Torsion and gravitational interaction in Riemann-Cartan space-time

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Abstract. The influence of space-time torsion on gravitational interaction at cosmological and astrophysical scales is discussed within the framework of gauge gravitation theory in Riemann-Cartan space-time. It is shown that the interaction of the vacuum torsion with proper angular momentums of gravitating objects leads to appearance of additional gravitational force which can be manifested at astrophysical scale.

1 Introduction

The gauge gravitation theory in 4-dimensional Riemann-Cartan spacetime U_4 (GTRC) is a necessary generalization of metric gravitation theory in the framework of gauge approach by including the Lorentz group into the gauge group corresponding to gravitational interaction. Gravitational equations of GTRC and their physical consequences depend on the choice of gravitational Lagrangian $\mathcal{L}_{\mathrm{g}}$ as function of gravitational field strengths - the curvature $F^{ik}{}_{\mu\nu}$ and torsion $S^{i}{}_{\mu\nu}$ tensors, and also on the coupling of matter with gravitational field. By using minimal coupling the energy-momentum and spin momentum tensors of gravitating matter play the role of sources of gravitational field. Pioneer works dedicated to GTRC were connected with investigation of Einstein-Cartan theory, gravitational Lagrangian of which is given in the form of scalar curvature of U_4 [1, 2, 3] (see also [4, 5]) [1]. In the frame of Einstein-Cartan theory the torsion tensor is linear algebraic function of spin momentum of gravitating matter and in the case of spinless matter the influence of torsion on dynamics of gravitating system vanishes. In connection with this the opinion that the torsion is created by spin momentum of matter and in the case of spinless matter has to vanish is widely extended in literature. However, such situation indicates an exceptional position of Einstein-Cartan theory. Really by taking into account that the torsion tensor is gravitational field strength corresponding to 4-translations subgroup of gauge group, which according to Noether theorem is connected with energy-momentum tensor, we have to conclude that generally torsion in the frame of GTRC can be created by energy-momentum tensor. Such situation takes place in the frame of GTRC based on gravitational Lagrangians including quadratic in the

[1] An important contribution to research of Einstein-Cartan theory was made by polish physicists (A. Trautman, W. Kopczynski, B. Kuchowicz, J. Tafel) in connection with investigation of the problem of cosmological singularity.

curvature and torsion terms [2]. In the frame of gauge approach the Lagrangian of gauge field usually is given as function quadratic in the gauge field strength; existence of many invariants quadratic in the curvature and torsion tensors is notable feature of GTRC, moreover there is linear in the curvature invariant - scalar curvature. Because the detailed form of gravitational Lagrangian is unknown, we will consider GTRC based on the following sufficiently general expression of $\mathcal{L}_{\rm g}$ used in a number of papers[3]:

$$\begin{aligned}\mathcal{L}_{\rm g} = f_0\, F + F^{\alpha\beta\mu\nu}\,(f_1\, F_{\alpha\beta\mu\nu} + f_2\, F_{\alpha\mu\beta\nu} + f_3\, F_{\mu\nu\alpha\beta}) + F^{\mu\nu}\,(f_4\, F_{\mu\nu} + f_5\, F_{\nu\mu}) \\ + f_6\, F^2 + S^{\alpha\mu\nu}\,(a_1\, S_{\alpha\mu\nu} + a_2\, S_{\nu\mu\alpha}) + a_3\, S^{\alpha}{}_{\mu\alpha} S_{\beta}{}^{\mu\beta}, \end{aligned} \tag{1}$$

where f_i $(i = 1, 2, \ldots, 6)$, a_k $(k = 1, 2, 3)$ are indefinite parameters, $f_0 = \frac{c^4}{16\pi G}$ (G is Newton's gravitational constant)[4]. If one supposes that GTRC corresponds to real Universe, we have to determine values of parameters f_i and a_k in expression (1). Restrictions on indefinite parameters of $\mathcal{L}_{\rm g}$ can be found on request that GTRC allows to solve some principal problems of general relativity theory (GR) and physical consequences of this theory are the most satisfactory. Some such restrictions were found from analysis of isotropic cosmology built in the frame of GTRC based on $\mathcal{L}_{\rm g}$ (1). It should be noted that in the case of spatially homogeneous isotropic matter contribution assuming in the frame of isotropic cosmology the average of spin momentum is equal to zero. The cosmological equations generalizing Friedmann cosmological equations of GR and equations for torsion functions were given in [9] (see also [8]) in general form without using any restrictions on parameters f_i and a_k. The investigation of these equations leads to the following restrictions:

