

# Beyond the speed of light on Finsler spacetimes

Christian Pfeifer\* and Mattias N.R. Wohlfarth†

*II. Institut für Theoretische Physik und Zentrum für Mathematische Physik,  
Universität Hamburg, Luruper Chaussee 149, 22761 Hamburg, Germany*

As a prototypical massive field theory we study the scalar field on the recently introduced Finsler spacetimes. We show that particle excitations exist that propagate faster than the speed of light recognized as the boundary velocity of observers. This effect appears already in Finsler spacetime geometries with very small departures from Lorentzian metric geometry. It switches on for sufficiently large ratios of the particle four-momentum components and mass. For specific Finsler spacetime models, we deduce the modified dispersion relation of Coleman–Glashow, and one known from quantum gravity phenomenology. If similar dispersion relations resulted for fermions on Finsler spacetimes, these generalized geometries could explain the very recent observations of the OPERA collaboration who found muon neutrinos propagating faster than light at very high energies, while being consistent with supernova observations.

Very recently the OPERA collaboration announced the stunning results [1] of an extremely carefully analyzed experiment that measures the time of flight of muon neutrinos along a baseline of around 730 km between CERN and the Gran Sasso laboratory, and compares it to the time expected using the speed of light. The measurement shows that the neutrinos are faster by 60.7 ns with high significance. Is this possible in fundamental theory?

In classical field theory particles like muon neutrinos are described by partial differential equations. The requirement that the Cauchy problem be well-posed implies that the leading second order differential operator in these equations must be hyperbolic. If the geometric structure of the spacetime background is given solely by a Lorentzian metric, as is the case in the Standard Model, hyperbolicity of the free field equations can only be related to the Lorentzian cone structure determined by this metric [2]. It follows that the support of the fields must propagate in timelike directions [3] which in turn holds for the particle excitations of the field. In other words, free massive particles in Lorentzian geometry are always slower than light.

Taking the OPERA result seriously hence tells us that the structure of spacetime cannot be described simply by Lorentzian geometry.

Various approaches to study deviations from Lorentzian geometry are studied in the literature, see e.g. the references in [2], often pointing towards modified dispersion relations of Finsler geometric origin [4–8]. Finsler geometry realizes the weak equivalence principle by providing the most general geometric clock postulate for which proper time  $T[x]$  depends locally on the position and four-velocity of an observer, or massive particle, moving along a worldline  $x(\tau)$  through the spacetime manifold  $M$ ,

$$T[x] = \int d\tau F(x(\tau), \dot{x}(\tau)). \quad (1)$$

The metric limit is given by the special tangent bundle function  $F(x, y) = |g_{ab}y^ay^b|^{1/2}$ . However, not all Finsler

functions  $F$  are suitable for physics. To see which are, we have developed the geometry of Finsler spacetimes in [9]. These are tailored to provide the notions of null geometry and causal structure which are not available in standard mathematical settings of Finsler geometry, but are required physically in order to describe the propagation of light and to define observers.

In this letter we will exploit the controlled geometrical framework of Finsler spacetimes to analyze the propagation behaviour of the massive scalar field. Already on Finsler backgrounds very mildly departing from Lorentzian geometry, this proves highly interesting. We will demonstrate the existence of particle excitations that propagate faster than the speed of light recognized as the boundary velocity that observers cannot reach. These particle modes are characterized by large ratios of their four-momentum components and mass. This effect has a fully Finsler geometrical origin, and is the direct consequence of a more complicated dispersion relation than appears on metric spacetimes. Since the massive scalar field is the prototypical massive field theory, a similar mechanism should be induced by Finsler spacetime geometry also on massive fermion fields.

In this way Finsler spacetime geometry could explain the measured puzzling neutrino results qualitatively, and answer why they are seen for the first time for very light particles and at collider energies around 17 GeV [1] that are much higher than those in supernova or reactor neutrinos in the MeV range [10, 11].

