A Five Dimensional Cosmological Model With A Fifth Dimension as Fundamental as Time

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Abstract

In five dimensional cosmological models, the convention is to include the fifth dimension in a way similar to the other space dimensions. In this work we attempt to introduce the fifth dimension in a way that a time dimension would be introduced. With such an internal space, we are able to obtain accelerated expansion without introducing dark energy. Moreover our work shows that all relevant cosmologies in four dimensions can be embedded in a flat cosmology in five dimensions.

1 Introduction

We considered ourselves to be living in a three dimensional space until Einstein changed our notion of time from a parameter to a dimension to explain electrodynamics of moving bodies and led us to think in terms of a four dimensional spacetime. The number of dimensions has been increasing ever since. With Kaluza[1] and Klein[2] the four dimensions were augmented to five in an attempt to unite electromagnetism and gravity. While we are plainly aware of our four dimensional surroundings, nobody has been able to observe a fifth dimension yet. Obviously extra dimensions are going to be helpful, but one needs to explain their observational absence. The natural attempt is to compactify them as very small, periodic internal dimensions.

The usefulness of a fifth dimension grew when Randall and Sundrum[3,4] used it to explain the hierarchy problem, which brought forth the concept of brane worlds. Although important steps were made with all these works and many others, it seems that there is still much to be done in order to completely understand internal extra dimensions. Today the number of dimensions have gone up to eleven or one can also say that they came down from twenty six via superstrings with string or M-theory's quest to understand quantum effects of gravity and unite all fundamental forces[5].

Something as mysterious as extra dimensions is dark energy. It was Hubble, who first observed galaxies to be receding from each other. Today we are certain that our universe is accelerating while expanding[6,7]. We have come up with the term dark energy as the source of this accelerated expansion, yet we are not certain what it really is, hence the name "dark". Perhaps the two mysterious concepts, dark energy and extra dimensions, are connected with each other[8]. A recent attitude towards dark energy is to explain it by a modification to the geometric side of Einstein's equations. One successful attempt which includes extra dimensions, is brane-world gravity, where at high energies massive modes of graviton dominate, gravity leaks off the brane where its weakening initiates acceleration[9].

In this work we want to approach this jungle of dimensions with purely cosmological concerns. We want to see what happens when we introduce an extra spacelike dimension into the cosmological metric, in the same way that a timelike dimension would be introduced. This way we will be putting forth symmetries between time and the internal space, which brings up the question whether internal space can be as fundamental as time. Our second motivation is to see if we can obtain the effects of dark energy from this five dimensional metric without having to introduce the cosmological constant. In the end we will achieve all relevant four dimensional cosmologies as a four dimensional slice of a flat five dimensional cosmology. Thus we will have pointed out that our internal space is just as fundamental as time and we will have obtained the expansion usually credited to dark energy, from an extra dimension.

2 The Metric and The Einstein Tensor

The Friedmann-Robertson-Walker metric has the following form

$$ds^2 = -dt^2 + a^2(t)d\Sigma^2 \tag{1}$$

where $d\Sigma^2$ is the metric of three spacelike dimensions all of which have uniform curvature. We use natural units with $c = \hbar = 1$. The spacelike sections, being scaled by $a(t)$, expand or contract in time. Therefore the scale factor $a(t)$ is what gives us the dynamics of this four dimensional spacetime. Because all three spatial dimensions have the same scale factor they all change by the same amount, hence this universe expands or contracts isotropically only with time. Here the time is proper time, which is what an observer who sees the universe expand around him measures as time. Since it doesn't have a factor dependent on any of the spacelike dimensions in front of it, it has the same value at every point. In other words the cosmological time is the proper time at every point in this spacetime. The role of time is fundamental here.

