

STRUCTURAL ISSUES IN QUANTUM GRAVITY^a

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A discursive, non-technical, analysis is made of some of the basic issues that arise in almost any approach to quantum gravity, and of how these issues stand in relation to recent developments in the field. Specific topics include the applicability of the conceptual and mathematical structures of both classical general relativity and standard quantum theory. This discussion is preceded by a short history of the last twenty-five years of research in quantum gravity, and concludes with speculations on what a future theory might look like.

1 Introduction

1.1 *Some Crucial Questions in Quantum Gravity*

In this lecture I wish to reflect on certain fundamental issues that can be expected to arise in almost all approaches to quantum gravity. As such, the talk is rather non-mathematical in nature—in particular, it is *not* meant to be a technical review of the who-has-been-doing-what-since-GR13 type: the subject has developed in too many different ways in recent years to make this option either feasible or desirable.

The presentation is focussed around the following *prima facie* questions:

1. Why are we interested in quantum gravity at all? In the past, different researchers have had significantly different motivations for their work—and this has had a strong influence on the technical developments of the subject.
2. What are the basic ways of trying to construct a quantum theory of gravity? For example, can general relativity be regarded as ‘just another field theory’ to be quantised in a more-or-less standard way, or does its basic structure demand something quite different?
3. Are the current technical and conceptual formulations of general relativity and quantum theory appropriate for the task of constructing a quantum theory of gravity? Specifically:

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- Is the view of spacetime afforded by general relativity adequate at the quantum level? In particular, how justified is (i) the concept of a ‘*spacetime point*’; (ii) the assumption that the set \mathcal{M} of such points has the cardinality of the *continuum*; and (iii) giving this continuum set the additional structure of a *differentiable manifold*. For example, in most discussions of quantum gravity a central role is played by the group $\text{Diff}\mathcal{M}$ of spacetime diffeomorphisms—which, of course, only makes sense if \mathcal{M} really is a smooth manifold.
- Are the technical formalism and conceptual framework of present-day quantum theory adequate for constructing a fully coherent theory of quantum gravity? In particular, can they handle the idea that ‘spacetime itself’ might have quantum properties in addition to those of the metric and other fields that it carries?

Note that if our existing views on spacetime and/or quantum theory are *not* adequate, the question then arises of the extent to which they are *relevant* to research in quantum gravity. Put slightly differently, are our current ideas about spacetime and quantum theory of fundamental validity, or are they only heuristic approximations to something deeper? And, if the latter is true, how *iconoclastic* do our research programmes need to be?

Of course, these are not the only significant issues in quantum gravity. For example, at a practical level it is important to find viable *perturbative* techniques for extracting answers to physically interesting questions. This is a major challenge to both superstring theory and the Ashtekar canonical-quantisation programme: two of the most promising current approaches to quantum gravity proper.

Another question—coming directly from the successful development of superstring theory—concerns the precise role of supersymmetry in a theory of quantum gravity. Note that supersymmetry is of considerable significance in modern unified theories of the *non*-gravitational forces, particularly in regard to having the running coupling constants all meet at a single unification point. Thus we are led naturally to one of the central questions of quantum gravity research: will a consistent theory necessarily unite all the fundamental forces—as is suggested by superstring theory—or is it possible to construct a quantum theory of the gravitational field alone—as is suggested by the Ashtekar programme?

1.2 *The Peculiar Nature of Research in Quantum Gravity*

Before discussing any of these matters in detail, it is prudent to point out that the subject of quantum gravity has some distinctly peculiar features when

viewed from the standpoint of most other branches of theoretical physics. At the end of the day, theoretical physics is supposed to be about the way things actually ‘are’ in the physical world: a situation that is reflected in the diagram

$$\text{theory} \longleftrightarrow \text{concepts} \longleftrightarrow \text{facts} \tag{1}$$

in which the theoretical (*i.e.*, mathematical) components are linked to the physical data (the ‘facts’) via a conceptual framework that depends to some extent on the subject area concerned. Of course, there is more to this than meets the eye: in particular, it is well understood these days that (i) the conceptual framework we use to analyse the world is partly determined by our prior ideas about the factual content of that world; and (ii) so-called ‘facts’ are not just bare, ‘given’ data but—in their very identification and isolation as ‘facts’—already presuppose a certain conceptual framework for analysing the world. However, with these caveats in mind, the simple diagram above does capture certain crucial aspects of how theoretical physicists view their professional activities.

The feature of quantum gravity that challenges its very right to be considered as a genuine branch of theoretical physics is the singular absence of any observed property of the world that can be identified *unequivocally* as the result of some interplay between general relativity and quantum theory. This problem stems from the fact that the natural Planck length—defined using dimensional analysis as $L_P := (G\hbar/c^3)^{\frac{1}{2}}$ —has the extremely small value of approximately 10^{-35} m; equivalently, the associated Planck energy E_P has a value 10^{28} eV, which is well beyond the range of any foreseeable laboratory-based experiments. Indeed, this simple dimensional argument suggests strongly that the only physical regime where effects of quantum gravity might be studied directly is in the immediate post big-bang era of the universe—which is not the easiest thing to probe experimentally.

This lack of obvious data means that the right hand side of diagram (1) is missing, and the shortened picture

$$\text{theory} \longleftrightarrow \text{concepts} \tag{2}$$

has generated an overall research effort that is distinctly lopsided when compared to mainstream areas of physics. In practice, most research in quantum gravity has been based on various *prima facie* views about what the theory *should* look like—these being grounded partly on the philosophical prejudices of the researcher concerned, and partly on the existence of mathematical techniques that have been successful in what are deemed, perhaps erroneously, to be closely related areas of theoretical physics, such as—for example—non-

abelian gauge theories. This procedure has lent a curious flavour to the whole field of quantum gravity.

In regard to the lack of experimental data that could act as a constraint, the situation resembles the one that—until relatively recently—faced those interested in foundational problems of quantum theory proper. In this light, it is curious that there has been such a sparsity of formal interactions between workers in quantum gravity, and workers in the foundations of quantum theory itself. This seems all the more remarkable when one recalls that some of the most basic problems that confront quantum cosmology are the same as those that have plagued the foundations of quantum theory in general. I am thinking in particular of the measurement problem, the meaning of probability, and the general issue of quantum entanglement in a closed system—in our case, the universe in its entirety.

2 Preliminary Remarks

2.1 *Motivations for Studying Quantum Gravity*

It is clear from the above that, strictly speaking, quantum gravity cannot be regarded as a standard scientific research programme, lacking as it does any well-established body of ‘facts’ against which putative theories can be verified or falsified in the traditional way. This does not mean there are no good reasons for studying the subject—there are—but they tend to be of a different type from those in all other branches of physics. Of course, some of the motivating factors do refer to potential observations or experiments—particularly in the area of cosmology—but most are of a more internal nature: for example, the search for mathematical consistency, the desire for a unified theory of all the forces, or the implementation of various quasi-philosophical views on the nature of space and time. It is important to appreciate these motivations in order to understand what people have done in the past, and to be able to judge if they succeeded in their endeavours: to be adjudged ‘successful’ a theory must either point beyond itself to new or existing ‘facts’ in the world, or else achieve some of its own internal goals.

It is useful pedagogically to classify research programmes in quantum gravity according to whether they originated in the community of elementary particle physicists and quantum field theorists, or in the community of those who work primarily in general relativity. This divide typically affects both the goals of research and the techniques employed.

A. Motivations from the perspective of elementary particle physics and quantum field theory

1. Matter is built from elementary particles that *are* described in quantum theoretical terms and that certainly interact with each other gravitationally. Hence it is necessary to say *something* about the interface between quantum theory and general relativity, even if it is only to claim that, ‘for all practical purposes’, the subject can be ignored (see below).
2. Relativistic quantum field theory might only make proper sense if gravity is included from the outset. In particular, the short-distance divergences present in most such theories—including those that are renormalisable, but not truly finite—might be removed by a fundamental cut-off at the Planck energy. Superstring theory is arguably the latest claimant to implement this idea.
3. A related claim is that general relativity is a necessary ingredient in any fully-consistent theory of the unification of the *non*-gravitational forces of nature (*i.e.*, the electromagnetic, the weak, and the strong forces). The opposing positions taken towards the *converse* claim—that a consistent quantum theory of gravity will necessarily include the other fundamental forces—is one of the most striking differences between superstring theory and the canonical quantum gravity programme.

B. Motivations from the perspective of a general relativist

1. Spacetime singularities arise inevitably in general relativity if the energy-momentum tensor satisfies certain—physically well-motivated—positivity conditions. It has long been hoped that the prediction of such pathological behaviour can be removed by the correct introduction of quantum effects.
2. Ever since Hawking’s discovery¹ of the quantum-induced radiation by a black-hole, a major reason for studying quantum gravity has been to understand the end state of gravitationally collapsing matter.
3. Quantum gravity should play a vital role in the physics of the very early universe. Possible applications include:
 - (a) understanding the very origin of the universe;
 - (b) finding an explanation of why spacetime has a macroscopic dimension of four (this does not exclude a Kaluza-Klein type higher dimension at Planckian scales);

- (c) accounting for the origin of the inflationary evolution that is felt by many cosmologists to describe the universe as it expanded from the initial big bang.

In addition to the above—relatively pragmatic—reasons for studying quantum gravity there remains what is, for many, the most alluring motivation of all. Namely, a consistent theory of quantum gravity may require a radical revision of our most fundamental concepts of space, time and substance. It was John Wheeler who first most clearly and consistently expounded this thesis over thirty years ago, and it is one that has fascinated generations of theoretical physicists ever since.

