Layered Uncertainty in Planetary Thermal History Models: Implications for Hypotheses Discrimination and Habitability Modeling

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Abstract

Multiple hypotheses/models have been put forward regarding the cooling history of the Earth. The search for life beyond Earth has brought these models into a new light as they connect to one of the two energy sources life can tap. The ability to discriminate between different Earth cooling models, and the utility of adopting such models to aid in the assessment of planetary habitability, has been hampered by a lack of uncertainty analysis. This motivates a layered uncertainty analysis for a range of thermal history models that have been applied to the Earth. The analysis evaluates coupled model input, initial condition, and structural uncertainty. Layered model uncertainty, together with data uncertainty and multiple working hypotheses (another form of uncertainty), means that results must be evaluated in a probabilistic sense even if the models are deterministic. For the Earth's cooling history uncertainty leads to ambiguity - multiple models, based on different hypotheses, can match data constraints. This has implications for using such models to forecast conditions for exoplanets that share Earth characteristics but are older than the Earth, i.e., it has implications for modeling the long-term life potential of terrestrial planets. Even for the most Earth-like planet we know of, the Earth itself, model uncertainty and ambiguity leads to large forecast spreads. Given that this comes from the planet with the most data constraints we should expect larger spreads for models of terrestrial planets in general. The layered uncertainty approach can be expanded by coupling planetary cooling models to climate models and propagating uncertainty between them to assess habitability from a probabilistic versus a binary view.

1 Introduction

The surface conditions of the Earth have evolved over our planet's history in response to two energy sources: solar energy and internal energy. Both energy sources have, themselves, evolved and continue to do so. Stellar models provide insights into the Sun's energetic evolution [Feulner, 2012]. Thermal history models provide insights into the cooling of the Earth's interior [Davies, 1980, Schubert et al., 1980]. Earth's internal energy comes from the decay of radioactive isotopes within its rocky interior and from heat retained from planetary formation and early differentiation. This internal energy drives volcanic and tectonic activity, both of which influence the cycling of life-essential elements and volatile elements, such as greenhouse gasses, between the Earth's interior and surface reservoirs (atmosphere, hydrosphere, biosphere). That connection to elemental cycling, along with the discovery of life that can tap into the Earth's internal energy and an expanding search for life beyond Earth, has rejuvenated interest in the cooling history of the Earth and, by association, thermal history models. This renaissance has moved thermal history modeling from the realm of geosciences into the realm of astronomy and astrophysics [Kite et al., 2009, Schaefer and Sasselov, 2015, Komacek and Abbot, 2016, Foley, 2015, Foley and Driscoll, 2016, Tosi et al., 2017, Foley and Smye, 2018, Rushby et al., 2018, Barnes et al., 2019].

When a modeling methodology moves from one discipline to another there is the potential for synergies and for misconceptions. The Earth has the largest observational data set that can constrain planetary models. This does not mean that model uncertainty has been minimized. This has not been communicated as well as it could be across communities. Even within the geosciences community itself the role of uncertainty and ambiguity for thermal history models has not received the level of attention it is given in other modeling endeavors (e.g., water resources, climate modelling [Loucks et al., 2005, Curry and Webster, 2011]). This provides the two-pronged motivation for this paper: 1) Given data and model uncertainties, what is the confidence level we can give to different hypotheses regarding the Earth's thermal evolution and, by association, are multiple hypotheses viable?; 2) What implications does uncertainty regarding the Earth's thermal history carry for modeling the habitability of terrestrial planets?

The cooling history of a planet depends on its tectonic mode [Lenardic, 2018]. The Earth's present mode is plate tectonics, with cold tectonic plates subducting back into its rocky and warmer interior. The simplest assumption is that this mode has operated over the Earth's geologic history. To date, the majority of Earth thermal history models have followed this path. To explore the effects of model uncertainties we will follow it as well. With knowledge of our conclusions we can say that observational data used to constrain the Earth's cooling cannot rule out this possibility. Models that allow for tectonic transitions may also be able to match data constraints, but that will only increase the effects of model uncertainty. By assuming a single tectonic mode we will not only follow an Occam's razor approach, but we will also be conservative in assessing model uncertainty.

Models that couple interior planet cooling to climate evolution, seeking to address long term habitability of terrestrial planets, consider the potential of different tectonic modes, with one example being a plate-tectonic cooling model [Driscoll and Bercovici, 2013, Foley and Driscoll, 2016]. A misconception that can follow is that there is a singular, agreed upon, plate tectonic cooling model.

That is not correct.

The range of proposed plate tectonic cooling models for the Earth differ significantly in terms of physical assumptions. The cooling rate associated with the convective overturn of tectonic plates depends on driving and resisting forces to plate motion. The earliest plate tectonic cooling models assumed that the dominant resistance to plate motions comes from the viscosity of the Earth's mantle - the rocky interior plates move over and subduct into [Tozer, 1972, Schubert et al., 1979, 1980]. Later models argued that the strength of plates needed to be considered as plate deformation and deformation at plate boundaries provided significant energy dissipation [Conrad and Hager, 1999a,b]. Those models assumed that plate strength would decrease under hotter conditions, i.e., in the Earth's past, or remain constant. That assumption was challenged by another plate tectonic cooling model that assumed plate strength increased in the Earth's past [Korenaga, 2003, 2006]. All of these models remain argued for to this day. Different authors argue with variable degrees of 'argumentative force' but the fact that debate remains is a sign that there is no singular, agreed upon, plate tectonic cooling model.