We will consider a metric of the form

$$ds^2 = f^2(t)g^2(w)[-dt^2 + dw^2] + a^2(t)b^2(w)\frac{dx^2 + dy^2 + dz^2}{(1 + \frac{\kappa(x^2+y^2+z^2)}{4})^2} \tag{2}$$

where κ is the curvature of spacelike sections with the values -1 for negatively curved, 0 for flat, $+1$ for positively curved, we can always make a coordinate transformation so that

$$dT = f(t)dt \tag{3a}$$

$$dW = g(w)dw \tag{3b}$$

$$ds^2 = -G^2(W)dT^2 + F^2(T)dW^2 + A^2(T)B^2(W)\frac{dx^2 + dy^2 + dz^2}{(1 + \frac{\kappa(x^2+y^2+z^2)}{4})^2}. \tag{4}$$

Here T may be called the cosmological time because it is the only coordinate that an observer will measure as time. But the value measured will change for different observers at different points in W, because we cannot get rid of the factor of W in front of time. We cannot get rid of the factor of time in front of W the internal space either. As such, the role of internal space in this five dimensional universe is as fundamental as the role time plays here. We will carry on our calculations in the coordinates where the metric is as it is in (2).

The observable three spacelike dimensions share the same scale factor and are again isotropic. Here they do not evolve only in time but in w as well. Although our internal space, w, is a spacelike dimension, it works as a timelike extra dimension would.

Our basis one forms are

$$e^4 = if(t)g(w)dt, \quad i = \sqrt{-1} \tag{5a}$$
$$e^5 = f(t)g(w)dw \tag{5b}$$
$$e^i = a(t)b(w)\frac{dx^i}{1+\frac{\kappa r^2}{4}} \tag{5c}$$

and we use the metric $g_{\mu\nu} = diag(1,1,1,1,1)$ with $i = 1,2,3$. Using Cartan's formalism we get the curvature two forms to be

$$\Omega^i{}_j = [\frac{\dot{a}^2(t)}{a^2(t)f^2(t)g^2(w)} - \frac{b'^2(w)}{b^2(w)f^2(t)g^2(w)} + \frac{\kappa}{a^2(t)b^2(w)}]e^i \wedge e^j \tag{6}$$

$$\Omega^i{}_4 = [\frac{\dot{a}(t)\dot{f}(t)}{a(t)f^3(t)g^2(w)} - \frac{\ddot{a}(t)}{a(t)f^2(t)g^2(w)} + \frac{b'(w)g'(w)}{b(w)f^2(t)g^3(w)}]e^4 \wedge e^i$$
$$+ [\frac{\dot{a}(t)b'(w)}{ia(t)b(w)f^2(t)g^2(w)} - \frac{\dot{a}(t)g'(w)}{ia(t)f^2(t)g^3(w)} - \frac{b'(w)\dot{f}(t)}{ib(w)f^3(t)g^2(w)}]e^5 \wedge e^i \tag{7}$$

$$\Omega^i{}_5 = [\frac{b'(w)\dot{a}(t)}{ia(t)b(w)f^2(t)g^2(w)} - \frac{b'(w)\dot{f}(t)}{ib(w)f^3(t)g^2(w)} - \frac{\dot{a}(t)g'(w)}{ia(t)f^2(t)g^3(w)}]e^4 \wedge e^i$$
$$+ [\frac{b''(w)}{b(w)f^2(t)g^2(w)} - \frac{b'(w)g'(w)}{b(w)f^2(t)g^3(w)} - \frac{\dot{a}(t)\dot{f}(t)}{a(t)f^3(t)g^2(w)}]e^5 \wedge e^i \tag{8}$$

$$\Omega^4{}_5 = [-\frac{\dot{f}(t)^2}{f^4(t)g^2(w)} + \frac{\ddot{f}(t)}{f^3(t)g^2(w)} + \frac{g'(w)^2}{g^4(w)f^2(t)} - \frac{g''(w)}{g^3(w)f^2(t)}]e^4 \wedge e^5 \tag{9}$$

where differentiation with respect to w and t are denoted as

$$\dot{h} = \frac{\partial h}{\partial t}$$

$$h' = \frac{\partial h}{\partial w}$$

We get the Riemann tensor $R_{\mu\nu\lambda x}$ from curvature two forms by

$$\Omega^\mu{}_\nu = \frac{1}{2}R^\mu{}_{\nu\lambda x}e^\lambda \wedge e^x$$

and the components of our Einstein tensor by

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R$$

where R is the Ricci Scalar $R = g^{\mu\nu}R_{\mu\nu}$. All this gives us the following

