The Nature and Origin of Time-asymmetric Spacetime Structures

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Abstract: Time-asymmetric spacetime structures, in particular those representing black holes and the expansion of the universe, are intimately related to other arrows of time, such as the second law and the retardation of radiation. The nature of the quantum arrow, often attributed to a collapse of the wave function, is also essential to understand the much discussed "black hole information loss paradox". The master arrow that would combine all arrows of time does not have to be identified with the direction of a formal time parameter that would allow us to formulate the dynamics as a succession of global states. It may even change direction with respect to a fundamental "physical" clock, such as the cosmic expansion parameter, if this were extrapolated to negative "pre-big-bang" values.

1. Introduction

Since gravity is attractive, most gravitational phenomena are asymmetric in time: objects fall down or contract under the influence of gravity. In General Relativity, this asymmetry leads to drastically asymmetric spacetime structures, such as future horizons and future singularities, which would occur, in particular, in black holes.* However, since the relativistic and nonrelativistic laws of gravitation are symmetric under time reversal, all asymmetries must arise as consequences of specific (seemingly "normal") initial conditions, for example a situation of rest that can be prepared by means of other arrows of time, such as friction. Otherwise the argument would apply in both directions of time. Indeed, the symmetry of the gravitational laws does allow objects to be thrown up, where their free motion could end by another external intervention, or the possible existence of "white holes", which would have to contain past singularities and past horizons.

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* By "lead to" I mean a (timeless) logical – not a (time-asymmetric) causal relation.

The absence of past horizons and past singularities from our universe (except for a possible big bang singularity) can be regarded as a time arrow characterizing our global spacetime (Sect. 2), while Einstein's field equations would not only allow the opposite situation, but also many solutions with mixed or undefined arrows of time – including closed time-like curves and non-orientable spacetimes. Therefore, the mere possibility of posing an "initial" condition is exceptional from a general point of view. I will here not discuss such mathematically conceivable solutions that do not seem to be realized in Nature, but instead concentrate on models that come close to our universe – in particular those which are globally of Friedmann type. A specific arrow characterizing a Friedmann universe is given by its expansion (unless this would be reversed at some time of maximum extension) – see Sect. 4.

In many cases, non-gravitational arrows of time remain relevant for gravitating bodies even after the latter have been prepared in an appropriate initial state. This applies, in particular, to strongly gravitating objects, such as stars, whose evolution is essentially controlled by thermodynamics (emission of heat radiation into the cold universe). The relation between the electrodynamic and thermodynamic arrows (retardation and the second law, respectively)[1] is quite obvious in this case.

Gravitating systems are nonetheless thermodynamically quite unusual: they possess negative specific heat.[2] This means, for example, that stars become hotter when losing energy through emitting heat, or that satellites accelerate as a consequence of friction in the earth's atmosphere. It can best be understood by means of the virial theorem, which states in the nonrelativistic limit that, for all forces varying with the second negative power of distance (that is, gravitational and Coulomb forces), bound states have to obey the relation $\overline{E_{pot}} = -2\overline{E_{kin}}$, where the overbar means averaging over (quasi) periods of time. Therefore,

$$E = E_{pot} + E_{kin} = \overline{E_{pot}} + \overline{E_{kin}} = \frac{1}{2}\overline{E_{pot}} = -\overline{E_{kin}} \propto -T \quad . \tag{1}$$

So these systems must gain twice as much (negative) gravitational energy than they are losing by radiation or by friction in order to keep up a state of equilibrium. Nonrelativistically, this negative heat capacity could be bounded by means of other (repulsive) forces that become relevant at high densities, or by the Pauli principle, which controls the density of electrons in white dwarfs or solid bodies, for example. Relativistically, even these limits will break down at a certain mass, since (1) relativistic degeneracy must ultimately lead to the creation of other particles, while (2) the potential energy of repulsive forces will itself gravitate, and for a sufficiently large mass overcompensate any repulsion. Therefore, it is the thermodynamic arrow that requires evolution of gravitating systems towards the formation of black holes. Classically, black holes thus define the final states in the evolution of such systems.

2. Black Hole Spacetimes

The metric of a spherically symmetric vacuum solution for non-zero mass is shown in Fig. 1 in Kruskal coordinates *u* and *v*. This diagram represents the uniquely completed Schwarzschild metric

$$ds^2 = \frac{32M^2}{r} e^{-r/2M} \left(-dv^2 + du^2\right) + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right) \quad , \tag{2}$$

where the new coordinates *u* and *v* are in the external region ($r > 2M$) related to conventional Scharzschild coordinates *r* and *t* by

$$u = e^{r/4M} \sqrt{\frac{r}{2M} - 1} \cosh\left(\frac{t}{4M}\right) \tag{3a}$$

$$v = e^{r/4M} \sqrt{\frac{r}{2M} - 1} \sinh\left(\frac{t}{4M}\right) \quad . \tag{3b}$$

Each point in the diagram represents a sphere with surface $4\pi r^2$. Note that *r* and *t* interchange their roles as space and time coordinates for $r < 2M$, where $2M$ is the Schwarzschild radius. All parameters are given in Planck units $h/2\pi = G = c = 1$.