$$2a_1 + a_2 + 3a_3 = 0, \qquad 2f_1 - f_2 = 0, \tag{2}$$

by which the solution of the problem of cosmological singularity and the dark energy problem was obtained [9, 10]. Then equations of isotropic cosmology include three indefinite parameters: parameter $\alpha = \frac{f}{3f_0^2}$ ($f = f_1 + \frac{f_2}{2} + f_3 + f_4 + f_5 + 3f_6 > 0$) with inverse dimension of energy density, parameter $b = a_2 - a_1$ with the same dimension as f_0 and dimensionless parameter $\omega = \frac{f_2 + 4f_3 + f_4 + f_5}{f}$. The solution of indicated cosmological problems together with fulfillment of the correspondence principle with GR leads to the following restrictions, which were defined more exactly by analysis of gravitational equations of GTRC [12, 13]:

$$0 < x = 1 - \frac{b}{f_0} \ll 1, \qquad 0 < \omega \ll 1 \tag{3}$$

and the value of parameter α^{-1} corresponds to some high energy density. There is a number of papers dedicated to isotropic cosmology with other restrictions on indefinite parameters

[2]First this was pointed in the case of homogeneous isotropic models (HIM) built in [6] and in series of papers dedicated to GTRC [7].

[3]The definitions and notations of our previous papers (see e.g. [8]) are used below. With the purpose to make quantitative estimations the light velocity c is conserved in formulas.

[4]In the case of GTRC, which is not invariant with respect to transformations of spatial inversions, a quantity of additional invariants can be built by using Levi-Civita discriminant tensor and added with indefinite parameters to $\mathcal{L}_{\rm g}$ (1). Such theories were studied in a number of papers (see e.g. [15, 16, 17]).

(see e.g. [18, 19, 20]) given in accordance with analysis of particle content of linearized GTRC fulfilled by supposition that physical space-time in the vacuum is Minkowski space-time [7]. However, as it was shown in [9] on the base of analysis of equations of isotropic cosmology the physical space-time in the vacuum in the frame of GTRC has the structure of Riemann-Cartan continuum with de Sitter metric and the strict analysis of particle content has to be connected with consideration of gravitational perturbations above the vacuum space-time of GTRC. It should be noted that the deviation of the structure of the vacuum space-time in the frame of GTRC from Minkowski space-time, which is essential at cosmological scale, can be unimportant by local analysis given in [7] because of smallness of corresponding characteristics of metric and torsion for the vacuum (see below). However, we have to consider corresponding results of [7] as approximative whose range of applicability is limited by weak fields.

The space-time torsion plays the principal role by the change of gravitational interaction by certain conditions. Unlike isotropic cosmology, where space-time torsion is created by spinless matter and spin momentum of gravitating matter is not demonstrated, the interaction of torsion with spinning matter can play principal role in astrophysics (galaxies, galactic clusters).

Isotropic cosmology in the frame of GTRC based on gravitational Lagrangian (1) was investigated in a number of our papers (see e.g. [8, 9, 10, 11, 12, 13, 14] and Refs. herein). In Section 2 some relations of isotropic cosmology are given in connection with consideration of the role of space-time torsion that is used by discussion of the influence of torsion on gravitational interaction at astrophysical scale in Section 3.

