Comment on "On the Origin of Gravity and the Laws of Newton" by Erik P. Verlinde

Sheldon Gao
Unit for History and Philosophy of Science
& Centre for Time, SOPHI, University of Sydney
Email: sgao7319@uni.sydney.edu.au

We show that Verlinde's idea of gravity as an entropic force is untenable. The main reason is that Verlinde made a mistake in identifying the causal relationship between physical variables when applying the first law of thermodynamics to the interacting process between a holographic screen and a particle. Moreover, it is shown that Verlinde's argument also has some other flaws.

Mathematical clarity has in itself no virtue… a complete physical explanation should absolutely precede the mathematical formulation.

----Niels Bohr[1]

1. Introduction

It is still a controversial issue whether gravity is fundamental or emergent. The solution of this problem may have important implications for a complete theory of quantum gravity. One remarkable indication for the nature of gravity comes from the deep study of black hole thermodynamics [2], which implies that there may exist general connections between gravity and thermodynamics. Inspired by these theoretical developments, Jacobson argued that the Einstein equation can be derived from the proportionality of entropy and horizon area together with the first law of thermodynamics [3]. Padmanabhan further showed that the equipartition argument can also provide a thermodynamic interpretation of gravity [4]. These results suggest that gravity may be explained as an emergent phenomenon and has an entropic origin (see, e.g. [5] for a review). Recently Verlinde proposed a new argument for emergent gravity [6], mainly based on the holographic principle. He argued and explicitly claimed that gravity is an entropic force caused by a change in the amount of information associated with the positions of bodies of matter[2]. This idea is interesting and, if right, may have important implications for the origin of gravity and its unification with the quantum. In this comment, we will critically examine Verlinde's argument, focusing more on the physical explanation.

---

1 Quoted by Heisenberg in Ref. [1].

2 Note that Ref. [4] also briefly discussed this idea, though it did not explicitly refer to entropic force. It said that the entropy gradient (due to the gradient in the microscopic degrees of freedom) presents over a region to give rise to a force.

2. The Achilles' heel of Verlinde's argument

In order to see whether Verlinde's idea of gravity as an entropic force is tenable or not, we need to not only simply check his mathematical formulae but also analyze their physical explanations, in particular, the causal relationship involved in the physical process. Concretely speaking, we need to analyze the causal relationship between the variables in the thermodynamics formula $F\Delta x = T\Delta S$ (i.e. the first law of thermodynamics) for the interacting process between a holographic screen and a particle he discussed. Even if Verlinde's mathematical derivation is wholly right, we still need to determine whether gravity results in the change in entropy or the change in entropy results in gravity.

We first see how Verlinde implicitly considers the causal relationship between the variables in the formula $F\Delta x = T\Delta S$. He studied a small piece of a holographic screen. A particle of mass *m* approaches it from the side at which space has already emerged. According to Verlinde, before the particle merges with the microscopic degrees of freedom on the screen, it already influences the amount of information that is stored on the screen. In other words, the change in entropy of the screen is due to the displacement of the particle (see [6], p.3 & p.7), i.e. the causal chain is $\Delta x \to \Delta S$, in which the change of position, Δx, is cause and the change in entropy, ΔS, is effect. Furthermore, Verlinde argued that the change in entropy with the change of position will lead to an entropic force, and it is just gravity (see, e.g. [6], p.12). Thus the whole causal chain is $\Delta x \to \Delta S \to F$ according to Verlinde. However, the causal chain should be $T\Delta S \to F\Delta x$ in order to explain gravity as an entropic force. Therefore, it seems that Verlinde already contradicts himself.

In the following, we will further show that neither Verlinde's causal chain nor the causal chain for entropic force is right for explaining the above interacting process between a holographic screen and a particle. First, we provide a proof by contradiction to show that Verlinde's causal chain is wrong. Assume the causal chain is $\Delta x \to \Delta S \to F$, as Verlinde implicitly argued. Then when $\Delta x = 0$, we have $\Delta S = 0$ and $F = 0$. This means that there will be no interaction between two masses being at rest relative to each other. But this result obviously contradicts experience, as there is a gravitational interaction between them. In fact, we can reach the same conclusion from another contradiction. The formula $F\Delta x = T\Delta S$ can only have two possible causal chains: one is $F\Delta x \to T\Delta S$, and the other is $T\Delta S \to F\Delta x$. But Verlinde's causal chain $\Delta x \to \Delta S \to F$ contradicts both, which also indicates that it is impossible and wrong.