2.2 Approaches to Quantum Gravity

The differing weights placed by individual researchers on the various motivations for studying quantum gravity have resulted in a plethora of views on how the subject should be tackled. As a consequence, we are far from having an ‘axiomatic’ framework, or—indeed—even a broad consensus on what to strive for beyond the minimal requirement that the theory should reproduce classical general relativity and normal quantum theory in the appropriate domains—usually taken to be all physical regimes well away from those characterised by the Planck length.

A. Can quantum gravity be avoided?

Perhaps there is no need for a quantum theory of gravity at all. In this context, we note the following.

1. The argument is sometimes put forward that the Planck length $L_P := (G\hbar/c^3)^{\frac{1}{2}} \simeq 10^{-35}\text{m}$ is so small that there is no need to worry about quantum gravity except, perhaps, in *recherché* considerations of the extremely early universe—*i.e.*, within a Planck time ($\simeq 10^{-42}\text{s}$) of the big-bang. However:

- Such a claim is only really meaningful if a theory exists within whose framework genuine perturbative expansions in L/L_P can be performed, where L is the length scale at which the system is probed: one can then legitimately argue that quantum effects are ignorable if $L/L_P \ll 1$. So we must try to find a viable theory, even if we promptly declare it to be irrelevant for anything other than the physics of the very early universe.
- The argument concerning the size of L_P neglects the possibility of *non-perturbative* effects—an idea that has often been associated with the

claim that quantum gravity produces an intrinsic cut-off in quantum field theory.

2. A somewhat different view is that it is manifestly *wrong* to attempt to ‘quantise’ the gravitational field in an active sense^c. The reasons advanced in support of this thesis include the following.

- The metric tensor is not a ‘fundamental’ field in physics, but rather a phenomenological description of gravitational effects that applies only in realms well away from those characterised by the Planck scale. One example is superstring theory, in which the basic quantum entities are far removed from those in classical general relativity. Another—somewhat different—example is Jacobson’s recent re-derivation of the Einstein field equations as an equation of state², which—presumably—it would be no more appropriate to ‘quantise’ than it would the equations of fluid dynamics^d.
- The gravitational field is concerned with the structure of space and time—and these are, *par excellence*, fundamentally classical in nature and mode of functioning.

3. If justified, the last position raises acutely the question of how matter—which presumably *is* subject to the laws of quantum theory—is to be incorporated in the scheme. Discussion of this issue has largely focussed on the posited equations for the ‘semi-classical’ spacetime metric γ ,

$$G_{\mu\nu}(\gamma) = \langle \psi | T_{\mu\nu}(g, \widehat{\phi}) | \psi \rangle \quad (3)$$

where $|\psi\rangle$ is some state in the Hilbert space of the quantised matter variables ϕ . In this context, we note the following:

- In the case of electromagnetism, the well-known analysis by Bohr and Rosenfeld³ of the analogue of Eq. (3) concluded that the electromagnetic field *had* to be quantised to be consistent with the quantised nature of the matter to which it couples. However—as Rosenfeld himself pointed

^cBy ‘active’ quantisation I mean a diorthotic scheme in which one starts with a classical system to which some quantisation algorithm is applied. This can be contrasted with approaches in which a quantum theory is defined in an intrinsic way—perhaps as a representation of some group or algebra—with no prior reference to a classical system that is being ‘quantised’.

^dIn 1971, I took part in a public debate with John Stachel in which he challenged me on this very issue. As a keen young quantum field theorist at the time, I replied that I was delighted to quantise everything in sight. These days I would be more cautious!

out⁴—the analogous argument for general relativity does not go through and—in spite of much discussion since then (for example, see Page and Geilker⁵)—there is arguably still no definitive proof that general relativity *has* to be quantised in some way.

- The right hand side of Eq. (3) generates a number of technical problems. For example, the expectation value is ultra-violet divergent, and regularisation methods only yield an unambiguous expression when the spacetime metric γ is static or stationary—but there is no reason why a semi-classical metric should have this property. In addition, there have been many arguments implying that solutions to Eq. (3) are likely to be unstable against small perturbations and—therefore—physically unacceptable.
- It is not clear how the state $|\psi\rangle$ should be chosen. In addition, if $|\psi_1\rangle$ and $|\psi_2\rangle$ are associated with a pair of solutions γ_1 and γ_2 to Eq. (3), there is no obvious connection between γ_1 and γ_2 and any solution associated with a linear combination of $|\psi_1\rangle$ and $|\psi_2\rangle$. Thus the quantum sector of the theory has curious non-linear features, and these generate many new problems of both a technical and a conceptual nature.

B. The four types of approach to quantum gravity

The four major categories in which existing approaches to quantum gravity can be classified are as follows.

1. *Quantise general relativity.* This means trying to construct an algorithm for actively quantising the metric tensor regarded as a special type of field. In practice, the techniques that have been adopted fall into two classes: (i) those based on a spacetime approach to quantum field theory—in which the operator fields are defined on a four-dimensional manifold representing spacetime; and (ii) those based on a canonical approach—in which the operator fields are defined on a three-dimensional manifold representing physical space.
2. *‘General-relativise’ quantum theory.* This means trying to adapt standard quantum theory to the needs of classical general relativity. Most work in this area has been in the context of quantising a matter field that propagates on a fixed, background spacetime (\mathcal{M}, γ) , where \mathcal{M} denotes the manifold, and γ is the spacetime metric.
3. *General relativity is the low-energy limit of a quantum theory of something quite different.* The most notable example of this type is the theory of closed superstrings.

4. *Start ab initio with a radical new theory.* The implication is that both classical general relativity and standard quantum theory ‘emerge’ from a deeper theory that involves a radical reappraisal of the concepts of space, time, and substance.

2.3 The Problem of Causality and Time

Until the onset of the superstring programme, most work in quantum gravity fell into the first category of ‘quantising’ general relativity. However, approaches of this type inevitably encounter the infamous ‘problem of time’ that lies at the heart of many of the deepest conceptual issues in quantum gravity. For this reason, I shall begin by briefly reviewing this problem before discussing the various specific approaches to quantum gravity.

A. The problem of time from a spacetime perspective

In the context of spacetime-oriented approaches to quantum theory, the problem of time and causality is easy to state: the causal structure of spacetime depends on the metric tensor γ —hence, if this is subject to quantum fluctuations, so is the causal structure. Similarly, if the metric is only a coarse-grained, phenomenological construct of some type, then so is the causal structure.

This situation poses severe technical problems since standard quantum field theory presupposes a fixed causal structure. For example, a quantum scalar field $\hat{\phi}(X)$ is normally required to satisfy the microcausal commutation relations

$$[\hat{\phi}(X), \hat{\phi}(Y)] = 0 \tag{4}$$

whenever the spacetime points X and Y are spacelike separated. However, the latter condition has no meaning if the spacetime metric is quantised or phenomenological. In the former case, the most likely scenario is that the right hand side of Eq. (4) never vanishes, thereby removing one of the foundations of conventional quantum field theory⁶. Replacing operator fields with C^* -algebras does not help—in so far as they have microcausal commutation properties—and neither does the use of functional integrals if the problem of their definition is taken at all seriously. In practice, the techniques that have been used to address the problem of time fall into one of the following categories:

1. Use a fixed background metric η —often chosen to be that of Minkowski spacetime—to define a fiducial causal structure with respect to which standard quantum field theoretical techniques can be employed. When

applied to the gravitational field itself, $\gamma_{\mu\nu}(X)$, this usually involves writing

$$\gamma_{\mu\nu}(X) = \eta_{\mu\nu} + \kappa h_{\mu\nu}(X) \tag{5}$$

and then regarding $h_{\mu\nu}(X)$ as the physical, ‘graviton’ field (here, $\kappa^2 = 8\pi G/c^2$ where G is Newton’s constant). The use of Eq. (5) strongly suggests a perturbative approach in which quantum gravity is seen as a theory of small quantum fluctuations around a background spacetime. The problems that arise include:

- finding a well-defined mathematical scheme (the obvious techniques give a non-renormalisable theory—see below);
 - knowing how to handle backgrounds that are other than Minkowski spacetime, both metrically and topologically—for example, to discuss cosmological issues;
 - understanding if—and how—the different possible backgrounds fit together into a single quantum scheme, and what becomes of the notion of ‘causality’ in such a scheme.
2. Start with a formalism in which the spacetime metric has a Riemannian (rather than Lorentzian) signature, and then worry at the end about making an ‘analytical continuation’ back to physical spacetime. Most work of this type has involved—rather heuristic—functional-integral approaches to quantisation.
 3. Forget about spacetime methods altogether and adopt a canonical approach to general relativity in which the basic ingredients are geometrical fields on a three-dimensional manifold. The problem then is to reconstruct some type of—possibly, only approximate—spatio-temporal picture within which the quantum calculations can be interpreted.

The absence of any fundamental causal structure also raises important conceptual issues. For example, the standard interpretation of quantum theory places much weight on the role of measurements made by an ‘observer’. But the simplest model for an observer is a time-like curve, and the notion of ‘time-like’ is dependent on the spacetime metric. Thus what it is to be an observer also becomes quantised—or phenomenological, as the case may be—which renders the standard interpretation distinctly problematic^e.

^eI am grateful to Steve Weinstein for emphasising this point to me, and for discussions on its significance.