We will now make these statements precise. The theory presented here, lives on the tangent bundle  $TM$  of the spacetime manifold  $M$  which is the union over all tangent spaces. Given coordinates  $(x^a)$  on some open neighbourhood  $U \subset M$ , one can write any vector  $Y$  in  $T_xM$  as  $Y = y^a \partial_a|_x$ ; regarding  $Y$  as a point in  $TM$ , one associates the induced coordinates  $(x^a, y^b)$ . The associated coordinate basis of  $TTM$  is denoted by  $\{\partial_a, \bar{\partial}_a = \partial/\partial y^a\}$ . We first review, and briefly comment on, our definition of Finsler spacetimes, see [9] for a detailed description, before we enter the discussion of the scalar field.

*Definition.* A Finsler spacetime  $(M, L, F)$  is a four-dimensional smooth manifold  $M$  with a continuous function  $L : TM \rightarrow \mathbb{R}$  on the tangent bundle which has the following properties: (a)  $L$  is smooth on  $TM \setminus \{0\}$ ; (b)  $L$  is positively homogeneous of real degree  $r \geq 2$  as  $L(x, \lambda y) = \lambda^r L(x, y)$  for all  $\lambda > 0$ ; (c)  $L$  is reversible in the sense  $|L(x, -y)| = |L(x, y)|$ ; (d) the Hessian  $g_{ab}^L$  of  $L$  with respect to the fibre coordinates is non-degenerate on  $TM \setminus A$  where  $A$  has measure zero and does not contain the null structure  $\{L(x, y) = 0\} \subset TM$ ,

$$g_{ab}^L(x, y) = \frac{1}{2} \bar{\partial}_a \bar{\partial}_b L; \quad (2)$$

(e) the unit timelike condition holds: for all  $x \in M$ ,

$$\Omega_x = \left\{ y \in T_x M \mid |L(x, y)| = 1, g_{ab}^L(x, y) \text{ has signature } (\epsilon, -\epsilon, -\epsilon, -\epsilon), \epsilon = \frac{|L(x, y)|}{L(x, y)} \right\} \quad (3)$$

contains a non-empty closed connected component  $S_x$ . The Finsler function associated to  $L$  is defined as  $F(x, y) = |L(x, y)|^{1/r}$ , the Finsler metric as

$$g_{ab}(x, y) = \frac{1}{2} \bar{\partial}_a \bar{\partial}_b F^2. \quad (4)$$

Observe that this definition is a direct generalization of Lorentzian metric spacetimes  $(M, \tilde{g})$  with metric  $\tilde{g}$  of signature  $(-, +, +, +)$ . To see this, set  $L(x, y) = \tilde{g}_{ab}(x) y^a y^b$ . This is homogeneous of order  $r = 2$ ; properties (a)–(c) are obvious; the metric  $g_{ab}^L = \tilde{g}_{ab}$  is invertible on  $TM$ , so (d) holds; the unit timelike condition (e) is satisfied by the set  $S_x$  of unit  $\tilde{g}$ -timelike vectors at  $x$ .

Our definition of Finsler spacetimes varies in several important aspects from the definitions of Finsler spaces in the literature, cf. the references in [9]. The central new ingredient is the function  $L$  that acts as the fundamental geometric background structure. Properties (a)–(d) ensure that the geometry of the null structure  $\{L = 0\} \equiv \{F = 0\}$  is fully under control, and that null Finsler geodesics are well-defined. These are essential to describe the effective motion of massless particles and light. The unit timelike condition (e) implies the existence of open convex cones of timelike vectors at all points  $x$ , constructed from the shell  $S_x$  of unit timelike vectors; these have a null boundary and are required to model causality and the four-velocities of physical observers. The Finsler function  $F$  that appears in the clock postulate (1) to measure time is a derived quantity.

Action principles for matter fields on Finsler spacetime can be obtained from the corresponding action integral over the spacetime manifold. The procedure was tested in [9] for electrodynamics, where we could prove that light indeed propagates along Finsler null geodesics.

