A new perspective on CP and T violation

Abhay Ashtekar*

Institute for Gravitation and the Cosmos & Physics Department,
Penn State, University Park, PA 16802, U.S.A.

It is shown that the results of CP and T violation experiments can be interpreted using a very general framework that does not require a Hilbert space of states or linear operators to represent the symmetries or dynamics. Analysis within this general framework brings out the aspects of quantum mechanics that are essential for this interpretation. More importantly, should quantum mechanics be eventually replaced by a new paradigm, this framework could *still* be used to establish violation of CP and T invariance from the already known experimental results.

This work arose as a 'formal response' to a talk *Three Merry Roads to T-Violation* by Dr. Bryan Roberts [1]. Both talks were given at a *Workshop on Cosmology and Time* held at Penn State in April 2013, which brought together physicists and philosophers of science.

I. INTRODUCTION

To interpret any experiment, one needs a theoretical paradigm. For experiments on violations of the CP and T symmetries in weak interactions, the standard choice has been the mathematical framework of quantum physics. This is natural because the best available description of the electroweak interactions is based on local quantum field theory. However, we will show that one can in fact analyze the T-violation experiments in a *much* more general setting. Classical mechanics, quantum mechanics and quantum field theory provide only specific examples of this general framework. As is often the case, because this framework has much less structure, the analysis becomes significantly simpler. As a result, aspects of quantum physics that are truly essential for interpreting experiments on CP and T-violation are brought to the forefront. They are clearly separated from other features which, though not central, have generally been treated in the literature as being equally significant. More importantly, the conceptual structure that is essential to the interpretation of experiments is so weak that, even if quantum physics were to be replaced by some more general framework —e.g., because of its unification with general relativity— the current experiments will still, in all likelihood, enable us to conclude that the fundamental laws of Nature fail to be invariant under time reversal.

In the tradition of joint physics and philosophy meetings, this is a formal 'response' to a paper [1] presented by Dr. Roberts in which he crystalized the conceptual underpinnings of experiments that established the CP and T violations. His clear presentation led me to sharpen my own understanding of the theoretical constructs that have been used to analyze these experiments. This communication is the result of that re-examination. In Dr. Roberts' terminology, the analysis of the Cronin-Fitch experiment on CP-violation can be

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*Electronic address: ashtekar@gravity.psu.edu

based on *the Curie principle* and that of the CPLEAR experiments on T-violation, on *the Kabir Principle.* Therefore I will discuss these principles in detail in sections II and III. I have attempted to make this discussion reasonably self-contained but, to fully appreciate its content, it would be helpful if the reader has already gone through Dr. Robert's paper [1]. (There is also interesting follow-up paper [2] in which Dr. Roberts has used the framework introduced in the next two sections to generalize a third approach to T-violation.)

The issue of time-reversal has two distinct facets: microscopic and macroscopic. The microscopic T-violation discussed in [1] and in the present communication is quite distinct from the manifest arrow of time we perceive in our everyday life and, more generally, in the physics of large systems. Since Dr. Roberts mentioned the macro-world only in passing, before entering the main discussion, let me begin with a brief detour to explain this point. For simplicity, I will use the framework of classical physics because the core of the argument is not sensitive to the distinction between classical and quantum mechanics. Consider a large box with a partition that divides it into two parts, say, the right and the left halves. Suppose there is some gas in the left half and vacuum in the right. Once the equilibrium is reached, the macroscopic state of this gas is described by the volume it occupies, V_i; the pressure it exerts on the walls of the box, P_i; and its temperature T_i; where i stands for 'initial'. If we open the partition slowly, the gas will fill the whole box and its macro-state in equilibrium will be described by new parameters, V_f, P_f, T_f. Thus, there has been a transition from the initial macro-state (V_i, P_i, T_i) to a final state (V_f, P_f, T_f). Our common experience tells us that the time reverse of this process is *extremely* unlikely. However, we also know that the microscopic variables for the system are the positions and momenta of some 10^{23} molecules in the box. These are subject just to Newton's laws which are *manifestly invariant under the time-reversal operation T*! Therefore, if we were to reverse the momentum $\vec{p}_{(\alpha)}(t)$ of each molecule (labeled by α) at a late time t, keeping the positions $\vec{x}_{(\alpha)}(t)$ the same, time evolution would indeed shift the gas back from its final macroscopic state to the initial one, confined just to the left half of the box. But in practice it is very difficult to construct this time-reversed initial state. Thus, there is indeed a macroscopic arrow of time but its origin is *not* in the failure of the microscopic laws to be invariant under T but rather in the fact that the initial conditions we normally encounter are very special. Specifically, in our example, there are vastly fewer micro-states compatible with the initial macro-state (V_i, P_i, T_i) than those that are compatible with the final macro-state (V_f, P_f, T_f).[1] Put differently, the entropy of the initial macro-state is much lower than that in the final macro-state. This is the arrow of time, commonly discussed in the literature.