Next, we will show that the causal chain for entropic force is not right either. The main reason is that, as rightly admitted by Verlinde, the change in entropy of the screen is due to the displacement of the particle (this means that the first part of Verlinde's causal chain, namely $\Delta x \to \Delta S$, is right). If the particle changes its position, then the entropy of the screen will change. But if the entropy of the screen changes, the particle needs not to change its position, as there may exist other causes that result in the change in entropy, e.g. other particles approaching the screen. Moreover, a particle in inertial motion can spontaneously change its position, and no external cause is needed. Therefore, the change in entropy is not the cause but the effect of the displacement of particle. If this process satisfies the first law of thermodynamics, then the causal chain can not be $T\Delta S \to F\Delta x$ but be $F\Delta x \to T\Delta S$. For the latter $F\Delta x$ is the work done on the screen by the particle and *F* is an external force exerted by the particle. For the former $F\Delta x$

is the work done by the screen and F is the so-called entropic force[3], defined as entropic gradient.

Although the above argument seems reasonable, one question still needs to be answered before we can reach a definite conclusion, namely why the force F, derived from the formula $F\Delta x = T\Delta S$, is just gravity for the interacting process between a holographic screen and a particle. In fact, this result can be readily understood once we know that the increase in entropy of the screen is an effect and the causal chain is $F\Delta x \to T\Delta S$. Why does the screen increase its entropy? And where does the heat or energy come from? Obviously it must come from the work done by the particle via some sort of interaction (in the emergent spacetime), while this interaction is just gravity. In short, it is just the work done by gravity that results in the increase in entropy. Since $F = ma = G\frac{Mm}{R^2}$ and $T = \frac{1}{2\pi k_B}\frac{\hbar a}{c} = \frac{1}{2\pi k_B}\frac{\hbar GM}{cR^2}$, there must exist the following relation $\Delta S = 2\pi k_B \frac{mc}{\hbar}\Delta x$ in accordance with the formula $F\Delta x = T\Delta S$ [4]. Then it is not surprising that when assuming $\Delta S = 2\pi k_B \frac{mc}{\hbar}\Delta x$ as Verlinde did, Newton's second law and law of gravity naturally follow. In short, why Verlinde's entropic force is gravity is because it is just gravity that results in the change in entropy. The first law of thermodynamics is the same, but the direction of causal chain is opposite.

To sum up, even though Verlinde's mathematical formulae are all right, his conclusion that gravity is an entropic force cannot be right[5]. The main reason is that he made a mistake in identifying the causal relationship between the variables in the formula $F\Delta x = T\Delta S$ when analyzing the physical process he discussed[6].

3. Other flaws of Verlinde's argument

Besides the above deadly flaw, there are also some other flaws in Verlinde's argument, part of which will be discussed in the following.

---

[3] It is worth noting that although an entropic force is independent of the details of the microscopic dynamics, it existence depends on the existence of interaction between the microscopic components of the studied system and environment. For example, there is no entropic force for an isolated polymer in vacuum. In fact, the laws of thermodynamics all depend on the existence of the interaction, without which no thermal equilibrium exists. In most familiar situations, the interaction has an eletromagnetic origin. For instance, the elastic force of a stretched rubber cord is due to the attraction between the rubber molecules, which is further due to the electromagnetic attraction between the electrons of one molecule and nuclei of the other. Therefore, it is not right to say that entropic force has a purely entropic origin.

[4] It should be stressed that this argument does not depends on the distance between the screen and the particle, and the particle needs not to be near the screen. The general formula will be $\Delta S = 2\pi k_B \frac{mcR^2}{\hbar R'^2}\Delta x$, where R is the radius of the spherical screen, and R' is the distance between the particle and the center of the screen.