As Nature seems to provide specific initial conditions, it may thereby exclude all past singularities, and hence all past horizons. This initial condition would immediately eliminate the Schwarzschild-Kruskal vacuum solution that is shown in the Figure, but we may instead consider a spherically symmetric initial mass distribution at rest, such as a dust cloud, which would freely collapse into a black hole, as quantitatively described by the Oppenheimer-Snyder scenario[3] (see left part of Fig. 2). The vacuum solution (1) is then valid only outside the surface of the dust cloud, but this surface must at some finite proper time fall through the arising horizon and a bit later onto the future singularity.

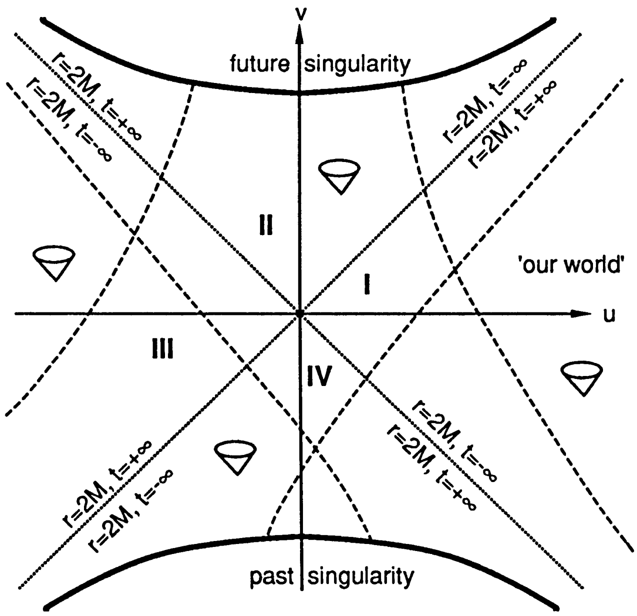

Fig. 1: Complete continuation of the Schwarzschild solution by means of unique Kruskal coordinates. Quadrants I and II represent external and internal parts of a black hole. III is another asymptotically flat region, while IV would describe the interior of a "white hole". In this diagram, fixed (non-unique) Schwarzschild coordinates r and t are represented by hyperbola or straight lines through the origin, respectively. Proper times of local objects could start at $t = -\infty$ in I or at $t = +\infty$ in III, or at $r = 0$ on the past singularity in IV, while they must end at $t = +\infty$ or $-\infty$ in I or III, ot at a second singularity with coordinate value $r = 0$ in II. On time-like or light-like curves intersecting one of the horizons at the Schwarzschild radius $r = 2M$, t jumps from $+\infty$ to $-\infty$ at the rim of quadrant I, or from $-\infty$ to $+\infty$ at the rim of quadrant III, where t decreases in the physical time direction.

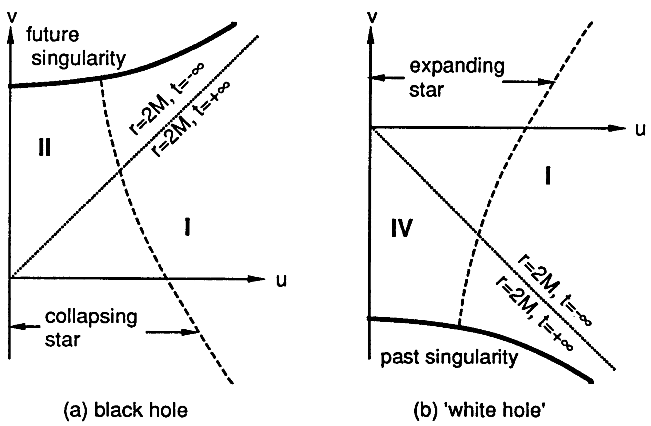

Fig. 2: Kruskal diagrams of a black and a "white" hole.

For an interacting gas cloud, this free collapse would be thermodynamically delayed by the arising pressure, as indicated in the Introduction. Gravitational radiation would finally lead to a further loss of order and structure, while whatever remains would become unobservable to an external observer. Although thermodynamic phenomena control the loss of energy by radiation during most of the time, the asymmetric absence of past singularities only represents a *cosmological* initial condition. However, a white hole initiated by a past singularity and *completely* representing a time-reversed black hole would require anti-thermodymics and coherently incoming advanced radiation. So one may wonder whether these various arrows are related to one another in order to define a common "master arrow".