To obtain the Finsler spacetime action of the massive scalar field, we start with the standard Lagrangian on

metric spacetime  $(M, \tilde{g})$ ,

$$\mathcal{L}[\tilde{g}, \tilde{\phi}, \partial\tilde{\phi}] = -\frac{1}{2} \tilde{g}^{ab}(x) \partial_a \tilde{\phi}(x) \partial_b \tilde{\phi}(x) - \frac{1}{2} m^2 \tilde{\phi}(x)^2. \quad (5)$$

Note that  $\mathcal{L}[\dots]$  can be seen as a prescription to form a scalar quantity from various tensorial objects. The same prescription can be used to generate a Lagrangian on  $TM$ , after promoting  $\tilde{\phi}(x)$  to a tangent bundle field  $\phi(x, y)$  of zero homogeneity in  $y$ , exchanging the partial derivatives on  $M$  for partial derivatives on  $TM$ , and replacing the metric  $\tilde{g}(x)$  by the Sasaki-type metric

$$G(x, y) = -g_{ab}(x, y) dx^a \otimes dx^b - \frac{g_{ab}(x, y)}{|L(x, y)|^{2/r}} \delta y^a \otimes \delta y^b. \quad (6)$$

Here  $\delta y^a = dy^a + N^a_b dx^b$ ; the coefficients of the Cartan non-linear connection are  $N^a_b = \frac{1}{4} \bar{\partial}_b [g^{L ap} (y^q \partial_m \bar{\partial}_p L - \partial_p L)]$ ; in the metric limit these essentially reduce to the Christoffel symbols  $N^a_b \rightarrow \Gamma[\tilde{g}]^a_{bc} y^c$ .

Action integrals on Finsler spacetimes can be formulated on the seven-dimensional subbundle  $\Sigma \subset TM$  defined by  $|L(x, y)| = 1$  and non-degenerate  $g^L(x, y)$ . Convenient coordinates  $(\hat{x}^a, u^\alpha)$  on  $\Sigma$  are constructed in [9]; a volume form is induced by the pullback  $G^*$  of  $G$ ,

$$G^* = -g_{ab|_\Sigma} d\hat{x}^a \otimes d\hat{x}^b - (g_{ab} \partial_\alpha y^a \partial_\beta y^b)|_\Sigma \delta u^\alpha \otimes \delta u^\beta, \quad (7)$$

where  $\delta u^\alpha = du^\alpha + \tilde{N}^\alpha_a d\hat{x}^a$  and  $\tilde{N}^\alpha_a = N^p_a \bar{\partial}_p u^\alpha - \partial_a u^\alpha$ . With these results the action for the massive scalar field on Finsler spacetimes becomes

$$\begin{aligned} S[\phi] &= \int_\Sigma d^4 \hat{x} d^3 u \sqrt{G^*} \mathcal{L}[G, \phi, \partial\phi]|_\Sigma \\ &= -\frac{1}{2} \int_\Sigma d^4 \hat{x} d^3 u \sqrt{G^*} \left[ G^{AB} \partial_A \phi \partial_B \phi + m^2 \phi^2 \right]|_\Sigma, \end{aligned} \quad (8)$$

where the capital indices  $A, B$  label the eight induced coordinates  $(x^a, y^b)$  on  $TM$ . We will see from the corresponding equations of motion below, that this field theory is designed to reduce to standard massive scalar field theory on  $M$  in the limit of metric geometry, i.e., for  $L(x, y) = \tilde{g}_{ab}(x) y^a y^b$  and  $\phi(x, y) = \tilde{\phi}(x)$ .

The equations of motion for  $\phi$  are obtained by variation. We first expand the action in the horizontal/vertical basis  $\{\delta_a = \partial_a - N^b_a \bar{\partial}_b, \bar{\partial}_a\}$  of  $TTM$ , where expression (6) can be applied,

$$S[\phi] = \frac{1}{2} \int_\Sigma \sqrt{G^*} \left[ g^{ab} \delta_a \phi \delta_b \phi + g^{ab} \bar{\partial}_a \phi \bar{\partial}_b \phi - m^2 \phi^2 \right]|_\Sigma. \quad (9)$$

