The Small Scale Structure of Spacetime

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Abstract

Several lines of evidence hint that quantum gravity at very small distances may be effectively two-dimensional. I summarize the evidence for such "spontaneous dimensional reduction," and suggest an additional argument coming from the strong-coupling limit of the Wheeler-DeWitt equation. If this description proves to be correct, it suggests a fascinating relationship between small-scale quantum spacetime and the behavior of cosmologies near an asymptotically silent singularity.

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Stephen Hawking and George Ellis prefaced their seminal book, *The Large Scale Struc*ture of Space-Time, with the explanation that their aim was to understand spacetime "on length-scales from 10^{-13} cm, the radius of an elementary particle, up to 10^{28} cm, the radius of the universe" [1]. While many deep questions remain, ranging from cosmic censorship to the actual topology of our universe, we now understand the basic structure of spacetime at these scales: to the best of our ability to measure such a thing, it behaves as a smooth (3+1)-dimensional Riemannian manifold.

At much smaller scales, on the other hand, the proper description is far less obvious. While clever experimentalists have managed to probe some features down to distances close to the Planck scale [2], for the most part we have neither direct observations nor a generally accepted theoretical framework for describing the very small-scale structure of spacetime. Indeed, it is not completely clear that "space" and "time" are even the appropriate categories for such a description.

But while a complete quantum theory of gravity remains elusive, we do have fragments: approximations, simple models, and pieces of what may eventually prove to be the correct theory. None of these fragments is reliable by itself, but when they agree with each other about some fundamental property of spacetime, we should consider the possibility that they are showing us something real. The thermodynamic properties of black holes, for example, appear so consistently that it is reasonable to suppose that they reflect an underlying statistical mechanics of quantum states.

Over the past several years, evidence for another basic feature of small-scale spacetime has been accumulating: it is becoming increasingly plausible that spacetime near the Planck scale is effectively two-dimensional. No single piece of evidence for this behavior is in itself very convincing, and most of the results are fairly new and tentative. But we now have hints from a number of independent calculations, based on different approaches to quantum gravity, that all point in the same direction. Here, I will summarize these clues, provide a further piece of evidence in the form of a strong-coupling approximation to the Wheeler-DeWitt equation, and discuss some possible implications.

1 Spontaneous dimensional reduction?

Hints of short-distance "spontaneous dimensional reduction" in quantum gravity come from a number of places. Here I will review some of the highlights:

Causal Dynamical Triangulations

As we have learned from quantum chromodynamics—and from our colleagues in condensed matter physics—lattice approximations to the Feynman path integral can give us valuable information about the nonperturbative behavior of theories that may otherwise be extremely difficult to analyze. Lattice approximations to quantum gravity are not quite typical: in contrast to QCD, where fields live on a fixed lattice, gravity *is* the lattice [3], which forms a discrete approximation of a continuous spacetime geometry. Despite this difference, though, we might hope that a suitable lattice formulation could tell us something important about quantized spacetime.

The idea of combining Regge calculus with Monte Carlo methods to evaluate the gravitational path integral on a computer dates back to 1981 [4]. Until fairly recently, though,



Figure 1: A spatial slice and a typical history contributing to the causal dynamical triangulations path integral (R. Kommu and M. Sachs, UC Davis)

no good continuum limit could be found. Instead, the simulations typically yielded two unphysical phases, a "crumpled" phase with very high Hausdorff dimension and a twodimensional "branched polymer" phase [5]. The causal dynamical triangulation program of Ambjørn, Jurkiewicz, and Loll [6–8] adds a crucial new ingredient, a fixed causal structure in the form of a prescribed time-slicing. By controlling fluctuations in topology, this added structure suppresses the undesirable phases, and appears to lead to a good four-dimensional continuum picture. Results so far are very promising; in particular, the cosmological scale factor appears to have the correct semiclassical behavior [8,9]. Figure 1 illustrates a typical time slice and a typical history contributing to the path integral in a simulation developed at UC Davis [10].