[5] Interestingly, Ref. [7] showed that if gravity is an entropic force as Verlinde argued, then Coulomb force should be also an entropic force. But it is well known that Coulomb force is a fundamental interaction transferred by photons. As we think, this apparent contradiction has also refuted Verlinde's argument and shown that his idea of gravity as an entropic force is wrong. In addition, this result can also be taken as a support for our conclusion that it is gravity (and Coulomb force) that results in the change in entropy, not the contrary.

[6] Note that our analysis also applies to the similar arguments proposed by other authors (e.g. [3,4]). Like Verlinde, Jacobson did not explicitly state the causal relationship between energy flux and entropy change either. He seemingly assumed the right causal chain, i.e., energy flux → entropy change (he said "the entropy is proportional to the horizon area" and "the area increase of a portion of the horizon will be proportional to the energy flux across it"). However, he also reached a wrong conclusion that the Einstein equation is an equation of state. Different from Verlinde, Jacobson stressed the stringent conditions on which his derivation of the Einstein equation relies.

First, the "derivation" of *F=ma* only means that the entropic force emergent in the specific thermodynamics process studied by Verlinde satisfies the second law of Newton. It does not demonstrate that all forces satisfy this law, and thus it is not proper to conclude that the second law of Newton has been derived based on this result. In fact, Newton's second law can be taken as a quantitative definition of inertial mass. In this sense, it can not be derived but be stipulated. Secondly, in Verlinde's "derivation" of Newton's law of gravity, there is also one obvious flaw. It is that Einstein's relativistic mass-energy relation is used to derive Newton's law of gravity, which is non-relativistic. In fact, Verlinde also used the Unruh temperature relation, which is a result of relativistic quantum field theory, to derive Newton's second law that is non-relativistic[7].

Thirdly, there may exist a serious mistake in Verlinde's argument for the connection between entropy and Newton's potential. Concretely speaking, Eqs. (3.15) and (3.16), $\frac{\Delta S}{n} = k_B \frac{a\Delta x}{2c^2}$ and $\frac{\Delta S}{n} = -k_B \frac{\Delta \Phi}{2c^2}$, probably do not hold true. According to Eq. (3.6), the energy increase of the screen during the particle approaching process should be $\Delta E = T\Delta S$, and thus the increased number of used bits on the screen, denoted by ΔN, should satisfy $T\Delta S = \frac{1}{2}\Delta N k_B T$ when assuming that each bit carries an energy $\frac{1}{2}k_B T$. Therefore we can only get $\frac{\Delta S}{\Delta N} = \frac{1}{2}k_B$, not $\frac{\Delta S}{\Delta N} = k_B \frac{a\Delta x}{2c^2} = -k_B \frac{\Delta \Phi}{2c^2}$, namely Eqs. (3.15) and (3.16). On the other hand, when the particle have merged with the screen, the change in entropy due only to the merger will not satisfy Eq. (3.6), and it should be $\Delta S = \Delta E / T = mc^2 / T$. Thus we also get $\frac{\Delta S}{\Delta N} = \frac{1}{2}k_B$. It seems that Verlinde mixed these two situations and thus obtained the wrong formulae Eqs. (3.15) and (3.16)[8]. Therefore, Verlinde's argument for the claimed connection between entropy and Newton's potential is not convincing[9]. In fact, since T and F are both proportional to the acceleration

[7] Note that these inconsistencies have been already pointed out in Ref. [8].

[8] It is worth noting that Ref. [9] seems to further misuse Eq. (3.16). In the paper the entropy of the screen is taken as a fixed quantity, independent of the radius of the screen, and the relationship between *N* and *A* is also different from that considered by Verlinde. Therefore, the formula Eq. (5) there is in fact a new assumption beyond Verlinde's derivation, which validness needs to be further studied. Based on Eq. (5) the authors derived a UV/IR relation and claimed that this relation is just that obtained by Cohen et al [10]. However, the latter is only an inequality, while the former is a strict equality. Moreover, the latter applies to the inside space enclosed by the screen, while the form applies to the screen itself. There is still a screen-bulk redshift factor between them. In fact, one can argue that the UV/IR relation is only necessary for the effective field theory describing the inside space in order to eliminate the redundant degrees of freedom due to the restriction of the holographic principle, while for the holographic screen no such restriction exists and thus no UV/IR relation is required. These problems indicate that Eq. (5) is at least questionable. In fact, as Verlinde has shown, the new relationship between *N* and *A* in terms of UV cutoff already contradicts Newton's law of gravity when assuming the equipartition of energy. In addition, the dark energy derivation in the paper is also debatable. First, since the cosmic horizon already includes all energy and degrees of freedom inside it, including those possessed by the particle, there will be no entropy change for the horizon when the particle approaches it. Thus there should not exist an entropic force between the test particle and the horizon, even according to Verlinde. Next, even if there is an entropic force, it should be the sum of the forces exerted by all parts of the horizon. As a result, it will be also zero according to Verlinde as the forces satisfy Newton's law of gravity. Besides, in the suggested derivation of dark energy Eq. (21) is only a mathematical trick, which lacks physical meaning.