How different are proposed plate tectonic cooling models in effect? That is, are the differences in terms of model outputs small relative to data uncertainty? Over the full range of the models that have been proposed to date, they are not. This is clearly demonstrated in the fact that the sign of the dominant feedback for planetary cooling varies from negative to positive over the full range of proposed models [Moore and Lenardic, 2015, Seales et al., 2019]. The implications for extrapolating Earth cooling models to "Earth-like" terrestrial planets is significant [Tozer, 1972, Korenaga, 2016].

To data, no study has systematically compared model outputs for the range of proposed plate tectonic thermal history models to observational data in light of uncertainty. The bulk of this paper sets out to provide such a comparison. First, the comparison is carried out for model evolutions over the Earth's geologic age. That exercise will isolate models that are consistent with Earth data constraints, within model and data uncertainties. From there, we will project this range of "successful" models forward in time to model Earth-like planets older than the Earth. This will provide insights into the level of certainty that exists for making statements regarding the thermal state of terrestrial planets assumed to operate in a plate tectonic cooling mode, an issue of interest to the planetary habitability community.

2 Methods

Uncertainties for thermal history models are of different types: 1) Uncertainty in initial conditions; 2) Uncertainty in model input values (e.g., internal heat production); 3) Model selection uncertainty (i.e. different models based on physically different assumptions); 4) Structural uncertainty (how sensitive are model results to unmodeled factors); 5) Uncertainty in the way uncertainty is evaluated. All of these will be considered in our layered uncertainty analysis.

In principle thermal history models can be formulated to solve for the full three dimensional evolution of a planetary interior over time [e.g., Zhong et al., 2000]. In practice such formulations (run over the Earth's full geological history) remain computationally expensive which limits the degree to which model space can be explored. For this reason, thermal history models of the Earth have been formulated to track the average internal temperature of the Earth's and the majority of thermal history models presented for the Earth are of this variety.

Thermal history models that track average internal temperatures are also referred to as parameterized thermal history models. Different parameterizations reflect different assumptions regarding the operation of plate tectonics (discussed more fully below). That difference being noted, thermal history models share a common underpinning: The average internal temperature of the Earth's upper mantle (T_p) evolves over time based on the balance between heat produced within (H) and lost from (Q) the mantle according to

$$C\dot{T_p} = H - Q. \tag{1}$$

Heat is produced within the mantle by the radiogenic decay of ^{238}U, ^{235}U, ^{232}Th and ^{40}K, and heat production over time is given by

$$H(t) = H_0 \sum_{n=0}^{4} h_n exp(\lambda_n t), h_n = \frac{c_n p_n}{\sum_n c_n p_n} \tag{2}$$

where H_0 is a reference heat production, h_n is the amount of heat produced by a given isotope, and t is time. We calculate relative isotopic concentrations by assuming present day proportions of $U : Th : K = 1 : 4 : (1.27x10^4$ and normalizing by total U [Turcotte and Schubert, 2002]. The values used in equation (2) are listed in Table 1.

Table 1: Radiogenic Heat Production

Isotope	P_n (W/kg)	c_n	h_n	λ_n $(1/Ga)$
^{238}U	9.37×10^{-5}	0.9927	0.372	0.155
^{235}U	5.69×10^{-4}	0.0072	0.0164	0.985
^{232}Th	2.69×10^{-5}	4.0	0.430	0.0495
^{40}K	2.79×10^{-5}	1.6256	0.181	0.555

Heat is lost from the interior by convective cooling. This cooling is parameterized according to the Nusselt-Rayleigh scaling law given by

$$Nu \sim Ra^{\beta}. \tag{3}$$

The Nusselt number Nu is a nondimensional heat flux calculated as the ratio of convective (Q) to conductive (q) heat flux across the convecting layer. Conductive heat flux is given by: $q = \frac{k\Delta T}{D}$, where k, D and ΔT are the thermal conductivity, convecting layer thickness and temperature drop across the convecting layer, respectively. To change this scaling to an equivalency requires a constant a be added to the right hand side of equation (3). The value of a is dependent on the geometry of the convecting system and the average aspect ratio of convection cells. To test the widest range of models, we follow Christensen [1985] and Korenaga [2003] and formulate the convective heat flux as

$$Q = Q_0 \left(\frac{T_p}{T_0}\right)^{1+\beta} \left(\frac{\eta(T_0)}{\eta(T_p)}\right)^{\beta}, \tag{4}$$

which has two free parameters Q_0 and T_0, a reference heat flux and reference mantle temperature, respectively. Equation (4) is also dependent on viscosity ($\eta(T)$) which is defined as

$$\eta(T_p) = \eta_0 \exp\left(\frac{A}{RT_p}\right) \tag{5}$$

where A, R and η_0 are the activation energy, universal gas constant and scaling constant [Karato and Wu, 1993], respectively. For comparison to previous work, we set η_0 so that the upper mantle has a viscosity of 10^{19} Pa$\cdot$ s at 1350 ^oC. Combining equations (3)-5 and using the definition of Nu leads to the governing equation

$$C\dot{T}_p = H_0 \sum_{n=0}^{4} h_n exp\left(-\lambda_n t\right) - Q_0 \left(\frac{T_p}{T_0}\right)^{1+\beta} \left(\frac{\eta(T_0)}{\eta(T_p)}\right)^{\beta}. \tag{6}$$