A crucial question for any such microscopic approximation is whether it can genuinely reproduce the four-dimensional structure we observe at "normal" distances. This is a subtle issue, which cannot be answered by merely looking at particular contributions to the path integral. As a first step, we need a definition of "dimension" for a discrete structure that may be very non-manifold-like at short distances. One natural choice—although by no means the only one—is the spectral dimension [11], the dimension as seen by a diffusion process or a random walker.

The basic idea of the spectral dimension is simple. In any structure on which a random walk can be defined, the associated diffusion process will gradually explore larger and larger regions of the structure. The more dimensions available for the random walk to explore, though, the longer this diffusion will take. Quantitatively, diffusion from an initial position x to a final position x' on a manifold M may be described by a heat kernel K(x, x', s) satisfying

$$\left(\frac{\partial}{\partial s} - \Delta_x\right) K(x, x'; s) = 0, \quad \text{with} \quad K(x, x', 0) = \delta(x - x'), \quad (1)$$

where Δ_x is the Laplacian on M at x, and s is a measure of the diffusion time. Let $\sigma(x, x')$ be Synge's world function [12], one-half the square of the geodesic distance between x and x'. Then on a manifold of dimension d_S , the heat kernel generically behaves as

$$K(x, x'; s) \sim (4\pi s)^{-d_S/2} e^{-\sigma(x, x')/2s} \left(1 + \mathcal{O}(s)\right)$$
(2)

for small s. In particular, the return probability K(x, x, s) is

$$K(x, x; s) \sim (4\pi s)^{-d_S/2}$$
. (3)

For any structure on which a diffusion process can be defined, be it a manifold or not, we can now use equation (3) to define an effective dimension d_S , the spectral dimension. On a lattice, in particular, we can determine the spectral dimension by directly simulating random walks. In the causal dynamical triangulation program, the ensemble average over the histories contributing to the path integral then gives a quantum spectral dimension. The results of such simulations yield a spectral dimension of $d_S = 4$ at large distances [8,11]. This is a promising sign, indicating the recovery of four-dimensional behavior. Similarly, (2+1)-dimensional causal dynamical triangulations yield a large-distance spectral dimension of $d_S = 3$ [10,13].

At short distances, though, the result is dramatically different. In both 3+1 dimensions and 2+1 dimensions, the small-scale spectral dimension falls to $d_S = 2$. This is the first, and perhaps the clearest, sign of spontaneous dimensional reduction at short distances.

As noted above, d_S is by no means the only definition of a generalized dimension, and one may worry about reading too much significance into this result. Note, though, that the heat kernel has a special significance in quantum field theory: propagators of quantum fields may be obtained as Laplace transforms of appropriate heat kernels. For a scalar field, in particular, the propagator is determined by the heat kernel (1), and the behavior of the spectral dimension implies a structure

$$G(x, x') \sim \int_0^\infty ds \, K(x, x'; s) \sim \begin{cases} \sigma^{-2} & \text{at large distances} \\ \ln \sigma & \text{at small distances.} \end{cases}$$
(4)

The logarithmic short-distance behavior is the standard result for a two-dimensional conformal field theory. If one probes short distances with a quantum field, the field will thus act as if it lives in an effective dimension of two.

Renormalization Group Analysis

General relativity is nonrenormalizable: conventional perturbative quantum field theory techniques yield an infinite number of higher derivative counterterms, each with its own coupling constant. Nevertheless, the renormalization group *flow* of these coupling constants may provide us with valuable information about quantum gravity. In particular, a renormalization group analysis, even if incomplete or truncated, may offer a method for probing the theory at high energies and short distances.

One particularly dramatic possibility, first suggested by Weinberg [14], is that general relativity may be "asymptotically safe." Consider the full effective action for conventional gravity, with its infinitely many coupling constants. The renormalization group describes the dependence of these constants on energy scale, and in principle allows us to compute the high-energy/small-distance ("ultraviolet") couplings in terms of their low-energy/largedistance ("infrared") values. Under the renormalization group flow toward high energies, some of these constants may blow up, indicating that the description has broken down and that new physics is needed. An alternative possibility, though, is that the coupling constants might remain finite and flow to an ultraviolet fixed point. In that case, the theory would continue to make sense down to arbitrarily short distances. If, in addition, the critical surface—the space of such UV fixed points—were finite dimensional, the long-distance coupling constants would be determined by a finite number of short-distance parameters: not quite renormalizability, but perhaps almost as good.