It is clear from the simple example that the fact that there is an obvious arrow of time in the macro-world does *not* imply that the microscopic or fundamental laws have to break T-invariance. Indeed, as Dr. Roberts emphasized in the beginning of his article [1], it was common to assume that the fundamental laws *are* invariant under the time-reversal operation T. It came as a shock that the weak force violates this 'self-evident' premise!

The rest of this article will focus on the T violation at the fundamental, microscopic level.

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[1] This is primarily because the volume V_f allowed for *each* molecule in the final macro-state is twice as large as V_i, allowed in the initial macro-state. Consequently, the number of microscopic configurations compatible with the final macroscopic state is about $2^{10^{23}}$ times the number of microscopic states compatible with the initial macroscopic configuration.

II.   WEAK INTERACTIONS AND THE CURIE PRINCIPLE

As Dr. Roberts has explained, what the Cronin-Fitch experiment [3] establishes directly is that the weak interactions are not invariant under CP, i.e., under the simultaneous operations of charge conjugation C and reflection through a mirror, P. As normally formulated, the parity operation is meaningful only if the underlying space-time is flat, i.e., represented by Minkowski space-time. This means one ignores curvature and therefore gravity. One further assumes that physics is described by a local quantum field theory on this Minkowski space, for which individual physical fields transform covariantly under the action of the Lorentz group, and dynamics is generated by a self-adjoint Hamiltonian obtained by integrating a scalar density (or a 3-form), constructed locally from the physical fields. Then, one has the *CPT theorem* that guarantees that the product CPT of charge conjugation, C, parity, P and time reversal T is an exact *dynamical* symmetry.[2] Therefore, although the Cronin-Fitch experiment does not *directly* imply T-violation, as Dr. Roberts explained, if we assume that weak interactions are described by a local quantum field theory in Minkowski space, then the observed breakdown of CP invariance implies that they violate T invariance as well. In the rest of this section, I will focus on just the CP symmetry and its violation observed in the Croin-Fitch experiment. Thus, I will not need to refer to the CPT at all.

In the current analysis of CP violation, one uses the following form of the Curie principle: If an initial state σ_i is in variant under CP but its time-evolved final state σ_f is not, then dynamics can not be CP invariant. As explained in section 2.4 of [1], the analysis has the remarkable feature that it does not assume a specific Hamiltonian H. Therefore, the argument will remain unchanged should we discover that the currently used Hamiltonian H in electro-weak interactions has to be modified, e.g., to accommodate future experiments, or to unify them with strong interactions.

However, the standard analysis *does* make a crucial use of the detailed kinematical structure of quantum physics (summarized below). If a future quantum gravity theory were to require that this structure has to be modified —e.g., by removing the emphasis on linearity— then the standard analysis cannot be used to conclude that the Cronin-Fitch experiment implies a violation of CP invariance in weak interactions. *The main point of this section is to show that this specific kinematical framework of Hilbert spaces and operators is not really necessary.* The Curie principle can be extended to a much more general framework than that offered by quantum physics.

