. Let the initial pure state of a system with the Hamiltonian H be ρ_0 and the survival probability be $P(t) = \mathrm{Tr}[\rho_0 \rho(t)]$. Consider the evolution of the system under the effect of an additional interaction. The total Hamiltonian reads

$$H_K = H + H_{\mathrm{meas}}(K),$$

where K is a set of parameters and $H_{\mathrm{meas}}(K = 0) = 0$. H is a full Hamiltonian of the system containing interaction terms, and $H_{\mathrm{meas}}(K)$ should be considered as an additional Hamiltonian performing the measurement. The term $H_{\mathrm{meas}}(K)$ may correspond to a chain of ideal discrete quantum measurements that are represented by reductions (collapses) of the state of the system as a special case of interaction. The system displays QZE if there exists an

E-mail address: a.panov@relcom.ru (A.D. Panov).

PII: S0375-9601(02)00577-7

interval $I^{(K)} = [t_1^{(K)}, t_2^{(K)}]$ such that

$$P^{(K)}(t) > P(t), \quad \forall t \in I^{(K)}, \tag{1}$$

and the system displays IZE if there exists an interval $I^{(K)}$ such that

$$P^{(K)}(t) < P(t), \quad \forall t \in I^{(K)}. \tag{2}$$

Here $P^{(K)}(t)$ and $P(t)$ are the survival probabilities under the action of H_K and H, respectively, and it is required

$$t_2^{(K)} \leqslant T_\mathrm{P}, \tag{3}$$

where T_P is the Poincaré time. The modification of this definitions is the following. In the addition to the definitions Eqs. (1)–(3) it is required:

(i) The measurement Hamiltonian $H_\mathrm{meas}(K)$ should be time-independent or periodical with the period less then the Poincaré time of the system.

The meaning of condition (i) is the following. Johann von Neumann proved the following proposition [9, Chapter V.2]. Using a sequence of frequently repeating measurements represented by time-dependent projections it is possible to force the quantum system pass through any arbitrary definite sequence of states. Particularly, it is possible to satisfy the conditions of Eqs. (2), (3) which define IZE. But such a situation is not actually IZE, rather it is a dynamical version of usual quantum Zeno effect (dynamical quantum Zeno effect, DQZE). Though DQZE is considered as "anti-Zeno paradox" sometimes [10], such interpretation seems to be misleading. The condition (i) is intended to avoid such misunderstands.

It was pointed out many times (see [7] and references herein) that both QZE and IZE can be obtained for a genuinely unstable system, whose Poincaré time is infinite. On the other hand, the possibility of IZE for an oscillating quantum mechanical system, whose Poincaré time is finite, was not reported up to now. Actually, only QZE is possible in two-level quantum mechanical oscillating systems. But it is not generally valid in multilevel systems with the number of levels more than two. In the present Letter a three-level oscillating system with finite Poincaré time exhibiting IZE is constructed.

2. Interaction picture for evolution interrupted by measurements

Before discussing of the main subject we introduce the interaction picture formalisms for the problem of quantum evolution interrupted by discrete measurements. Let $H = H_0 + V$ be a Hamiltonian of a system S and $\rho(t)$ be the density operator of the system. Let $\{P_i\}$, $P_i^2 = P_i$, $\sum_i P_i = 1$ be a complete set of projection operators. This set of projectors represents an instantaneous reduction of the system state following an ideal quantum measurement. Change of state during the measurement is

$$\rho' = \sum_i P_i \rho P_i \equiv \hat{R}\rho, \tag{4}$$

where ρ is the state before the measurement and ρ' is the state after the measurement. Let $D(t)\,dt$ be mean number of measurements on the system during time interval $(t, t+dt)$. Then, it is easy to prove that the state of the system is governed by the Lindblad equation

$$\frac{d\rho}{dt} = -\frac{i}{\hbar}[H_0 + V, \rho] - \frac{1}{2}D(t)\sum_i [P_i, [P_i, \rho]]. \tag{5}$$