Distinguishing among these possibilities (and others) is extremely difficult, and we are still a long way from knowing whether quantum general relativity is asymptotically safe. But there is growing evidence for a UV fixed point, coming from various truncations of the effective action and from exact calculations in dimensionally reduced models [15–17]. For our present purposes, the key result of these calculations is that field operators acquire large anomalous dimensions; that is, under a change in mass scale, they scale differently than one would expect from dimensional analysis based on their classical "engineering dimension." In fact, these operators scale precisely as one would expect for the corresponding quantities in a *two-dimensional* field theory [15]. Moreover, the spectral dimension near the putative fixed point can be computed using field theoretical techniques, and the result is again $d_S = 2$ [18].

There is, in fact, a fairly general argument that if quantum gravity is asymptotically safe, it must be effectively two-dimensional at very short distances [17, 19]. Consider the dimensionless coupling constant $g_N(\mu) = G_N \mu^{d-2}$, where G_N is Newton's constant and μ is the mass scale that appears in the renormalization group flow. Under this flow,

$$\mu \frac{\partial g_N}{\partial \mu} = [d - 2 + \eta_N(g_N, \dots)]g_N, \tag{5}$$

where the anomalous dimension η_N depends upon both g_N and any other dimensionless coupling constants in the theory. Evidently a free field (or "Gaussian") fixed point can occur at $g_N = 0$. For an additional non-Gaussian fixed point g_N^* to be present, though, the right-hand side of (5) must vanish: $\eta_N(g_N^*, \ldots) = 2 - d$.

But the momentum space propagator for a field with an anomalous dimension η_N has a momentum dependence $(p^2)^{-1+\eta_N/2}$. For $\eta_N = 2 - d$, this becomes p^{-d} , and the associated position space propagator depends logarithmically on distance. As I noted earlier, such a logarithmic dependence is characteristic of a two-dimensional conformal field. A variation of this argument shows that arbitrary matter fields interacting with gravity at a non-Gaussian fixed point exhibit a similar two-dimensional behavior [17].

Loop quantum gravity

A third hint of short-distance dimensional reduction comes from the area spectrum of loop quantum gravity [20]. States in this proposed quantum theory of gravity are given by spin networks, graphs with edges labeled by half-integers j (which represent holonomies of a connection) and vertices labeled by SU(2) intertwiners, that is, generalized Clebsch-Gordan coefficients. Given such a state, the area operator for a surface counts the number of spin network edges that puncture the surface, with each such edge contributing an amount

$$A_j \sim \ell_p^2 \sqrt{j(j+1)},$$

where

$$\ell_p = \sqrt{\frac{\hbar G}{c^3}} \tag{6}$$

is the Planck length.

While area and volume operators in loop quantum gravity are well understood, it has proven rather difficult to define a length operator. But since j is, crudely speaking, a quantum of area, one can (equally crudely) think of \sqrt{j} as a sort of quantum of length. If we therefore define a length $\ell_j = \sqrt{j}\ell_p$, we can rewrite the spectrum as

$$A_j \sim \sqrt{\ell_j^2 (\ell_j^2 + \ell_p^2)} \sim \begin{cases} \ell_j^2 & \text{for large areas} \\ \ell_p \ell_j & \text{for small areas.} \end{cases}$$
(7)

Like the propagator (4), this spectrum undergoes a change in scaling at small distances. Indeed, one can define a scale-dependent effective metric that reproduces the behavior (7), and use it to compute an effective spectral dimension. One again finds a dimension that decreases from four at large scales to two at small scales.