The value used for β in equation 6 relates to different assumptions regarding the dynamics of plate tectonics. The earliest thermal history models used a value of 0.33 [Schubert et al., 1980, Spohn and Schubert, 1982, Jackson and Pollack, 1984]. This assumes that the dominant resistance to convective motion comes from mantle viscosity [Tozer, 1972]. Later models, that more directly incorporated model analogues to tectonic plates, showed that this scaling would be recovered provided that plate boundaries zones were very weak [e.g., Gurnis, 1988]. The next generation of models argued that β should be less than the classic value because plate margins offer resistance to plate motion – they are not so weak that energy dissipation can be neglected – and there is an added component of dissipation associated with the strength of plates themselves [Christensen, 1985, Giannandrea and Christensen, 1993, Conrad and Hager, 1999b,a, Höink et al., 2013]. These models argued for β values between 0.15 and zero. An argument for $\beta < 0$ has also been made [Korenaga, 2003]. The physical basis for this last class of models is that at hotter mantle temperatures enhanced melting would generate a thicker dehydrated layer below oceanic crust. This layer would be responsible for the bulk of plate strength. As temperatures are hotter in the Earth's past, this layer would have been thicker, plates would have been stronger, plate velocities would have been slower and, thus, mantle cooling rates would have been lower. Given that different β values represent different assumptions as to the physics of plate tectonics, uncertainties in β fall under model selection uncertainties. Different values of β represent different hypotheses. As such, they are not the same as uncertainties in other model input values (e.g., mantle heat production) which do not imply different fundamental assumptions as to the workings of the process being modeled. For assessing model selection uncertainty we will allow for β values between of -0.15 and 0.3.

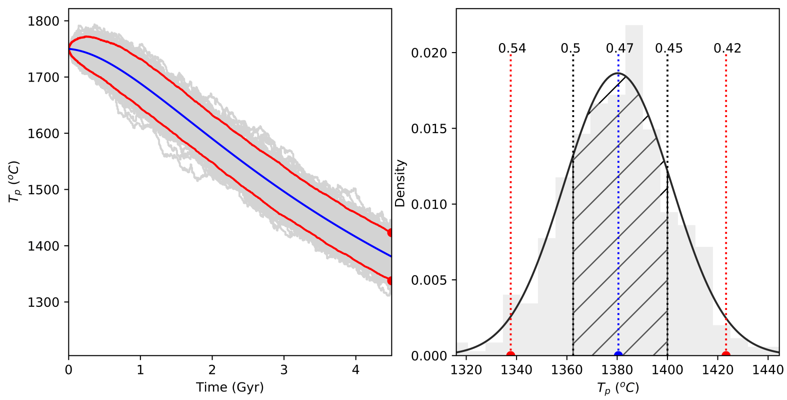

Figure 1: Ensemble model runs for a single thermal history model (a) and an uncertainty measure for a single ensemble (b). An ensemble of 100 perturbed paths (gray lines) is plotted in (a), along with the ensemble mean (blue line) and ensemble two standard deviation limits (red lines). In (b) a present day time slice is taken through the ensemble evolution. This ensemble has an approximately 50% probability of satisfying present day constraints, which are temperature (1300^oC-1400^oC) and a Urey ratio between 0.2 and 0.5. Urey ratios are listed are labeled for the the mean, uncertainty limit and acceptable bound solutions.

Another form of uncertainty is intrinsic/structural. It is the uncertainty associated with unmodeled factors (i.e., factors assumed to have relatively low effects on model outputs). Seales et al. [2019] show that this uncertainty correlates with the sign and strength of the dominant model feedback. A model with a strong negative feedback (the most positive β values) will have smaller structural uncertainties. These models can damp perturbations/fluctuations associated with physical factors not directly incorporated into them. Reducing the strength of the negative feedback (reducing β towards zero) increases the damping time, which increases structural uncertainty. A positive feedback ($\beta < 0$) allows models to amplify perturbations/fluctuations. If the positive feedback becomes strong enough, a model can become structurally unstable. One method to quantify this structural uncertainty is via a perturbed physics approach. During integration of equation 6 the solution is perturbed (see Seales et al. [2019] for full details). The perturbation is randomly drawn from a normally distributed set. By fixing initial conditions and parameter values and integrating equation 6 many times we get output distribution of potential cooling paths (henceforth called an ensemble) as shown in Figure 1a. At a time slice of the evolution, such as the present day, the T_p ensemble (Figure 1b) can be characterized by a mean and a two standard deviation (2σ) uncertainty bound. In performing this analysis, we found that increasing the standard deviation of the perturbation set did not significantly effect the accumulation of uncertainty (Figure 2) - an assessment of the uncertainty associated with the particular uncertainty metric itself.