Since it would take infinite Schwarzschild coordinate time for matter to reach the horizon, any message from the close vicinity of the horizon would not only be strongly redshifted, but also be dramatically delayed by the gravitational field. The message could reach an observer only at increasingly later stages of the universe. (An observer falling into a galactic size black holes could live happily and send messages for a considerable length of proper time before he would approach the horizon.) So all objects falling into the black hole must completely disappear from the point of view of mortal external observers and their descendents, even though these objects never seem to reach the horizon. The only exceptions are some conserved properties that have consequences for the asymptotic metric or other asymptotic fields, namely angular momentum and electric charge. This time-asymmetric conclusion is known as the "no-hair theorem" for black holes. Over very long times, a black hole accumulating ionized interstellar matter may even lose its charge and angular momentum, too, for statistical and dynamical reasons.[4] Only its mass and its center of mass motion would then remain observationally meaningful. A black hole is usually characterized in its center of mass system and in terms of its long-lasting properties: its mass M, charge Q, and angular momentum J, in which case its "Kerr-Newman metric" is explicitly known. The internal topological structures of these metrics for $J \neq 0$ and/or $Q \neq 0$ are radically different from that of the Kruskal geometry, thus raising doubts in the reality of these unobservable consequences of the classical theory of General Relativity.

It is important, though, to keep in mind the causal structure of a Schwarzschild black hole: its interior spacetime region II never enters the past of any external observer, that is, it will never become a historical "fact" for him. While the exterior region r

$> 2M$ can be completely foliated in terms of space-like slices according to increasing Schwarzschild or similar time coordinates with $-\infty < t < +\infty$, the interior can then be regarded as its *global future* continuation beyond the horizon, where it can be labeled by the Schwarzschild coordinate r decreasing from $r = 2M$ to $r = 0$. This structure must be essential for all causal considerations that include black holes. In this classical scenario, the internal state of a black hole would be completely determined by the infalling matter, which could even depend on our "free" decisions. Nonetheless, all properties of this matter become "irrelevant" for external observers, an adjective that is related to the generalized concept of coarse graining, which is used in the definition of physical entropy in statistical thermodynamics.[5]

3. Black Hole Thermodynamics

In the classical picture described so far, a black hole would be a perfect absorber at zero temperature. This picture had to be corrected when Bekenstein and Hawking demonstrated,[6] the latter by taking into account quantum fields, that black holes must possess finite temperatures and entropies proportional to their surface gravity κ and surface area A, respectively:

(4a) $$T = \frac{\hbar\kappa}{2\pi k_B} \rightarrow \frac{\hbar c^3}{8\pi G k_B}\frac{1}{M} \quad ,$$

(4b) $$S = \frac{k_B c^3 A}{4\hbar G} \rightarrow \frac{4\pi k_B G}{\hbar c} M^2 \quad .$$

Here, κ and A are functions of M, Q and J, while the explicit expressions given on the right hand sides hold for Schwarschild black holes ($Q = J = 0$) and with respect to spatial infinity (that is, taking into account the gravitational redshift). This means, in particular, that a black hole must emit thermal radiation (Hawking radiation) proportional to $T^4 A$ according to Stefan-Boltzmann's law, and therefore, that it could live only for a finite time of about $10^{65}(M/M_{sun})^3$ years. For astrophysical objects this is very much longer than the present age of the universe of 10^{10} years.

Even these large evaporation times would apply only after the black hole has for a very long time *grown* in mass by accreting matter[7] – at least until the cosmic background temperature has dropped below the corresponding black hole temperature because of its increasing Hubble redshift. Although the resulting time scales are extremely large, they are always less than the infinite Schwarzschild coordinate time required for a horizon to form. These coordinate times represent physical times for asymptotic observers in the rest frame of the black hole, and they may be consistently used as global dynamical time parameters in the external region. However, according to black hole causality, the interior region will *never* form, since its mass must have been radiated away before this could ever happen. Schwarzschild simultaneities may be counterintuitive. For example, one may use time translation invariance to define a Kruskal diagram (Figs. 1 or 2) by fixing the time coordinate $v = t = 0$ to coincide with an external time close to the peak of the Hawking radiation (in the very distant future from our point of view). Assuming that one can neglect any quantum uncertainty of the metric (which must in principle arise in quantum gravity) for this purpose, all infalling matter that had survived so far would at this coordinate time $v = 0$ be in the very close vicinity of the origin at $u = 0$, although most of the remaining black hole mass must already exist in the form of outgoing Hawking radiation for all kinds of fields.

Black hole radiation is again based on the radiation arrow, but it also depends on the quantum arrow in using a statistical description for the emission. Accordingly, a pure quantum state gravitationally collapsing into a black hole would be transformed into a mixed state in a way quite similar to an unread measurement, or to the statistical description of the decay of a highly excited pure quantum state of a complex object.[8] Such a "representative ensemble" may be defined for a pure state again by means of some appropriate coarse graining. In quantum theory, one usually neglects (that is, one regards as irrelevant) the entanglement that would arise between all decay products according to a unitary description. This irrelevance is not just a matter of an arbitrarily chosen macroscopic point of view, but it must be dynamically consistent in allowing the application of a corresponding master equation. It does then not only permit the definition of an objective concept of non-trivial "physical" entropy,[5] but it also justifies the concept of decoherence. In contrast to the global ensemble entropy $-trace(\rho \ln \rho)$ that would be conserved under unitary dynamics, physical entropy is additive (that is, in accordance with an entropy density). The major difference between the decay of highly excited complex matter states and the evaporation of black holes is that the latters' unitary description is not explicitly known (and often even questioned to exist), while the fundamental interpretation of quantum probabilities forms an open problem by its own.