High temperature strings

A fourth piece of evidence for small-scale dimensional reduction comes from the behavior of string theory at high temperatures. At a critical temperature, the Hagedorn temperature, the string theory partition function diverges, and the theory (probably) undergoes a phase transition. As early as 1988, Atick and Witten discovered that at temperatures far above the Hagedorn temperature, string theory has an unexpected thermodynamic behavior [21]: the free energy in a volume V varies with temperature as

$$F/VT \sim T.$$
 (8)

For a field theory in d dimensions, in contrast, $F/VT \sim T^{d-1}$. So even though string theory lives in 10 or 26 dimensions, at high temperatures it behaves thermodynamically as if spacetime were two-dimensional.

Anisotropic scaling models

A fifth sign of short-distance spontaneous dimensional reduction in quantum gravity comes from "Hořava-Lifshitz" models [22]. These are new models of gravity that exhibit anisotropic scaling, that is, invariance under constant rescalings

$$\mathbf{x} \to b\mathbf{x}, \ t \to b^3 t.$$

This scaling property clearly violates Lorentz invariance, breaking the symmetry between space and time and picking out a preferred time coordinate. In fact, it is this breaking of Lorentz invariance that makes the models renormalizable: the field equations may now contain many spatial derivatives, leading to high inverse powers of spatial momentum in propagators that can tame loop integrals, while keeping only second time derivatives, thus avoiding negative energy states or negative norm ghosts. This might seem an unlikely way to quantize gravity—and indeed, these models may face serious low-energy problems [23]—but the hope is that Lorentz invariance might be recovered and conventional general relativity restored at low energies and large distances.

Hořava has calculated the spectral dimension in such models [24], and again finds $d_S = 2$ at high energies. In this case, the two-dimensional behavior can be traced to the fact that

the propagators contain higher inverse powers of momentum; the logarithmic dependence on distance comes from integrals of the form

$$\int \frac{d^4p}{p^4} e^{ip \cdot (x-x')}.$$

Whether this is "real" dimensional reduction becomes a subtle matter, which may depend on how one operationally defines dimension. As before, though, the logarithmic behavior of propagators implies that quantum fields used to probe the structure of spacetime will act as if they are in a two-dimensional space.

Other hints

Hints of two-dimensional behavior come from several other places as well. In the causal set approach to quantum gravity, spacetime is taken to be fundamentally discrete, with a "geometry" determined by the causal relationships among points. In this setting, a natural definition of dimension is the Myrheim-Meyer dimension, which compares the number of causal relations between pairs of points to the corresponding number for points randomly sprinkled in a d_M -dimensional Minkowski space (see, for instance, [25]). For a small enough region of spacetime, one might guess that the causal structure is generic, coming from a random causal ordering. In that case, the Myrheim-Meyer dimension is approximately 2.38—not quite 2, but surprisingly close [26].

Dimensional reduction has also appeared in an analysis of quantum field theory in a background "foam" of virtual black holes [27], although the effective dimension depends on the (unknown) black hole distribution function. A dimension of 2 also appears in Connes' noncommutative geometrical description of general relativity [28]; it is not clear to me whether this is related to the dimensional reduction considered here.

2 Strong coupling and small-scale structure

Let us suppose these hints are really telling us something fundamental about the smallscale structure of quantum gravity. We then face a rather bewildering question: which two dimension? How can a four-dimensional theory with no background structure or preferred direction pick out two "special" dimensions at short distances? To try to answer this question, it is worth looking at one more approach to Planck scale physics: the strongcoupling approximation to the Wheeler-DeWitt equation.

