DOES TIME EXIST IN QUANTUM GRAVITY?

Claus Kiefer

Institute for Theoretical Physics,
University of Cologne,
Zülpicher Strasse 77, 50937 Köln, Germany.
`http://www.thp.uni-koeln.de/gravitation/`

Abstract

Time is absolute in standard quantum theory and dynamical in general relativity. The combination of both theories into a theory of quantum gravity leads therefore to a "problem of time". In my essay I shall investigate those consequences for the concept of time that may be drawn without a detailed knowledge of quantum gravity. The only assumptions are the experimentally supported universality of the linear structure of quantum theory and the recovery of general relativity in the classical limit. Among the consequences are the fundamental timelessness of quantum gravity, the approximate nature of a semiclassical time, and the correlation of entropy with the size of the Universe.

1 Time in Physics

On December 14, 1922, Albert Einstein delivered a speech to students and faculty members of Kyoto University in which he summarized how he created his theories of relativity [1]. As for the key idea in finding special relativity in 1905, he emphasized: "An analysis of the concept of time was my solution." He was then able to complete his theory within five weeks.

An analysis of the concept of time may also be the key for the construction of a quantum theory of gravity. Such a hope is supported by the fact that a change of the fundamental equations in physics is often accompanied by a change in the notion of time. Let me briefly review the history of time in physics.

Before Newton, and thus before the advent of modern science, time was associated with periodic motion, notably the motion of the 'Heavens'. It was therefore a countable time, each tick corresponding to one period; there was no idea of a continuum.

It was Newton's great achievement to invent the notion of an absolute and continuous time. Such a concept was needed for the formulation of his laws of mechanics and universal gravitation. Although Newton's concepts of absolute space and absolute time were heavily criticized by some contemporaries as being unobservable, alternative relational formulations were only constructed after the advent of general relativity in the 20th century [2].

In Einstein's theory of special relativity, time was unified with space to form a four-dimensional spacetime. But this "Minkowski spacetime" still constitutes an absolute background in the sense that there is no *reactio* of fields and matter – Minkowski spacetime provides only the rigid stage for their dynamics. Einstein considered this lack of back reaction as very unnatural.

Minkowski spacetime provides the background for relativistic quantum field theory and the Standard Model of particle physics. In the non-relativistic limit, it yields quantum mechanics with its absolute, Newtonian time t. This is clearly seen in the Schrödinger equation,

$$\mathrm{i}\hbar \frac{\partial \psi}{\partial t} = \hat{H}\psi \,. \tag{1}$$

It must also be noted that the presence of t occurs on the left-hand side of this equation together with the imaginary unit, i; this fact will become important below. In relativistic quantum field theory, (1) is replaced by its functional version.

The Schrödinger equation (1) is, with respect to t, deterministic and time-reversal invariant. As was already emphasized by Wolfgang Pauli, the presence of both t and i are crucial for the probability interpretation of quantum mechanics, in particular for the conservation of probability *in* time.

But the story is not yet complete. It was Einstein's great insight to see that gravity is a manifestation of the geometry of spacetime; in fact, gravity is geometry. This led him to his general theory of relativity, which he completed in 1915. Because of this identification, spacetime is no longer absolute, but dynamical. There *is* now a *reactio* of all matter and fields onto spacetime and even an interaction of spacetime with itself (as is e.g. the case in the dynamics of gravitational waves).

So, time is absolute in quantum theory, but dynamical in general relativity. What, then, happens if one seeks a unification of gravity with quantum theory or, more precisely, seeks an accommodation of gravity into the quantum framework? Obviously, time cannot be both absolute and non-absolute: this dilemma is usually referred to as the "problem of time" [3, 4, 5]. One can also rephrase it as the problem of finding a background-independent quantum theory.

But does one really have to unify gravity with quantum theory into a theory of quantum gravity? In the next section, I shall give a concise summary of the main reasons for doing so. I shall then argue that one can draw important conclusions about the nature of time in quantum gravity *without* detailed knowledge of the full theory; in fact, all that is needed is the semiclassical limit – general relativity. I shall then describe the approximate nature of any time parameter and clarify the relevance of these limitations for the interpretation of quantum theory itself. I shall finally show how the direction of time can be understood in a theory which is fundamentally timeless.