The thus described situation is nonetheless known and much discussed as the "information loss paradox for black holes".[9] A unitary description would require that the "information" that defined the initial pure state is transformed into non-local entanglement (formally analogous to the statistical correlations arising in deterministic Boltzmann collisions), which in the quantum case must give rise, in particular, to a superposition of "many worlds". Its replacement by an ensemble of many *possible* worlds according to a fundamental statistical interpretation (a collapse of the wave function) would not only neglect the information contained in their unobservable (even though formally existing) relative phases, but also *change* the quantum state in a stochastic and time-asymmetric way. Note that this argument does not specifically depend on the quantization of gravity. If the two entropy concepts (black hole and thermodynamic) are to be compatible, the entropy of the final (thermal) radiation must be greater than that of the black hole, while the latter has to exceed that of any kind of infalling matter. Smaller than solar mass "primordial" black holes might also have formed from exceptional density fluctuations during the radiation era of the Friedmann universe, but they seem to be rare and, because of their origin, can hardly have stored much "information".

Roger Penrose compared black hole entropy numerically with that of matter under normal conditions.[10] Since the former is proportional to the square of the black hole mass, macroscopic black hole formation leads to a tremendous increase of entropy. As thermodynamic entropy is proportional to the particle number, it is dominated in the universe by photons from the primordial cosmic radiation (whose number exceeds baryon number by a factor of 10^9). If our observable part of the universe of about 10^{79} baryons consisted completely of solar mass black holes, it would possess an entropy of order 10^{98} (in units of k_B^{-1}), that is, 10^{10} times as much as the present matter entropy represented by the 10^{88} photons. Combining all black holes into one huge one would even raise this number to 10^{121}, the highest conceivable entropy for this (perhaps partial) universe unless its volume would tremendously increase.[4,7,11] If entropy is indeed a measure of probability, any approximately homogenous matter distribution would statistically be extremely improbable except at a very late stage of an eternally expanding universe. For this reason, the homogeneity of the initial universe is usually regarded as "the fundamental improbable initial condition" that would give rise to a global master arrow of time for statistical reasons (see Sect. 4). However, its relationship to the thermodynamically important condition of absent or "dynamically irrelevant" non-local initial correlations (or entanglement in the quantum case) is as yet not fully understood.

The information loss paradox has mostly been discussed by using other foliations than those according to Schwarzschild time (appropriately continued into the collapsing matter region but remaining outside the horizon). They are often assumed to consist of space-like slices of the black hole spacetime of Fig. 2 (left) that intersect the horizon at some point, so it would become questionable how the information and energy of infalling matter may ever be transferred back into the outgoing Hawking radiation. However, if neither the horizon nor the interior region ever entered existence, the presumption of such a foliation would simply be wrong. Recall that the Oppenheimer-Snyder model does not dynamically take into account any mass loss by Hawking radiation. Although the corresponding ("back") reaction of the metric in response to the radiation loss may in principle require quantum gravity, the argument is here based on no more than the conservation of energy in a situation where it does not have to be questioned.

Instead of using a vacuum metric when calculating the arising Hawking radiation, one should take into account the presence of infalling matter, in which case some kind of internal conversion may lead to the annihilation of this matter. Note that the *local* Bekenstein-Hawking temperature diverges close to the horizon, while the high-energy tail of the external Hawking radiation may turn out to be asymmetric with respect to particles and antiparticles in order to preserve lepton and baryon numbers. As long as such a rather conventional possibility does not have to be excluded, there is no really convincing motivation for speculating about black hole remnants, a gravitational violation of conservation rules, superluminal tunneling through an existing horizon, or a fundamental violation of unitarity that would go beyond conventional quantum measurements or phase transitions.[12] Authors talking about "black hole complementarity" seem to have overlooked that the "non-concept" of complementarity can and must be avoided if quantum theory is assumed to be universally valid, as done in decoherence theory. Note also that the concept of an S-matrix is inapplicable to macroscopic objects because of their permanent and unavoidable interaction with the environment. Because of the extreme lifetimes of black holes, the information loss problem is at any rate a rather academic one: any "lost" information would remain hidden at least for the next 10^{65} years, and it could hardly ever be used even if it finally came out in the form of entangled radiation.