In the conventional Dirac quantization of canonical general relativity, the configuration space variables are given by the spatial metric g_{ij} on a time slice Σ , while their canonically conjugate momenta are related to the extrinsic curvature of the slice. The Hamiltonian constraint, which expresses invariance under diffeomorphisms that deform the slice Σ , then acts on states $\Psi[g]$ to give the Wheeler-DeWitt equation [29]

$$\left\{16\pi\ell_p^2 G_{ijkl}\frac{\delta}{\delta g_{ij}}\frac{\delta}{\delta g_{kl}} - \frac{1}{16\pi\ell_p^2}\sqrt{g}\,^{(3)}R\right\}\Psi[g] = 0\tag{9}$$

where

$$G_{ijkl} = \frac{1}{2}g^{-1/2} \left(g_{ik}g_{jl} + g_{il}g_{jk} - g_{ij}g_{kl} \right)$$
(10)

is the DeWitt metric on the space of metrics. The Wheeler-DeWitt equation is not terribly well-defined—the operator ordering is ambiguous, the product of functional derivatives must be regularized, and the wave functions $\Psi[g]$ should really be functions of spatial diffeomorphism classes of metrics—and it is not at all clear how to find an appropriate inner product on the space of solutions [30]. Nevertheless, the equation is widely accepted as a heuristic guide to the structure of quantum gravity.

Note now that spatial derivatives of the three-metric appear only in the scalar curvature term ⁽³⁾R in (9). As early as 1976, Isham [31] observed that this structure implied an interesting strong-coupling limit $\ell_p \to \infty$. As the Planck length becomes large—that is, as we probe scales near or below ℓ_p [32,33]—the scalar curvature term becomes negligible. Neighboring spatial points thus effectively decouple, and the equation becomes ultralocal.

The absence of spatial derivatives greatly simplifies the Wheeler-DeWitt equation, which becomes exactly solvable. Its properties in this limit have been studied extensively [34–38], and preliminary attempts have been made to restore the coupling between neighboring points by treating the scalar curvature term as a perturbation [33, 39–41]. The same kind of perturbative treatment of the scalar curvature is important classically in a very different setting: it is central to the Belinskii-Khalatnikov-Lifshitz (BKL) approach to cosmology near a spacelike singularity [42, 43]. We can therefore look to this classical setting for clues about the small-scale structure of quantum gravity.

To understand the physics of the strong-coupling approximation, it is helpful to note that the Planck length (6) also depends on the speed of light. In fact, this approximation can also be viewed as a small c ("anti-Newtonian") approximation. As the Planck length becomes large, particle horizons shrink and light cones collapse to timelike lines, leading to the decoupling of neighboring points and the consequent ultralocal behavior [44]. In the completely decoupled limit, the classical solution at each point is a Kasner space,

$$ds^{2} = dt^{2} - t^{2p_{1}}dx^{2} - t^{2p_{2}}dy^{2} - t^{2p_{3}}dz^{2}$$

$$(-\frac{1}{3} < p_{1} < 0 < p_{2} < p_{3}, \quad p_{1} + p_{2} + p_{3} = 1 = p_{1}^{2} + p_{2}^{2} + p_{3}^{2}).$$

$$(11)$$

More precisely—see, for example, [45]—the general solution is an arbitrary GL(3) transformation of a Kasner metric. This is still essentially a Kasner space, but now with arbitrary, not necessary orthogonal, axes.

For large but finite ℓ_p , the classical solution exhibits BKL behavior [42,43]. At any given point, the metric spends most of its time in a nearly Kasner form. But as the metric evolves, the scalar curvature can grow abruptly. The curvature term in the Hamiltonian constraint the classical counterpart of the Wheeler-DeWitt equation—then acts as a potential wall, causing a Mixmaster-like "bounce" [46] to a new Kasner solution with different axes and exponents. In contrast to the ultralocal behavior at the strong-coupling limit, neighboring points are now no longer completely decoupled. But the Mixmaster bounces are chaotic [47], and the geometries at nearby points quickly become uncorrelated, with Kasner exponents occurring randomly with a known probability distribution [48].

We can now return to the problem of dimensional reduction. Consider a timelike geodesic in Kasner space, starting at $t = t_0$ with a random initial velocity. The geodesic equation is exactly integrable, and in the direction of decreasing t, the proper spatial dis-

tance traveled along each Kasner axis asymptotes to

$$s_x \sim t^{p_1}$$

$$s_y \sim 0$$

$$s_z \sim 0.$$
(12)

Particle horizons thus shrink to lines, and geodesics effectively explore only one spatial dimension. In the direction of increasing t, the results are similar, though a bit less dramatic:

$$s_{x} \sim t s_{y} \sim t^{\max(p_{2}, 1+p_{1}-p_{2})} s_{z} \sim t^{p_{3}}.$$
(13)

Since p_2 , $1 + p_1 - p_2$, and p_3 are all less than one, a random geodesic again predominantly sees only one spatial dimension.