Let $\rho_I(t)$ and $V_I(t)$ be the state and the Hamiltonian of the system in the interaction picture:

$$\rho_I(t) = \exp\left(\frac{i}{\hbar}H_0 t\right)\rho(t)\exp\left(-\frac{i}{\hbar}H_0 t\right), \tag{6}$$

$$V_I(t) = \exp\left(\frac{i}{\hbar}H_0 t\right)V\exp\left(-\frac{i}{\hbar}H_0 t\right). \tag{7}$$

Further, let

$$[P_i, H_0] = 0, \quad \forall i. \tag{8}$$

By substitution of Eqs. (6), (7) in Eq. (5) and accounting for Eq. (8) it is not hard to prove that

$$\frac{d\rho_I}{dt} = -\frac{i}{\hbar}[V_I, \rho_I] - \frac{1}{2}D(t)\sum_i \big[P_i, [P_i, \rho_I]\big]. \tag{9}$$

Eq. (9) is a generalization of Lindblad equation (5) for the interaction picture.

Let us consider the evolution of the system during the time interval $(0, t)$. Let $t_0, t_1, \ldots, t_n$ be moments of time such that $t_0 = 0 < t_1 < \cdots < t_{n-1} < t_n = t$. Then Eq. (5) and Eq. (9) are also valid for the singular distribution $D(t)$:

$$D(t) = \sum_{k=0}^{n} \delta(t - t_k),$$

where $\delta(\cdot)$ is Dirac's delta-function. This special distribution $D(t)$ describes the sequence of measurements at the definite moments of time $t_0, t_1, \ldots, t_n$. It is not hard to understand that the solution of Eq. (9) for this special $D(t)$ may be written as

$$\rho_I(t) = \hat{R}\hat{U}_I(t_n, t_{n-1}) \cdots \hat{R}\hat{U}_I(t_1, t_0)\hat{R}\rho(t_0), \tag{10}$$

where the superoperator of reduction $\hat{R}$ is defined by Eq. (4) and the superoperator of evolution

$$\hat{U}_I(t'', t')\rho = U_I(t'', t')\rho U_I^+(t'', t'), \tag{11}$$

is defined by the solution of Schrödinger equation in the interaction picture without measurements:

$$\frac{d\rho_I}{dt} = -\frac{i}{\hbar}[V_I, \rho_I].$$

If the system was prepared in the pure eigenstate $|\Psi_0\rangle$ of Hamiltonian H_0 at the initial moment of time $t_0 = 0$, then it is easily shown that the survival probability $P(t)$ reads

$$P(t) = \langle\Psi_0|\rho_I(t)|\Psi_0\rangle, \tag{12}$$

where $\rho_I(t)$ is defined by Eq. (10).

3. Model system

Let us consider three-level atom with free Hamiltonian H_0 and eigenstates $|0\rangle, |1\rangle, |2\rangle$:

$$H_0|j\rangle = \hbar\omega_j|j\rangle, \quad j = 0, 1, 2, \qquad \omega_{ij} = \omega_i - \omega_j, \quad i \neq j. \tag{13}$$

Let the initial state of the atom be $|\Psi_0\rangle = |0\rangle$. The atom interacts with classical electric field consisting of two components being in resonance with the transitions ω_{10} and ω_{21}, respectively:

$$\mathbf{E}(t) = \mathbf{E}_{10}e^{i\omega_{10}t} + \mathbf{E}_{10}^* e^{-i\omega_{10}t} + \mathbf{E}_{21}e^{i\omega_{21}t} + \mathbf{E}_{21}^* e^{-i\omega_{21}t}. \tag{14}$$

where $\mathbf{E}_{10}$ and $\mathbf{E}_{21}$ are complex amplitudes of fields. The interaction of electric field with the atom is

$$V = -\mathbf{DE}, \tag{15}$$

where $\mathbf{D}$ is the operator of dipole moment of the atom. The following relations are admitted to be valid:

$$|\omega_{ij}| \gg |V_{mn}|, \qquad \forall i, j, m, n, \tag{16}$$

$$|\omega_{ij} - \omega_{kl}| \gg |V_{mn}|, \quad \forall i, j, k, l\colon \{i, j\} \neq \{k, l\},\ \forall m, n, \tag{17}$$

where $V_{mn} = \langle m|V|n\rangle$.