B. The problem of time in canonical quantisation

The canonical approach to quantum gravity starts with a reference foliation of spacetime that is used to define the appropriate canonical variables. These are the 3-metric $g_{ab}(x)$ on a spatial manifold Σ , and a canonical conjugate $p^{ab}(x)$ that—from a spacetime perspective—is related to the extrinsic curvature of Σ as embedded in the four-dimensional spacetime. However, these variables are not independent, but satisfy the constraints

$$\mathcal{H}_a(x) = 0 \quad (6)$$

$$\mathcal{H}_\perp(x) = 0 \quad (7)$$

where ^f

$$\mathcal{H}_a(x) := -2p_a{}^b{}_{|b}(x) \quad (8)$$

$$\mathcal{H}_\perp(x) := \kappa^2 \mathcal{G}_{abcd}(x) p^{ab}(x) p^{cd}(x) - \frac{|g|^{\frac{1}{2}}(x)}{\kappa^2} R(x) \quad (9)$$

in which $R(x)$ denotes the curvature scalar of the 3-metric $g_{ab}(x)$, and where the ‘DeWitt supermetric’ on the space of three-metrics⁷ is given by

$$\mathcal{G}_{abcd}(x) := \frac{1}{2} |g|^{-\frac{1}{2}}(x) (g_{ac}(x)g_{bd}(x) + g_{bc}(x)g_{ad}(x) - g_{ab}(x)g_{cd}(x)). \quad (10)$$

The functions \mathcal{H}_a and \mathcal{H}_\perp of the canonical variables (g, p) play a key role, centered on the fact that their Poisson bracket algebra

$$\{\mathcal{H}_a(x), \mathcal{H}_b(x')\} = -\mathcal{H}_b(x) \partial_a^{x'} \delta(x, x') + \mathcal{H}_a(x') \partial_b^x \delta(x, x') \quad (11)$$

$$\{\mathcal{H}_a(x), \mathcal{H}_\perp(x')\} = \mathcal{H}_\perp(x) \partial_a^x \delta(x, x') \quad (12)$$

$$\begin{aligned} \{\mathcal{H}_\perp(x), \mathcal{H}_\perp(x')\} &= g^{ab}(x) \mathcal{H}_a(x) \partial_b^{x'} \delta(x, x') - \\ &g^{ab}(x') \mathcal{H}_a(x') \partial_b^x \delta(x, x') \end{aligned} \quad (13)$$

is that of the spacetime diffeomorphism group projected along, and normal to, the spacelike hypersurfaces.

The way in which the problem of time appears depends very much on the approach taken to quantising this classical canonical system. One possibility is to (i) impose a gauge for the invariance associated with the algebra Eqs. (11–13); (ii) solve the constraints Eqs. (6–7) classically; and (iii) quantise the resulting ‘true’ canonical system in a standard way. The final equations are intractable in anything other than a perturbative sense, where they promptly

^fThe ‘|’ sign denotes covariant differentiation using the Christoffel symbol of the 3-metric.

succumb to ultraviolet divergences. However, if the formalism could be given a proper mathematical meaning, the problem of time would involve relating the different choices of gauge and associated classical notions of time. Simple model calculations suggest that this is far from trivial.

Most approaches to canonical quantum gravity do not proceed in this way. Instead, the full set of fields $(g_{ab}(x), p^{cd}(x))$ is quantised via the canonical commutation relations

$$[\widehat{g}_{ab}(x), \widehat{g}_{cd}(x')] = 0 \quad (14)$$

$$[\widehat{p}^{ab}(x), \widehat{p}^{cd}(x')] = 0 \quad (15)$$

$$[\widehat{g}_{ab}(x), \widehat{p}^{cd}(x')] = i\hbar \delta_{(a}^c \delta_{b)}^d \delta(x, x') \quad (16)$$

of operators defined on the 3-manifold Σ . Following Dirac, the constraints are interpreted as constraints on the allowed state vectors Ψ , so that $\widehat{\mathcal{H}}_a(x)\Psi = 0 = \widehat{\mathcal{H}}_{\perp}(x)$ for all $x \in \Sigma$. In particular, on choosing the states as functions of the three-geometry g —and with operator representatives $(\widehat{g}_{ab}(x)\Psi)[g] := g_{ab}(x)\Psi[g]$ and $(\widehat{p}^{cd}(x)\Psi)[g] := -i\hbar\delta\Psi[g]/\delta g_{ab}(x)$ —the constraints $\widehat{\mathcal{H}}_a\Psi = 0$ imply that $\Psi[g]$ is constant under changes of g induced by infinitesimal diffeomorphisms of the spatial 3-manifold Σ .

The crucial constraint is $\widehat{\mathcal{H}}_{\perp}(x)\Psi = 0$, which—in this particular representation of states and canonical operators—becomes the famous Wheeler-DeWitt equation

$$-\hbar^2\kappa^2\mathcal{G}_{abcd}(x)\frac{\delta^2\Psi[g]}{\delta g_{ab}(x)\delta g_{cd}(x)} - \frac{|g|^{\frac{1}{2}}(x)}{\kappa^2}R(x)\Psi[g] = 0 \quad (17)$$

where \mathcal{G}_{abcd} is the DeWitt metric defined in Eq. (10).

The most obvious manifestation of the problem of time is that the Wheeler-DeWitt equation Eq. (17) makes no reference to time, and yet this is normally regarded as the crucial ‘dynamical’ equation⁹ of the theory! This situation is usually understood to mean that ‘time’ has to be reintroduced as the values of special *physical* entities in the theory—either gravitational or material—with which the values of other physical quantities are to be correlated. It is a major unsolved problem whether (i) this can be done at all in an exact way; and (ii) if it can, how the results of two different such choices compare with each other, and how this can be related to concepts of a more spatio-temporal nature.

Note that even the starting canonical commutation relations Eqs. (14–16) are suspect. For example, the vanishing of a commutator like Eq. (14) would

⁹Essentially because in the classical theory—when viewed from a spacetime perspective— \mathcal{H}_{\perp} is associated with the canonical generators of displacements in time-like directions.

normally reflect the fact that the points x and x' are ‘spatially separated’. But what does this mean in a theory with no background casual structure? Questions of this type have led many people to question the whole canonical approach to quantum gravity, and have generated searches for a new—essentially ‘timeless’—approach to quantum theory itself. However, the problem of time is very complex and is still the subject of much debate. Two recent extensive reviews are by Kuchař⁸ and Isham⁹.

3 A Brief History of Quantum Gravity

Rather than just summarising recent developments in quantum gravity I would like to start by presenting a short history^h of the subject as it has developed over the last twenty-five years: if we are interested in speculating on where quantum gravity is going, it is not unreasonable to reflect first on where it has come from!

3.1 The Situation Twenty-Five Years Ago

Let me begin by recalling the status of quantum gravity studies in the year 1970 (*i.e.*, twenty-five years before this present GR14 conference) in particular, the state of quantum gravity proper, and the way elementary particle physics and general quantum field theory were viewed at that time.

A. Quantum gravity before 1970

1. The canonical analysis of classical general relativity was well understood by this time. The pioneering work of Dirac¹⁰ had been developed by many people, with one line of research culminating in the definitive treatment by Arnowitt, Deser and Misner¹¹ of how to isolate the physical degrees of freedom in classical general relativityⁱ. The classical constraint algebra Eqs. (11–13) was also well-known, and its broad implications for the quantum theory were understood. In particular, the Wheeler-DeWitt equation Eq. (17) had been written down (by Wheeler^{14,15} and DeWitt⁷, of course).

2. The first studies of quantum cosmology had been made. In particular, DeWitt⁷ and Misner¹⁶ had introduced the idea of *minisuperspace* quantisation:

^hPlease note that had I tried to give full references, the bibliography would have consumed the entire page allowance for this article! Therefore, in several places I have had to be satisfied with merely citing review papers containing full bibliographies for specific subjects.

ⁱA fairly comprehensive bibliography of papers on canonical general relativity can be found in my review paper¹² on the problem of time, and in a review by Kuchař¹³.

a truncation of the gravitational field to just a few degrees of freedom so that the—rather intractable—functional differential equation Eq. (17) becomes a partial differential equation in a finite number of variables, which one can at least contemplate attempting to solve exactly.

3. There was a fair appreciation of many of the conceptual problems of quantum gravity. In particular:

- the instrumentalist concepts central to the Copenhagen interpretation of quantum theory were understood to be inappropriate in the context of quantum cosmology (recall that the many-worlds interpretation dates back to papers by Everett¹⁷ and Wheeler¹⁸ that were published in 1957!);
- there had been a preliminary analysis of the problem of time, especially in the context of canonical quantum gravity;
- doubts had been expressed about the operational meaning of a ‘space-time point’ in quantum gravity—for example, the twistor programme has its genesis in this era¹⁹.

4. A number of studies had been made of the spacetime approach to quantum gravity centered around the expansion Eq. (5). When substituted into the Einstein-Hilbert action $S = \int d^4X |\gamma|^{\frac{1}{2}} R(\gamma)$, this gives a bilinear term that describes massless spin-2 gravitons, plus a series of higher-order vertices describing graviton-graviton interactions. The Feynman rules for this system were well understood, including the need to introduce ‘ghost’ particles to allow for the effects of non-physical graviton modes propagating in internal loops^{20,21,7,22,23,24}. And—most importantly—the theory was widely expected to be non-renormalisable, although only a simple power-counting argument was available at that time.

B. Elementary particle physics and quantum field theory before 1970

The attitude in the 1960s towards quantum field theory was very different from that of today. With the exception of quantum electrodynamics, quantum field theory was poorly rated as a fundamental way of describing the interactions of elementary particles. Instead, this was the era of the S-matrix, the Chew axioms, Regge poles, and—towards the end of the period—the dual resonance model and the Veneziano amplitude that led eventually to string theory.