Since the heat kernel describes a random walk, one might expect this behavior of Kasner geodesics to be reflected in the spectral dimension. The exact form of heat kernel for Kasner space is not known, but Futamase [49] and Berkin [50] have evaluated K(x, x'; s) in certain approximations. Both find behavior of the form

$$K(x,x;s) \sim \frac{1}{4\pi s^2} \left[1 + \frac{a}{t^2} s + \dots \right].$$
 (14)

The interpretation of this expression involves an order-of-limits question. For a fixed time t, one can always find s small enough that the first term in (14) dominates. This is not surprising: the heat kernel is a classical object, and we are still looking at a setting in which the underlying classical spacetime is four-dimensional. For a fixed return time s, on the other hand, one can always find a time t small enough that the second term dominates, leading to an effective spectral dimension of two. One might worry about the higher order terms in (14), which involve higher inverse powers of t and might dominate at smaller times. But these terms do not contribute to the singular part of the propagator (4); rather, they give terms that go as positive powers of the geodesic distance, and are irrelevant for short distance singularities, light cone behavior, and the like.

One can investigate the same problem via the Seeley-DeWitt expansion of the heat kernel [51-53],

$$K(x, x, s) \sim \frac{1}{4\pi s^2} \left([a_0] + [a_1]s + [a_2]s^2 + \dots \right).$$
(15)

The "Hamidew coefficient" $[a_1]$ is proportional to the scalar curvature, and vanishes for an exact vacuum solution of the field equations. In the presence of matter, however, the scalar curvature will typically increase as an inverse power of t as $t \to 0$ [42]; this growth is slow enough to not disrupt the BKL behavior of the classical solutions near t = 0, but it will nevertheless give a diverging contribution to $[a_1]$.

This short-distance BKL behavior of the strongly coupled Wheeler-DeWitt equation may thus offer an explanation for the apparent dimensional reduction of quantum gravity at the Planck scale. We now have a possible answer to the question, "Which two dimensions?" If this picture is right, the dynamics picks out an essentially random timelike plane at each point. This choice, in turn, is reflected in the behavior of the heat kernel, and hence in the propagators and the consequent short-distance properties of quantum fields.

3 Spacetime foam?

The idea that the "effective infrared dimension" might differ from four goes back to work by Hu and O'Connor [54], but the relevance to short-distance quantum gravity was not fully appreciated at that time. To investigate this prospect further, though, we should better understand the physics underlying BKL behavior.

The BKL picture was originally developed as an attempt to understand the cosmology of the very early Universe near an initial spacelike singularity. Near such a singularity, light rays are typically very strongly focused by the gravitational field, leading to the collapse of light cones and the shrinking of particle horizons. This "asymptotic silence" [43] is the key ingredient in the ultralocal behavior of the equations of motion, from which the rest of the BKL results follow.

Small-scale quantum gravity has no such spacelike singularity, so if a similar mechanism is at work, something else must account for the focusing of null geodesics. An obvious candidate is "spacetime foam," small-scale quantum fluctuations of geometry. Seeing whether such an explanation can work is very difficult; it will ultimately require that we understand the full quantum version of the Raychaudhuri equation. As a first step, though, let us start with the classical Raychaudhuri equation for the focusing of null geodesics,

$$\frac{d\theta}{d\lambda} = -\frac{1}{2}\theta^2 - \sigma_{\alpha}{}^{\beta}\sigma_{\beta}{}^{\alpha} + \omega_{\alpha\beta}\omega^{\alpha\beta} - R_{\alpha\beta}k^{\alpha}k^{\beta}.$$
 (16)

Here, θ is the expansion of a bundle of light rays, essentially $(1/A)(dA/d\lambda)$ where A is the cross-sectional area; σ and ω are the shear and rotation of the bundle. Negative terms on the right-hand side of (16) decrease expansion, and thus focus null geodesics; positive terms contribute to defocusing.