To find  $\delta S[\phi]$  we then require the following formulae for integration by parts which can be proven using the coordinate transformation rules detailed in [9]. For non-homogeneous functions  $A^a(x, y)$  we have the identities

$$0 = \int_\Sigma \sqrt{G^*} \left[ \delta_a A^a + (\Gamma^{\delta p}_{pa} + S^p_{pa}) A^a \right]|_\Sigma, \quad (10)$$

$$0 = \int_\Sigma \sqrt{G^*} \left[ \bar{\partial}_a A^a + (g^{pq} \bar{\partial}_a g_{pq} - (n+3) y^p g_{pa}) A^a \right]|_\Sigma,$$

with the shorthand notation  $\Gamma^{\delta a}_{bc} = \frac{1}{2}g^{ap}(\delta_b g_{pc} + \delta_c g_{pb} - \delta_p g_{bc})$  and  $S^a_{bc} = \Gamma^{\delta a}_{bc} - \bar{\partial}_b N^a_c$ . With these technical preparations we find the equations of motion

$$\left[ -g^{ab} \left( \delta_a \delta_b - \Gamma^{\delta p}_{ab} \delta_p + \bar{\partial}_a \bar{\partial}_b + S^p_{pa} \delta_b \right) \phi - m^2 \phi \right]_{|\Sigma} = 0. \quad (11)$$

In the limit of metric geometry this reduces to the standard Klein–Gordon equation  $(\tilde{g}^{ab} \nabla[\tilde{g}]_a \partial_b - m^2) \tilde{\phi} = 0$ , where  $\nabla[\tilde{g}]$  denotes the Levi–Civita connection.

We will now study the propagation behaviour of the field  $\phi$  using techniques from the analysis of partial differential equations [3]. For this purpose we first deduce the principal symbol from the second order derivatives. In the coordinates  $(x^{\hat{M}}) = (\hat{x}^a, u^\alpha)$  of  $\Sigma$  with associated coordinate basis  $\{\partial_{\hat{M}}\} = \{\hat{\partial}_a, \partial_\alpha\}$  of  $T\Sigma$  one finds

$$\begin{aligned} -g^{ab} |_{\Sigma} \left[ \hat{\partial}_a \hat{\partial}_b \phi - 2\tilde{N}^\alpha_a \partial_\alpha \hat{\partial}_b \phi \right. \\ \left. + (\tilde{N}^\alpha_a \tilde{N}^\beta_b + \bar{\partial}_a u^\alpha \bar{\partial}_b u^\beta) \partial_\alpha \partial_\beta \phi \right]_{|\Sigma}. \end{aligned} \quad (12)$$

It can be shown that these terms can be rewritten with the help of the pullback metric  $G^*$  as

$$G^*{}^{\hat{M}\hat{N}} \partial_{\hat{M}} \partial_{\hat{N}} \phi. \quad (13)$$

Below we will specify the underlying Finsler spacetime geometry to be a very small departure from Lorentzian geometry with two almost identical lightcones. We shall confirm for this case that the pullback metric  $G^*$  on  $\Sigma$  is of Lorentzian signature  $(-, +, +, +, +, +)$ . This implies that the field equations of the massive scalar field  $\phi$  are hyperbolic, and so have a well-posed Cauchy problem. Allowed Cauchy surfaces of initial data are conormal to the momenta  $P \in T^*\Sigma$  inside the lightcone,  $G^{*-1}(P, P) < 0$ . Moreover, it follows that the support of the field  $\phi$  on  $\Sigma$  propagates into timelike directions inside the lightcones of  $G^*$  in  $T\Sigma$ . These are obtained from the momenta by raising the indices with the map

$$X = \frac{1}{m} G^{*-1}(P, \cdot). \quad (14)$$

On the level of classical field theory it is clear that only those excitations of the field  $\phi$  can be interpreted as particles which move along worldlines tangent to the spacetime manifold  $M$ . These tangents to  $M$  all sit in  $TM$  which in turn can be identified with the so-called horizontal vector fields over the bundle  $\Sigma$ . Any such horizontal vector field can be expressed in the local coordinate basis as  $X = X^a \hat{\partial}_a = X^a (\hat{\partial}_a - \tilde{N}^\alpha_a \partial_\alpha)$ . Using the map between momentum and velocity above and the fact that  $G^*$  preserves the horizontal and vertical structure according to (7), shows that also particle momenta must be horizontal, i.e.,  $P = P_a d\hat{x}^a$ .