Let us begin by introducing this framework, which we will call *general mechanics*. It will incorporate quantum as well as classical mechanics, but only as specific special cases. The basic assumptions of *general mechanics* are:

- i) We have a set $\mathcal{S}$ of states;

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[2] This is a summary of the discussion one finds in quantum field theory text books (see, e.g. [4]). More rigorous versions based on Weightman axioms [5] and the algebraic approach [6] are also available in the literature. However, the self-adjoint Hamiltonian required in the text book version is not rigorously defined in four dimensions beyond the quantum theory of free fields. Similarly, we still do not have a single example of a 4-dimensional, interacting quantum field theory satisfying either the Wightman axioms or the axioms of the algebraic quantum field theory. Thus, there is a curious mis-match between the mathematical statements of CPT theorems and theories of direct physical interest.

- ii) There is a 1-1, onto dynamical map S —the 'S-matrix'— from $\mathcal{S}$ to itself. This S could refer to finite time evolution, say from time t_1 to t_2 or, alternatively, to the time evolution in the infinite past to the infinite future. In practice is it convenient to consider two copies $\mathcal{S}_i$ and $\mathcal{S}_f$ of $\mathcal{S}$, representing initial and final states, and regard S as a map from $\mathcal{S}_i$ to $\mathcal{S}_f$;

$$S : \mathcal{S}_i \to \mathcal{S}_f; \qquad S(\sigma_i) = \sigma_f \qquad \forall \sigma_i \in \mathcal{S}_i \tag{2.1}$$

- iii) Potential symmetries are represented by a 1-1, onto maps $R : \mathcal{S} \to \mathcal{S}$, from $\mathcal{S}$ to itself. We will first consider the case in which R maps $\mathcal{S}_i$ to itself and $\mathcal{S}_f$ to itself. This is the case if R is, for example, the discrete symmetry represented by C, or P or CP.

Note that framework is truly minimalistic. *The space $\mathcal{S}$ of states is just a set*; no further structure is needed. To draw the contrast, let me summarize the structure we use in classical and quantum mechanics.[3] In classical mechanics, $\mathcal{S}$ is the phase space which is assumed to be a smooth, even dimensional differentiable manifold Γ, equipped with a non-degenerate, closed 2-form Ω, called the symplectic structure. In quantum mechanics, one first introduces an auxiliary structure, a Hilbert space, H: it is a complex vector space that is endowed with an Hermitian scalar product and is complete in the sense that the limit point of each Cauchy sequence in H is again in H. Pure states are then represented by rays in H and mixed states by density matrices, i.e., bounded, self-adjoint operators with finite trace. As for dynamics, in classical mechanics, it is given by the flow generated by a Hamiltonian vector field $X = \Omega^{-1} \lrcorner\, dH$ for a smooth function H on Γ, called the Hamiltonian. Now the map S corresponds either to the finite S-matrix between a time t_1 and t_2, obtained by moving along the Hamiltonian flow an affine parameter distance $t_2 - t_1$, or its limit as $t_1 \to -\infty$ and $t_2 \to \infty$. In quantum mechanics, as mentioned above, dynamics is represented by a unitary flow $\mathcal{U}_t = \exp -itH$ generated by a self-adjoint operator. The map S corresponds again either to a finite S matrix, $\mathcal{U}_{t_2-t_1}$, or its limit as $t_1 \to -\infty$ and $t_2 \to \infty$. Finally, in classical mechanics the potential symmetries R are diffeomorphisms on Γ that preserve the symplectic structure Ω and those in quantum mechanics are linear, unitary mappings on H.