$$G_{ii} = -2\frac{b''(w)}{b(w)f^2(t)g^2(w)} + 2\frac{\ddot{a}(t)}{a(t)f^2(t)g^2(w)} + \frac{\dot{a}^2(t)}{a^2(t)f^2(t)g^2(w)} - \frac{b'^2(w)}{b^2(w)f^2(t)g^2(w)}$$
$$+ \frac{\kappa}{a^2(t)b^2(w)} - \frac{\dot{f}^2(t)}{f^4(t)g^2(w)} + \frac{\ddot{f}(t)}{f^3(t)g^2(w)} + \frac{g'^2(w)}{f^2(t)g^4(w)} - \frac{g''(w)}{f^2(t)g^3(w)} \tag{10}$$

$$G_{44} = 3\frac{\dot{a}(t)\dot{f}(t)}{a(t)f^3(t)g^2(w)} + 3\frac{b'(w)g'(w)}{b(w)f^2(t)g^3(w)} - 3\frac{b''(w)}{b(w)f^2(t)g^2(w)}$$
$$3\frac{\dot{a}^2(t)}{a^2(t)f^2(t)g^2(w)} - 3\frac{b'^2(w)}{b^2(w)f^2(t)g^2(w)} + 3\frac{\kappa}{a^2(t)b^2(w)} \tag{11}$$

$$G_{55} = 3\frac{\ddot{a}(t)}{a(t)f^2(t)g^2(w)} + 3\frac{\dot{a}^2(t)}{a^2(t)f^2(t)g^2(w)} + 3\frac{\kappa}{a^2(t)b^2(w)}$$
$$- 3\frac{\dot{a}(t)\dot{f}(t)}{a(t)f^3(t)g^2(w)} - 3\frac{b'^2(w)}{b^2(w)f^2(t)g^2(w)} - 3\frac{b'(w)g'(w)}{b(w)f^2(t)g^3(w)} \tag{12}$$

$$G_{54} = 3[\frac{b'(w)\dot{a}(t)}{ia(t)b(w)f^2(t)g^2(w)} - \frac{b'(w)\dot{f}(t)}{ib(w)f^3(t)g^2(w)} - \frac{\dot{a}(t)g'(w)}{ia(t)f^2(t)g^3(w)}] \tag{13}$$

3 Vacuum Solutions in 5 Dimensions

Now let us consider the vacuum solutions for flat spacelike sections, that is solutions to $G_{\mu\nu} = 0$ with $\kappa = o$.

From $G_{ii} = 0$ we get

$$2\frac{\ddot{a}(t)}{a(t)} + \frac{\dot{a}^2(t)}{a^2(t)} - \frac{\dot{f}^2(t)}{f^2(t)} + \frac{\ddot{f}(t)}{f(t)} = 2\frac{b''(w)}{b(w)} + \frac{b'^2(w)}{b^2(w)} - \frac{g'^2(w)}{g^2(w)} + \frac{g''(w)}{g(w)} \tag{14}$$

The right hand side of this equation is purely $w-$dependent, and the left hand side purely $t-$dependent. The only way these two sides are equal to one another is if they are equal to the same constant k. Thus out of G_{ii} we get the following two equations

$$2\frac{\ddot{a}(t)}{a(t)} + \frac{\dot{a}^2(t)}{a^2(t)} - \frac{\dot{f}^2(t)}{f^2(t)} + \frac{\ddot{f}(t)}{f(t)} = k \tag{15}$$

and

$$2\frac{b''(w)}{b(w)} + \frac{b'^2(w)}{b^2(w)} - \frac{g'^2(w)}{g^2(w)} + \frac{g''(w)}{g(w)} = k \tag{16}$$

With the same reasoning we get from $G_{44} = 0$

$$\frac{\dot{a}(t)\dot{f}(t)}{a(t)f(t)} + \frac{\dot{a}^2(t)}{a^2(t)} = l \tag{17}$$

$$\frac{b''(w)}{b(w)} + \frac{b'^2(w)}{b^2(w)} - \frac{b'(w)g'(w)}{b(w)g(w)} = l \tag{18}$$

and from $G_{55} = 0$

$$\frac{\ddot{a}(t)}{a(t)} + \frac{\dot{a}^2(t)}{a^2(t)} - \frac{\dot{a}(t)\dot{f}(t)}{a(t)f(t)} = m \tag{19}$$

$$\frac{b'^2(w)}{b^2(w)} + \frac{b'(w)g'(w)}{b(w)g(w)} = m \tag{20}$$

Thus we have two sets of equations, one set related to t and the other related to w. We will solve these two sets first and check whether the solutions satisfy $G_{54} = 0$, which gives

$$1 - \frac{a(t)\dot{f}(t)}{\dot{a}(t)f(t)} = \frac{g'(w)b(w)}{g(w)b'(w)} = constant. \tag{21}$$

