Free will, undecidability, and the problem of time in quantum gravity

Rodolfo Gambini

Instituto de Física, Facultad de Ciencias,

Universidad de la República, Iguá 4225,

CP 11400 Montevideo, Uruguay

rgambini@fisica.edu.uy

Jorge Pullin

Department of Physics and Astronomy,

Louisiana State University,

Baton Rouge, LA 70803-4001

pullin@lsu.edu

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Abstract

In quantum gravity there is no notion of absolute time. Like all other quantities in the theory, the notion of time has to be introduced "relationally", by studying the behavior of some physical quantities in terms of others chosen as a "clock". We have recently introduced a consistent way of defining time relationally in general relativity. When quantum mechanics is formulated in terms of this new notion of time the resolution of the *measurement problem* can be implemented via decoherence without the usual pitfalls. The resulting theory has the same experimental results of ordinary quantum mechanics, but every time an event is produced or a measurement happens two alternatives are possible: a) the state collapses; b) the system evolves without changing the state. One therefore has two possible behaviors of the quantum mechanical system and physical observations cannot decide between them, not just as a matter of experimental limitations but as an issue of principle. This first-ever example of fundamental undecidability in physics suggests that nature may behave sometimes as described by one alternative and sometimes as described by another. This in particular may give new vistas on the issue of free will.

In general relativity there is no notion of absolute time. In fact, there is no absolute notion. All physical predictions have to be formulated as relations between physical quantities. This has been recognized since the early days of general relativity through Einstein's "hole argument" [1]. In particular the notion of time has to emerge "relationally". One possible way of attacking this was introduced by Page and Wootters [2]. In their proposal one takes any physical quantity one is interested in studying and chooses another physical variable that will act as "clock". One then studies how the first variable "evolves as a function of the second one". In this view, time does not play any preferred role among other physical variables. This is in contrast to ordinary quantum mechanics where one has to unnaturally assume that time is supposed to be the only variable in the universe not subject to quantum fluctuations. In spite of the simplicity and naturalness of this proposal to tackle the problem of time in quantum gravity, technical problems arise. The problems are related with what one considers to be physical quantities in a theory like general relativity. Usual things that one may consider physical quantities, like "the scalar curvature of space-time at a given point" are not well defined objects in general relativity. The problem is what is "a given point"? Points in space have to be defined physically in general relativity. One can characterize a point as a "place where something physical happens" (for instance, a set of physical fields takes certain values). Then one could ask "how much is the curvature at that point". The end result is indeed physical. But it is again a relation between the values of curvatures and fields. Such relation is given and immutable. How could one construct a clock out of something immutable? It appears that the only things that are physical are immutable relations and the only things that evolve are the members of the relations, like the curvatures and fields. In technical terms, what one can consider as physical observable in general relativity is a quantity that is left invariant under the symmetries of the theory, or in the canonical language, that commutes with the constraints. Since one of the constraints is the Hamiltonian, physical quantities do not evolve. Therefore they cannot work as clocks. This created problems [3] for the Page–Wootters proposal. A way out was sought by trying to establish relations not between physical quantities but between mathematical quantities one uses to describe the theory that are not directly measurable (like for instance, the components of the metric at a point). Far from helping, this led to significant technical problems since one ends formulating the theory in terms of unobservable quantities. Ultimately it was shown in model systems that the proposal cannot be used to compute elementary things, for instance quantum probabilities of transition [3].

The observation we have recently made [4] is that the Page–Wootters construction can be rescued by using Rovelli's proposal of "evolving constants of the motion" [5], a concept that can be traced back to DeWitt, Bergmann and Einstein himself. This idea is to introduce genuinely observable physical quantities, i.e. relations between magnitudes as we highlighted above, but that depend on a continuous parameter. If one imagines evolution as changes in such a parameter, one can actually construct the relational description of Page and Wootters and show that it actually leads to the correct quantum probabilities of transition, at least in model systems [4]. The beauty of the complete construction is that the continuous parameter in the evolving constants completely drops out at the end of the day and the formulation remains entirely written in terms of truly observable physical quantities, even in the extreme situations that can develop in physics when quantum gravity effects become important.

Remarkably, taking seriously this relational solution to the problem of time in quantum gravity has implications for all of physics. We will concentrate on the the changes that this new description introduces in ordinary quantum mechanics. When one has a relational description where one's "clock" is a physical variable subject to quantum fluctuations like any any other, the description of ordinary quantum mechanics is different from the traditional one. In the ordinary formulation there exists an absolute time that is *not quantum mechanical*, it is represented by a completely classical parameter that is not subject to quantum fluctuations. This is clearly an idealization since all clocks we use in the real world are subject to such fluctuations. Although very small to be observed in practice, they are there.