This rather detailed discussion serves to brings out three points. First, both classical and quantum mechanics are based on very rich but specific and intricate mathematical frameworks. From the summary of the literature contained in section 2.5 of [1], it would appear that the specifics of quantum mechanics —the Hilbert space structure of $\mathcal{H}$ and linearity of the candidate symmetries R— are all essential for a formulation of the Curie principle that can be used to interpret the Cronin-Fitch experiment. Second, the mathematical underpinnings of the classical and the quantum frameworks are *very different from each other*. Therefore, it is natural to anticipate that the next paradigm shift (that may emerge, e.g., from a successful quantum gravity theory) could bring with it another rich yet *different* mathematical framework. Would one then have to reconstruct an appropriate formulation of the Curie principle and re-examine experiments using the new framework? The third point is that this would not be necessary if one could develop a formulation of the Curie principle (and also the Kabir principle, discussed in section III) using just the three

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[3] But the material till the end of this paragraph can be skipped without loss of continuity. It brings to forefront the intricacy of the structure underlying classical and quantum mechanics, in contrast to *general mechanics*.

assumptions of *general mechanics* which have survived the transition from classical physics to quantum. Indeed, the assumptions are so minimalistic that it is reasonable to hope that they will also be realized in the future, more general mechanics.

Let us then investigate if the Curie principle can be formulated using just the three assumptions of *general mechanics*. In the spirit of this principle, suppose that there exists some initial state $\sigma_i \in \mathcal{S}$ such that

$$R\sigma_i = \sigma_i \qquad \text{but} \qquad R\sigma_f \neq \sigma_f \tag{2.2}$$

Then,

$$R\left(S\sigma_i\right) = R\sigma_f \neq \sigma_f, \quad \text{but} \quad \sigma_f = S(\sigma_i) = S\left(R\sigma_i\right). \tag{2.3}$$

Therefore, we conclude

$$SR \neq RS. \tag{2.4}$$

Thus, the dynamical map S does not commute with the candidate symmetry R: *R can not a dynamical symmetry.* To summarize, we have derived the Kabir principle in the framework of *general mechanics*: *If there exists a state σ_i such that $R\sigma_i = \sigma_i$ and $S\sigma_i = \sigma_f$ but $R\sigma_f \neq \sigma_f$ then R is not a dynamical symmetry of the system.*[4] By repeating the argument of [1], now it is easy to analyze the Cronin-Fitch experiment in the setting of *general mechanics*. Even if the theory describing weak interaction is more general than quantum mechanics, the experiment would still imply that the dynamics of weak interactions violates T-invariance, so long as the underlying framework of this theory meets the three basic postulates of *general mechanics*.

Note that this general formulation in particular enables one to discuss in one go all the symmetries R in classical mechanics and linear symmetries used in quantum mechanics. Furthermore, since one does not even need to refer to the underlying Hilbert space structure, one can also apply the argument within standard quantum mechanics even if R or S were represented by *non-linear* mappings. In particular, for the argument to hold, the CP symmetry does not have to be unitarily realized. By contrast, as summarized in [1], the current literature on the Cronin-Fitch experiment often emphasizes that the assumption of unitarity is essential to conclude that the experiment implies CP violation.

We will conclude this discussion of the Curie principle with three remarks.

1) Since $\mathcal{S}$ is only a set, we cannot speak of continuous evolution in the framework introduced above. But one could trivially extend the framework by endowing $\mathcal{S}$ with a topology and replacing S with a continuous evolution map $E(t)$, where t is to be thought of as a time parameter. The argument given above will then imply that $E(t)$ will not commute with R.

2) However, we *did* assume that R maps the space $\mathcal{S}_i$ of initial states to itself and the space $\mathcal{S}_f$ of final states to itself. This assumption is *not* satisfied by the time-reversal operation T, which maps initial states to final states (and vice versa): $T : \mathcal{S}_i \to \mathcal{S}_f$ is a 1-1 onto map from S_i to S_f. Therefore, in this case, T is a dynamical symmetry if and only if

$$S\sigma_i = \sigma_f \quad \Longrightarrow \quad S^{-1}(T\sigma_i) = T^{-1}(\sigma_f) = T^{-1}(S\sigma_i) \tag{2.5}$$

i.e., the time reverse of σ_i (which is in $\mathcal{S}_f$) is mapped by dynamics to the time reverse of σ_f. The generalization of the Curie principle discussed above does not have any implication

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[4] It is straightforward to alter the argument to obtain the other desired conclusion of fact 1 in section 2.4 of [1].

in this case. Therefore to directly test T-invariance we need a generalization of the Kabir principle, discussed in section III. In this respect, the situation is the same as in section 2 of [1].