In so far as it was invoked at all in strong interaction physics, quantum field theory was mainly used as a phenomenological tool to explore the predictions of current algebra, which was thought to be more fundamental. When quantum

field theory was studied seriously, it was largely in the context of an ‘axiomatic’ programme—such as the Wightman²⁵ axioms for the n -point functions.

This general down-playing of quantum field theory influenced the way quantum gravity developed. In particular—with a few notable exceptions—physicists trained in particle physics and quantum field theory were not interested in quantum gravity, and the subject was left to those whose primary training had been in general relativity. This imparted a special flavour to much of the work in that era. In particular, the geometrical aspects of the theory were often emphasised at the expense of quantum field theoretic issues—thereby giving rise to a tension that has affected the subject to this very day.

3.2 *The Highlights of Twenty-Five Years of Quantum Gravity*

My personal choice of the key developments in the last twenty-five years is as follows.

1. *The general renaissance of Lagrangian quantum field theory.* I make no apologies for beginning with the discovery by t’Hooft in the early 1970s of the renormalisability of quantised Yang-Mills theory. Although not directly connected with gravity, these results had a strong effect on attitudes towards quantum field theory in general and reawakened a wide interest in the subject. One spin-off was that many young workers in particle physics became intrigued by the challenge of applying the new methods to quantum gravity—a trend that has continued to the present time.

2. *Black hole radiation.* I was present at the Oxford conference²⁶ in 1974 at which Hawking announced his results on black hole radiation¹, and I remember well the amazement engendered by his lecture. His seminal work triggered a series of research programmes that have been of major interest ever since. For example:

(a) The most obvious conclusion at the time was that there is some remarkable connection between thermodynamics—especially the concept of entropy—quantum theory, and general relativity. The challenge of fully elucidating this connection has led to some of the most intriguing ideas in quantum gravity (see later).

(b) Hawking’s work generated an intense—and ongoing—interest in the general problem of defining quantum field theory on a curved space-time background^{*j*}.

^{*j*}For a recent overview and bibliography see the paper by Bob Wald in this volume.

(c) Hawking’s results were quickly rederived using thermal Green’s functions²⁷ which—in normal quantum field theory—are closely connected with replacing time by an imaginary number whose value is proportional to the temperature. This led Hawking to propose his ‘Euclidean’ quantum gravity programme^k in which the central role is played by Riemannian, rather than Lorentzian, metrics (this being the appropriate curved-space analogue of replacing time t with $\sqrt{-1}t$). In particular, Hawking proposed to study functional integrals of the form

$$Z(\mathcal{M}) := \int \mathcal{D}\gamma e^{-\int_{\mathcal{M}} |\gamma|^{\frac{1}{2}} R(\gamma)} \quad (18)$$

where the integral is over all Riemannian metrics γ on a four-manifold \mathcal{M} . It is not easy to give a rigorous mathematical meaning to this object but, nevertheless, the idea has been extremely fertile. For example:

- Solving the problem of time involves the ‘analytical continuation’ of manifolds with a Riemannian signature to those whose signature is Lorentzian. This procedure is of considerable mathematical interest in its own right.
- Saddle-point approximations to Eq. (18) have been widely used as a gravitational analogue of the instanton techniques developed in Yang-Mills theory. This has generated considerable mathematical interest in classical solutions to the Riemannian version of general relativity.
- The expression Eq. (18) generalises naturally to include a type of ‘quantum topology’ in which each four-manifold \mathcal{M} contributes with a weight $\chi(\mathcal{M})$ in an expression of the type

$$Z := \sum_{\mathcal{M}} \chi(\mathcal{M}) Z(\mathcal{M}). \quad (19)$$

- If applied to a manifold with a single three-boundary Σ , the expression Eq. (18) gives rise to a functional $\Psi[g]$ if the functional integral is taken over all 4-metrics γ on \mathcal{M} that are equal to the given 3-metric g on Σ . Furthermore, the functional of g thus defined satisfies (at least, in a heuristic way) the Wheeler-DeWitt equation Eq. (17). This is the basis of the famous Hartle-Hawking²⁹ ‘wave-function of the universe’ in quantum cosmology.

3. *The non-renormalisability of quantum gravity.* Around 1973, a number of calculations were performed^l confirming that perturbative quantum gravity is

^kA convenient recent source for many of the original articles is Gibbons and Hawking²⁸.

^lFull references can be found in reviews written around that time; for example, in the proceedings of the first two Oxford conferences on quantum gravity^{30,26}.

indeed non-renormalisable^m. There have been three main reactions to this situation, each of which still has many advocates today:

- Continue to use standard perturbative quantum field theory but change the classical theory of general relativity so that the quantum theory becomes renormalisable. Examples of such attempts include (i) adding higher powers of the Riemann curvature $R_{\beta\mu\nu}^{\alpha}(\gamma)$ to the action; and (ii) supergravity (see later).
- Keep classical general relativity as it is, but develop quantisation methods that are intrinsically non-perturbative. The two best examples of this philosophy are (i) the use of Regge calculus and techniques based on lattice gauge theory; and (ii) the Ashtekar programme for canonical quantisation (see below).
- Adopt the view that the non-renormalisability of perturbative quantum gravity is a catastrophic failure that can be remedied only by developing radically new ideas in the foundations of physics. Research programmes of this type include (i) twistor theory; (ii) non-commutative geometry, as developed by Connes³², Dubois-Violetteⁿ and others; and (iii) a variety of ideas involving discrete models of space and time^o.

4. *Supergravity and superstrings.* The development of supergravity^p (in the mid 1970s) and superstring theory (mainly from the mid 1980s onwards) has had a major impact on the way the problem of quantum gravity is viewed. For example:

- These theories provide a concrete realisation of the old hope that quantum gravity necessarily involves a unification with other fundamental forces in nature.
- Superstring theory shows clearly how general relativity can occur as a small part of something else—thereby removing much of the fundamental

^mMore precisely, it was shown that—in a variety of matter-plus-gravity systems—the one-loop counter-term is ultraviolet divergent (in the background-field method, the one-loop counter-term for pure gravity vanishes for kinematical reasons). In 1986, Goroff and Sagnotti³¹ showed that the two-loop contribution in pure gravity is also infinite. Thus—barring a very improbable, fortuitous cancellation of all higher infinities—the theory is non-renormalisable.

ⁿA comprehensive recent review is by Djemai³³.

^oA bibliographic review of some of the lesser known schemes has been written recently by Gibbs³⁴.

^pA full bibliography is contained in a definitive review written in 1980 by van Nieuwenhuizen³⁵.

significance formerly ascribed to the notions of space and time. Not unsurprisingly, this radical implication of the superstring programme tends not to be overwhelmingly popular with the general relativity community!

- There is clear evidence in string theory of the existence of non-local structure at around the Planck length. Indeed, there have been frequent suggestions that the very notion of ‘length’ becomes meaningless below this scale—an idea that turns up frequently in many other approaches to quantum gravity^{36,37}.

5. *The new canonical variables.* A major advance in the development of the canonical quantisation of gravity occurred in 1986 when Abhay Ashtekar³⁸ found a set of canonical variables in terms of which the structure of the constraints is dramatically simplified. Ever since, there has been a very active programme to exploit these new variables in both classical and quantum gravity. Some of the more striking implications include the following.

- For the first time, there is real evidence in support of the old idea that non-perturbative methods must play a key role in constructing a quantum theory of gravity.
- The new variables involve complex combinations of the standard canonical variables. Thus the use of *complexified* general relativity moves to the front of the quantum stage.
- One of the new variables is a spin-connection, which suggests the use of a gravitational analogue of the gauge-invariant loop variables introduced by Wilson in Yang-Mills theory. Seminal work in this area by Rovelli and Smolin³⁹ has produced many fascinating ideas, including a claim that the area and volume of space are quantised^q in discrete units.

This concludes my list of the main highlights of the last twenty-five years of research. However, this by no means exhausts the topics on which people have worked. In particular:

1. There has been a very successful research programme^r aimed at understanding quantum gravity in 2 + 1-dimensional space-time. This has been particularly valuable for illustrating the relations between different approaches to quantum gravity. It also provides a viable platform for

^qFor full references on this—and other—aspects of the Ashtekar programme, see the paper by Abhay in this volume.

^rFor a recent review see the lecture notes⁴⁰ by Carlip.

analysing some of the many conceptual problems that plague quantum gravity, in a way that is free of the—often intractable—mathematical problems that infest the theory in $3 + 1$ dimensions.

2. There has been a steady growth of interest in ‘topological quantum field theories’ (see later).
3. Many people have continued to think long and hard about the conceptual issues^s in quantum gravity. In particular, there have been intensive studies of (i) the problem of time in canonical quantum gravity, and the associated problem of the nature of physical observables; (ii) the possibility of finding new interpretations of quantum theory that avoid the instrumentalism of the standard approaches; and (iii) the possibility that quantum gravity can solve some of the conceptual problems in normal quantum theory—in particular, the idea that the ‘reduction of the state vector’ can be associated with the non-linear nature of general relativity (see later).

At this point—having completed my brief historical survey—it might seem natural to list the research areas that are currently active. However, in fact this is hardly necessary since almost every topic mentioned above is still being pursued in one way or another. Indeed, reflecting on these topics suggests that—like old soldiers—ideas in quantum gravity do not die but merely fade away—in some cases, over quite a long time scale! As far as quantum gravity proper is concerned, there is currently much activity in all three of the major approaches: (i) superstring theory; (ii) the Ashtekar programme; and (iii) the Euclidean quantum gravity programme. These three programmes complement each other nicely and enable the special ideas of any one of them to be viewed in a different perspective by invoking the other two—a feature that is rather useful in a subject that so singularly lacks any unequivocal experimental data.