Several authors (including myself) have seen a problem in the equivalence principle, which seems to require that observers or detectors freely falling into the black hole should *not* register any black hole radiation. They have therefore concluded that the disappearance of black holes, too, must be observer-dependent. However, this conclusion appears to be wrong. The equivalence between a black hole and a uniformly accelerated detector is not complete, since an observer or detector external to a black hole is not immersed in *isotropic* heat radiation. Radiation comes only from the black hole surface. Even if the infalling detector does not register it, its effect on detectors at fixed distance from the black hole, or its flux through a fixed sphere around the black hole, must exist just as objectively as the clicks of an accelerated detector in an inertial vacuum (attributed to Unruh radiation) can be observed by an inertial observer, too. When using Schwarzschild simultaneities, the infalling observer would have to regard these events as happening in an extreme quick motion movie with respect to his proper time – including an extreme blue-shift of the outgoing radiation. Nonetheless, the phenomenon of black holes from the point of view of external observers must be compatible with the fate of an infalling observer, who may either soon (in his proper time) himself have to be affected by the internal conversion process, or otherwise have to experience the black hole radius as very rapidly shrinking and disappearing before he would arrive at the expected horizon. If he could survive, he would have travelled far into the future by this spacetime detour through the vicinity of a black hole. However, no theory that is compatible with the equivalence principle can explain baryon number nonconservation in the absence of a singularity, although all symmetries can in principle be broken by the stochastic evolution of an *individual* Everett branch (an experienced "world").

4. Expansion of the Universe

The expansion of the universe (or its observable part) is time-asymmetric, but in contrast to all other arrows it forms an individual process rather than a whole class, such as black holes, radiation emitters, or steam engines. It may even change its direction at some time of maximum extension, although present astronomical observations seem to indicate that the expansion will last forever. A homogeneous and isotropic Friedmann universe is described by the dynamics of the expansion parameter $a(t)$ according to the time-symmetric "energy theorem"

(5) $$\frac{1}{2}\left(\frac{1}{a}\frac{da}{dt}\right)^2 = \frac{4\pi}{3}\rho(a) + \frac{\Lambda}{6} - \frac{k}{2a^2} \quad ,$$

where ρ is the energy density of matter, Λ the cosmological constant, and k the sign of the spatial curvature. The "total energy" in Equ. (5) is fixed in general-relativistic cosmology. Penrose's estimates demonstrate that this homogeneity requires an extremely improbable initial condition at least from a classical statistical point of view.[10] Therefore, it must be highly unstable under the influence of gravity.

In addition to a homogeneous initial matter distribution, Penrose postulated that free gravitational fields vanished at the Big Bang. These free fields are described by the *Weyl tensor*, that is, the trace-free part of the curvature tensor. The trace itself (the Ricci tensor) is locally fixed by the stress-energy tensor of matter by means of the Einstein field equations. The Weyl tensor, on the other hand, is analogous to the divergence-free part of the electrodynamic field tensor $F^{\mu\nu}$, since its divergence $\partial_\mu F^{\mu\nu}$ (the trace of the tensor of its derivatives) is given by the charge current j^ν. Therefore, the *Weyl tensor hypothesis* is analogous to the requirement of the absence of any free initial electromagnetic radiation, a condition that would leave only retarded electromagnetic fields of all sources of the past in the universe. This retardation had indeed been proposed by Planck (in a discussion with Boltzmann)[13] and again by Ritz (in a discussion with Einstein)[14] as an explanation of the thermodynamic arrow. Here, Boltzmann and Einstein turned out to be right, since the retardation observed in reality is instead a conseqence of thermodynamic absorbers[1] – cosmologically of an absorber formed by the radiation era, which does not allow us to observe any conceivable earlier radiation. In contrast, the universe seems to be "transparent" to gravitational fields, including those that might have been present at the Big Bang.

Note that the initial homogeneity can *not* be explained by an early cosmic inflation (as has occasionally been claimed) if this inflation is described as a unitary process that would have to conserve the number of states in an effective ensemble that may represent entropy.

Although our universe seems to expand forever, the idea of a recontracting one is at least conceptually interesting. Thomas Gold first proposed that the low entropy condition should not be based on an absolute direction of time, and hence be valid at a conceivable Big Crunch as well.[15] The latter would then be observed as another Big Bang by observers living during the contraction era. This scenario would not only require a transition era without any arrow in our distant future, but it would also pose serious consistency problems, since the extremely low initial probability would have to be squared if the two conditions are statistically independent.[16] If nonetheless true, it would have important consequences for the unobservable fate of matter falling into massive black holes. If such black holes survived the thermodynamical transition era that must accompany the turning point, they would enter an era with reversed arrows of time. Because of the transparency of the late universe to light, they would receive coherent advanced radiation from their formal future even before that happens. This advanced radiation must then "retro-cause" black holes to expand again in order to approach a state of homogeneity in accordance with the final condition.[17]