2 Gravitational interaction at cosmological scale and vacuum torsion

Any HIM in Riemann-Cartan space-time is described by three functions of time: the scale factor of Robertson-Walker metric $R(t)$ and two torsion functions - scalar function $S_1(t)$ and pseudoscalar function $S_2(t)$. Cosmological equations generalizing Friedmann cosmological equations of GR by using restrictions (2) take the form [10]

$$\frac{k}{R^2} + (H - 2S_1)^2 - S_2^2 = \frac{1}{6f_0 Z}\left[\rho c^2 - 6bS_2^2 + \frac{\alpha}{4}\left(\rho c^2 - 3p - 12bS_2^2\right)^2\right], \tag{4}$$

$$\dot{H} - 2\dot{S}_1 + H(H - 2S_1) = -\frac{1}{12f_0 Z}\left[\rho c^2 + 3p - \frac{\alpha}{2}\left(\rho c^2 - 3p - 12bS_2^2\right)^2\right], \tag{5}$$

where $H = \dot{R}/R$ is the Hubble parameter (a dot denotes the differentiation with respect to $x^0 = ct$), $k = +1, 0, -1$ for closed, flat and open models respectively, ρ is mass density, p is pressure and $Z = 1 + \alpha\,(\rho c^2 - 3p - 12bS_2^2)$. The torsion functions S_1 and S_2 are:

$$S_1 = -\frac{\alpha}{4Z}[\dot{\rho}c^2 - 3\dot{p} + 12f_0\omega H S_2^2 - 12(2b - \omega f_0)S_2\dot{S}_2], \tag{6}$$

$$S_2^2 = \frac{\rho c^2 - 3p}{12b} + \frac{1 - (b/2f_0)(1 + \sqrt{X})}{12b\alpha(1 - \omega/4)}, \tag{7}$$

where

$$X = 1 + \omega(f_0^2/b^2)[1 - (b/f_0) - 2(1 - \omega/4)\alpha(\rho c^2 + 3p)] \geq 0. \tag{8}$$

In accordance with gravitational equations of GTRC functions ρ and p satisfy the equation as in GR:

$$\dot{\rho} + 3H(\rho + p/c^2) = 0. \tag{9}$$

By given equation of state of gravitating matter eqs. (4)-(7) allow to find cosmological solutions of GTRC. Behaviour of cosmological solutions in GTRC differs essentially from that of GR at the beginning of cosmological expansion and at asymptotics, where the torsion plays the important role. This follows directly from expression (7) for torsion function S_2^2. The presence of $\sqrt{X}$ in formula (7) leads to appearance of limiting (i.e. maximum allowable) energy density. Gravitational interaction near limiting energy density has repulsive character and corresponding cosmological solutions describe regular transition from compression to expansion ("Big Bounce"). In the case of HIM with restrictions (3) filled with gravitating matter with equation of state $p = p(\rho)$ the Hubble parameter with its time derivative near a bounce in the first approximation with respect to $\sqrt{X}$ are:

$$\begin{aligned} H_\pm &= \pm\frac{2b^2}{3f_0^2\omega\alpha}\frac{\sqrt{X}[(1/4b)(\rho_m c^2 + p_m) - (k/R^2) - \frac{1-b/(2f_0)}{24f_0\alpha}]^{1/2}}{(3\frac{1}{c^2}\frac{dp_m}{d\rho_m} + 1)(\rho_m c^2 + p_m)}, \\ \dot{H} &= \frac{4b^2}{3f_0^2\omega\alpha}\frac{(1/4b)(\rho_m c^2 + p_m) - (k/R^2) - \frac{1-b/(2f_0)}{24f_0\alpha}}{(3\frac{1}{c^2}\frac{dp_m}{d\rho_m} + 1)(\rho_m c^2 + p_m)}. \end{aligned} \tag{10}$$

H_-- and H_+-solutions describe the stages of compression and expansion correspondingly, and the transition from compression to expansion takes place by reaching limiting energy density determined from equality $X = 0$: $(\rho_{max}c^2) \sim (\omega\alpha)^{-1}$, which in the frame of our classical theory has to be less than the Planckian one. Because energy density near a bounce is close to $(\rho_{max}c^2)$, the constant term $\frac{1-b/(2f_0)}{24f_0\alpha}$ in (10) is not essential.