4 Structural Issues Concerning Space and Time

I shall focus on four issues concerning the use in quantum gravity of the picture of spacetime suggested by classical general relativity. Namely (i) the representation of spacetime as a C^∞ -manifold; (ii) the role of spacetime diffeomorphisms; (iii) the role of black holes; and (iv) the implications of recent developments in superstring theory.

^sMany of these are discussed at length in the notes of my 1991 Schladming lectures¹²; see also the proceedings⁴¹ of the 1988 Osgood Hill conference on conceptual problems in quantum gravity.

4.1 Is Spacetime a C^∞ -Manifold?

In classical general relativity, the basic mathematical structure is a pair (\mathcal{M}, γ) where the smooth differentiable manifold \mathcal{M} represents spacetime, and γ is a Lorentzian-signature metric defined on \mathcal{M} . Similarly, space is modeled by a three-dimensional manifold Σ equipped with a Riemannian metric g .

In the context of a quantum theory of gravity, the first crucial question is whether it is still correct to base everything on an underlying set \mathcal{M} of ‘spacetime points’. If so, is the correct mathematical structure still differential geometry, or should a different—perhaps broader—category like general point-set topology be used? Implicit in such questions is the idea that—in addition to the fields it carries—the structure of ‘spacetime itself’ may be subject to quantum effects. The broad options seem to be the following.

1. The theory of quantum gravity requires a fixed set of spacetime (or, if appropriate, spatial) points equipped with a fixed topological and differential structure.

Thus spacetime (or space) itself is the same as in classical general relativity. This is the view adopted by the canonical quantum gravity programme. It is also inherent in spacetime oriented quantisation schemes based on the expansion Eq. (5), and in simple versions of perturbative superstring theory (see later).

2. The theory of quantum gravity requires a fixed set of spacetime (or spatial) points. However, the topology and/or differential structure on this set is subject to quantum effects.

For example, in the context of canonical quantisation, Wheeler¹⁵ suggested that large quantum fluctuations in the metric tensor could induce fluctuations in the spatial topology—what I shall call ‘metric-driven’ topology changes. Another—but not necessarily unrelated—possibility is that the topological structure of space or spacetime is ‘*actively* quantised’ in some way; an example in the Euclidean quantum gravity programme is the sum Eq. (19) over manifolds \mathcal{M} . In either case, the question arises whether the resulting quantum effects could move from the category of differentiable manifolds to something more general. If so, one might be cautious about starting with a formalism (*i.e.*, classical general relativity) in which differential geometry plays such a fundamental role from the outset.

3. The notion of a spacetime point is not meaningful at a fundamental level.

In particular, the language of differential geometry employed in classical general relativity is a *phenomenological* tool that applies only at scales well away from the Planck length or energy.

A. The possibility of asymptotic freedom

The idea of metric-driven topology change is based on the intuition that the effects of quantum gravity become more pronounced at decreasing distances, resulting eventually in a ‘foamlike’ structure at around the Planck length. However, an alternative view is that quantum gravity could be *asymptotically-free*—in which case the effects become smaller, not larger, as the scale reduces. Under these circumstances, there would be no metric-driven topology changes.

Asymptotic freedom would also mean that semi-classical methods could give physically useful predictions at very small scales: a possibility that has been exploited recently by Brandenburger⁴² in a cosmological context. The idea that gravity might be asymptotically-free was studied some years ago by Fradkin and Tseytlin⁴³ in the context of $R + R^2$ theories of gravity. More recently, Tseytlin⁴⁴ has emphasised the importance of the analogous effect in superstring theory.

B. Spacetime as a phenomenological construct

The notion of active quantisation of topology raises a number of fundamental questions that will be addressed in Section 5.2 in the course of a general discussion of the applicability of present-day quantum theory. However, my personal leanings are towards the more iconoclastic view that the concept of ‘spacetime’ is not a fundamental one at all, but only something that applies in a ‘phenomenological’ sense when the universe is not probed too closely. Of course, in modern quantum field theory we have become accustomed to the idea of phenomenological schemes that only work with some degree of coarse-graining of the physical world. However, all existing theories of this type employ a strictly classical view of the fundamental nature of the manifold of spacetime (or spatial) points, whereas what is being suggested now is that spacetime itself is also a concept of strictly limited applicability.

The most obvious thing to regard as phenomenological in this sense is probably the topology or differential structure on a fixed set of spacetime points. However, the phenomenological status might extend to the notion of a spacetime point itself. Note that, if correct, ideas of this type imply that the first two options above are definitely incorrect: it is wrong to work with a fixed spacetime manifold—because that is not a meaningful concept at a

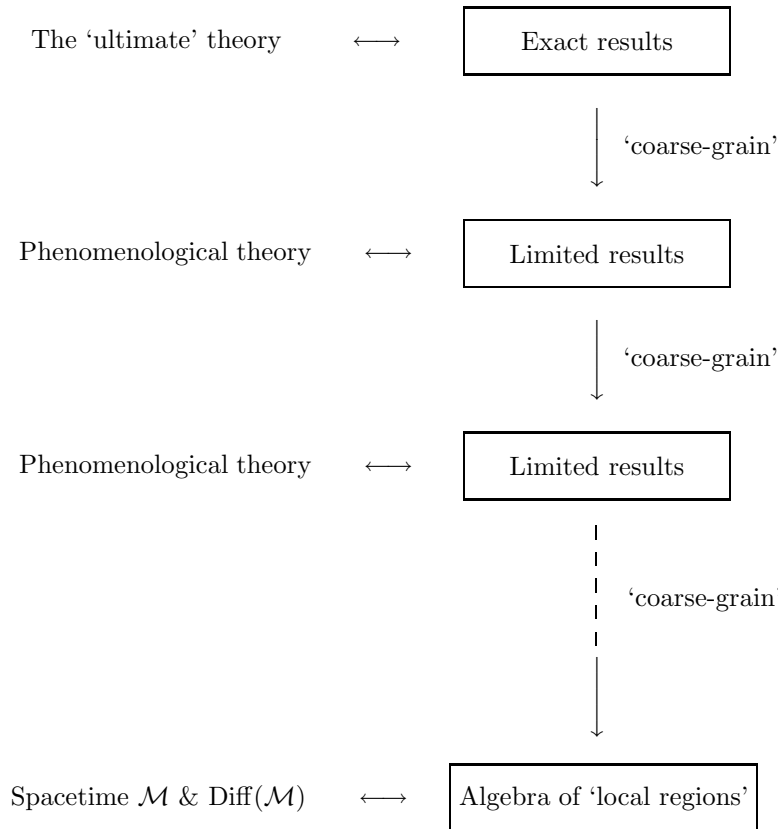


Figure 1. The hierarchy of phenomenological theories

fundamental level—but it is also wrong to talk about ‘actively quantising’ such things if there is no such thing to quantise.

The general idea behind this view of spacetime is sketched in Figure 1. At the top of the tower is the ‘ultimate’ theory of physics, whose fundamental categories—we are supposing—do not include continuum ideas of spacetime or, indeed, ideas of spacetime points at all. A ‘phenomenological theory’ means a mathematical structure that replicates only certain coarse-grained features of the fundamental theory. This structure may itself be coarse-grained further, and so on, leading eventually to a mathematical model in which our conventional ideas of space and time can be recognised. This could happen in various ways. For example, one result of coarse-graining might be the idea of a ‘local

region’—not regarded as a subset of something called ‘spacetime’, but rather as an emergent concept ^t (like pressure, for example) in its own right—plus an algebra of such regions that specifies their theoretical use, and that can be identified mathematically as the algebra of a certain open covering^{45,46,47} of a genuine continuum manifold \mathcal{M} . Hence—as long as one keeps to the phenomena appropriate to this level—it is *as if* physics is based on the spacetime manifold \mathcal{M} , even though this plays no fundamental role in the ‘ultimate’ theory with which we started.

Of course, many philosophical—as well as mathematical and physical—issues are involved in a picture of this type. For example, the idea of an ‘ultimate’ theory may be meaningless; in which case the tower in Figure 1 has no upper member at all. And many different towers may branch off from the same level, thus raising general issues of realism and instrumentalism. However, the central idea—that concepts of spacetime point, topology, and differential structure—have no fundamental status, is one that could well form an important ingredient in future theories of quantum gravity.

4.2 The Group of Spacetime Diffeomorphisms

When thinking about the role of spacetime \mathcal{M} (or space Σ) in quantum gravity, one key issue is the status of the associated group of diffeomorphisms $\text{Diff}(\mathcal{M})$ (resp. $\text{Diff}(\Sigma)$) that is such a central feature of the classical theory of general relativity. There are at least three ways in which $\text{Diff}(\mathcal{M})$ or $\text{Diff}(\Sigma)$ could appear in the quantum theory: (i) as an *exact* covariance group; (ii) as a *partial* covariance group; (iii) as a limited concept associated with a phenomenological view of spacetime (or space). Note that the third option flows naturally from the view of spacetime promulgated above: if spacetime is a phenomenological concept of limited applicability, then so will be the diffeomorphisms of the manifold that models spacetime in this limited sense. All three potential roles of the diffeomorphism group merit attention, and each will be discussed briefly in what follows.

A. Diffeomorphisms as an exact covariance group

The idea that the group of spacetime diffeomorphisms is an exact covariance group plays a key role in many existing approaches to quantum gravity. For example, it is one of the defining properties of a topological quantum field theory. It also plays a major role in canonical quantum gravity via the classical

^tOf course, this could also exist already in the top level of the tower as a central ingredient in the fundamental theory.