3) While R invariance of dynamics is captured by the property $RS = SR$ of the S-matrix S, the T invariance is captured by $S^{-1}T = T^{-1}S$. As we just saw, the difference arises simply because, while R preserves each of $\mathcal{S}_i$ and $\mathcal{S}_f$, T maps one to the other. At a fundamental level, then, *the difference is not because R is linear while T is anti-linear,* as is often stated in the literature: If one works entirely in the quantum framework, one misses the broader perspective and is then tempted to focus only on the anti-linearity of T.

III. THE KABIR PRINCIPLE

By using the Curie principle in conjunction with the Cronin-Fitch experiment we have concluded that CP cannot be a dynamical symmetry in any theory of weak interactions that conforms to the principles of *general mechanics*. But this provides only an indirect evidence for T-violation because one needs to assume, in addition, validity of the CPT theorem. As explained in section 3 of [1], more direct evidence for T-violation comes from another idea, called the Kabir principle by Roberts, which is again formulated in the framework of quantum mechanics: If a transition $\sigma_i \rightarrow \sigma_f$ occurs with a different probability from the transition $T\sigma_f \rightarrow T\sigma_i$ then the dynamics underlying this process cannot be invariant under the time-reversal operation, T. Again this argument is deeply rooted in the detailed structure of quantum mechanics. A natural question for us is then: Can we extend the Kabir principle to *general mechanics*? The answer is in the affirmative. However, since it refers to probabilities, even to state Kabir's principle one needs to introduce some additional structure in *general mechanics*.

Let us then introduce the necessary structure, in addition to the three postulates stated in section II:

- An *overlap map* $\varnothing$ on the space of states $\mathcal{S}$ (and therefore on each of $\mathcal{S}_i$ and $\mathcal{S}_f$): $\varnothing : \mathcal{S} \times \mathcal{S} \rightarrow [0, 1] \in R$, such that

$$\varnothing(\sigma, \sigma') = \varnothing(\sigma', \sigma), \qquad \forall\; \sigma, \sigma' \in \mathcal{S}. \tag{3.1}$$

$\varnothing(\sigma, \sigma')$ is to be thought of as the overlap between states σ and σ'.

In classical mechanics, general states are represented by non-negative distributions ρ on the phase space Γ which are normalized so that $\int_\Gamma \rho \, dV_\Omega = 1$, where dV_Ω is the Liouville volume element on Γ. Then the overlap function $\varnothing(\rho, \rho')$ can be the obvious overlap between the two states: $\varnothing(\rho, \rho') = \int_\Gamma (\rho\rho')^{\frac{1}{2}} \, dV_\Omega$.[5] In quantum mechanics, a general state is represented by a normalized density matrix $\hat{\rho}$ —a non-negative, bounded, self-adjoint operator on the Hilbert space $\mathcal{H}$ with $\mathrm{Tr}\hat{\rho} = 1$, where Tr is the trace operation. The overlap function can be then taken to be $\varnothing(\hat{\rho}, \hat{\rho}') = \mathrm{Tr}\,\hat{\rho}\,\hat{\rho}'$. Pure states —projection operators on one

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[5] The square-root in the integrand ensures that the overlap function is bounded by 1. (It is not needed in quantum theory.) A pure state $\rho = \rho_\gamma$, is just the Dirac distribution peaked at a point γ of Γ. For these states, natural regularizations yield : $\varnothing(\rho_\gamma, \rho_{\gamma'}) = \delta_{\gamma, \gamma'}$. Thus the overlap between a pure state and a distinct pure state simply vanishes while its overlap with itself is perfect.

dimensional subspaces of $\mathcal{H}$— can be represented by (equivalence classes of) normalized elements ψ of $\mathcal{H}$ (where two elements are equivalent if they differ by a phase). Then the overlap function $\emptyset(\psi, \psi')$ is simply the standard overlap probability of quantum mechanics: $\emptyset(\psi, \psi') = |\langle \psi, \psi' \rangle|^2$.