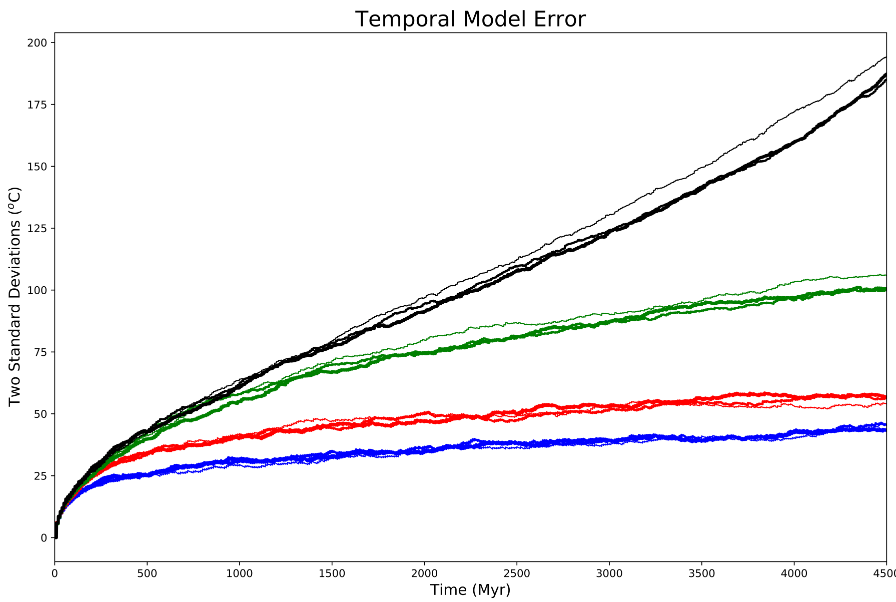

Figure 2: As larger perturbations were tested to evaluate structural uncertainty, model error tended to saturate (thicker lines indicate larger allowable maximum perturbations). This provides a measure of the uncertainty associated with our perturbed physics approach (i.e., the uncertainty associated with a particular structural uncertainty metric).

The ensemble in Figure 1 is for a unique combination of model inputs and initial conditions, which introduce further uncertainty as their values are not perfectly known. Equation 6 has a total of three free parameters for any choice of β: H_0, Q_0 and T_0. A set of values for these parameters will be tested (see Table 2). The success or failure of an ensemble will be determined by comparing it to paleo and present day constraints. Ganne and Feng [2017] suggest mantle potential temperatures were between 1428 ^{o}C and 1666 ^{o}C at 3.5 Ga and between 1321 ^oC and 1553 ^oC at 100 Ma. We use their results to set our paleo constraints. At present, the mantle potential temperature is approximately 1350 ^oC $\pm$50 ^oC [Herzberg and Asimow, 2008]. A second present day constraint is the mantle Urey ratio, Ur, which is the the ratio of H to Q. Jaupart et al. [2007] estimate it to be between 0.3 and 0.5. Allowing for continents, the Ur upper bound extends to 0.7 [Grigné and Labrosse, 2001, Lenardic et al., 2011].

We assume both present day constraints are of equal importance. Using these present day constraints, we define the ensemble probability for successful models. This involves identifying the upper and lower most bound on the ensemble probability distribution that fall within constraints and calculating the probability that an ensemble member falls between these two points. For example, in Figure 1b the mean of the ensemble distribution is ~1380 ^oC. The upper temperature bound occurs at 1400 ^oC, where the present day Ur is 0.45, within present day constraints. The lower temperature bound is not set to 1300 ^oC because at this temperature Ur is greater than 0.5. An Ur value of 0.5 occurs at 1365 ^oC. Therefore, for this ensemble, any output temperature between 1365 and 1400 ^oC satisfies present day constraints with a probability of 0.5.

Table 2: Model Parameters

Parameter	Values	Units	Description
T_i	1000, 1250, 1500, 1750, 2000	^oC	Initial Temperature
T_0	1300, 1350, 1400	^oC	Scaling Temperature
Q_0	3.0e13, 3.5e13, 4.0e13	TW	Scaling Heat Flow
H_0	2.19e13, 2.55e13, 2.92e13, 4.38e13, 5.12e13, 5.84e13, 6.57e13, 7.66e13, 8.76e13, 1.02e14, 1.09e14, 1.17e14, 1.28e14, 1.46e14	TW	Initial Radiogenics
η_0	2.21e9	Pa·s	Viscosity constant
A	300	kJ mol^{-1}	Activation Energy
R	8.314	J / (mol·K)	Universal Gas Constant

3 Results

Figure 3 shows mean cooling paths for models with different β values, input values, and initial conditions. We leave off the structural uncertainty bounds for clarity but they were calculated for all the paths plotted. The mean paths that satisfy the present day T_p constraints are shown as red lines. Mean paths that fall outside the constraint but are associated with models that can match the constraint within structural uncertainty bounds are shown as light red lines. Solutions that do match the constraint even allowing for structural uncertainty bounds are shown as grey lines. A model with $\beta < 0$ is very initial condition and input value dependent. This leads to a wide model solution space. Models with $\beta \geq 0$ had weaker initial condition and input value dependencies, resulting in a more concentrated solution space.