Now that we understand the propagation of the field  $\phi$  and have defined its particle excitations, we are in the position to study the corresponding dispersion relation. To

do so we specify a Finsler spacetime very mildly departing from flat Lorentzian metric geometry, where standard dispersion relations are well-defined. We consider the simple bimetric background structure

$$L(x, y) = \eta_{ab} y^a y^b (\eta_{cd} + h_{cd}) y^c y^d \quad (15)$$

which has no  $x$ -dependence. The  $\eta_{ab}$  denote the usual Minkowski metric, and  $h_{cd}$  is chosen so that the metric  $\eta_{ab} + h_{ab}$  is Lorentzian and has a timelike cone that contains the cone of  $\eta_{ab}$ . The null structure of  $L$  is the union of the light cones of  $\eta$  and  $\eta + h$ . The function  $L$  produces a Finsler spacetime according to our definition above with homogeneity  $r = 4$ ; for small components  $h_{ab}$ , it is not hard to check that the closed connected component  $S_x$  of unit timelike vectors is given by  $\eta(y, y) = -1$  up to a perturbation. Hence observers in this Finsler spacetime can only move along worldlines with  $\eta$ -timelike tangents; their boundary velocity is given by the lightcone of  $\eta$ . These characteristics are shown in figure 1.

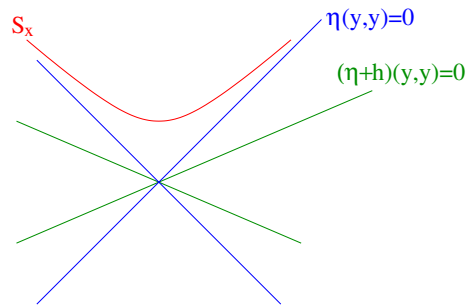


FIG. 1. Null structure and unit timelike shell in  $T_x M$  at any point  $x$  of a simple bimetric Finsler spacetime.

The dispersion relation is now derived from the field equation of  $\phi$ , see (11). This equation simplifies on the specific Finsler spacetime defined by  $L$ , where the geometry implies both  $N^a_b = 0$  and  $\Gamma^{\delta a}_{bc} = 0$ , to

$$-g^{ab} |_{\Sigma} \left( \hat{\delta}_a \hat{\delta}_b + \bar{\partial}_a u^\alpha \bar{\partial}_b u^\beta \partial_\alpha \partial_\beta + \bar{\partial}_a \bar{\partial}_b u^\alpha \partial_\alpha \right) \phi = m^2 \phi. \quad (16)$$

We perform a Fourier decomposition into modes of momentum  $P = P_a d\hat{x}^a + P_\alpha \delta u^\alpha = (P_a + \tilde{N}^\alpha_a P_\alpha) d\hat{x}^a + P_\alpha \delta u^\alpha$ . These have the form  $\exp[i(P_a + \tilde{N}^\alpha_a P_\alpha) \hat{x}^a + i P_\alpha u^\alpha]$ , so that  $\hat{\delta}_a$  and  $\partial_\alpha$  act as multiplication by  $i P_a$  and  $i P_\alpha$ , respectively. As explained above, particle modes must correspond to Fourier modes with  $P_\alpha = 0$  so that only horizontal momentum remains. Hence we find the following dispersion relation for the particle excitations of the massive scalar field:

$$-g^{ab} P_a P_b = -m^2. \quad (17)$$

This dispersion relation is governed by the Finsler metric  $g$ . We will now interpret this result.

The Finsler metric of the bimetric spacetime geometry  $L$  in equation (15) is calculated from the definition

as  $g_{ab}(x, y) = (\text{sign } \eta(y, y))(\eta_{ab} + h_{ab}/2)$  to first order in the components  $h_{ab}$ . According to the interpretation of fields on Finsler spacetimes proposed in [9], the tangent direction  $y$  in the Finsler metric has to be identified with the four-velocity of the observer studying the scalar field. Since all observers here are  $\eta$ -timelike, we thus find  $-g_{ab}(x, y) = \eta_{ab} + h_{ab}/2$ . The signature of  $-g_{ab}$  is  $(-, +, +, +)$ ; comparing (6) and (7) similarly as in [9] then shows that  $G^*$  has the Lorentzian signature  $(-, +, +, +, +, +, +)$  required for a well-posed propagation of  $\phi$ . Substituting  $g_{ab}$  into the dispersion relation, and using the momentum–velocity map (14) finally yields

$$\eta_{ab}X^aX^b = -1 - \frac{h^{ab}P_aP_b}{2m^2} \quad (18)$$

where  $h^{ab} = \eta^{ap}\eta^{bq}h_{pq}$ , and  $X^a$  and  $P_a$  are the horizontal components of particle velocity and momentum, i.e., those components interpreted as velocity and momentum on the spacetime manifold.