[9] Another indication is that one can deduce from Eq. (3.16) that the entropy of a spherical holographic screen equals to the Bekenstein bound [11]. But for some weakly gravitating matter systems in asymptotically flat space, their entropy can be smaller than this bound. This inconsistency also indicates that Eq. (3.16) is probably wrong. Certainly, one can still assume a different relationship between entropy and Newton's potential, and then examine its consistency with existing theories. However, even if there is such a relationship, gravity cannot be an entropic

$a = \nabla\Phi$, the change in entropy ΔS will not relate to the acceleration a and Newton's potential Φ according to the formula $T\Delta S = F\Delta x$.

Lastly, even if Verlinde's derivations of Newton's second law and law of gravity are all right, he only derived the equivalence of inertial and passive gravitational mass (see also [12]). This is only one part of weak equivalence principle (WEP), the other part of which is the equivalence of inertial and active gravitational mass. Furthermore, even WEP is not enough to derive the Einstein equations. In order to derive Einstein's law of gravity (i.e. general relativity), a more powerful equivalence principle, the Einstein equivalence principle (EEP), is needed[10]. Besides WEP, it also includes local Lorentz invariance and local position invariance [13]. In fact, Verlinde already used the second part of EEP in his "derivation" of the Einstein equations, as he assumed that the microscopic theory knows about Lorentz symmetry, or even has the Poincare group as a global symmetry (see [6], p.16).

4. Further discussions

According to general relativity, gravity is a "curved spacetime" phenomenon. If spacetime is emergent, then gravity must be also emergent. But even so, they should have corresponding microscopic elements in the pre-spacetime theory. No doubt there is still a long road to walk in order to reach the final theory, to which the holographic principle may be one helpful guide. On the other hand, gravity is probably fundamental in the emergent spacetime, and we may finally find its origin after we have understood the nature of spacetime. In this regard, it is worth noting that the existence of a minimum size of spacetime, which is generally considered as an indispensable element in a complete theory of quantum gravity (see, e.g. [14] for a review), may imply the fundamental existence of gravity as a geometric property of spacetime [15]. The argument can be briefly introduced as follows.

The Heisenberg uncertainty principle in quantum theory requires $\Delta x \geq \frac{\hbar}{2\Delta p}$. As a result, the uncertainty of the position of a particle Δx can be arbitrarily small by increasing its momentum uncertainty Δp. This holds true in continuous spacetime. However, the discreteness of spacetime will demand that the uncertainty of the position of a particle should have a minimum value L_U, namely Δx should satisfy the limiting relation $\Delta x \geq L_U$. In order to satisfy this relation, the Heisenberg uncertainty relation in discrete spacetime should at least contain another term proportional to the momentum uncertainty, namely it should be $\Delta x \geq \frac{\hbar}{2\Delta p} + \frac{L_U{}^2 \Delta p}{2\hbar}$ in the first order of Δp. This generalized uncertainty principle can satisfy the limiting relation

force, as the holographic screens describing the same system do not co-exist, and the entropic gradient, unlike the potential gradient, does not really exist in space.

[10] It is possible to argue convincingly that if EEP is valid, then gravitation must be a "curved spacetime" phenomenon, in other words, the effects of gravity must be equivalent to the effects of living in a curved spacetime [13].

imposed by the discreteness of spacetime[11]. It can be seen that the new term, which is required by the discreteness of spacetime, means that the momentum and energy uncertainty of a particle will introduce an inherent spacetime uncertainty for the position of the particle. This further implies that the energy-momentum of a particle will change the geometry of spacetime it moves in (e.g. in each momentum-energy branch of a quantum superposition). Concretely speaking, the energy $E \approx pc$ contained in a region with size L will change the proper size of the region, and the change is $\Delta L \approx \frac{L_U T_U E}{2\hbar}$, where $T_U = L_U / c$. This means that a flat spacetime will be curved by the energy-momentum contained in it.