If we naively treat (16) as an operator equation in the Heisenberg picture and take the expectation value, ignoring for the moment the need for renormalization, we see that quantum fluctuations in the expansion and shear should focus geodesics. Indeed,

$$\langle \theta^2 \rangle = \langle \theta \rangle^2 + (\Delta \theta)^2, \tag{17}$$

with a similar equation for σ , so the uncertainties $\Delta \theta$ and $\Delta \sigma$ contribute negative terms to the right-hand side of (16). We can estimate the size of these fluctuations by noting that the expansion θ is roughly canonically conjugate to the cross-sectional area A [55]. Indeed, θ is the trace of an extrinsic curvature, which is, as usual, conjugate to the corresponding volume element. Keeping track of factors of \hbar and G, we find an uncertainty relation of the form

$$\Delta\theta\Delta A \sim \ell_p,\tag{18}$$

where θ is the expansion averaged over a Planck distance along the congruence. In many approaches to quantum gravity—for instance, loop quantum gravity—we expect areas to be quantized in Planck units. It is thus plausible that $\Delta A \sim \ell_p^2$, which would imply fluctuations of θ of order $1/\ell_p$. This would mean very strong focusing at the Planck scale, as desired.

As I have presented it, this argument is certainly inadequate. To begin with, I have not specified which congruence of null geodesics I am considering. In the BKL analysis, a spacelike singularity determines a special null congruence. In short-distance quantum gravity, no such structure exists, and we will have to work hard to define θ as a genuine observable.

Moreover, while the classical shear contributes a negative term to the right-hand side of (16), the operator product $\sigma_{\alpha}{}^{\beta}\sigma_{\beta}{}^{\alpha}$ in a quantum theory must be renormalized, and need not remain positive. Indeed, it is known that this quantity becomes negative near the horizon of a black hole [56]; this is a necessary consequence of the fact that Hawking radiation decreases the horizon area. On the other hand, there are circumstances in which a particular average of this operator over a special null geodesic must be positive to avoid violations of the generalized second law of thermodynamics [57]. The question of whether quantum fluctuations and "spacetime foam" at the Planck scale can lead to something akin to asymptotic silence thus remains open.

4 What next?

In short, the proposal I am making is this: that spacetime foam strongly focuses geodesics at the Planck scale, leading to the BKL behavior predicted by the strongly coupled Wheeler-DeWitt equation. If this suggestion proves to be correct, it leads to a novel and interesting picture of the small-scale structure of spacetime. At each point, the dynamics picks out a "preferred" spatial direction, leading to approximately (1+1)-dimensional local physics. The preferred directions are presumably determined classically by initial conditions, but because of the chaotic behavior of BKL bounces, they are quickly randomized; in the quantum theory, they are picked out by an initial wave function, but again one expects evolution to scramble any initial choices. From point to point, these preferred directions vary continuously, but they oscillate rapidly [58]. Space at a fixed time is thus threaded by rapidly fluctuating lines, and spacetime by two-surfaces; the leading behavior of the physics is described by an approximate dimensional reduction to these surfaces.

There is a danger here, of course: the process I have described breaks Lorentz invariance at the Planck scale, and even small violations at that scale can be magnified and lead to observable effects at large scales [2]. Note, though, that the symmetry violations in this scenario vary rapidly and essentially stochastically in both space and time. Such "nonsystematic" Lorentz violations are harder to study, but there is evidence that they lead to much weaker observational constraints [59].

The scenario I have presented is still very speculative, but I believe it deserves further investigation. One avenue might be to use results from the eikonal approximation [60–62]. In this approximation, developed to study very high energy scattering, a similar dimensional reduction takes place, with drastically disparate time scales in two pairs of dimensions. Although the context is very different, the technology developed for this approximation could prove useful for the study of Planck scale gravity.

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