At this point it is good to introduce another element that will be modified by the use of real clocks and rods to describe quantum mechanics[1]. The evolution of states in ordinary quantum mechanics is technically called "unitary". This in particular implies that information is preserved in evolution. In a sense "evolution is trivial" and everything is determined given the initial state. The only place where something non-trivial happens in ordinary quantum mechanics is when a measurement takes place. There the quantum states are supposed to evolve in a non-trivial way. Why this type of evolution happens in the measurement process and what justifies such change in the state is one of the open conceptual problems of quantum mechanics called *the measurement problem* [7].

Acknowledging that the real clocks and rods that one may use to measure space-time are not arbitrarily accurate requires reformulating the theory in terms of such clocks and rods. We have carried out such reformulation in some detail in ref. [8]. It is not too surprising that in the resulting

[1] We concentrate in this essay on the use of real clocks, but similar effects appear in the measurement of distances that add on to the ones we discuss here. See [6] for details.

picture one does not have a unitary evolution: although the underlying theory *is unitary*, our clocks and rods are not accurate enough to give a depiction of evolution that *appears unitary*. It is also no more the case that evolution is only non-trivial at the measurement of quantities. This also implies that there is a steady loss of information in the evolution of quantum states.

The reader at this point may ask: sure, a classical non-fluctuating time is an idealization. But one uses idealizations frequently in physics. Can I not always find a clock such that its quantum fluctuations are small enough to ignore this effect altogether? The answer is negative. And it is quantum mechanical and gravitational in nature. It is well known that in quantum mechanics one needs to expend energy in order to achieve accurate measurements [9]. On the other hand, gravity puts fundamental limits on how much energy can be concentrated in a measuring device before it turns into a black hole. Coupling together these two observations, one concludes that there exist fundamental limits, imposed by quantum mechanics and gravity, on how accurately we can measure distances and time [10, 11]. A detailed calculation shows that these limits imply that quantum states described in terms of a realistic clock variable T lose coherence (quantum information) exponentially. The exponent is given by $T_{\text{Planck}}^{4/3}T^{2/3}\omega^2$ where T is the time elapsed ω is the Bohr frequency associated with the energy difference between components of the quantum state (to have evolution one needs a superposition of states with different energies) and $T_{\text{Planck}} \sim 10^{-44}s$ is Planck's time[2]. The effect is too small to be observed with current technologies, but might be within the reach of technologies of the relatively near future [14]. To give an idea of the meaning of these numbers, the loss of coherence is larger the larger the energy difference of the quantum states in superposition is and the longer one waits. To have something visible in typical times in the lab, one would require states involving about $10^{12} - 10^{14}$ atoms in coherence. Current Bose–Einstein condensates have 10^6 atoms in coherence. Notice that although the effect is too small to be observed today, its existence is not controversial, since one can magnify the behavior arbitrarily just by choosing bad clocks. In fact, experiments in cavities can be reinterpreted in this way and the effect is readily measurable [15].

Although experimentally not detectable today, this fundamental effect has important conceptual implications, in particular for the *measurement problem in quantum mechanics* we mentioned above. As stated, the latter refers to the fact that the state of the system being measured changes abruptly during the process of measurement. Technically it falls into an *eigenstate of the measured quantity*

[2] Although the particular time dependence has been questioned [12], for what follows one only needs an effect that grows with time and other mechanisms have been proposed that yield such behavior, albeit with a weaker time dependence [13].

right after a measurement has been performed. This is usually referred to as the "reduction process". The conceptual problem is: how can one explain this abrupt change of state? Notice that this problem is quite pervasive because in quantum mechanics "measurement" has a more general meaning than in common parlance. Namely, in quantum mechanics the theory describes probabilities of events. Every time an event happens, a "measurement" takes place. For instance, every element of the reality we see around us is constituted by a network of "events" and therefore may be considered as a result of many "measurements" (Tegmark has a nice discussion of this point [16]).

A widely accepted explanation of the abrupt change in the wavefunction in a measurement is to consider that there exists an interaction between the system being measured and the environment in general (in particular with the measuring device). Both the environment and the measuring device are typically systems with a vastly larger number of degrees of freedom than those of the quantum system being studied. Also, the measuring device has many more degrees of freedom than the one displaying the measurement (e.g. "the needle in a gauge"). The interaction of the quantum system with the environment and measuring device leads to the quantum system losing information that in particular is not registered in "the needle of the gauge". The end result is that the evolution apparently loses unitarity, and information appears to be lost. Because the interaction is with a vast number of degrees of freedom, the rate of information loss is very quick and the phenomenon therefore appears as "abrupt".