This dispersion relation is derived from a clear theoretical framework; it differs fundamentally from the standard relation  $\eta_{ab}X^aX^b = -1$  on flat Minkowski spacetime. The key feature is that an observer who regards  $\eta$  as the relevant spacetime structure can no longer conclude that the massive particle is always timelike; this depends on the ratio of  $h^{ab}P_aP_b$  and its squared mass. If this ratio is negative which depends on the components of  $h_{ab}$ , and if the  $P_a/m$  are sufficiently large, then the massive particles will be faster than the boundary velocity for observers given by light on  $\{\eta(y, y) = 0\}$ . A closer look at the full background null structure, see figure 1, reveals that there still exists even faster light on  $\{(\eta + h)(y, y) = 0\}$ . We remark that the massive scalar field in this situation will have particle excitations moving in  $\eta$ -timelike,  $\eta$ -null and  $\eta$ -spacelike directions.

Finsler spacetime models realizing this scenario can be easily obtained. It remains to be seen whether such spacetimes can be obtained as solutions of generalized gravitational field equations [12]. We now discuss two examples. First, consider a diagonal matrix  $h_{ab}$  with entries  $(0, -\kappa^2, -\kappa^2, -\kappa^2)$  for small  $|\kappa|$ . Then  $\eta_{ab} + h_{ab}$  is a Lorentzian metric, and its timelike cone contains the cone of  $\eta_{ab}$  as required. The right hand side of (18) becomes  $-1 + \kappa^2|\vec{p}|^2/(2m^2)$  which may become positive for sufficiently large spatial momenta; then the particle moves faster than light on  $\{\eta(y, y) = 0\}$ . The dispersion relation in momentum space follows from (17) using the expression for the inverse Finsler metric  $-g^{ab} = \eta^{ab} - h^{ab}/2$ :

$$\eta^{ab}P_aP_b = -m^2 - \frac{1}{2}\kappa^2|\vec{p}|^2. \quad (19)$$

This precisely agrees with the modified dispersion relation of Coleman–Glashow [13] which was derived in a Lorentz symmetry violating extension of the Standard Model. Here it is derived as the effect of a generalized spacetime geometry on classical field theory.

As a second example consider setting  $h_{00} = -\kappa^2 \neq 0$  for small  $|\kappa|$  and let all other components  $h_{ab}$  vanish. Again the timelike cone of  $\eta_{ab}$  is contained in the timelike cone of  $\eta_{ab} + h_{ab}$ . The observer regarding  $\eta$  as the sole geometric background structure identifies  $X^0 = -P_0/m = -E/m$  where  $E$  is the particle energy. From (18) one calculates the particles' three-velocity as

$$|\vec{v}| = 1 - \frac{m^2}{2E^2} + \frac{\kappa^2}{4}. \quad (20)$$

This corresponds to one of the modified dispersion relations discussed in [14], derived here from a fundamental theoretical setup for the first time.

*Discussion.* The OPERA observations on neutrinos faster than light have already inspired a number of very different, mostly phenomenological, explanations [15–24]. In contrast, our explanation is of a fundamental, purely geometric, nature. We argued that particle velocities beyond the speed of light require modified principal symbols in the free field equations; these must be given by a generalization of Lorentzian spacetime geometry. The only way around this argument is the effective modification of the standard metric principal symbol for the muon neutrino by other fields, so that the muon neutrino is always interacting [25–28].

Our geometric approach is based on Finsler spacetimes [9]. We proved for the massive scalar field that particle modes with superluminal velocities appear even on simple spacetimes with small departures from metric geometry. The effect switches on abruptly when  $0 < -h^{ab}P_aP_b/(2m^2) \sim \mathcal{O}(1)$ ; so it neither occurs for particles with too small energies nor with too large masses. Our central result is the dispersion relation (18) that covers the Coleman–Glashow relation and other special cases. As an inherently geometric effect this result should carry over to other massive field theories on Finsler spacetimes. Then the OPERA neutrino results could be explained, while being consistent with the absence of superluminality in supernova neutrinos in contrast to [29]. All these features arise from a fundamental spacetime picture in a remarkably simple way.