The number of cases that satisfy the present day T_p constraint for variable β are shown in Figure 4. The number of cases where the mean matches the present day constraint (dark green) is small at the most negative β endmember. The number of mean paths remained below 10% until the positive feedback was removed by switching β to positive values. Once a negative feedback was present, the number of successful cases began to grow. At a β value near 0.2 the number

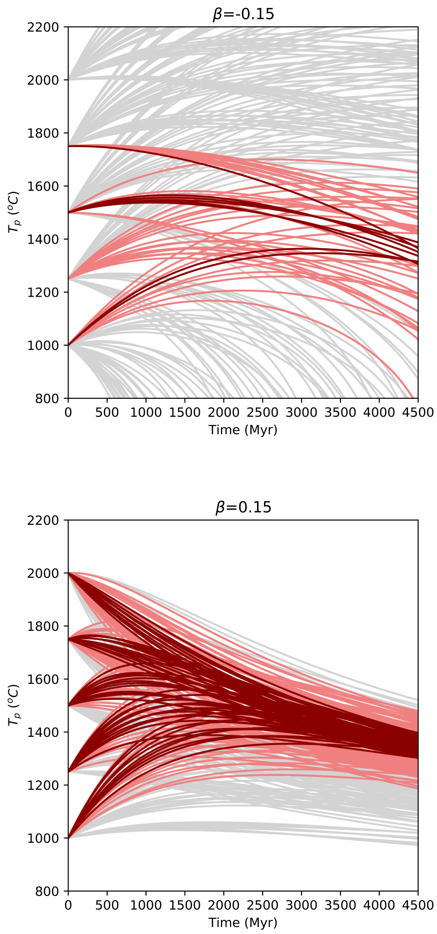

Figure 3: Probability distribution for models that satisfy all observational constraints (a). In (b we tabulate the total fraction of models that match all constraints by calculating the probability of success of each ensemble as shown in Figure 1.

of successful cases plateaued around 30%. Accounting for intrinsic/structural uncertainty increases the number of successful cases for all β values (lighter green). The rise in successful cases occurred while β was still negative, around $\beta = -0.1$, and plateaued at a β slightly greater than 0.1.

The number of cases matching present day Ur are shown in Figure 5. The color scheme is the same as Figure 4 with mean solutions in green and those that include structural uncertainty in lighter green. Results are shown for cases that match present day Ur without accounting for the effect of present day continen-

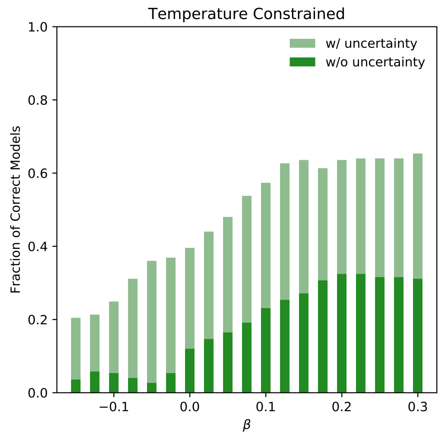

Figure 4: Probability distribution for models that satisfy the present day temperature constraint.

tal distribution (mid-green) and for cases in which the effect of continents, on the present day Urey ratio [Grigné and Labrosse, 2001, Lenardic et al., 2011], is accounted for (lightest-green). The distribution of successful cases peaked around 60% for $\beta = 0.05$. Accounting for structural uncertainty had little effect for the Ur constraint. Accounting for continents increased the number of successful cases. At its peak, near $\beta = 0.1$, the number of successful matches was greater than 90%. Including continents disproportionately benefited more positive β values.

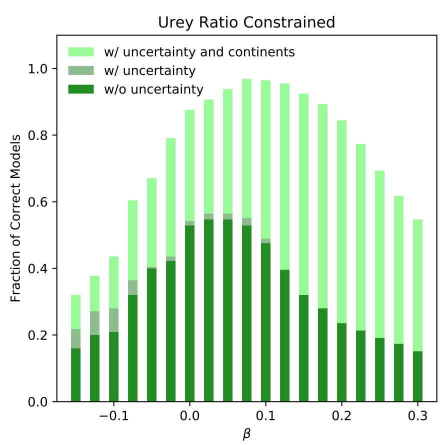

Figure 5: Probability distribution for models that satisfy the present day Urey ratio constraint.

Figure 6 shows the number of successful cases when assigning equal weight to the present day T_p and Ur constraints. The distribution is non-normal. Mean paths resulted in less than 10% of successful cases across the board. The peak for the mean solutions is at a β value slightly less than 0.1. Below this value successful models fall to nearly zero before increasing slightly when values of $\beta < 0$ were considered. Increasing the upper Ur bound to 0.7, to account for the potential effect of continents, shifted the peak β value to be greater than 0.1 and increased the number of successful cases to nearly 60% at the peak. Considering intrinsic/structural uncertainty preferentially benefited the lower half of the tested β space. A very low percentage of models could match both constraints for β values greater than 0.2 unless the effects of continents were considered (and it should be kept in mind that doing so adds its own layer of uncertainty as the continental correction comes from models [Grigné and Labrosse, 2001, Lenardic et al., 2011]).

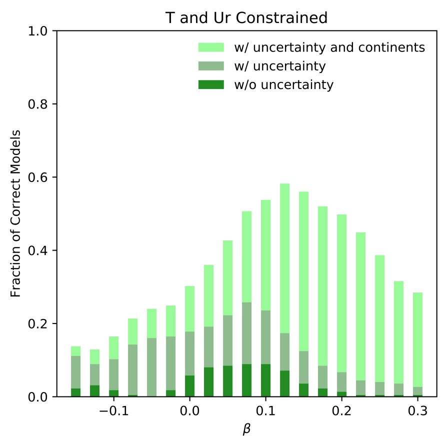

Figure 6: Probability distribution for models that satisfy the present day temperature and Urey ratio constraint.