2 The Disappearance of Time

The main arguments in favour of quantizing gravity have to do with the *universality* of both quantum theory and gravity. The universality of quantum theory is encoded in the apparent universality of the superposition principle, which has passed all experimental tests so far [6, 7]. There is, of course, no guarantee that this principle will not eventually break down. However, I shall make the conservative assumption, in accordance with all existing experiments, that the superposition principle does hold universally: arbitrary linear combinations of physical quantum state do again lead to a physical quantum state; in general, the resulting quantum states describe highly entangled quantum systems. If the superposition principle holds universally, it holds in particular for the gravitational field.

The universality of the gravitational field is a consequence of its geometric nature: it couples equally to all forms of energy. It thus interacts with all quantum states of matter, suggesting that it is itself described by a quantum state. This is not a logical argument, though, but an argument of naturalness [8].

A further argument for the quantization of gravity is the incompleteness of general relativity. Under very general assumptions one can prove singularity theorems that force us to conclude that time must come to an end in regions such as the Big Bang and the interior of black holes. This is, of course, only possible because time in general relativity is dynamical. The hope, then, is that quantum gravity will be able to deal with these situations.

It is generally argued that quantum-gravity effects can only be seen at a remote scale – the Planck scale, which originates from the combination of the three fundamental constants c (speed of light), G (gravitational constant), and $\hbar$ (quantum of action). The Planck length, for example, is given by

$$l_{\mathrm{P}} = \sqrt{\frac{\hbar G}{c^3}} \approx 1.62 \times 10^{-35}\,\mathrm{m}\,, \tag{2}$$

and is thus much smaller than any length scale that can be probed by the Large Hadron Collider (LHC).

This argument is, however, misleading. One may certainly expect that quantum effects of gravity are always important at the Planck scale. But they are not restricted to this scale *a priori*. The superposition principle allows the formation of non-trivial gravitational quantum states at any scale. Why, then, is such a state not being observed? The situation is analogous to quantum mechanics and the non-observability of states such as a Schrödinger-cat state. And the reason why such states are not found is the same: decoherence [6, 7]. The interaction of a quantum system with its ubiquitous environment (that is, with unaccessible degrees of freedom) will usually lead to its classical appearance, except for micro- or mesoscopic situations. The process of decoherence is founded on the standard quantum formalism, and it has been tested in many experiments [7].

The emergence of classical behaviour through decoherence also holds for most states of the gravitational field. But there may be situations where the quantum nature of gravity is visible – even far away from the Planck scale. We shall encounter such a situation in quantum cosmology. It is directly related to the concept of time in quantum gravity.

Due to the absence of a background structure, the construction of a quantum theory of gravity is difficult and has not yet been accomplished. Approaches are usually divided into two classes. The more conservative class is the direct quantization of general relativity; path-integral quantization and canonical quantum gravity belong to it. The second class starts from the assumption that a consistent theory of quantum gravity can only be achieved within a unified quantum theory of all interactions; superstring theory is the prominent (and probably unique) example for this class.

In this essay I want to put forward the view that the concept of time in quantum gravity can be discussed without having the final theory at one's disposal; the experimentelly tested part of physics together with the above universality assumptions suffice.

The arguments are similar in spirit to the ones that led Erwin Schrödinger in 1926 to his famous equation (1). Motivated by Louis de Broglie's suggestion of the wave nature of matter, Schrödinger tried to find a wave equation which yields the equations of classical mechanics in an appropriate limit, in analogy to the recovery of geometric optics as a limit to the fundamental wave optics. To achieve this, Schrödinger put classical mechanics into the so-called Hamilton–Jacobi form from which the desired wave equation could be easily guessed [9].

The same steps can be followed for gravity. One starts by casting Einstein's field equations into Hamilton–Jacobi form. This was already done by Asher Peres in 1962 [10]. The wave equation behind the gravitational

Hamilton–Jacobi equation is then nothing but the Wheeler–DeWitt equation, which was derived by John Wheeler [11] and Bryce DeWitt [12] in 1967 from the canonical formalism. It is of the form

$$\hat{H}_{\rm tot}\Psi = 0\ , \tag{3}$$

where $\hat{H}_{\rm tot}$ denotes here the full Hamilton operator for gravity plus matter. The wave functional Ψ depends on the *three-dimensional* metric plus all non-gravitational fields.[1]

The Wheeler–DeWitt equation (3) may or may not hold at the fundamental Planck scale (2). But as long as quantum theory is universally valid, it will hold at least as an approximate equation for scales much bigger than $l_{\rm P}$. In this sense, it is the most reliable equation of quantum gravity, even if it is not the most fundamental one.