The principal influence of torsion on gravitational interaction becomes apparent also when energy density is small and its influence on geometrical structure of space-time in the vacuum is essential. Unlike GR (without cosmological constant) where space-time in the vacuum (in the case of flat models with $k = 0$) is Minkowski space-time, in the frame of GTRC space-time in the vacuum has the structure of Riemann-Cartan continuum with de

Sitter metric [9]. It is connected with the presence of constant term - vacuum torsion - in expression (7) of S_2^2:

$$S_2^{2(vac)} = \left[1 - \frac{b}{2f_0} [1 + \left(1 - \omega (1 - b/f_0) \frac{f_0^2}{b^2} \right)^{1/2}] \right] [12\alpha b(1 - \omega/4)]^{-1}. \tag{11}$$

Then in accordance with eqs. (4)-(7) the vacuum value of H^2 (in the case $k = 0$) is:

$$H^{2(vac)} = \frac{6b^2}{f_0} \alpha S_2^{4(vac)} [1 - 6\alpha(2b + \omega f_0) S_2^{4(vac)}]^{-1}. \tag{12}$$

At asymptotics when energy density is small ($\alpha \rho c^2 \ll 1$), by using the restriction $0 < x = 1 - \frac{b}{f_0} \ll 1$ the expression (7) for S_2^2 in the lowest approximation with respect to x takes the form:

$$S_2^2 = \frac{1}{12b} \left[\rho c^2 - 3p + \frac{1 - b/f_0}{\alpha} \right], \tag{13}$$

and as a result cosmological equations (4)-(5) at asymptotics are:

$$\frac{k}{R^2} + H^2 = \frac{1}{6b} \left[\rho c^2 + \frac{1}{4\alpha} \left(1 - \frac{b}{f_0} \right)^2 \right], \tag{14}$$

$$\dot{H} + H^2 = -\frac{1}{12b} \left[(\rho c^2 + 3p) - \frac{1}{2\alpha} \left(1 - \frac{b}{f_0} \right)^2 \right]. \tag{15}$$

According to (13)-(14) and in compliance with (11)-(12) we have:

$$S_2^{2(vac)} = \frac{1 - b/f_0}{12b\alpha}, \qquad H^{2(vac)} = \frac{\left(1 - \frac{b}{f_0} \right)^2}{24b\alpha}. \tag{16}$$

The effective cosmological constant in (14)-(15) is induced by the vacuum torsion $S_2^{2(vac)}$. Unlike standard ΛCDM-model effective cosmological constant appears in (14)-(15) as a result of solution of gravitational equations for HIM that leads to the change of gravitational interaction when energy density is small and comparable with cosmological constant - the vacuum gravitational repulsion effect leading to accelerating cosmological expansion at present epoch.

Because of restriction $0 < x = 1 - \frac{b}{f_0} \ll 1$ the vacuum value of H and the vacuum torsion function $|S_1|$ are negligibly small in comparison with $|S_2^{(vac)}|$. Owing to this the curvature tensor (see [11, 8]) has the following vacuum components:

$$F^{12}{}_{12} = F^{13}{}_{13} = F^{23}{}_{23} = -S_2^{2(vac)}. \tag{17}$$

Unlike the torsion function S_1, the influence of which on gravitational interaction is essential only at extreme conditions near a bounce, the torsion function S_2^2 plays important role also at asymptotics. Connection between parameters b and α can be found by supposition that the value of effective cosmological constant in eqs. (14)-(15) corresponds to observable accelerating cosmological expansion. Parameter ω together with α play important role at extreme conditions near a bounce. By energy densities, which are much smaller than limiting energy density and greater than constant term in (14) $\frac{1}{4\alpha}\left(1-\frac{b}{f_0}\right)^2$ the behaviour of cosmological solutions of eqs. (14)-(15) practically coincides with that of Friedmann cosmological equations of GR.We deal with such energy densities in astrophysics in the case of various objects in galaxies and galactic clusters. In general case the description of such systems in the frame of GTRC is difficult problem because of complexity of gravitational equations. The situation is simplifying, when minimum GTRC was determined [13]. This theory includes three indefinite parameters, which can be expressed through parameters of isotropic cosmology [5]. By neglecting terms with small parameter ω equations of minimum GTRC lead to gravitation equations for metric in the form of Einstein gravitation equations with cosmological constant, which are valid for spinless gravitating systems at wide range of energy density - beginning with extremely high energy densities defined by α^{-1}:

$$G^{\mu}{}_{\lambda} = -\frac{1}{2b}\left[T_{\lambda}{}^{\mu} + \delta^{\mu}_{\lambda}\frac{(1-\frac{b}{f_0})^2}{12\alpha}\right], \tag{18}$$

where $G^{\mu}{}_{\lambda}$ is Einstein tensor. The influence of torsion appears in eq. (18) via formation of effective cosmological constant and the change of gravitational constant. We see that the correspondence principle with GR will be fulfilled if parameter b satisfies the condition $0 < 1-\frac{b}{f_0} \ll 1$. Nonvanishing components of torsion tensor $S_{\alpha\mu\nu}$ satisfy the relation [13]:

$$S_{\lambda\mu\nu}(S^{\lambda\mu\nu} - 2S^{\mu\nu\lambda}) = \frac{1}{2b}\left[T + \frac{1-b/f_0}{\alpha}\right], \tag{19}$$

where $T = T_{\mu}{}^{\mu}$. This relation corresponds to formula (13) of isotropic cosmology, if we take into account that $S_2^2 = -\frac{1}{6}S_{\alpha\mu\nu}S^{\alpha\mu\nu}$ with $\alpha,\mu,\nu = 1,2,3$ ($\alpha\neq\mu$, $\alpha\neq\nu$).

3 Vacuum torsion and gravitational interaction at astrophysical scale

By neglecting spin effects the description of various astrophysical objects in the frame of minimum GTRC practically coincides with that in GR [6], because the influence of cosmological constant in (18) is negligibly small at astrophysical scale. In the case of spinning matter corrections of the metric determined by eq. (18) are also sufficiently small. However

[5] The gravitational Lagrangian of minimum GTRC is given by (1), if we assume that $f_1 = f_2 = f_3 = f_4 = 0$, $a_1 = b, a_2 = 2b, a_3 = -\frac{4}{3}b, f_5 = 3f_0^2\alpha\omega, f_6 = f_0^2\alpha(1-\omega)$ and $0<\omega\ll 1$.

[6] We don't discuss here objects with extremely high energy densities, consideration of which is possible by taking into account terms with parameter ω in gravitational equations.

spin effects can be manifested as a result of interaction of vacuum torsion $S_2^{2(vac)}$ defined according to (16) with proper angular momentums of astrophysical objects (stars in galaxies, galaxies in galactic clusters) that can have an influence on their movement. Because the value of vacuum torsion is much greater than cosmological constant (by virtue of restriction $0 < x = 1 - \frac{b}{f_0} \ll 1$), its influence quantitatively can become apparent at non-relativistic approximation.

With the purpose to study movement of an object with proper angular momentum in gravitational field we will use equations of motion of particle with momentum in Riemann-Cartan space-time [21] generalizing Papapetrou's equations for rotating particle in GR [22] [7]. In the case of rotating particle with angular velocity tensor Ω_{ik} corresponding equations of motion by conserving terms, which are essential at non-relativistic approximation, are:

$$\frac{DP_i}{d\tau} = \frac{1}{2} I \Omega_{mn} F^{mn}{}_{il} v^l \qquad (i, l, m, n = 1, 2, 3), \tag{20}$$

where $\frac{D}{d\tau}$ denotes riemannian absolute derivative with respect to proper time τ, P_i is generalized momentum, I is inertia momentum and v^l is velocity of particle. In non-relativistic approximation $P_i = mv_i$ (m is particle mass), $\Omega_{mn} = const$ and influence of curvature tensor in right side of (20) can become apparent by means of vacuum curvature (17). The right side of equation (20) determines additional gravitational force connected with interaction of vacuum torsion with proper angular momentum of particle.