Poisson bracket algebra Eqs. (11)–(13) of the constraint functions \mathcal{H}_a and \mathcal{H}_\perp that can be interpreted as the algebra of spacetime diffeomorphisms projected along, and normal to, spacelike hypersurfaces. In approaches to the quantum theory in which all twelve functions $g_{ab}(x)$ and $p^{cd}(x)$ are quantised (with the canonical algebra Eqs. (14)–(16)), it is natural to suppose that Eqs. (11)–(13) are to be replaced with the analogous commutator algebra of the quantum operators $\hat{\mathcal{H}}_a$ and $\hat{\mathcal{H}}_\perp$; indeed, if the classical algebra is *not* preserved by the quantum theory there is a great danger of anomalous quantum excitations of non-physical modes of the gravitational field.

Of course, even to talk of such things requires the operators $\hat{\mathcal{H}}_a$ and $\hat{\mathcal{H}}_\perp$ to be defined rigorously—a task that is highly non-trivial, not least because this is the point at which ultraviolet divergences are likely to appear. Indeed, the possibility of addressing this issue properly only arose fairly recently following significant advances in the Ashtekar programme, and the issue of anomalies there will clearly be a crucial one in the next few years; particularly in the supergravity version where extra structure is available^{48,49}. Note also Jackiw’s⁵⁰ recent demonstration that Dirac quantisation and BRST quantisation can give *different* results concerning anomalies: a warning shot for all those involved in canonical quantum gravity. At the very least, the possibility arises that the ‘true’ group in the quantum theory is some deformation $\text{Diff}(\mathcal{M})_q$ of the classical group $\text{Diff}(\mathcal{M})$.

B. Diffeomorphisms as a partial covariance group

Another possibility is that the classical group $\text{Diff}(\mathcal{M})$ of spacetime diffeomorphisms arises only as a ‘partial’ covariance group. The two obvious options of this type are:

- *Injective*: $\text{Diff}(\mathcal{M})$ could appear as a *subgroup* of a larger covariance group G , as summarised in the exact sequence

$$0 \rightarrow \text{Diff}(\mathcal{M}) \rightarrow G. \quad (20)$$

An example is superstring theory which—at least, in the perturbative domain—appears to have a much bigger gauge structure than $\text{Diff}(\mathcal{M})$ alone.

- *Projective*: $\text{Diff}(\mathcal{M})$ could be related to a bigger group G in a projective way: *i.e.*, there is some normal subgroup K of G so that $G/K \simeq \text{Diff}(\mathcal{M})$, as summarised in the exact sequence

$$0 \rightarrow K \rightarrow G \rightarrow \text{Diff}(\mathcal{M}) \rightarrow 0. \quad (21)$$

In this case, the relation between the full covariance group G and the group $\text{Diff}(\mathcal{M})$ is analogous to that between $SU2$ and $SO3 \simeq SU2/Z_2$, *i.e.*, like the relation between fermions and bosons.

4.3 *The Role of Black Holes in Quantum Gravity*

Ever since Hawking's discovery of black hole radiation, a major issue has been the precise role of black holes in the quantum theory of gravity. For example, it is tempting to speculate that spacetime at the Planck scale has a foam-like structure built from 'virtual' black holes. Of course, the use of such language presupposes that differential geometry is still applicable at this scale, which—as argued above—is perhaps debatable. However, the idea has many attractive features and could make sense in a semi-classical approximation. Some recent work on the production of virtual black-hole pairs is reported in Hawking's paper⁵¹ in these proceedings.

Another subject of much debate has been the final state of a collapsing black hole—particularly the fate of the information that is apparently lost across the event horizon. There are three different views on this: (i) the information is truly lost—signalling a fundamental breakdown of conservation of probability (this is the option that Hawking himself prefers); (ii) the information is returned in some way in the late stages of the Hawking radiation; and (iii) the black hole leaves long-lived, Planck-mass size remnants.

These ideas are of considerable importance and interest. However, the feature of black holes on which I wish to focus here is their possible effect on the nature of quantum physics in a bounded region. This has been emphasised recently by several people and goes back to an old remark of Bekenstein⁵²: any attempt to place a quantity of energy E in a spatial region with boundary area A —and such that $E > \sqrt{A}$ —will cause a black hole to form, and this puts a natural upper bound on the value of the energy in the region (the argument is summarised nicely in a recent paper by Smolin⁵³). The implication is that in any theory of quantum gravity whose semi-classical states contain something like black-hole backgrounds, the quantum physics of a bounded region will involve only a *finite*-dimensional Hilbert space.

This intriguing possibility is closely related to the so-called 'holographic' hypothesis of t'Hooft⁵⁴ and Susskind⁵⁵ to the effect that physical states in a bounded region are described by a quantum field theory on the *surface* of the region, with a Hilbert space of states that has a finite dimension—a hypothesis that is itself echoed by recent ideas in topological quantum field theory, especially the work of Barrett⁵⁶, Crane⁵⁷, and Smolin^{58,53} concerned with the role of topological quantum field theory in quantum cosmology.

Ideas of this type could have profound implications for quantum gravity. In terms of the tower in Figure 1, the implication is that at one level of phenomenological theory the idea of local spacetime regions makes sense, and in those regions the quantum theory of gravity is finite-dimensional. However, in the—presumably different—tower of phenomenological approximations that includes weak-field perturbative approaches to quantum gravity, the effective theory uses an infinite-dimensional Hilbert space to describe the states of weakly-excited gravitons.

It is worth remarking that—even in normal quantum theory—an infinite-dimensional Hilbert space can arise as an approximate quantisation of a system whose real quantum state space is finite-dimensional. For example, consider a classical system whose (compact) phase space is the two-sphere S^2 . The group $SO(3)$ acts as a transitive group of symplectic transformations of this phase space, and one can argue that quantising the system consists in finding irreducible representations of this group, all of which—of course—are finite-dimensional. On the other hand, if one fixes a point $p \in S^2$ and studies only fluctuations around this point, it is natural to describe the quantum theory by quantising the system whose classical state space is the tangent space \mathbb{R}^2 at p . However, the appropriate group of transformations of \mathbb{R}^2 is the familiar Weyl-Heisenberg group of standard wave mechanics, and the (essentially unique) irreducible representation of this group has an *infinite* dimension.

4.4 Lessons From String Theory

At this point, it is appropriate to say something about how the nature of spacetime is seen from the perspective of the various currently active approaches to quantum gravity.

The Euclidean programme works mainly with the classical picture of spacetime as a differentiable manifold. Indeed—in so far as the formalism does not transcend its own putative semi-classical limit—the categories used are precisely those of standard general relativity.

Similarly, the starting point for canonical quantum gravity is the structure of a fixed 3-manifold that represents physical space. Indeed, it is arguable that the use of classical categories is inevitable in *any* approach to quantum gravity that is based on the idea of ‘quantising’ some version of the classical theory of general relativity. Of course, it is a different question whether or not the classical picture of space or spacetime is maintained throughout the development of the theory. For example, the recent ideas in the Ashtekar programme about the quantisation of area and volume suggest that the ‘ultimate’ picture of space may have an essentially discrete aspect. However, I shall say nothing further

here about such matters, but refer to the comprehensive review by Ashtekar in the present volume.

The situation concerning superstring theory is rather different. True, the perturbative (‘ σ -model’) approach to string theory does involve quantising a given classical system, but the system concerned is not general relativity—and hence the role of space or spacetime is certainly different from that in the Euclidean or canonical programmes. However—and perhaps more importantly—there have been major recent developments in unraveling non-perturbative aspects of the theory, and these could have dramatic implications for our understanding of the nature of space and time. We shall consider perturbative and non-perturbative aspects in turn.

A. Spacetime and perturbative string theory

Perturbative string theory is based on a ‘ σ -model’ approach in which the string is viewed as a map $X : \mathcal{W} \rightarrow \mathcal{M}$ from a two-dimensional ‘world-sheet’ \mathcal{W} to spacetime \mathcal{M} (the ‘target’ space). The famous Polyakov action for the simplest theory is

$$S = \int_{\mathcal{W}} d^2\sigma \sqrt{f} f^{ab}(\sigma) \partial_a X^\mu(\sigma) \partial_b X^\nu(\sigma) \gamma_{\mu\nu}(X(\sigma)) \quad (22)$$

where f^{ab} and $\gamma_{\mu\nu}$ are metric tensors on \mathcal{W} and \mathcal{M} respectively. This action is conformally invariant, and to preserve this invariance when X and f are quantised, it is necessary that (i) the spacetime \mathcal{M} has a certain critical dimension D (the exact value depends on what other fields are added to the simple bosonic string described by Eq. (22)); and (ii) the background spacetime metric γ satisfies the field equations

$$R_{\mu\nu} + \frac{\alpha'}{2} R_{\mu\alpha\beta\tau} R_{\nu}^{\alpha\beta\tau} + O(\alpha'^2) = 0 \quad (23)$$

where α' is a coupling constant with value around L_P^2 . It is by these means that Einstein’s equations enter string theory.

The most relevant observation concerning the nature of spacetime is that it is still represented by a smooth manifold, although its dimension may not be four—hence requiring some type of Kaluza-Klein picture. In addition, it is extremely important to note that the more realistic superstring theories involve a massless ‘dilaton’ scalar field ϕ , and a massless vector particle described by a three-component field strength $H_{\mu\nu\rho}$, whose low-energy field equations can be obtained from the effective spacetime action

$$S_{\text{rm}} = \int d^D X \sqrt{\gamma} e^{-2\phi} [R - 4\gamma^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + \frac{1}{12} H_{\mu\nu\rho} H^{\mu\nu\rho} + O(\alpha')]. \quad (24)$$

The presence of these extra fundamental fields has a major effect on the classical solutions of the field equations; in particular, there have been many studies recently^u of black-hole and cosmological solutions. The unavoidable presence of these extra basic fields in superstring theory—plus the central role of supersymmetry itself—contrasts sharply with competing approaches in which just the metric field is quantised.