Let's first look at the set related to t, whose solution will give us $a(t)$ and $f(t)$

$$2\frac{\ddot{a}(t)}{a(t)} + \frac{\dot{a}^2(t)}{a^2(t)} - \frac{\dot{f}^2(t)}{f^2(t)} + \frac{\ddot{f}(t)}{f(t)} = k \tag{22}$$

$$\frac{\dot{a}(t)\dot{f}(t)}{a(t)f(t)} + \frac{\dot{a}^2(t)}{a^2(t)} = l \tag{23}$$

$$\frac{\ddot{a}(t)}{a(t)} + \frac{\dot{a}^2(t)}{a^2(t)} - \frac{\dot{a}(t)\dot{f}(t)}{a(t)f(t)} = m \tag{24}$$

We can get an equation for $a(t)$ by adding the last two equations,

$$\frac{\ddot{a}}{a} + 2\frac{\dot{a}^2}{a^2} = m + l. \tag{25}$$

If we consider a solution of the form $a(t) = a_0 e^{\nu t}$ and plug this in (25) we get

$$a(t) = a_0 \exp[\sqrt{\frac{(m+l)}{3}}t]. \tag{26}$$

By imposing this solution on equation (25) we obtain

$$f(t) = f_0 \exp[\frac{2l - m}{\sqrt{3(m+l)}}t] \tag{27}$$

When the solutions (27) and (26) are inserted into equations (22), (23), (24) we find that (23) and (24) are satisfied identically where as (22) imposes the condition

$$m + l = k. \tag{28}$$

A similar approach to the w related set of equations,

$$2\frac{b''(w)}{b(w)} + \frac{b'^2(w)}{b^2(w)} - \frac{g'^2(w)}{g^2(w)} + \frac{g''(w)}{g(w)} = k \tag{29}$$

$$\frac{b''(w)}{b(w)} + \frac{b'^2(w)}{b^2(w)} - \frac{b'(w)g'(w)}{b(w)g(w)} = l \tag{30}$$

$$\frac{b'^2(w)}{b^2(w)} + \frac{b'(w)g'(w)}{b(w)g(w)} = m, \tag{31}$$

gives

$$b(w) = b_0 \exp[\sqrt{\frac{(m+l)}{3}}w] \tag{32}$$

and

$$g(w) = g_0 \exp[\frac{\sqrt{3}(2m - l)}{\sqrt{m+l}}w] \tag{33}$$

where equation(29) imposes the same condition $m+l=k$. Moreover our solutions imply that

$$k = m + l \geq 0 \tag{34}$$

since they each contain a $\sqrt{(m+l)}$ term. With these solutions $G_{54} = 0$ is satisfied as well.

Thus the vacuum solutions of our five dimensional metric with flat spacelike sections is

$$ds^2 = f_0^2 g_0^2 exp[\frac{4l-2m}{\sqrt{3(m+l)}}t + \frac{4m-2l}{\sqrt{3(m+l)}}w](-dt^2+dw^2) + a_0^2 b_0^2 exp[2\sqrt{\frac{m+l}{3}}(t+w)][dx^2+dy^2+dz^2] \tag{35}$$

4 The Effective 4 Dimensional Solution

We will now consider the above solution of the vacuum five dimensional spacetime at some $w = w_0$ where w_0 is a constant. At $w = w_0$ spacetime metric becomes

$$ds^2 = f_0^2 g_0^2 exp[\frac{4m-2l}{\sqrt{3(m+l)}}w_0] exp[\frac{4l-2m}{\sqrt{3(m+l)}}t](-dt^2) + a_0^2 b_0^2 exp[2\sqrt{\frac{m+l}{3}}w_0] exp[2\sqrt{\frac{m+l}{3}}t][dx^2+dy^2+dz^2] \tag{36}$$