As example we will consider the circular motion of rotating particle in spherically symmetric gravitational field created by mass M in non-relativistic approximation. By taking into account that $g_{00} = 1 + \frac{2\phi}{c^2}$ (ϕ is newtonian potential), components of angular velocity $\Omega_i = \epsilon_{ikl}\Omega^{kl}$ and relation (17) we obtain in the case of motion in plane XOY (centrum of mass M is in origin of coordinates, vector of orbital angular momentum is directed along the axe OZ) equation of motion in usual form $m\frac{d\mathbf{v}}{dt} = \mathbf{F}$ with the following expression of the force vector:

$$\mathbf{F} = -m\frac{d\phi}{d\mathbf{r}} + I\Omega_3 S_2^{2(vac)} v \frac{\mathbf{r}}{r}. \tag{21}$$

The force (21) includes besides Newtonian term additional force, direction of which depends on relative orientation of proper and orbital angular momentums. We have the force of attraction or repulsion depending on $\Omega_3 < 0$ or $\Omega_3 > 0$, as result its value is:

$$F = G\frac{mM}{r^2} \pm I\Omega S_2^{2(vac)} v, \tag{22}$$

where $\Omega = |\Omega_3|$. By taking into account that the force (22) is centripetal force we obtain the following dependence of velocity on distance from centrum and parameters of particle and gravitational field:

$$v = \pm\frac{I}{2m}\Omega S_2^{2(vac)} r + \left[(\frac{I}{2m}\Omega S_2^{2(vac)} r)^2 + \frac{GM}{r} \right]^{\frac{1}{2}}. \tag{23}$$

[7] In [21] the curvature tensor was defined with opposite sign and signature (+2) was used.

By given parameters of particle and gravitational field values of the force (22) and velocity (23) depend on parameter $x = 1 - \frac{b}{f_0}$. By taking into account that average mass density in the Universe at present epoch $\rho_1 = \frac{x^2}{4c^2\alpha}$ is of order $10^{-26}\frac{\mathrm{kg}}{\mathrm{m}^3}$, we obtain that at the first approximation

$$S_2^{2(vac)} = \frac{16\pi G}{3c^2 x}\rho_1 \sim \frac{10^{-52}}{x}(\mathrm{m}^{-2}). \tag{24}$$

Now we will consider the application of (23) in the case of attraction force to star similar to Solar ($I/m \sim 10^{18}$ m^2, $\Omega \sim 0.5 \cdot 10^{-6}$ s^{-1}) moving in galaxy similar to Andromeda ($M = 2 \cdot 10^{41}$ kg) by taking $x = 10^{-25}$ and consequently $S_2^{2(vac)} = 10^{-27}(\mathrm{m}^{-2})$ that corresponds to high energy density scale $\alpha^{-1} = 10^7 \rho_{nucl} c^2$ (ρ_{nucl} is nuclear mass density). As numerical analysis shows at distances $r < 9$ kpc (1kpc$= 0,3086 \cdot 10^{20}$m) Newtonian term in (23) plays the definitive role, by growth of r from 9 kpc to 25 kpc the velocity v according to Newtonian law decreases from $219 \cdot 10^3$ km/s to $132 \cdot 10^3$ km/s, but according to (3.4) the velocity v changes only from $256 \cdot 10^3$ km/s to $259 \cdot 10^3$ km/s. By further increase of r essential growth of velocity v takes place according to (23); this effect can be essential in galactic clusters, where we deal with vast space scale of order 10 Mpc and more. We see that the force of interaction of the vacuum torsion with proper angular momentums of stars can be essential by formation of rotation curves in galaxies that can be interesting in connection with dark matter problem. Effects discussed above at galactic scale (at $x \sim 10^{-25}$) are negligible when moving the planets in the Solar system due to the smallness of the additional force in (22) in comparison with the Newtonian force. For example, in the case of Earth the additional force of attraction is only 10^{-12} part of the Newtonian force.

Effects connected with interaction of vacuum torsion with proper angular momentums can be important in astrophysics also in the case of systems of stars with high angular velocity of proper rotation, for example systems of double pulsars.