The above "solution" to the measurement problem has been criticized. We cannot do justice to the full extent of the problem and its associated vast literature in the confines of this essay (see for instance the book of d'Espagnat [17] for references). Our claim is that the fact that non-trivial evolution happens all the time due to our imperfect clocks and measuring rods contributes to surmounting a significant portion of these obstacles [18]. Let us briefly review how.

There are two main criticisms to the solution of the problem of measurement by invoking the environment (decoherence). The first criticism is that that the "system plus environment" evolves unitarily and therefore all information is still present at the end and could in principle be retrieved. The second type of criticism is related to the fact that the system is left in a superposition of states through the interaction with the environment and therefore it would not generate a definite event (or measurement) but a superposition of them.

Let us address the first of the two criticisms. No matter how large the number of degrees of freedom of the environment, one can *in principle* recover the information by realizing a measurement of the joint "system plus measuring device plus environment" system [17]. Another way of recovering the information, assuming the "system plus measuring device plus environment" is a closed system is to just wait for a long time. Eventually the interactions of the system with the environment and measuring device will bring back the information to the "system plus measuring device" degrees of freedom. These phenomena are called "revivals" in the literature [17]. But let us now reconsider the fundamental loss of coherence that arises due to our inability to measure space-time with arbitrary precision. It has the attractive feature of killing off the possibility of "revivals" in a fundamental, inescapable way. If one attempted to "wait longer" to see revivals, the effect we discussed just becomes larger, as we pointed above. Therefore the detailed study of concrete examples like Zurek's model (see [18] for details) lead us to conclude that the longer one waits the more information the system loses and the chances of revivals actually diminish. One can also see that the effect does not allow to measure observables for the complete system including the environment and attempting to recover the information that way. The reason for this is that the fundamental loss of coherence that we discuss in this essay, although typically minute, is magnified —in examples we have studied— due to the large number of degrees of freedom that get involved in a measurement process [19].

Before addressing the second objection, let us note that the above behavior naturally leads to the main point of the essay: undecidability. Since examples have led us to the conjecture that one cannot measure observables of the whole system plus environment nor we can observe revivals one cannot decide if the quantum state has suffered reduction or it evolved unitarily. In fact, it could even be conceivable that sometimes there might be reduction, sometimes not, and we do not have reasons to expect one or the other in a given instance. The difference between these two views of nature can be very significant. In one extreme case quantum states are given "once and for all" as initial conditions. The evolution is unitary, and what we perceive as loss of unitarity is due to our inability to access the underlying variables of the theory, due to gravitational limitations. In the other extreme view, quantum states are evolving all the time due to reduction processes. There can also be combinations of the two scenarios where in some events evolution is unitary and in others is not. Undecidability does not imply that the difference between these two scenarios is irrelevant. For instance, there may exist complex systems (an intriguing example are living organisms) for which it is impossible to prepare their initial state or consider ensembles. For such systems, undecidability may occur among widely different states. No matter what is the outcome, reduction or unitary evolution, the choice between these alternatives could produce observable phenomena later on. The specific outcome may have important consequences on the occurrence of future events.

Let us get back to the second objection to the decoherence solution to the measurement problem: that at the end of the interaction with the environment, the measuring apparatus is generically left in a superposition of (eigen)-states corresponding to different "positions of the needle of the gauge". That would not correspond to what one usually calls a "measurement" in which the apparatus is in a given (eigen-)state, corresponding to the "needle of the gauge" taking a definite position. This is what John Bell called the "and-or" problem [20]. Namely, What explains that the needle took a definite position from within the superposition of states? In particular, did a further change in the state occur to select the given position of the needle? This does not have to be the case. For instance, in the many-worlds interpretation of quantum mechanics it is assumed that the state has a unitary evolution all the time but it does not describe a particular universe but the whole set of alternatives. In our world only one of the states in the superposition is realized and in each world a different event is observed. Also, in the modal interpretations [21] the occurrence of events is associated to the "actual properties" that the system can acquire without changing its state, which evolves unitarily. It turns out that our effect may help avoid this problem since it allows to define the appearance of events without necessarily implying a change in the quantum state. We can assume that an event occurs when the distinction between the "system plus apparatus plus environment" being in a superposition or in a given state becomes undecidable. This phenomenon is typical of interaction with an environment or a measuring device, i.e. it does not occur in quantum systems in isolation. In such case a unitary evolution or an abrupt change as the one given by a collapse would be obviously distinguishable.