Let us consider evolution of the atom during the time interval $(0, t)$. Suppose the measurement $\hat{R}$ to be carried out on the atom at the moments $t_0, t_1, \ldots, t_n$ where $t_k = k\Delta t$, $\Delta t = t/n$, $n = 1, 2, 3, \ldots$. The measurement $\hat{R}$ is intended to find the atom on the level $|2\rangle$. Therefore, the superoperator $\hat{R}$ reads

$$\hat{R}\rho = P_{01}\rho P_{01} + P_2\rho P_2, \tag{18}$$

where

$$P_{01} = \mathrm{diag}(1, 1, 0), \qquad P_2 = \mathrm{diag}(0, 0, 1). \tag{19}$$

Let us find the probability to find the atom in the state $|0\rangle$ at the time t. This probability is the survival probability $P(t)$ Eq. (12) for $|\Psi_0\rangle = |0\rangle$. To calculate $\rho_I(t)$ one can use Eq. (10). Further, $\hat{R}$ in Eq. (10) is already known from Eq. (18). Consequently, $\hat{U}_I(t'', t')$ is should be calculated.

Let $a(t)$ be a three-dimensional complex vector, $a = [a_0, a_1, a_2]$. Consider the equation

$$\frac{da(t)}{dt} = -\frac{i}{\hbar} V_I(t) a(t), \tag{20}$$

with initial conditions defined for the time t': $a(t') = [a_0^0, a_1^0, a_2^0]$. In Eq. (20) $V_I(t)$ is the interaction picture Hamiltonian for the free Hamiltonian Eq. (13) and the interaction Eq. (15). The solution of Eq. (20) for the time t'' can be written as

$$a(t'') = U_I(t'', t') a(t'), \tag{21}$$

where $U_I(t'', t')$ is connected with the superoperator $\hat{U}_I(t'', t')$ by Eq. (11). Using Eqs. (14), (15) and the rotating wave approximation (which is right under the conditions (16), (17)), one can rewrite Eq. (20) as

$$\frac{d}{dt}\begin{pmatrix} a_0 \\ a_1 \\ a_2 \end{pmatrix} = -i \begin{pmatrix} 0 & \Omega_{01} e^{i\varphi_{01}} & 0 \\ \Omega_{01} e^{-i\varphi_{01}} & 0 & \Omega_{12} e^{i\varphi_{12}} \\ 0 & \Omega_{12} e^{-i\varphi_{12}} & 0 \end{pmatrix} \begin{pmatrix} a_0 \\ a_1 \\ a_2 \end{pmatrix}, \tag{22}$$

where the notations

$$\Omega_{01} e^{i\varphi_{01}} = -\langle 0|\mathbf{D}|1\rangle \mathbf{E}_1/\hbar, \qquad \Omega_{12} e^{i\varphi_{12}} = -\langle 1|\mathbf{D}|2\rangle \mathbf{E}_2/\hbar,$$

were introduced. The values Ω_{01} and Ω_{12} are considered to be positive real numbers. The evolution operator $U_I(t'', t')$ can be obtained by solution of Eq. (22) and by comparison the results with Eq. (21):

$$U_I(t'', t') = \begin{pmatrix} \dfrac{\Omega_{12}^2 + \Omega_{01}^2 \cos\alpha}{\Omega^2} & -i\dfrac{\Omega_{01}}{\Omega} e^{i\varphi_{01}} \sin\alpha & -\dfrac{\Omega_{01}\Omega_{12}}{\Omega^2} e^{i(\varphi_{01}+\varphi_{12})}(1 - \cos\alpha) \\ -i\dfrac{\Omega_{01}}{\Omega} e^{-i\varphi_{01}} \sin\alpha & \cos\alpha & -i\dfrac{\Omega_{12}}{\Omega} e^{i\varphi_{12}} \sin\alpha \\ -\dfrac{\Omega_{01}\Omega_{12}}{\Omega^2} e^{-i(\varphi_{01}+\varphi_{12})}(1 - \cos\alpha) & -i\dfrac{\Omega_{12}}{\Omega} e^{-i\varphi_{12}} \sin\alpha & \dfrac{\Omega_{01}^2 + \Omega_{12}^2 \cos\alpha}{\Omega^2} \end{pmatrix}, \tag{23}$$

where

$$\Omega = \sqrt{\Omega_{01}^2 + \Omega_{02}^2}, \qquad \alpha = \Omega(t'' - t').$$