Although given above consideration was realized in the frame of minimum GTRC, similar effects take place in other GTRC because of existence of the vacuum torsion.

4 Conclusion

Research of gravitation theory in Riemann-Cartan space-time shows that satisfying the correspondence principle with general relativity theory GTRC leads to certain principal differences concerning gravitational interaction at cosmological and astrophysical scales. Distinctions are connected with geometrical structure of physical space-time, namely with space-time torsion. The torsion created by spinless matter changes the character of gravitational interaction at extreme conditions and leads to possible existence in the nature of limiting energy density. Gravitational repulsion at extreme conditions ensures the regular behaviour of all HIM, including inflationary cosmological models. The deviation of the structure of physical space-time in the vacuum from that of Minkowski space-time leads also to important physical consequences concerning gravitational interaction at cosmological and astrophysical scales. The vacuum torsion generates effective cosmological constant by changing gravitational interaction at cosmological scale when energy density is small that allows to explain accelerating cosmological expansion at present epoch. The interaction of the vacuum torsion with proper angular momentums of gravitating objects leads to corrections of gravitational interaction at astrophysical scale, namely to appearance of additional gravitational force, which can have an influence on movement of stars in galaxies and galaxies in galactic clusters. The search of possible observational demonstrations of this phenomenon is of direct physical interest.

It should be noted that discussed phenomena connected with the change of gravitational interaction have essentially non-linear origin. Because of non-linear character of gravitating vacuum approximative analysis of GTRC based on investigation of linearized theory and perturbations of gravitational field above Minkowski space-time [7] has to be re-examined. In particular this concerns the analysis of particle content of GTRC, where it would be taken into account not only deviation of space-time metric in the vacuum from that of Minkowski space-time, but also presence of the vacuum torsion (compare with [23]) [8].

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References

[1] T.W.B Kibble, *Lorentz Invariance and the Gravitational Field, J. Math. Phys.* **2** (1961) 212.

[2] A.M. Brodskii, D.D. Ivanenko, H.A. Sokolik, *A New Conception of the Gravitational Field, Zh. Eksp. Teor. Fiz.* **41** (1961) 1307; *Acta Phys.Hungar.* **14** (1962) 21.

[3] D.W. Sciama, *On the analogy between charge and spin in GR*, in: *Recent Developments in General Relativity, Festschrift for Infeld*, Pergamon Press, Oxford, PWN, Warsaw (1962), pg. 415.

[4] A. Trautman, *On the structure of the Einstein-Cartan equations*, *Bull. Acad. Pol. Sci., Ser. sci. math., astr., phys.* **21**, No 3 (1973) 345; A. Trautman, *The Einstein-Cartan theory*, in: *Encyclopedia of Mathematical Physics* **2**, J.-P. Francoise et al. (eds.), Elsevier, Oxford (2006), pg. 189.

[8] The applying of [7] to minimum GTRC shows that weak gravitational field (by neglecting terms with ω in gravitational equations) in the frame of this theory besides massless graviton includes massive particles with spin-parity 2^+.

[5] F.W. Hehl, P. von der Heyde, G.D. Kerlik, and J.M. Nester, *General relativity with spin and torsion: Foundations and prospects, Rev. Mod. Phys.* **48** (1976) 393.

[6] A.V. Minkevich, *Gravitation and limiting mass density*, *Vestsi Akad. Navuk BSSR, ser. fiz.-mat. navuk*, No 2 (1980) 87; A.V. Minkevich, *Generalized cosmological Friedmann equations without gravitational singularity*, *Phys. Lett.* **A 80**, No 4 (1980) 232.

[7] K. Hayashi and T. Shirafuji, *Gravity from Poincaré Gauge Theory of the Fundamental Particles, 1. General Formulation*, *Progress of Theoretical Physics*, **64** (1980) 866 [*Erratum ibid.*, **65** (1981) 2079]; *Gravity from Poincaré Gauge Theory of the Fundamental Particles, 2. Equations of motion for test bodies and various limits*, *Progress of Theoretical Physics*, **64** (1980) 883 [*Erratum ibid.*, **65** (1981) 2079]; *Gravity from Poincaré Gauge Theory of the Fundamental Particles, 3. Weak field approximation*, *Progress of Theoretical Physics*, **64** (1980) 1435 [*Erratum ibid.*, **66** (1981) 741]; *Gravity from Poincaré Gauge Theory of the Fundamental Particles, 4. Mass and energy of particle spectrum*, *Progress of Theoretical Physics*, **64** (1980) 2222.