The distributions that resulted from using only paleo temperature constraints are shown in Figure 7. The trends are similar to those in Figure 3. One difference is the uniform decrease in the fraction of ensembles able to match the paleo constraints. This intuitively makes sense in that to be successful an ensemble must stay within a temperature window over 2.5 Gyr of evolution rather than match a value at a single time. A subtle, but noteworthy difference between Figure 3 and Figure 7 is that the fraction of successful ensembles matching the paleo constraints increased to a greater degree as β was increased. Positive β models tend to lessen initial condition dependence. Nearer to the model start time there is less time to eliminate the influence of the initial condition. As a result, some models that converge to present day temperatures were too hot or too cold at 2.5 Gyr and thus considered unsuccessful. Even with this change in slope, the distribution peaked around 0.2.

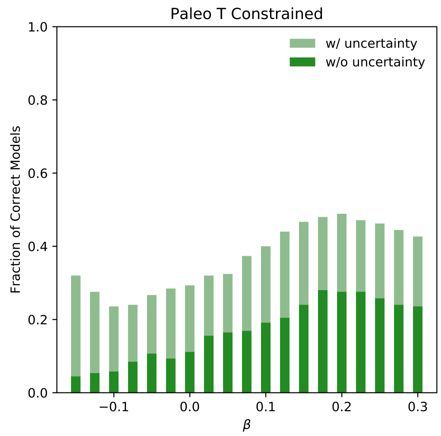

Figure 7: Probability distribution for models that satisfy the paleo temperature constraints.

Figure 8 shows models that can match paleo and present day constraints. Figure 8a shows the fraction of models for each β that have some portion of the ensemble that satisfies all three constraints. Distributions are bi-modal, having one peak in the negative β domain and one peak in the positive domain. Accounting for intrinsic/structural uncertainty increased the fraction of successful solutions across the board and produced nearly identical peaks in both the positive and negative domains. Allowing for continental effects shifted the largest peak close to a β value of 0.2, but a peak just less than zero remained. A representation of the total probability is shown in Figure 8b. For each β, the total probability is the sum of each ensemble probability divided by the total number of initial conditions and model input combinations assessed. For a constraint on present day Ur that does not account for continents, a peak probability of approximately 10% occurred at a β value of 0.1. This distribution has a single peak with a heavy left tail, which is caused by the hard upper Ur limit of 0.5, which cast out a large portion of the more positive β ensembles. Relaxing this constraint resulted in more normal distribution peaked around 0.15. This is close to the value argued for by Conrad and Hager [1999a]. We have given all data constraints equal weight. If one of the constraints is found to be more reliable than the others, then distribution peak will shift towards the β values that coincide with matching that constraint.

Using only the paths that matched paleo and present day constraints, we projected mantle potential temperature out to 10 Gyr (Figure 9). Figure 9a projects only those mean paths that matched Earth constraints (dark green models in Figure 7a). The differing feedbacks within the models becomes apparent as time evolves with negative β models reaching far cooler mantle temperatures. These models lead to cooling runaways and once temperatures drop too low the models are cut off as they have lost structural stability [Seales et al., 2019].

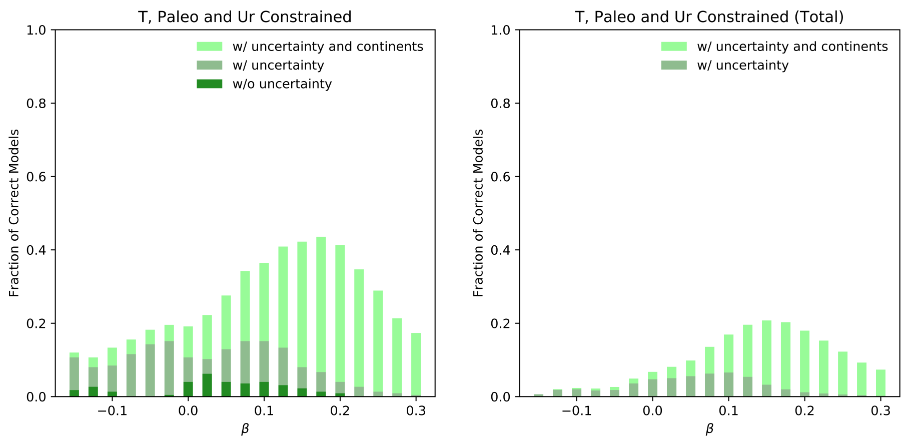

Figure 8: Probability distribution for models that satisfy all observational constraints (a). In (b we tabulate the total fraction of models that match all constraints by calculating the probability of success of each ensemble as shown in Figure 1.

Models with $\beta > 0.1$ cooled more slowly, maintaining temperatures above 1000 ^oC throughout. Figure 9b shows the projected models that match paleo and present day constraints with intrinsic/structural uncertainty now accounted for. Projections were limited to those models that matched Ur values between 0.3 and 0.5. Including structural uncertainties allowed for run away cooling behavior to occur nearly one billion years nearer to present day for models with the most negative β values. If we take into account the total probabilities, which peak between β values of 0.1 and 0.2, and only use those cases, then projected temperature vary between 1000 and 1200 ^{o} C at 10 Gyr of model evolution. However, as each of the model paths plotted match Earth constraints, they all remain possible. Stated another way, there is no reason why the evolution path of a particular planet, the Earth, needs to follow a most probable path within a model solutions space.