Let us return to *general mechanics.* Since the overlap map is part of the kinematical structure, the notion of symmetries and dynamics, introduced in section II, have to be compatible with it. The compatibility conditions are all natural and obvious. But since they will be used in the proof of the Kabir principle, let us spell them out explicitly.

- To qualify as symmetry, the map $R : \mathcal{S}_i \to \mathcal{S}_i$ introduced in section II has to satisfy

$$\emptyset_i(R\sigma_i, R\sigma_i') = \emptyset_i(\sigma_i, \sigma_i') \tag{3.2}$$

  on $\mathcal{S}_i$ (and similarly on $\mathcal{S}_f$). Similarly to qualify as symmetry, the time reversal map $T : \mathcal{S}_i \to \mathcal{S}_f$ must satisfy

$$\emptyset_f(T\sigma_i, T\sigma_i') = \emptyset_i(\sigma_i, \sigma_i') \tag{3.3}$$

  for all $\sigma_i \in \mathcal{S}_i$.

- The dynamical map $S : \mathcal{S}_i \to \mathcal{S}_f$ should be compatible with the overlap map, i.e., satisfy

$$\emptyset_i(\sigma_i, \sigma_i') = \emptyset_f(S\sigma_i, S\sigma_i') \equiv \emptyset_f(\sigma_f, \sigma_f'). \tag{3.4}$$

Finally, using the overlap map $\emptyset$ and the dynamical map S, we introduce the notion of transition probability:

- Given a dynamical map S, the *transition probability* between an initial state $\sigma_i \in \mathcal{S}_i$ and *any* given final state σ_f' is defined to be

$$P(\sigma_f', \sigma_i) := \emptyset_f(\sigma_f', S\sigma_i) \equiv \emptyset_f(\sigma_f', \sigma_f). \tag{3.5}$$

This concludes the additional kinematical structure we need on our *general mechanics* to formulate the Kabir principle. Note that the notions of the overlap function and transition probability are already available in classical as well as quantum mechanics. In both cases symmetries and dynamics satisfy the compatibility conditions we have introduced. Thus, as in section II, *general mechanics* continues to be minimalist: it simply extracts certain notions that are common to both classical and quantum paradigms, which we can reasonably expect to be present also in the future generalizations of these settings.

Recall that T is a dynamical symmetry if and only if $S^{-1}T = T^{-1}S$, or, equivalently, $ST^{-1} = TS^{-1}$. *Suppose a dynamical map S satisfies this condition.* As before, let us use the notation $\sigma_f = S\sigma_i$ and $S\sigma_i' = S\sigma_f'$. Then,

$$\emptyset_f(T\sigma_i, S(T^{-1}\sigma_f')) = \emptyset_f(T\sigma_i, T(S^{-1}\sigma_f')) = \emptyset_f(T\sigma_i, T\sigma_i') = \emptyset_i(\sigma_i, \sigma_i') \tag{3.6}$$

where in the last step we used (3.3). On the other hand, (3.4) and (3.1) imply

$$\emptyset_i(\sigma_i, \sigma_i') = \emptyset_f(S\sigma_i, S\sigma_i') = \emptyset_f(S\sigma_i', S\sigma_i) = \emptyset_f(\sigma_f', S\sigma_i). \tag{3.7}$$