[8] A.V. Minkevich, A.S. Garkun and V.I. Kudin, *On some physical aspects of isotropic cosmology in Riemann-Cartan spacetime*, *JCAP* , 03 (2013) 040 [arXiv:1302.2578 [gr-qc]].

[9] A.V. Minkevich, *De Sitter spacetime with torsion as physical spacetime in the vacuum and isotropic cosmology*, *Mod. Phys. Lett. A* , **26** (2011) 259 [arXiv:1002.0538v2 [gr-qc]].

[10] A.V. Minkevich, *Limiting Energy Density and a Regular Accelerating Universe in Riemann-Cartan Spacetime, J. Exp. Theor. Phys. Lett.* **94** (2011) 831.

[11] A.V. Minkevich, A.S. Garkun and V.I. Kudin, *Regular accelerating universe without dark energy in Poincaré gauge theory of gravity*, *Clas. Quant. Grav.* **24** (2007) 5835 [arXiv:0706.1157 [gr-qc]].

[12] A.V. Minkevich, *Gauge gravitation theory in Riemann-Cartan space-time and gravitational interaction*, *Grav. Cosmol.* **22** (2016) 148.

[13] A.V. Minkevich, *Relationship of gauge gravitation theory in Riemann-Cartan spacetime and general relativity theory*, *Grav. Cosmol.* **23**, No 4 (2017) [arXiv:1609.05285v2 [gr-qc]].

[14] A.V. Minkevich, *Towards the theory of regular accelerating Universe in Riemann-Cartan space-time*, *Int. J. Mod. Phys. A* **31**, Nos. 2-3 (2016) 1641011.

[15] V.I. Kudin, A.V. Minkevich and F.I. Fedorov, *On homogeneous isotropic cosmological models with torsion, Vestsi Akad. Navuk BSSR, ser. fiz.-mat. navuk*, No 4 (1981) 59.

[16] F. Karakura, V.I. Kudin and A.V. Minkevich, *About gravitational equations for homogeneous isotropic cosmological models with torsion*, VINITI No. 4512-82, Minsk (1982) pg. 18.

[17] P. Baekler, F.W. Hehl and J.M. Nester, *Poincaré Gauge Theory of Gravity Friedman cosmology with even and odd parity modes. Analytic part*, *Phys. Rev.*, **D 83** (2011) 024001 [arXiv:1009.5112].

[18] K.-F. Shie, J.M.Nester and H.-J. Yo, *Torsion Cosmology and the Accelerating Universe, Phys. Rev.* **D78** 023522 (2008) [arXiv:0805.3834].

[19] H.Chen, F.-H. Ho, J.M.Nester, Ch.-H.Wang, H.-J. Yo, *Cosmological dynamics with propagating Lorentz connection modes of spin zero, JCAP* **0910** (2009) 027 [arXiv:0908.3323].

[20] Xi-Chen Ao and Xi Zhou Li, *Torsion cosmology of Poincaré gauge theory and the constraints of its parameters via SNela data, JCAP* **1202** (2012) 003.

[21] A.V. Minkevich, A.A. Sokolsky, *General-relativistic variational formalism for test-particle with momentums*, *Vestsi Akad. Navuk BSSR, ser. fiz.-mat.* No 4 (1975) 72 (in rus.).

[22] A. Papapetrou, *Spinning test-particles in GR. I. Proc. Roy. Soc. A*, **209** (1951) 248.

[23] V.P. Nair, S. Randjbar-Daemi, V. Rubakov, *Massive spin-2 fields of geometric origin in curved spacetimes*, *Phys. Rev.* **D80** 104031 (2009) [arXiv:0811.3781 [hep-th]].