A reversal of the arrow of time may not only occur in the distant future, but also in the past. Several *pre-big-bang* scenarios have been dicussed in novel and so far speculative theories, which are not the subject of this article. In them, one usually identifies the direction of the formally continued time parameter with the direction of all physical arrows of time. For example, according to arguments first used in loop quantum gravity,[18] the configuration space for Friedmann type universes may be doubled by interpreting negative values of the cosmic expansion parameter a as representing formally negative volume measures. The cosmic expansion can then be continued backwards in time beyond the Big Bang into its mirror image by "turning space inside out" (turning right-handed triads into left-handed ones) while going through $a = 0$ even in a classical picture. For this purpose, the classical dynamical description (5) would have to be modified close to the otherwise arising singularity at $a = 0$ – as it is indeed suggested by loop quantum gravity. However, if the "initial" conditions responsible for the arrow of time are still assumed to apply at this former singularity, this arrow would change direction at this time, and $|a|$ rather than a would represent a cosmic clock. Observers on both temporal sides of the Big Bang could only remember events in the direction towards $a = 0$. Another possibility to avoid the singularity is a repulsive force acting at small values of a,[19] which would lead to a Big Bounce with similar conceivable consequences for the arrow of time as the above model that involves space inversion.

In cosmology, quantum aspects of the arrow of time must again play an important role. According to the Copenhagen interpretation, there is no quantum world – so any consistent cosmic history would end when quantum aspects become relevant. In other orthodox interpretations, the unitary evolution of the quantum state is repeatedly interrupted by measurements and similar events, when the wave function "collapses" indeterministically. The consequences of such stochastic events on quantum cosmology would be enormous, but as long as no general dynamical formulation of a collapse has been found and confirmed one has again arrived at a dead end. Going forward in time may be conceptually simple in such theories, since one just has to "throw away" certain components of the wave function, while going backwards would require all those lost components. So one has at least to keep them in the cosmic bookkeeping – regardless of whether they are called "real" (as in the Everett interpretation) or not. Going back to the Big Bang would require *all* those "many worlds" that have ever been thrown away in the orthodox description during the past of our universe, while one would then have to throw away others when going beyond the Big Bang in order to obtain an individual quasi-classical "pre-big-bang history". In other words, a unitary continuation beyond the Big Bang can only relate the complete Everett superpositions of worlds on both sides of the Big Bang, but *not* any individually observed worlds. The master arrow of time does not only affect all realms of physics, but it is truly universal in a much deeper sense: it can only have "multiversal" meaning. The same multiversality was required in a unitary black hole evolution, and it does, in fact, apply to the unitary quantum description of all macroscopic objects, when irreversible decoherence mimics a collapse of the wave function, and thereby explains classicality.

The time direction of this branching of the wave function requires a highly symmetric initial state (presumably at $a = 0$), which does not contain any nonlocal entanglement that could later have local effects. Quantum dynamics will then lead to growing decoherence (the in practice irreversible dislocalization of superpositions), and it would "intrinsically" break various global symmetries – thus producing a (symmetric) superposition of many different asymmetric "landscapes", in modern terminology.

5. Quantum Gravity

General Relativity has traditionally been considered in a block universe picture, but because of the hyperbolic type of Einstein's field equations it is a dynamical theory just as any other field theory. An explicit dynamical description, which requires a non-Lorentzinvariant *form*, has been derived by Arnowitt, Deser and Misner (ADM).[20] This Hamiltonian formulation is a prerequisite for the canonical quantization of the theory.

The ADM formalism is based on an arbitrary foliation of spacetime that has to be chosen "on the fly", that is, while solving an initial value problem. The spatial metric on these space-like slices represents the dynamical variables of the theory, and it has to be described by a symmetric matrix $h_{kl}(x_m)$ (with k,l,m running from 1 to 3). Three of its six independent matrix elements represent the choice of physically meaningless coordinates, two would in the linear limit correspond to the spin components of a gravitational wave (±2 with respect to the direction of propagation for a plane wave), while the remaining one can be regarded as a measure of "many-fingered" physical time (metric distance between slices as a function on space). The corresponding canonical momenta π^{kl} define the embedding of the spatial metric into spacetime and the propagation of spatial coordinates. The dynamics can then be formulated by means of the Hamiltonian equations with respect to an arbitrary time parameter t that formally distiguishes between different slices in a given foliation. They are equivalent to Einstein's field equations. In contrast to metric time, the parameter t is geometrically or physically meaningless, and can therefore be arbitrarily replaced by any monotonic function $t' = f(t)$, including its inversion.

Note that when one says that Special Relativity abandoned the concept of absolute time, this statement refers only to the concept of absolute simultaneity, while proper times, which control all motion according to the principle of relativity, are here assumed to be given absolutely by the Lorentz metric. This remaining absoluteness is dropped only in General Relativity, where the metric itself becomes a dynamical object, as indicated above. The absence of an absolute time parameter (here represented by its reparametrizability) had already been required by Ernst Mach. Julian Barbour, who studied its consequences in much historical detail,[21] called it "timelessness". A complete absence of time would remove any possibility to define its arrow, while a remaining time parameter (characterizing a one-dimensional succession of states) still allows one to define time asymmetric trajectories (histories).