4 Discussion

Ours is not the first thermal history study to consider uncertainty. For example, Korenaga [2011] performed a parameter estimation analysis for one unique thermal history model, whereas McNamara and Van Keken [2000] performed a model selection study. Our analysis is unique in that we considered a layered uncertainty analysis, for a range of models, to assess the probability that any given model fits Earth constraints. Any model with a probability greater than zero is capable of explaining Earth's thermal history. One model being less probable

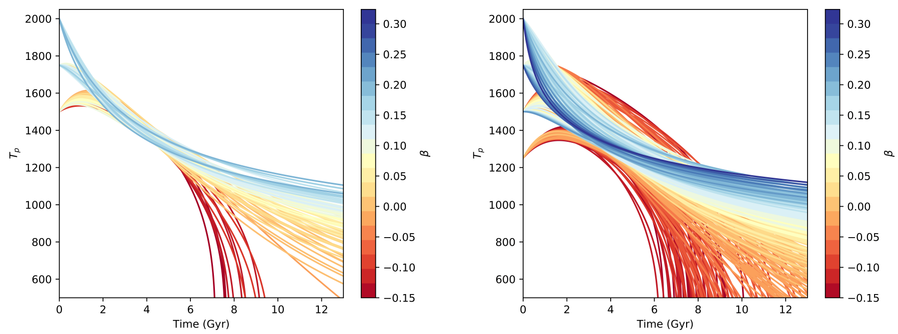

Figure 9: Forward in time projections for successful models. In (a) only successful mean solution paths are plotted (i.e., model structural uncertainty is not accounted for). In (b) successful ensemble paths are plotted (i.e., this accounts for structural uncertainty).

than another, in model solution space, does not eliminate the possibility that the lower probability model captures the essential physics of plate tectonics, as related to planetary cooling. Having said that, we can also weigh probabilities to assess which models can match Earth constraints over the widest range of uncertainties. Figure 8b indicates models with β between 0.1 and 0.2 fall into this category. High β models can match present day temperature over a wide input range but struggle to match the lower Ur constraint. Lower β models can match the Ur constraint but struggle with present day temperature constraints if β drops too low as they then run hot [McNamara and Van Keken, 2000]. That a "sweet spot" could exist between the two end-members is not, in hindsight, qualitatively surprising.

Uncertainties in the data constraints we used influenced the calculated probability of successful models. We assumed equal weighting for each constraint. Of the two present day constraints, the present day mantle temperature is a harder constraint. This is because there is less uncertainty in estimating its value than there is in estimating the Urey ratio Jaupart et al. [2007]. The difficulty of considering different weightings is that, although the distribution of uncertainties associated with temperature data has been calculated [Condie et al., 2016, Ganne and Feng, 2017], the same is not true for the Urey ratio (to date, the implicit assumption has been a boxcar distribution). At this stage, we did not consider it warranted to apply different weightings but this could be done in the future.

Our analysis explored a slice of potential model space. A more extensive exploration would change quantitative results but key qualitative results are likely to be robust. The qualitative differences between positive and negative β models comes from the fact that the former is dominated by a negative system feedback and the latter by a positive feedback [Moore and Lenardic, 2015]. More sophisticated models can be constructed (e.g., fully 3-D models) but the dominant feedback will still dictate end-member behavior and uncertainty structure. Uncertainties will be smaller in models dominated by negative feedbacks as will model solution space. The latter means that model cases may be less likely to match any given data constraints but if they can match constraints, then a narrow solution space will lead to a larger percentage of model paths being successful. Models with high positive feedbacks will have greater uncertainty and an associated larger solution space. A large solution space increases the potential that at least some cases can match a given data constraint and, at the same time, it favors a smaller percentage of potential model paths being successful.

The connection between uncertainty and successful models relates to another conclusion we argue is robust: Models based on different hypotheses, regarding the dynamics of plate tectonics, are consistent with constraints on the Earth's thermal history, i.e., competing hypotheses remain viable. Phrased another way, model and data uncertainties lead to ambiguity - more that one model is viable. Considering more sophisticated (complex) models will not, we argue, change this conclusion, provided full model uncertainties are assessed. Increasing complexity can increase model uncertainty and model solution space [Saltelli, 2019]. With a larger solution space, the potential that some combinations of model inputs, initial conditions, and ensemble paths will match constraints will increase, as will the computational time needed to find them. It is possible that new and/or more certain data constraints could bridle this to a degree, though historical data from the Earth will always have uncertainty. As such, we argue that multiple hypotheses will likely remain viable into the near future, particularly if there is a trend toward developing more complex models.

Although, to the best of our knowledge, model ambiguity has not previously been laid out explicitly for thermal history models, ambiguity for Earth models in general is not new [e.g., Richards and Lenardic, 2018]. Hypotheses discrimination can continue, but it must proceed in a statistical manner. This, we argue, is another robust conclusion. We can ask which models come with higher probabilities of success in light of uncertainties. This is the utility of Figure 8. The degree to which one is willing to push this further depends on a question that cannot be scientifically answered at present: of all the possible evolution paths, consistent with physical and chemical principles, did a single planet, the Earth, follow what is the most likely path in that potentiality space? The conservative stance is to say 'We don't know,' which means we consider all models with greater than zero probability as viable.