Now both values, $\hat{R}$ (from Eqs. (18), (19)) and $\hat{U}_I(t'', t')$ (from Eqs. (11), (23)), are known, and the survival probability for state $|0\rangle$ can be calculated by Eqs. (10), (12).

4. Results of calculations and discussion

First of all let us discuss the mechanisms of IZE in this system qualitatively. For the beginning suppose that the measurements are absent at all. Since the initial state of system $|\Psi_0\rangle = |0\rangle$ is pure at the moment $t_0 = 0$ so it will be pure in future, and the evolution will be governed only by the operator $U_I(t'', t')$: $|\Psi_I(t)\rangle = U_I(t, 0)|0\rangle$. Suppose $\Omega_{12} = 0$. It is seen from Eq. (23) that the evolution of the atom is reduced to the usual Rabi oscillations:[1]

$$a_0(t) = \cos \Omega_{01} t, \qquad a_1(t) = -i \sin \Omega_{01} t, \qquad a_2(t) = 0.$$

In the converse case, $\Omega_{12} \gg \Omega_{01}$, it follows from Eq. (23) that the initial state is "frozen":

$$\forall t: \quad a_0(t) \approx 1, \qquad a_1(t) \approx 0, \qquad a_2(t) \approx 0.$$

The transition between states $|1\rangle$ and $|2\rangle$ hinders the transition between states $|0\rangle$ and $|1\rangle$. This is well-known phenomenon [8,11,12] which is considered as a Zeno-like effect. However, if the transition $|1\rangle \to |2\rangle$ is continuously observed by frequent measurements, then this transition is "frozen" by usual QZE, and the mentioned Zeno-like effect will be hindered by this usual Zeno effect. Rabi transition $|0\rangle \to |1\rangle$ is restored and this phenomenon is IZE.

To represent the detailed calculations, $\Omega_{12} = \Omega_{01}\sqrt{15}$ is chosen. Hence $\Omega = 4\Omega_{01}$ and it is seen from Eq. (23) that the Poincaré time of the system is

$$T_P = 2\pi/\Omega = \pi/(2\Omega_{01}). \tag{24}$$

Thick solid line on Fig. 1 represents "free" evolution of the atom during one Poincaré time: both resonant components of electric field are switched on, but the measurements are switched off. Thin solid lines represent the survival probability of state $|0\rangle$ with different numbers of measurements during the interval $t \in (0, T_P)$; the number near the line is the number n like in Eq. (10). It is seen from Fig. 1 that IZE takes place in the exact accordance with the definition of IZE by Eqs. (2), (3) and condition (i). Moreover, the evolution of the atom tends to the free Rabi transition between levels $|0\rangle$ and $|1\rangle$ with frequency $2\Omega_{01}$ as $n \to \infty$ (dashed line on Fig. 1), as one should expect.

Note that there are no discontinuities of the derivative $dP(t)/dt$ at the moments of the measurements (see Fig. 1). This proposition could be proven by using the relation

$$\frac{dP(t)}{dt} = -\frac{i}{\hbar}\langle 0|\big[V_I(t), \rho_I(t)\big]|0\rangle,$$

and Eq. (18). Behavior of the survival probability $P(t)$ under two-dimensional projection measurement Eq. (18) is different from the behavior of survival probability under usual one-dimensional projection measurement

$$\rho' = P_0 \rho P_0 + P_1 \rho P_1.$$

The derivative $dP(t)/dt$ has discontinuity at the moment of the measurement in the last case.