The invariance of the theory under spatial coordinate transformations and time reparametrization is warranted by four constraints for the matrix $h_{kl}(t)$, called momentum and Hamiltonian constraints, respectively. They may be regarded as initial conditions, but are conserved in time. In particular, the Hamiltonian constraint assumes the form

(6) $$H(h_{kl},\pi_{kl}) = 0 \quad .$$

When quantized,[22] and when also taking into acount matter variables, this constraint translates into the Wheeler-DeWitt equation,

(7) $$H\ \Psi(h_{kl},matter) = 0 \quad ,$$

which means that the corresponding Schrödinger equation becomes

(8) $$\frac{\partial\Psi}{\partial t} = 0 \quad .$$

Even the time parameter t has now disappeared, because there are no parametrizable trajectories representing cosmic histories in quantum theory any more. Only this drastic quantum consequence of classical reparametrizability can be regarded as *genuine timelessness.*

The timelessness of the Wheeler-DeWitt wave function has been known at least since 1967, but it seems to have originally been regarded as a mainly formal aspect. Time was often smuggled in again in various ways – for example in terms of parametrizable Feynman paths, by means of semiclassical approximations, or by attempts of reintroducing a Heisenberg picture in spite of the Hamiltonian constraint.[23] The problem became pressing in connection with realistic interpretations of the wave function.[24]

The wave functional $\Psi(h_{kl},matter)$ describes a general entanglement of geometry and matter. If we still had a succession of such quantum states (forming a quantum trajectory or quantum history), an appropriate, unentangled initial state could explain the arrow of growing entanglement and decoherence. The resulting branching of the wave function according to a corresponding parameter t would then include branching spacetime geometries. We do *not*, but the metric h_{kl} does contain a measure of metric time regardless of the existence of any trajectories. Therefore, it describes a *physical* time dependence by means of the entanglement of this measure with all other degrees of freedom even for a solution of (8).[25] For Friedmann universes, the expansion parameter a, which is part of the metric h_{kl}, is such an appropriate measure of time, but how does that help us to define an initial value problem for the static wave equation (7)? The surprising answer is that this equation is globally hyperbolic for Friedmann type universes – now not on spacetime as for classical fields, but on its infinite-dimensional configuration space. The expansion parameter a appears as a time-like variable because of the unusual negative sign of its formal kinetic energy component.[26] Therefore, the Wheeler-DeWitt equation allows one to define an initial value problem at a small value of a, for example. For a modified Wheeler-DeWitt equation this possibility may even be extended to $a = 0$. There is then no difference between a (multiversal) Big Bang and a Big Crunch any more, since in the absence of a time parameter the wave function can only be a standing wave on configuration space.

A metric tensor and other fields defined on a Friedmann sphere may be represented by a four-dimensional multipole expansion, which is quite useful for describing the very early, approximately homogeneous and isotropic universe.[27] In this case, one may conveniently model matter quantum mechanically by a massive scalar field $\Phi(x_k)$. The wave functional of the universe then assumes the form $\Psi(a,\Phi_0,\{x_n\})$, where Φ_0 is the homogeneous part of the scalar field, while $\{x_n\}$ are all higher monopoles of geometry and matter. For the metric, only the tensor modes are geometrically meaningful, while the rest represents gauge degrees (spatial coordinates). The global hyperbolic nature then becomes obvious.

In a simple toy model one neglects all higher multipoles in order to solve the Wheeler-DeWitt equation on the remaining two-dimensional "mini-superspace" formed by the two monopoles only. The remaining Hamiltonian represents an a-dependent harmonic oscillator for Φ_0, which allows one to construct adiabatically stable Gaussian wave packets ("coherent states").[28] Figure 3 depicts the propagation of such a wave packet with respect to the "time" variable $\alpha = \ln a$. This standing wave on mini-superspace mimics a timeless classical trajectory. However, the complete wave functional has to be expected to form a broad superposition of many such dynamically separate wave packets (a cosmologically early realization of "many worlds"). If the higher multipoles are also taken into account, the Wheeler-DeWitt equation may describe decoherence progressing with a – at first that of the monopoles Φ_0 and a itself, although this requires effective renormalization procedures.[29]