The question above relates to the extension of thermal history studies from Earth to planetary application, habitability in particular [Kite et al., 2009, Schaefer and Sasselov, 2015, Komacek and Abbot, 2016, Foley, 2015, Foley and Driscoll, 2016, Tosi et al., 2017, Foley and Smye, 2018, Barnes et al., 2019]. Thermal history models applied to the Earth are postdictive: they set out to match historical data. In the context of habitability studies, thermal history models are used in a predictive mode to determine whether liquid water may be present on the surface of terrestrial planets with variable planetary and orbital properties. Using models in this mode increases the potentiality space of model outputs. With the thought of limiting the vastness of this space, many studies have focused on planets commensurate with Earth as a starting point [e.g., Foley, 2015, Foley and Driscoll, 2016, Rushby et al., 2018]. Implicit to this is the thought that uncertainties will be lowest for modeling this subset of planets. Our analysis suggests that even if we consider the most Earth-like planet possible, with the most observational data (the Earth), significant uncertainty remains [Fig. 9].

The above leads to a few suggestions on moving forward. First, even if we focus on a plate tectonic mode of planetary cooling, we should consider all viable models [Figure 8]. To date, habitability models have considered a single plate tectonic model [e.g., Driscoll and Bercovici, 2013, Foley, 2015, Foley and Driscoll, 2016, Rushby et al., 2018]. The particular model adopted (a classic high β model) is not the most probable model for matching Earth data. This is not damning but it is inconsistent with the idea that many of the studies are based on: given a large model space, let's start with models that best account for Earth data. It also bypasses model selection uncertainty. Second, all models should be subjected to a layered uncertainty analysis. Typically only a range of initial conditions and input values are tested. An ensemble approach is generally not employed, which leaves out structural uncertainty. One uncertainty measure is not a substitute for another and all need to be evaluated before model implications can be assessed and/or before a model can be validated. A corollary is that model implications need to be viewed in a probabilistic manner by presenting results as probability distributions. This becomes particularly important for models used to make forecasts.

All of the projections in Figure 9b should, we argue, be considered as potentialities. In that view, they are all counterfactuals [Taleb, 2012] with very different implications if used as forecasts. For example, a family of paths imply that plate tectonics could end in about 1.5 billion years as the mantle becomes too cold. This family of paths is consistent with a study that did forecast the end of plate tectonics in 1.45 billion years [Cheng, 2018]. Such a forecast has implications for life beyond Earth, and, given that, it's no surprise that popular science media latched on [Nace, 2018]. The fact that some of our projections are in line with the study of Cheng [2018] speaks to model reproducibility, as that study used a negative β model, which is also the one we found leads to cold runaways. However, it is the negative β models that are associated with the largest uncertainty and are prone to structural instability Seales et al. [2019]. Not being clear about uncertainty, especially for a provocative conclusion, only invites misinformation (e.g., presenting a highly uncertain model forecast as a singular "result"). We would suggest that if full uncertainty analysis was as strong a component of planetary modeling studies as, for example, a methods section, then the odds of unintentionally making conclusions that can send misinformation would be reduced. We will add a corollary, the greater a modeling study moves toward the prediction end of the postdiction/description-prediction/forecast spectrum, the greater the responsibility of the modelers to present a full uncertainty analysis. That corollary applies to essentially all modeling studies of terrestrial exoplanets. Adhering to it could, we argue, prevent red-herring debates of the type that have surfaced in the past [e.g., Chorost, 2013].

In the exoplanet modeling field, thermal history models are being coupled to other models to explore how interior planet evolution co-evolves with other systems – stellar, orbital, volatile cycling, climate, weathering and life [Barnes et al., 2019]. Each system sub-model is subject to the types of uncertainty we have presented for thermal history models. This can make a grid search approach, to map out the coupled model solution space in light of uncertainties, intractable. However, the full model potentiality space is often not of primary interest. A more primary driver behind the coupled models is mapping the subspace that allows water to exist at the surface of a planet over geologic time (this connects the models to the search for life beyond Earth – life as we know it relies on water). Having a search target, within model potentiality space, can reduce the computational work load, but a grid search, akin to that of this paper, would still be impractical given the large dimensionality of the problem. More efficient computational methods can bring the modeling back to a tractable level (e.g., machine learning based methods [P. Fleming and VanderPlas, 2018]). This will introduce another layer of uncertainty that will need to be evaluated – the uncertainty associated with the particular search method. All of this will increase the workload and the move toward a probabilistic framework. Such a framework, in turn, would move the field beyond a binary assessment habitability and towards assessing the potential of a planet to host life that requires a particular type of environment. Given that all of this is being done in the prediction/forecast mode, uncertainty analysis will need to play a larger role than it has to date.

5 Conclusions

We applied a layered uncertainty analysis to solid Earth cooling models. The analysis accounted for the combined effects of: 1) Model selection uncertainty; 2) Model structural uncertainty; 3) Uncertainty in initial conditions; 4) Uncertainty in model input values. Accounting for model and observational uncertainties allows for model validation (testing the degree to which model outputs can match data constraints). Validation, once full uncertainty measures are evaluated, requires a probabilistic approach and results are presented as probability distributions. Given we only have one planet evolutionary path, the Earth, we have argued that any models that maintain finite probabilities of accounting for observational data, over model potentiality space and in light of uncertainties, remain viable. For the thermal history models we examined this leads to ambiguity (multiple hypotheses remain viable for the Earth's thermal history). When thermal history models move from a postdictive mode (accounting for existing Earth data) into a predictive mode designed to constrain conditions that allow for clement surface environments on terrestrial planets, the role of a layered uncertainty analysis becomes more critical.

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