B. Duality and the target manifold

In an action of the type Eq. (22), the differential structure and topology of \mathcal{M} are fixed in advance, and there seems to be no room for any deviation from the classical view of spacetime. However, there have recently been significant developments in non-perturbative aspects of string theory, and these have striking implications for our understanding of the nature of space and time at the Planck scale. It is appropriate, therefore, to say something about this here, even though I can only touch on a few of the relevant ideas.

The advances under discussion are based on various types of ‘duality’ transformations or symmetries: specifically, (i) T -duality; (ii) S -duality; and (iii) mirror symmetry. I shall say a little about each of these in turn.

I. T -duality: The simplest example of target-space duality (‘ T -duality’) arises when the target space is a five-dimensional manifold of the form $\mathcal{M}_4 \times S^1$. It can be shown that the physical predictions of the theory are invariant under replacement of the radius R of the fifth dimension with $2\alpha'/R$. Thus we cannot differentiate physically between a very small, and a very large, radius for the additional dimension—indeed, there is a precise sense in which they are ‘gauge’ equivalent to each other. This invariance suggests the existence of a minimum length of $R_{\min} = \sqrt{2}\alpha'$, and can be generalised to more than one extra dimension and with a topology that is more complex than just a product of circles. This phenomenon is often cited in support of the claim that strings do not ‘see’ spacetime in the same way as do point particles: a point that has been strongly emphasised by Horowitz and collaborators in their studies of the operational definition of spacetime singularities in string theory^{59,60}.

II. S -duality: This concept goes back to an old suggestion by Montonen and Olive⁶¹ that the electric-magnetic duality of source-free electromagnetic theory has an analogue in non-abelian Yang-Mills theory whereby the physics in the large-coupling limit is given by the weak-coupling limit of a ‘dual’ theory whose fundamental entities can be identified with solitonic excitations of the original theory. The full implementation of this idea requires the addition of

^uFor references, consult the paper by Brandenburger in these proceedings.

supersymmetry, and it is only relatively recently that there has been definitive evidence that pairs of such gauge theories (with $N = 2$ supersymmetry) really do arise⁶². For a recent survey see Olive⁶³.

Considerable excitement has been generated recently by the idea that a similar phenomenon may arise in string theories. In particular, there have been well-supported claims^{64,65} that the strong-coupling limit of a type II, ten-dimensional, superstring theory is a supergravity theory in *eleven* dimensions—a duality that is associated naturally with the introduction of extended objects (‘membranes’) of dimension greater than one⁶⁶. Ideas of this type are attractive because (i) they provide a real possibility of probing the—physically interesting, but otherwise rather intractable—high-energy limits of these theories; and (ii) the results suggest that the apparent plethora of technically viable superstring theories is not as embarrassingly large as had once been feared: an important step in securing the overall credibility of the superstring programme.

III. Mirror symmetry: This is another mechanism whereby two—apparently very different—string theories can be physically equivalent. The main developments have been in the context of pairs of Calabi-Yau manifolds that can be continuously deformed into each other via an operation involving conifold singularities. From a physical perspective, these singular points have been identified with massless black holes^{67,68}. The net effect is that string theories with different compactifications can now be seen as part of a smoothly connected set, even though the topological structures of the extra dimensions may be quite different from one theory to another.

These developments all suggest rather strongly that the classical ideas of space and time are not applicable at the Planck length. Indeed, these new results in string theory are very compatible with the general idea espoused earlier that spacetime is not a fundamental category in physics but only something that applies in a ‘phenomenological’, coarse-grained sense. At a more technical level, the new ideas suggest that Lagrangian field-theoretic methods (as represented by the Polyakov action Eq. (22)) are reaching the limit of their domain of applicability and should be replaced by—for example—a more algebraic approach to theory construction that places less emphasis on an underlying classical system of fields.

5 Structural Issues Concerning Quantum Theory

5.1 *The Key Question*

The question of concern is whether present-day quantum theory can cope with the demands of quantum gravity. There are several aspects to this: one is the

conceptual problems associated with quantum cosmology; another concerns the possibility that spacetime itself should be quantised in some way—an idea that arguably stretches the current quantum formalism to its limits, both technically and conceptually.

It has often been remarked that the instrumentalist tendencies of the Copenhagen interpretation are inappropriate in quantum cosmology. Much to be preferred would be a formalism in which no fundamental role is ascribed to the idea of ‘measurement’ construed as an operation external to the domain of the formalism. Of course—setting aside the needs of quantum cosmology—there has been extensive debate for many years about finding a more ‘realist’ interpretation of quantum theory. Two such programmes are the Bohm approach^{69,70,71,72} to quantum theory, and the ‘decoherent histories’ approach⁷³, both of which have been actively investigated in the last few years for their potential application to quantum cosmology.

From a more technical perspective, the main current approaches to quantum gravity proper—the Euclidean programme, the Ashtekar scheme, and superstring theory—all use what are, broadly speaking, standard ideas in quantum theory. In particular—as discussed above—they work with an essentially classical view of space and time—something that, arguably, is a *prerequisite* of the standard quantum formalism. This raises the important question of whether quantum theory can be adapted to accommodate the idea that spacetime itself (*i.e.*, not just the metric tensor) is subject to quantum effects: surely one of the most intriguing challenges to those working in quantum gravity.

5.2 Quantising Space-Time

Some of the many issues that arise can be seen by contemplating how one might try to quantise spacetime ‘itself’ by analogy with what is done for—say—the simple harmonic oscillator, or the hydrogen atom. Of course, this may be fundamentally misguided—for example, the concept of ‘quantum topology’ may be meaningful only in the coarse-grained sense of belonging to a hierarchy of the type signified by Figure 1. Nevertheless, it is instructive to think about the types of problem that occur if one does try to actively quantise spacetime itself—if nothing else, it reveals the rather shaky basis of the whole idea of ‘quantising’ a given classical structure.

One approach to spacetime quantisation is the ‘sum over manifolds’ method employed in the Euclidean programme, as in Eq. (19). Another is to treat spatial topology as some type of canonical variable⁴⁷. And then there are ideas about using discrete causal sets^{46,74}, or non-commutative geometry³² and the like.

In reflecting on these—and related—schemes for quantising spacetime, two major issues are seen to arise. First, a sophisticated mathematical concept like a differentiable manifold appears at one end of a hierarchical chain of structure, and it is necessary to decide at what point in this chain quantum ideas should be introduced. Second, a given mathematical structure can often be placed into more than one such chain, and then a decision must be made about which one to use.

Two chains leading to (\mathcal{M}, γ)

For example, if γ is a Lorentzian metric on a spacetime manifold \mathcal{M} , the pair (\mathcal{M}, γ) fits naturally into the chain

$$\text{set of spacetime points} \rightarrow \text{topology} \rightarrow \text{differential structure} \rightarrow (\mathcal{M}, \gamma) \quad (25)$$

where the lowest level (*i.e.*, the left hand end) is a set \mathcal{M} of bare spacetime points (with the cardinality of the continuum), which is then given the structure of a topological space, which in turn is given the structure of a differentiable manifold (only possible—of course—for very special topologies) which is then equipped with a Lorentzian metric to give the final pair (\mathcal{M}, γ) . Note that a variety of intermediate stages can be inserted: for example, the link ‘differential structure $\rightarrow (\mathcal{M}, \gamma)$ ’ could be factored as

$$\text{differential structure} \rightarrow \text{causal structure} \rightarrow (\mathcal{M}, \gamma). \quad (26)$$

A quite different scheme arises by exploiting the fact that a differentiable manifold \mathcal{M} is uniquely determined by the algebraic structure of its commutative ring of differentiable functions, $\mathcal{F}(\mathcal{M})$. A ring is a complicated algebraic structure that can be analysed into a hierarchy of substructures in several ways. Thus one alternative chain to Eq. (25) is

$$\text{set} \rightarrow \text{abelian group} \rightarrow \text{vector space} \rightarrow \mathcal{F}(\mathcal{M}) \rightarrow (\mathcal{M}, \gamma). \quad (27)$$

5.3 Three Quantisation Modes

For any given hierarchical chain that underpins a specific classical mathematical structure there are at least three different ways in which quantum ideas might be introduced⁷⁵.

1. *Horizontal quantisation.* By this I mean the active quantisation of one level of the chain whilst keeping fixed all the structure below. Thus quantum fluctuations occur within a fixed classical category. For example—in

the context of the first chain above—most approaches to quantum gravity keep fixed the set of spacetime points, its topology and its differential structure—only the metric γ is quantised (consistent with the fact that, in *classical* general relativity, only the metric is a dynamical variable). One example is the first stage Eq. (18) of the Euclidean programme; another is canonical quantisation of the 3-metric $g_{ab}(x)$ (with space replacing spacetime). More adventurous would be a scheme in which the set \mathcal{M} and its topology are fixed but the differential structure is quantised^v; or perhaps—as in Eq. (19)—quantum fluctuations may be restricted to topologies that are compatible with \mathcal{M} being a differentiable manifold.