The last two equations and the definition (3.5) of transition probability now yield

$$P(\sigma_f', \sigma_i) = P(T\sigma_i, T^{-1}\sigma_f'). \tag{3.8}$$

Thus, we have shown that if T is a symmetry of the dynamical map S then the transition probability between the states σ_i and σ_f' must equal that between the two states $T^{-1}\sigma_f'$ and $T\sigma_i$, obtained by a time-reversal. Therefore if the transition probability between *any* two states and their time reversed versions differ observationally, then the time reversal symmetry is broken by dynamics.[6] This is the generalization of the Kabir principle from quantum mechanics to *general mechanics* we were seeking. As Dr. Roberts has explained, it has been used to show that the *kaon oscillations*, $K^0 \leftrightarrows \bar{K}^0$ observed at the CPLEAR detector provide a direct violation of T-invariance (see, e.g., [7–9]). Using the same arguments as in section 3.2 of [1], we can now conclude that the CPLEAR observations imply that the time-reversal symmetry T is violated in *any* theory of weak interactions *so long as it falls in the broad framework of general mechanics* developed in this section.

As with the discussion of the generalized Curie principle of section II, this generalization of the Kabir criterion does not refer to the Hilbert space structure of the space of states or linearity (or anti-linearity) of various maps. In particular, in the case of quantum mechanics, while it incorporates the standard treatment neatly summarized in section 3 of [1], the result would hold even if, say, the S-matrix were anti-unitary (or, indeed, non-linear!). By contrast, as emphasized in [1], it is generally believed that unitarity of the S matrix is essential to conclude that we have experimental evidence of direct T-violation in weak interactions.

IV. DISCUSSION

Already in the context of quantum mechanics, it is interesting to analyze the interplay between symmetries and dynamics using only that structure which is available in *general mechanics*. The analysis becomes much simpler and, furthermore, brings to forefront the concepts and structures that are essential in the discussions of CP and T violations, separating them from inessential notions that have added unnecessary complexity to the analyses one finds in the literature. In particular, neither the linear structure nor the details of the Hermitian inner product on the space of quantum mechanical states is essential. Nor is it necessary that symmetries have to be represented by unitary or anti-unitary operators; indeed, they need not even be linear operators! Secondly, the primary distinction between T on the one hand, and C, P, or CP on the other, is only that while the spaces of 'in' and 'out' states are separately invariant under C, P and CP, the map T sends the incoming states to the outgoing ones. In standard quantum mechanics, C, P and CP are represented by linear, unitary maps, while T is represented by an anti-linear, anti-unitary map. However, if one uses only that structure which is provided by *general mechanics* one finds that, contrary to a wide-spread belief, this difference is not primary to the distinction between the Curie and the Kabir criteria.

While these points are illuminating, the main strength of the more general perspective presented here is that it broadens the reach of the Cronin-Fitch and CPLEAR experiments enormously. We are no longer constrained to analyze them in the strict framework of

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[6] It is worth noting that, in practice, one measures is the transition rate. This is not determined solely by the transition probability. In the leading order approximation ("Fermi's golden rule") the transition probability has to be multiplied by the density of final states. But in practice one can easily take care of this issue and verify whether or not the transition probability is symmetric under time-reversal.

quantum mechanics. This is a significant advantage because future advances, particularly in quantum gravity, may well lead us to a mathematical framework which is quite different from that offered by quantum mechanics. Just as in the passage from special relativity to general relativity we had to abandon flatness of space-time geometry, the future theory may well force us to abandon the linear structure that underlies quantum mechanics. For example, in quantum mechanics the pure states are represented by rays in the Hilbert space $\mathcal{H}$ and constitute a trivial Kähler manifold. It may be replaced by *general, non-trivial* Kähler manifolds which do *not* refer to a Hilbert space. Or, dynamics and symmetries may be represented by *non-linear* maps. Indeed, there are concrete ideas as to how this may come about (see, e.g., [10] and references therein). But such generalizations are likely to retain the minimal structure of *general mechanics*, introduced in sections II and III. If so, then the current experiments will still continue to imply that the dynamics of weak interactions violates CP and T invariance even in these much more general theories. Thus, at a conceptual level the reach of the current experiments on CP and T violations seems to be extraordinarily long, going well beyond the standard quantum mechanics framework in which they have generally been interpreted.

Acknowledgments

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