The above proposal leads naturally to a revision of the ideas of natural laws in physics. In philosophy there are different attitudes that have been taken towards the physical laws of nature (see for instance [22]). One of them is the "regularity theory", many times attributed to Hume [23]; in it, the laws of physics are statements about uniformities or regularities of the world and therefore are just "convenient descriptions" of the world. Ernest Nagel in *The Structure of Science* [24] describes this position in the following terms: *"Hume proposed an analysis of causal statements in terms of constant conjunctions and de facto uniformities.. —according to Hume [physical laws consist] in certain habits of expectation that have been developed as a consequence of the uniform but de facto conjunctions of [properties]."* The laws of physics are dictated by our experience of a preexisting world and are a representation of our ability to describe the world but they do not exhaust the content of the physical world.

A second point of view sometimes taken is the "necessitarian theory" [22], which states that laws of nature are "principles" which govern the natural phenomena, that is, the world "necessarily obeys" the laws of nature. The laws are the cornerstone of the physical world and nothing exists without a law. The presence of the undecidability we point out suggests strongly that the "regularity theory" point of view is more satisfactory since the laws do not dictate entirely the behavior of nature.

Let us turn now our attention to the issue of freedom. We have seen that after the occurrence of an event the system may choose between behaving as if there has been a reduction process or not. That is, after the observation of the event either the system simply behaves as if it were part of the universe and its state were that of the universe or if as its state would be given by the reduction postulate. In the first case the system would keep its entanglement with the rest of the universe (i.e. the environment), in the second it will lose its entanglement. The availability of this choice opens the possibility of the existence of free acts. This type of act of the system will not imply any violation whatsoever of the laws of physics, understanding the latter as regularities in the observation of nature. It should be noted that this freedom in the system is not even ruled by a law of probabilities for the possible outcomes.

It is worthwhile pointing out that the notion of free will introduced here is different from the one introduced by Conway and Kochen (CK) [25]. We have started from quantum mechanics and gravitation and concluded that there exists undecidability and as a consequence free will. CK, on the other hand, start by considering a human observer conducting an experiment, which can make one of a set of possible observations. Starting from this and a limited set of assumptions that do not involve assuming that quantum mechanics holds, they conclude that elementary particles and other microscopic systems must also behave with "free will". This observation is attractive because it has as almost inevitable result that physics is indeterministic in the sense that is usually understood in quantum mechanics.

Freedom affects the causal structure of the world and therefore it does not belong in the realm of psychology but the ultimate discussion about its existence belongs in the realm of physics. The great philosopher Spinoza was the first in successfully building a complete philosophical system consistent with the laws of physics of his time. Those laws were completely deterministic. In his point of view, *"in nature there is nothing contingent, but all things have been determined from the necessity of the divine nature to exit and produce an effect"* [26].

We now have more advanced physical laws than the ones available to Spinoza, and they seem to imply undecidability and allow for free will. We live in a contingent world. In it, the transition from "what could be" to "what is" results either from mere chance or from a meaningful choice of free will. It is surprising that the freedom stemming from the undecidability yields two alternatives, the choice between which is meaningful: either the systems involved in events conserve their entanglement with the universe or break that entanglement. We would like to put forward the proposal that adopting the regularist point of view together with the idea of undecidability may allow to confront important objections to the libertarian[3] stance. These types of objections have been repeatedly leveled against attempts to substantiate free will based on the probabilistic nature of quantum mechanics. In fact if quantum mechanics only implies a mere lack of causal determination in the occurrence of events, this is not sufficient to ensure that it makes sense to consider a free act for which responsibility is possible. These objections stem from the potential fallacy of considering that only two exclusive alternatives exist: the deterministic and the random, excluding the possibility that the agent have any capability to control or self-determination over her acts. Implicit in this argument is the necessitarian point of view which excludes all aspects of reality not controlled by physical laws.

To conclude, we have observed that inherent limitations in the measurement process introduced by the use of a relational notion of time in quantum gravity appear to imply undecidability in the laws of physics. This strengthens the regularist vision of physical laws and opens the door to an essential difference through which free acts lead to different possible evolutions of the quantum state when an event takes place. The ability to act freely we discuss stems from quantum mechanics and therefore has a universal character. It is not entirely clear that it is connected with the decision making process of humans. It is currently widely contentious if quantum mechanics plays any role in processes in the human brain [28]. It would be quite disappointing if a universe that naturally includes in the laws of physics the capability for free acts will end up disallowing them for human beings.

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