The wave function Ψ in the Wheeler–DeWitt equation (3) does not contain any time parameter t. Although at first glance surprising, this is a straightforward consequence of the quantum formalism. In classical mechanics, the trajectory of a particle consists of positions q in time, $q(t)$. In quantum mechanics, only probability amplitudes for those positions remain. Because time t is external, the wave function in (1) depends on both q and t, but not on any $q(t)$. In gravity, three-dimensional space is analogous to q, and the classical spacetime corresponds to $q(t)$. Therefore, upon quantization spacetime vanishes in the same manner as the trajectory $q(t)$ vanishes. But as there is no absolute time in general relativity, only space remains, and one is left with (3).

We can thus draw the conclusion that quantum gravity is timeless solely from the validity of the Einstein equations at large scales and the assumed universality of quantum theory. Our conclusion is independent of additional modifications at the Planck scale, such as the discrete features that are predicted from loop quantum gravity and string theory.

3 Time Regained

In August 1931, Neville Mott submitted a remarkable paper to the Cambridge Philosophical Society [13]. He discussed the collision of an alpha-particle with an atom. The remarkable thing is that he considered the time-independent Schrödinger equation of the total system and used the state of the alpha-particle to *define* time and to derive a time-dependent Schrödinger equation for the atom alone. The total quantum state is of the form

$$\Psi(\mathbf{r},\mathbf{R}) = \psi(\mathbf{r},\mathbf{R})\mathrm{e}^{\mathrm{i}\mathbf{kR}}\ , \tag{4}$$

[1]There also exist the so-called diffeomorphism constraints, which state that Ψ is independent of the choice of spatial coordinates, see e.g. [4] for details.

where $\mathbf{r}$ ($\mathbf{R}$) refers to the atom (alpha-particle). The time t is then defined from the exponential in (4) through a directional derivative,

$$\mathrm{i}\frac{\partial}{\partial t} \propto \mathrm{i}\mathbf{k}\cdot\nabla_{\mathbf{R}}\ . \tag{5}$$

This leads to the time-dependent Schrödinger equation for the atom. Such a viewpoint of time as a concept derived from a fundamental timeless equation is seldom adopted in quantum mechanics. It is, however, the key step to understanding the emergence of time from the timeless Wheeler–DeWitt equation (3). While the alpha-particle in Mott's example corresponds to the gravitational part, the atom corresponds to the non-gravitational degrees of freedom. The time t of the Schrödinger equation (1) is then *defined* by a directional derivative similar to (5). Various derivations of such a "semiclassical time" have been given in the literature (reviewed e.g. in [4]), but the general idea is always the same. Time emerges from the separation into two different subsystems: one subsystem (here: the gravitational part) defines the time with respect to which the other subsystem (here: the non-gravitational part) evolves.[2] Time is thus only an approximate concept. A closer investigation of this approximation scheme then reveals the presence of quantum-gravitational correction terms [14].

I have remarked above that the Hilbert-space structure of quantum theory is related to the probability interpretation, and that the latter seems to be tied to the presence of t. In the light of the fundamental absence of t, one may speculate that the Hilbert-space structure, too, is an approximate structure and that different mathematical structures are needed for full quantum gravity.

I have also remarked above that the time t in the Schrödinger equation (1) occurs together with the imaginary unit i. The quantum-mechanical wave functions are thus complex, which is an essential feature for the probability interpretation. Since the Wheeler–DeWitt equation is real, the complex numbers emerge together with the time t [15, 16]. Hasn't this been put in by hand through the i in the ansatz (4)? Not really. One can start with superpositions of complex wave functions of the form (4), which together give a real quantum state. But now again decoherence comes into play. Irrelevant degrees of freedom distinguish the complex components from each other, making them dynamically independent [6]. In a sense, time is "measured" by irrelevant degrees of freedom (gravitational waves, tiny density fluctuations). Some time ago I estimated the magnitude of this effect for a simple cosmological model [17] and found that the interference terms between the complex components can be as

[2] More precisely, some of the gravitational degrees of freedom can also remain quantum, while some of the non-gravitational variables can be macroscopic and enter the definition of time.

small as

$$\exp\left(-\frac{\pi mc^2}{128\hbar H_0}\right) \sim \exp\left(-10^{43}\right) \ , \tag{6}$$

where H_0 is the Hubble constant and m the mass of a scalar field, and some standard numbers have been chosen. This gives further support for the recovery of time as a viable semiclassical concept.