This new analysis based on quantum principle and the discreteness of spacetime may provide a deeper basis for the Einstein equivalence principle. It implies that gravity is essentially a geometric property of spacetime, which is determined by the energy-momentum contained in that spacetime, not only at the classical level but also at the quantum level. Moreover, the Einstein gravitational constant can also be determined in terms of the discreteness of spacetime. The result is $\kappa = 2\pi \frac{L_U T_U}{\hbar}$. Note that this formula itself also suggests that the discreteness of spacetime may result in the existence of gravity. In continuous spacetime where $T_U = 0$ and $L_U = 0$, we have $\kappa = 0$, and thus gravity does not exist.

The above argument for the fundamental existence of gravity not only holds true for the microscopic particles, but also may probably apply to the bits living on the holographic screen as well. This provides a further support for the conclusion that gravity is not an emergent force, and especially gravity is not an entropic force.

References

[1] S. Rozental (eds). Niels Bohr: His Life and work as seen by his friends and colleagues. Amsterdam: North-Holland Publishing Company, (1967). p.98.

[2] J. M. Bardeen, B. Carter and S. W. Hawking, Commun. Math. Phys. 31, 161 (1973). J. D. Bekenstein, Phys. Rev. D 7, 949 (1973). J. D. Bekenstein, Phys. Rev. D 7, 2333 (1973). S. W. Hawking, Commun. Math. Phys. 43, 199 (1975) [Erratum-ibid. 46, 206 (1976)].

[3] T. Jacobson, Phys. Rev. Lett. 75, 1260 (1995).

[4] T. Padmanabhan, Class. Quant. Grav. 21, 4485 (2004). T. Padmanabhan, Equipartition of energy in the horizon degrees of freedom and the emergence of gravity, arXiv:0912.3165 [gr-qc]

[5] T. Padmanabhan, Thermodynamical Aspects of Gravity: New Insights, arXiv:0911.5004 [gr-qc].

---

[11] Similarly, we will also have $\Delta t \geq \frac{\hbar}{2\Delta E} + \frac{T_U^{\,2} \Delta E}{2\hbar}$ for time uncertainty. The argument here may be regarded as a reverse application of the generalized uncertainty principle. But it should be stressed that the existing arguments for the generalized uncertainty principle are based on the analysis of measurement process, which conclusion is that it is impossible to *measure* positions to better precision than a fundamental limit. On the other hand, in the above argument, the uncertainty of position is objective and real, and the discreteness of spacetime means that the objective uncertainty of the position of a particle has a minimum value, which is independent of measurement.

[6] E. P. Verlinde, On the Origin of Gravity and the Laws of Newton, arXiv:1001.0785 [hep-th].
[7] T. Wang, The Coulomb Force as an Entropic Force, arXiv:1001.4965 [hep-th].
[8] L. Zhao, Hidden symmetries for thermodynamics and emergence of relativity, arXiv:1002.0488 [hep-th].
[9] M. Li and Y. Wang, Quantum UV/IR Relations and Holographic Dark Energy from Entropic Force, arXiv:1001.4466 [hep-th].
[10] A. Cohen, D. Kaplan, and A. Nelson, Phys. Rev. Lett. 82 (1999) 4971.
[11] Y. Tian and X. N. Wu, Thermodynamics of Black Holes from Equipartition of Energy and Holography, arXiv:1002.1275 [hep-th].
[12] L. Smolin, Newtonian gravity in loop quantum gravity, arXiv:1001.3668 [gr-qc].
[13] C. M. Will, The Confrontation between General Relativity and Experiment. http://relativity.livingreviews.org/Articles/lrr-2006-3/
[14] L. J. Garay. Int. J. Mod. Phys. A 10, 145 (1995).
[15] S. Gao, Why gravity is fundamental, arXiv:1001.3029 [physics.gen-ph].