*Acknowledgments.* CP and MNRW thank Rutger Boels, Claudio Dappiaggi, Andreas Degner, Klaus Fredenhagen and Manuel Hohmann for helpful discussions; the authors gratefully acknowledge full financial support from the German Research Foundation through the Emmy Noether grant WO 1447/1-1.

- 
- \* christian.pfeifer@desy.de  
† mattias.wohlfarth@desy.de
- [1] OPERA, [arXiv:1109.4897 [hep-ex]].
- [2] D. Ratzel, S. Rivera, F. P. Schuller, Phys. Rev. **D83** (2011) 044047.
- [3] S. Benzoni-Gavage and D. Serre, Multi-dim. hyperbolic partial differential equations, Oxford Univ. Press 2007.
- [4] G. Amelino-Camelia, J. R. Ellis, N. E. Mavromatos, D. V. Nanopoulos, Int. J. Mod. Phys. A **12** (1997) 607-624. [hep-th/9605211].
- [5] R. Gambini, J. Pullin, Phys. Rev. D **59** (1999) 124021. [gr-qc/9809038].
- [6] F. Girelli, S. Liberati and L. Sindoni, Phys. Rev. D **75** (2007) 064015 [arXiv:gr-qc/0611024].
- [7] G. W. Gibbons, J. Gomis, C. N. Pope, Phys. Rev. D **76** (2007) 081701. [arXiv:0707.2174 [hep-th]].
- [8] F. P. Schuller, C. Witte and M. N. R. Wohlfarth, Annals Phys. **325** (2010) 1853 [arXiv:0908.1016 [hep-th]].
- [9] C. Pfeifer and M. N. R. Wohlfarth, Phys. Rev. D **84** (2011) 044039 [arXiv:1104.1079 [gr-qc]].
- [10] K. Hirata *et al.* [ KAMIOKANDE-II Collaboration ], Phys. Rev. Lett. **58** (1987) 1490-1493.
- [11] T. Lasserre, H. W. Sobel, Comptes Rendus Physique **6** (2005) 749-757. [nucl-ex/0601013].
- [12] C. Pfeifer and M. N. R. Wohlfarth, in preparation.
- [13] S. R. Coleman and S. L. Glashow, Phys. Rev. D **59** (1999) 116008 [arXiv:hep-ph/9812418].
- [14] G. Amelino-Camelia, G. Gubitosi, N. Loret, F. Mercati, G. Rosati, P. Lipari, [arXiv:1109.5172 [hep-ph]].
- [15] G. Cacciapaglia, A. Deandrea, L. Panizzi, [arXiv:1109.4980 [hep-ph]].
- [16] D. Autiero, P. Migliozzi, A. Russo, [arXiv:1109.5378 [hep-ph]].
- [17] K. Svozil, [arXiv:1109.5411 [gen-ph]].
- [18] F. Tamburini, M. Laveder, [arXiv:1109.5445 [hep-ph]].
- [19] J. Ciborowski, J. Rembielinski, [arXiv:1109.5599 [hep-ex]].
- [20] S. S. Gubser, [arXiv:1109.5687 [hep-th]].
- [21] R. B. Mann, U. Sarkar, [arXiv:1109.5749 [hep-ph]].
- [22] A. Drago, I. Masina, G. Pagliara, R. Tripiccone, [arXiv:1109.5917 [hep-ph]].
- [23] M. Li, T. Wang, [arXiv:1109.5924 [hep-ph]].
- [24] J. Alexandre, J. Ellis and N. E. Mavromatos, arXiv:1109.6296 [hep-ph].
- [25] F. R. Klinkhamer, [arXiv:1109.5671 [hep-ph]].
- [26] G. F. Giudice, S. Sibiryakov, A. Strumia, [arXiv:1109.5682 [hep-ph]].
- [27] G. Dvali, A. Vikman, [arXiv:1109.5685 [hep-ph]].
- [28] Z. Lingli, B. -Q. Ma, [arXiv:1109.6097 [hep-ph]].
- [29] D. Fargion, [arXiv:1109.5368 [astro-ph.HE]].