[1] Hereafter we suppose $\varphi_{01} = \varphi_{12} = 0$ since if the initial state of the atom is $|0\rangle$, these phases do not effect the probabilities to find any atomic state at any time $t > 0$.

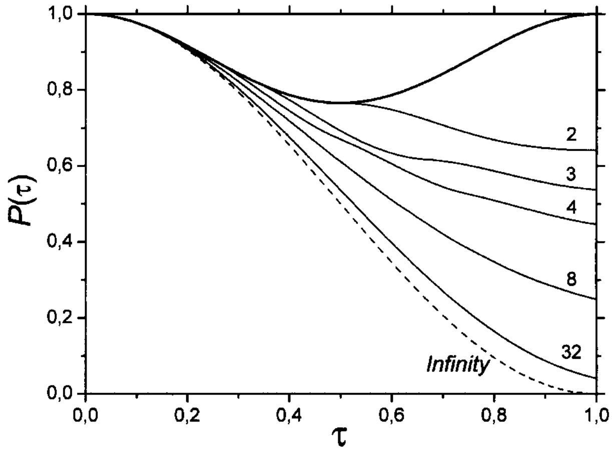

Fig. 1. Inverse Zeno effect in three-level atom with double Rabi transition. $\tau = t/T_P$, where Poincaré time T_P is defined by Eq. (24); $P(\tau)$ is the probability to find the atom in the initial state $|0\rangle$. Thick solid line represents the evolution of atom without measurements, thin solid lines represent the evolution of atom with different number of measurements during the interval $\tau \in (0, 1)$.

The model three-level system with double Rabi transition and measurements described in the present Letter might be realized in an experiment similar to Itano and collaborators QZE-experiment with simple Rabi transition [3]. The levels $|0\rangle$, $|1\rangle$, $|2\rangle$ of the atom might correspond to fine or hyperfine structure. Rabi transitions between these levels might be forced by ultra high frequency or radio frequency fields. In the addition the forth level $|3\rangle$ should be involved such that it should be higher than level $|2\rangle$ and the transition $|3\rangle \to |2\rangle$ should be a non-forbidden optical transition. The state $|3\rangle$ is to decay onto the state $|2\rangle$ much faster than all the Rabi transitions involved to the experiment. A short π-pulse of laser tuned into resonance with the transition $|3\rangle \to |2\rangle$ will simulate the measurement $\hat{R}$, Eq. (18). The probability to find the atom in the state $|0\rangle$ at the end of evolution might be measured by the usual way [3].

Acknowledgements

The author acknowledges the fruitful discussions with P. Facchi, M.B. Mensky, and S. Pascazio and is grateful to V.A. Arefjev for the help in preparation of the paper.

References

[1] B. Misra, E.C.G. Sudarshan, J. Math. Phys. 18 (1977) 756.
[2] C.B. Chiu, E.C.G. Sudarshan, B. Misra, Phys. Rev. D 16 (1977) 520.
[3] W.M. Itano, D.J. Heinzen, J.J. Bollinger, D.J. Wineland, Phys. Rev. A 41 (1990) 2295.
[4] M.C. Fisher, B. Gutiérrez-Medina, M.G. Raizen, quant-ph/0104035.
[5] G. Bernardini, L. Maiani, M. Testa, Phys. Rev. Lett. 71 (1993) 2687.
[6] P. Facchi, S. Pascazio, Phys. Lett. A 241 (1998) 139.
[7] P. Facchi, H. Nakazato, S. Pascazio, Phys. Rev. Lett. 86 (2001) 2699.
[8] P. Facchi, S. Pascazio, in: E. Wolf (Ed.), Progress in Optics, Vol. 42, Elsevier, Amsterdam, 2001, pp. 147–217, Chapter 3.
[9] J. von Neumann, Mathematische Grundlagen der Quantenmechanik, Springer, Berlin, 1932.
[10] A.P. Balachandran, S.M. Roy, quant-ph/0102019.
[11] A. Peres, Am. J. Phys. 48 (1980) 931.
[12] K. Kraus, Found. Phys. 11 (1981) 547.