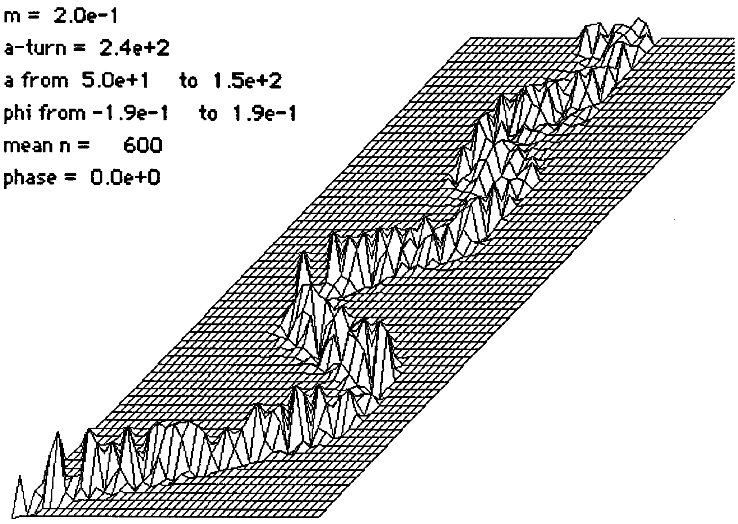

Fig. 3: Wave packet for a homogeneous massive scalar field amplitude Φ_0 (plotted along the horizontal axis) dynamically evolving as a function of the time-like parameter $\alpha = \ln a$ that is part of the metric (second axis in this mini-superspace). The classical trajectory possesses a turning point above the plot region – at about $a = 240$ in this example that represents an expanding and recontracting universe. Wave mechanically, this corresponds to a reflection of the wave packet by means of a repulsive potential in (5) at this value of a (reflected part not shown in the Figure). This reflection leads to considerable spreading of the "initial" wave packet. The causal order of the two legs of the trajectory is quite arbitrary, however, and the phase relations defining coherent wave packets could alternatively be chosen to fit the second leg instead. So this spreading does *not* represent an arrow of time (From Ref. 1, Sect. 6.2.1.)

This intrinsic dynamics with respect to the expansion parameter a has nothing as yet to do with the local dynamics in spacetime (controlled by proper times along time-like curves) that must be relevant for matter as soon as the metric becomes quasi-classical (except for microscopic gravitational waves). In order to understand the relation between these two dynamics, one may apply a Born-Oppenheimer expansion in terms of the inverse Planck mass, which is large compared to all particle masses, in order to study the Wheeler-DeWitt wave function.[30,22] The Planck mass occurs in the kinetic energy terms of all geometric degrees of freedom (their part in the Laplacean of the Hamiltonian constraint). The formal expansion in terms of powers of $m_{Planck}^{-1/4}$ defines an "adiabatic approximation" in analogy to the theory of molecular motion (light electrons versus heavy nuclei). In most regions of configuration space (depending on the boundary conditions) one may then further apply a WKB approximation to the "heavy" degrees of freedom Q, and in this way one obtains an approximate solution of the type

$$\Psi(h_{kl},matter) = \Psi(Q,q) = e^{iS(Q)}\chi(Q,q) \quad , \tag{9}$$

where $S(Q)$ is a solution of the Hamilton-Jacobi equations for Q, while $\chi(Q,q)$ depends only slowly on Q, and q describes all "light" (matter) variables. Under these approximations one may derive from the Wheeler-DeWitt equation the adiabatic dependence of $\chi(q)$ on Q in the form

$$i\nabla_Q S \cdot \nabla_Q \chi(Q,q) = h_Q \chi(Q,q) \quad . \tag{10}$$

The operator h_Q is the weakly Q-dependent Hamiltonian for the matter variables q. This equation defines a new time parameter t_{WKB} separately along all WKB trajectories (classical spacetimes) by the directional derivative

$$\frac{\partial}{\partial t_{WKB}} := \nabla_Q S \cdot \nabla_Q \quad . \tag{11}$$

In this way one obtains from (10) a time-dependent global Schrödinger equation for matter with respect to the *derived* WKB time t_{WKB}.[24] This parameter defines a time coordinate in spacetime, since the classical trajectories $Q(t)$ in the superspace of spatial geometries Q define spacetime geometries. Equ. (10) would also lead to the decoherence of superpositions of different WKB components, such as they would be present in a real wave function $e^{iS}\chi_+ + e^{-iS}\chi_-$ that has to be expected for the real Wheeler-DeWitt equation.

In order to solve this derived time dependent Schrödinger equation along a given WKB trajectory, hence with respect to a classical spacetime that does in turn adiabatically depend on the evolving matter, one needs a (low entropy) initial condition in the region where the WKB approximation applies. For this purpose one would in principle first have to solve the exact Wheeler-DeWitt equation (or its generalized version that may apply to some as yet elusive unified theory) with respect to a by using its fundamental cosmic initial condition at $a = 0$. This might be done, for example, by using the multipole expansion until one enters the WKB region (at some distance from $a = 0$), where this solution must provide initial conditions for the matter wave functions χ at all arising WKB trajectories. The derived time-dependent Schrödinger equation with respect to t_{WKB} may then be expected to describe further decoherence (the emergence of classical properties), and thereby explain the origin of all other arrows of time. In particular, it must enforce decoherence of superpositions of different spacetimes, which would form separate quasi-classical "worlds".[22] This decoherence would even affect conceivable *CPT* symmetric superpositions of black and white holes – in analogy to parity eigenstates of chiral molecules if they had ever existed.[12]

Acknowledgement: I wish to thank Claus Kiefer for his comments on an early draft of this manuscript.

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