$f_0 g_0 exp[\frac{2m-l}{\sqrt{3(m+l)}}w_0]$ is just a constant so we can set it equal to another constant F_0. With

$$F_0 = f_0 g_0 exp[\frac{2m-l}{\sqrt{3(m+l)}}w_0],$$

$$A_0 = a_0 b_0 exp[\sqrt{\frac{m+l}{3}}w_0]$$

we can write our solution as

$$ds^2 = -F_0^2 exp[\frac{4l-2m}{\sqrt{3(m+l)}}t]dt^2 + A_0^2 exp[2\sqrt{\frac{m+l}{3}}t][dx^2+dy^2+dz^2]. \tag{37}$$

To write this in terms of the cosmological proper time consider the following coordinate transformation

$$d\tilde{t} = F_0 exp[\frac{2l-m}{\sqrt{3(m+l)}}t]dt. \tag{38}$$

To simplify the notation we will define

$$\beta = \sqrt{\frac{m+l}{3}},$$

and

$$\alpha = \frac{3\beta}{2l-m}.$$

With all this our coordinate transformation gives,

$$\tilde{t} = F_0 \alpha e^{[\frac{t}{\alpha}]} \tag{39}$$

and

$$e^{2\beta t} = (\frac{\tilde{t}}{F_0 \alpha})^{2\beta\alpha}. \tag{40}$$

This coordinate transformation has turned our solution into

$$ds^2 = -d\tilde{t}^2 + A_0^2\tilde{t}^{2\alpha\beta}[dx^2 + dy^2 + dz^2]. \tag{41}$$

We can always absorb A_0 into $\vec{r}$ by a coordinate transformation. So if we drop the tilde, define $\alpha\beta = n$ our metric in its simplest form becomes

$$ds^2 = -dt^2 + t^{2n}[dx^2 + dy^2 + dz^2]. \tag{42}$$

Metric (42) contains all the relevant four dimensional cosmologies. For $n = \frac{2}{3}$ we have matter dominated universe, for $n = \frac{1}{2}$ we have radiation dominated universe.

Furthermore by setting $m = 2l$ in (36) we get

$$ds^2 = f_0^2 g_0^2 e^{2\sqrt{l}w_0}[-dt^2] + a_0^2 b_0 e^{2\sqrt{l}w_0} e^{2\sqrt{l}t}[dx^2 + dy^2 + dz^2]. \tag{43}$$

Before explaining what we have obtained let us simplify this metric further first. The factor $e^{2\sqrt{l}w_0}$ is just a constant which can be set to c_0^2. We can also absorb all the constants into dt^2 by the coordinate transformation,

$$\begin{aligned} d\tau &= f_0 g_0 c_0 dt \\ \frac{\tau - \tau_0}{f_0 g_0 c_0} &= t \end{aligned} \tag{44}$$

and define $a_0 b_0 c_0 exp[-\frac{\tau_0}{f_0 g_0 c_0}] = A_0^2$ so that we have

$$ds^2 = -d\tau^2 + A_0^2 e^{[\frac{2\sqrt{l}}{f_0 g_0 c_0}\tau]} d\vec{r}^2. \tag{45}$$

Let us denote τ by t and set $\alpha = \frac{\sqrt{l}}{f_0 g_0 c_0}$, the constant A_0 can also be absorbed into $d\vec{r}$

$$ds^2 = -d\tau^2 + e^{2\alpha t} d\vec{r}^2. \tag{46}$$

Thus we have obtained an exponential scale factor, a behavior attributed to dark energy with $\alpha = H_0$ where H_0 is approximately today's value of Hubble's parameter.

5 The Curvature and The Weyl Tensors

Components of the Weyl tensor in our convention of Ricci tensor $R_{\nu\lambda} = R^{\mu}{}_{\nu\lambda\mu}$, metric sign $(-,+,+,+)$, are calculated as

$$\begin{aligned} C_{\rho\sigma\mu\nu} = R_{\rho\sigma\mu\nu} &+ \frac{1}{d-2}(g_{\rho\mu}R_{\nu\sigma} - g_{\rho\nu}R_{\mu\sigma} - g_{\sigma\mu}R_{\nu\rho} + g_{\sigma\nu}R_{\mu\rho}) \\ &- \frac{1}{(d-1)(d-2)}(g_{\rho\mu}g_{\nu\sigma} + g_{\rho\nu}g_{\mu\sigma})R \end{aligned} \tag{47}$$

where d is the number of dimensions.