There are, of course, situations where the recovery of semiclassical time breaks down. They can be found through a study of the full Wheeler–DeWitt equation (3). One can, for example, study the behaviour of wave packets: semiclassical time is only a viable approximation if the packets follow the classical trajectory without significant spreading. One may certainly expect that a breakdown of the semiclassical limit occurs at the Planck scale (2). But there are other situations, too. One occurs for a classically recollapsing Universe and is described in the next section. Other cases follow from models with fancy singularities at large scales. The "big brake", for example, corresponds to a Universe which classically comes to an abrupt halt with infinite deceleration, leading to a singularity at large scale factor. The corresponding quantum model was recently discussed in [18]. If the wave packet approaches the classical singularity, the wave function will necessarily go to zero there. The time t then loses its meaning, and all classical evolution comes to an end before the singularity is reached. One might even speculate that not only time, but also space disappears [19].

The ideas presented here are also relevant to the interpretation of quantum theory itself. They strongly suggest, for example, that the Copenhagen interpretation is not applicable in this domain. The reason is the absence of a classical spacetime at the most fundamental level, which in the Copenhagen interpretation is assumed to exist from the outset. In quantum gravity, the world is fundamentally timeless and does not contain classical parts. Classical appearance only emerges for subsystems through the process of decoherence – with limitations dictated by the solution of the full quantum equations.

4 The Direction of Time

A fundamental open problem in physics is the origin of irreversibility in the Universe, the recovery of the arrow of time [20]. It is sometimes speculated that this can only be achieved from a theory of quantum gravity. But can statements about the direction of time be made if the theory is fundamentally timeless?

The answer is yes. The clue is, again, the semiclassical nature of the time parameter t. As we have seen in the last section, t is defined via fundamental gravitational degrees of freedom. The important point is that the Wheeler–DeWitt equation (3) is *asymmetric* with respect to the scale factor that describes the size of the Universe in a given state. It assumes a simple form for a small universe, but a complicated form for a large universe. For small scale factor there is only a minor interaction between most of the degrees of freedom. The equation then allows the formulation of a simple initial condition [20]: the absence of quantum entanglement between global degrees of freedom (such as the scale factor) and local ones (such as gravitational waves or density perturbations). The local variables serve as an irrelevant environment in the sense of decoherence.

Absence of entanglement means that the full quantum state is a product state. Tracing out the environment has then no effect; the state of the global variables remains pure. There is then no entropy (as defined by the reduced density matrix) connected with them: all information is contained in the system itself. The situation changes with increasing scale factor; the entanglement grows and the entropy for the global variables increases, too. As soon as the semiclassical approximation is valid, this growth also holds with respect to t; it is inherited from the full equation. The direction of time is thus defined by the direction of increasing entanglement. In this sense, the expansion of the universe would be a tautology.

There are interesting consequences for a classically recollapsing universe [21]. In order to produce the correct classical limit, the wave function of the quantum universe must go to zero for large scale factors. Since the quantum theory cannot distinguish between the different ends of a classical trajectory (such ends would be the Big Bang and the Big Crunch), the wave function must consist of many quasi-classical components with entropies that increase in the direction of a larger Universe; one could then never observe a recollapsing universe. In the region where the classical turning point would be found, all components have to interfere destructively in order to fulfill the final boundary condition of the wave function going to zero. This is a drastic example of the relevance of the superposition principle far away from the Planck scale – with possible dramatic consequences for the fate of our Universe: the classical evolution would come to an end in the future.

Let me emphasize again that all the consequences presented in this essay result from a very conservative starting point: the assumed universality of quantum theory and its superposition principle. Unless this assumption breaks down, these consequences should hold in every consistent quantum theory of gravity. We are able to understand from the fundamental picture of a timeless world both the emergence and the limit of our usual concept of time.

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