Thus, when using the first chain Eq. (25) we are led naturally to talk of ‘quantum geometry’, ‘quantum topology’, and the like. However, if applied to the second chain Eq. (27), quantising within a level leads naturally to considerations of—for example—the algebraic approach to classical general relativity pioneered by Geroch⁷⁷ (‘Einstein algebras’) and non-commutative analogues thereof⁷⁸. Of course, the idea of a non-commutative version of the ring $\mathcal{F}(\mathcal{M})$ was one of the motivating factors behind Connes’ seminal ideas on non-commutative geometry³².

2. *Trickle-down effects.* This refers to the type of situation envisaged by Wheeler in his original ideas of quantum topology in which large quantum fluctuations in a quantised metric $g_{ab}(x)$ generate changes in the spatial topology. Thus active quantisation at one level ‘trickles down’ to produce quantum effects further down the chain.

Another example is Penrose’s thesis that a projective view of spacetime structure is more appropriate in quantum gravity, so that—for example—a spacetime point should be identified with the collection of all null rays that pass through it. Quantising the spacetime metric will then induce quantum fluctuations in the null rays, which will therefore no longer intersect in a single point. In this way, quantum fluctuations at the top of the first chain Eq. (25) trickle right down to the bottom of the chain, so that the very notion of a ‘spacetime point’ acquires quantum overtones.

3. *Transcendental quantisation.* From time to time, a few hardy souls have suggested that a full theory of quantum gravity requires changing the foundations of mathematics itself. A typical argument is that standard mathematics is based on set theory, and certain aspects of the latter (for example, the notion of the continuum) are grounded ultimately in our

^vPercival⁷⁶ has recently applied the ideas of ‘primary state diffusion’ to quantise the differential structure of spacetime.

spatial perceptions. However, the latter probe only the world of classical physics, and hence we feed into the mathematical structures currently used in *all* domains of physics, ideas that are essentially classical in nature. The ensuing category error can be remedied only by thinking quantum mechanically from the very outset—in other words, we must look for ‘quantum analogues’ of the categories of standard mathematics.

How this might be done^w is by no means obvious. One approach is to claim that, since classical logic and set theory are so closely linked (a proposition P determines—and is determined by—the class of all entities for which P can be rightly asserted), one should start instead with the formal structure of *quantum* logic and try to derive an analogous ‘non-Boolean set theory’. Transcending classical categories in this way is a fascinating idea, but it is also very iconoclastic and—career-wise—it is probably unwise to embark on this path before securing tenure!

5.4 *General-Relativity Driven State Reduction*

A very different perspective on the adequacy of standard quantum theory is given by the idea that the thorny problem of state-vector reduction itself requires the introduction of general relativity^{81,82,83}. This position is often associated with a general view that spacetime is the ‘ultimate classical object’ and—as such—is not subject to quantum fluctuations in any way^x—a position that is diametrically opposite to the one explored in the previous section.

This approach to the reduction problem is attractive for several reasons: (i) gravity is the only *universal* force we know, and hence the only force that can be guaranteed to be present in all physical interactions; and (ii) gravitational effects grow with the size of the objects concerned—and it is in the context of macroscopic objects that entangled quantum states are particularly problematic.

From a technical perspective, most concrete implementations of GR-driven state reduction involve variants of the ‘spontaneous reduction’ theories of the type pioneered by Ghirardi, Rimini and Weber⁸⁴, and Pearle⁸⁵. A recent example is the paper by Pearle and Squires⁸⁶, which also contains a good bibliography.

^wRecent examples of this type of thinking can be found in a book by Finkelstein⁷⁹ and a paper⁸⁰ of Krause and French.

^xHowever, Roger Penrose—one of the principal advocates of this view—has wavered between the idea that (i) superpositions of spacetimes never occur; and (ii) superpositions may occur, but they decay in a very short time—a notion that itself encounters the problem of time (private communication).

As with so many of the other ideas I have discussed in this article, a key question is whether the notion of gravity-induced reduction should be built into the theory from the very beginning, or if it could ‘emerge’ in a phenomenological sense within a tower of the type in Figure 1.

6 Where Are We Going?

I would like to summarise some of the preceding discussion by speculating on what a future theory of quantum gravity might look like, especially in regard to the way it deals with the basic categories of space and time.

A. Desirable properties of the new theory at a basic level

At a basic level (or—at least—as high up the tower in Figure 1 as I am willing to speculate) a future theory of quantum gravity might have the following features:

- There will be no fundamental use of the continuum. Applied in general, this proscribes the use of any set whose cardinality is greater than countably infinite. Applied in particular, it excludes a continuum of spacetime points. Indeed, there should probably be no fundamental use of the idea of a ‘spacetime point’ at all.
- At a basic level, the interpretation of the theory must not involve instrumentalist ideas of the type used in the Copenhagen view; in particular, there must be no invocation of external ‘observers’—conscious or otherwise.
- The quantum aspects of the theory will not be grounded in the use of Hilbert spaces; not even those over a finite field. This reflects the old idea of Bohr that the wave function of a system does not refer to the object itself but only to the range of results that could be obtained by a measurement process for a specific observable quantity. This view has been resurrected recently by several authors—in particular, Rovelli⁸⁷, Crane⁵⁷, and Smolin⁵⁸—in the context of developing a relational view of quantum theory. The implication ‘no observer, hence no Hilbert space’, is not logically inevitable, but it is one that I find quite attractive.

As to what should replace Hilbert space, I currently favour the types of algebraic structure adopted in the past by those working in quantum logic—especially orthoalgebras and manuals⁸⁸. This fits in well with an interpretative framework based on the consistent histories approach to quantum theory^{89,90}.

B. Emergent structure in the theory at a phenomenological level

In the spirit of Figure 1, let me now list some of the features that might be expected to emerge from the basic structure in a phenomenological sense.

- *A continuum spacetime.* At some stage, the familiar ideas of a continuum spacetime should emerge—perhaps via the mechanism of an ‘algebra of local regions’ discussed earlier.
- *Standard quantum theory.* The formalism of standard quantum theory should also emerge in an appropriate limit. This would include the usual mathematical framework of Hilbert spaces, but perhaps augmented with the ‘holographic hypothesis’ that the state space for physics in a bounded space-time region has a finite dimension (assuming, of course, that the notions of ‘space-time region’ and ‘bounded’ make sense at the phenomenological level concerned).

I am also very attracted by the idea that state-reduction is associated with general relativity. However—in the type of theory being discussed—this would probably be in a ‘phenomenological’ sense rather than appearing as one of the basic ingredients.

- *A theory of quantum gravity.* What we would currently regard as a ‘theory of quantum gravity’ should also appear at a phenomenological level once both standard quantum theory and general relativity have emerged. If our present understanding of quantum gravity is any guide, this effective quantisation of the gravitational field will involve a non-local—possibly string-like—structure.

The last point raises the intriguing question of whether superstring theory and the loop-variable approach to canonical quantisation can both be regarded as different modes—or phases—of a more basic, common structure. This fascinating possibility is a strong motivating factor^y behind Smolin’s recent work aimed at relating canonical quantisation and topological quantum field theory^{58,53}. A central issue, presumably, is whether supersymmetry can be assigned some significant role in the Ashtekar programme.

C. The Key Questions

It is clear that certain key questions will arise in any attempt to build a structure of the type envisaged above. Specifically:

^yPrivate communication

- What is the *basic* structure (if any) in the theory, and what emerges as ‘*effective*’ structure in a more phenomenological sense?
- How *iconoclastic* do we have to be to construct a full theory of quantum gravity? In particular, is it necessary to go as far as finding ‘quantum analogues’ of the categories of normal mathematics?

The first question provides a useful way of categorising potential theoretical frameworks. The second question is the most basic of all, addressing as it does the challenge of finding the ingredients for a theory that can head a tower of phenomenological approximations of the type under discussion. The key problem is to identify the correct choice of such building blocks among the myriad of possibilities. This is no easy task, although—as illustrated by the list of desirable features in a future theory—certain broad ideas are suggested by the existing research programmes. Certainly, the momentum behind these approaches—the Ashtekar programme, superstring theory, and the Euclidean programme—is such that each is likely to be developed for the foreseeable future, and—in the process—may yield further ideas for a more radical approach to the problem of quantum gravity.

On the other hand, it is possible that none of the current programmes is on the right track, in which case we need to look elsewhere for hints on how to proceed. What is missing, of course, is any hard *empirical* data that would enforce a fundamental shift in approach—which brings us back to a question raised at the very beginning of the paper: is it possible to find experimental tests to resolve some of the many obscure issues that cloud the subject of quantum gravity?

The obvious problem is the simple dimensional argument suggesting that effects of quantum gravity will appear at energies of around $E_P \simeq 10^{-28}\text{eV}$, which is well beyond the range of terrestrial experiments. Of course, there are subtler possibilities than this. For example:

- There may be non-perturbative effects of the type mentioned earlier in the context of a quantum-gravity induced ultraviolet cutoff in quantum field theory.
- Qualitative as well as quantitative properties of the theory should be considered—some examples of this type are discussed in a recent paper by Smolin⁹¹.
- There can be unexpected predictions from a theory—for example, the results reported by Hawking⁵¹ in this volume in his discussion of virtual black-hole production and its possible implications for the vanishing θ -angle in QCD.

- The physics of the immediate post big-bang era may provide an important testing ground. For example, Hawking has suggested⁹² that the anisotropies in the microwave background originate in quantum fluctuations around the Hartle-Hawking ground state⁹³; while Grischuk⁹⁴ has sought to explain the same phenomena in terms of his work on squeezed graviton states.

So the situation concerning experimental tests is not completely hopeless. But it is something we must continually strive to improve if studies in quantum gravity are not to become the 20th century equivalent of the medieval penchant for computing the cardinality of angels on pinheads: an ever-present danger at this extreme edge of modern theoretical physics!

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