For our five dimensional solution, in equation (35),

$$R_{ijij} = [f_0^2 g_0^2 exp(\frac{4l - 2m}{\sqrt{3(m+l)}}t + \frac{4m - 2l}{\sqrt{3(m+l)}}w)]^{-1}(\frac{m+l}{3} - \frac{m+l}{3}) = 0$$

$$R_{i44i} = [f_0^2 g_0^2 exp(\frac{4l - 2m}{\sqrt{3(m+l)}}t + \frac{4m - 2l}{\sqrt{3(m+l)}}w)]^{-1}(\sqrt{\frac{m+l}{3}}\frac{l+m}{\sqrt{3(m+l)}} - \frac{m+l}{3}) = 0$$

$$R_{i45i} = R_{i54i} = \frac{1}{if^2(t)g^2(w)}[\frac{m+l}{3} - \sqrt{\frac{m+l}{3}}\frac{m+l}{\sqrt{3(m+l)}}] = 0$$

$$R_{i55i} = \frac{1}{f^2(t)g^2(w)}[\frac{m+l}{3} - \sqrt{\frac{m+l}{3}}\frac{m+l}{\sqrt{3(m+l)}}] = 0$$

$$R_{4545} = \frac{1}{f^2(t)g^2(w)}[-\frac{\dot{f}^2}{f^2} + \frac{\ddot{f}}{f} + \frac{g'^2}{g^2} - \frac{g''}{g}] = 0$$

all the components of Riemann curvature tensor are zero. Therefore the Ricci Scalar, all components of $R_{\mu\nu}$, and the Weyl tensor for the Ricci flat five dimensional metric are all zero. Our five dimensional universe is Ricci flat, meaning it contains no energy nor momentum density, and conformally flat, it contains no gravitational fields, in short it is flat and empty. It is not surprising that our universe turned out to be conformally flat, since we were handling the Ricci flat solution. What is good is that this flat five dimensional universe is able to contain all relevant four dimensional cosmologies.

It is a well established fact that the Friedmann-Robertson-Walker (FRW) metric can be put in a conformally flat form[10,11]. It has been further pointed out that[12] calculations on the age of the universe and its matter density carried out in conformally flat spacetime (CFS) coordinates agree better with the observations then those carried out in FRW coordinates. With such emphasis on the conformal flatness of our universe, it is an achievement to be able to embed standard four dimensional cosmology in a conformally flat five dimensional cosmology in this work on higher dimensional cosmologies.

6 Conclusion

We have obtained all relevant cosmologies, including dark energy dominated cosmology, as four dimensional slices of a flat five dimensional metric. We have been able to bring together all these different cases of four dimensional cosmologies in a five dimensional version because we allowed the internal dimension to be fundamental, like time. We know that at early times universe goes through different phases, radiation dominated, dust dominated and so forth by a power law, $a(t) = t^n$, where the value of n changes from one era to another. In our model this power depends on m and l, which are free parameters obtained from the separability of variables in $G_{44} = 0$ and $G_{55} = 0$.

We can say that one can assume the five dimensional universe to be flat and still obtain all relevant four dimensional cosmologies. The key point is to introduce the extra dimension, although spacelike, in a way a timelike dimension would be introduced. As such the internal dimension plays a fundamental role and the four dimensional cases can be put inside a five dimensional flat universe. The extra dimension can be compactified in the usual manner by introducing an orbifold.

The common intuition would be to imagine a four dimensional space expanding along time. Instead what we have introduced here is the three dimensional space expanding along both time and internal space. So in a sense we should visualize this as a four dimensional spacetime evolving along the extra dimension. Thus we conclude by saying, it is possible that we live in a vacuum, five dimensional universe where what we consider to be the effects of energy and momentum in four dimensions, are actually the effects of a five dimensional flat geometry.

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