On the Role of Einstein-Cartan Gravity in Fundamental Particle Physics

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Two of the major open questions in particle physics are: (1) Why are the elementary fermionic particles that are so far observed have such low mass-energy compared to the Planck energy scale? And (2), what mechanical energy may be counterbalancing the divergent electrostatic and strong force energies of point-like charged fermions in the vicinity of the Planck scale? In this paper, using a hitherto unrecognized mechanism derived from the non-linear amelioration of Dirac equation known as the Hehl-Datta equation within Einstein-Cartan-Sciama-Kibble extension of general relativity, we present detailed numerical estimates suggesting that the mechanical energy arising from the gravity-induced self-interaction in the ECSK theory can address both of these questions in tandem.

I. INTRODUCTION

For over a century Einstein's theory of gravity has provided remarkably accurate and precise predictions for the behaviour of macroscopic bodies within our cosmos. For the elementary particles in the quantum realm, however, Einstein-Cartan theory of gravity may be more appropriate, because it incorporates spinors and associated torsion within a covariant description [1][2]. For this reason there has been considerable interest in Einstein-Cartan theory, in the light of the field equations proposed by Sciama [3] and Kibble [4]. For example, in a series of papers Poplawski has argued that Einstein-Cartan-Sciama-Kibble (ECSK) theory of gravity [5] solves many longstanding problems in physics [6][7][8][9]. His concern has been to avoid singularities endemic in general relativity by proposing that our observed universe is perhaps a black hole within a larger universe [7]. Our concern, on the other hand, is to point out using numerical estimates that ECSK theory also offers solutions to two longstanding problems in particle physics.

The first problem we address here concerns the well known fact that in the limit of vanishing radii (or point limit) the electrostatic and strong force self-energies of point-like fermions become divergent. We will show, however, that torsion contributions within the ECSK theory resolves this difficulty as well, at least numerically, by counterbalancing the divergent electrostatic and strong force energy densities near the Planck scale. In fact, the negative torsion energy associated with the spin angular momentum of elementary fermions may well be the long sought after mechanical energy that counteracts the divergent positive energies stemming from their electrostatic and strong nuclear charges. Because of this counterbalancing, however, our suggestion does not have anything to do with high energy physics.

The second of these problems can be traced back to the fact that gravity is a considerably weaker "force" compared to the other forces. When Newton's gravitational constant is combined with the speed of light and Planck's constant, one arrives at the energy scale of $\sim 10^{19}$ GeV, which is some 17 orders of magnitude larger than the heaviest known elementary fermion (the top quark) observed at the mass-energy of ~ 172 GeV. Thus there is a difference of some 17 orders of magnitude between the electroweak scale and the Planck scale. There have been many attempts to explain this difference, but none is as simple as our explanation based on the torsion contributions within the ECSK theory.

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II. EVALUATION OF THE CHARGED FERMIONIC SELF-ENERGY WITHIN ECSK THEORY

The ECSK theory of gravity is an extension of general relativity allowing spacetime to have torsion in addition to curvature, where torsion is determined by the density of intrinsic angular momentum, reminiscent of the quantum-mechanical spin [1][2][3][4][5][6][7][8][9][10][11][12][13][14][15][16]. As in general relativity, the gravitational Lagrangian density in the ECSK theory is proportional to the curvature scalar. But unlike in general relativity, the affine connection $\Gamma_{i\,j}^{\,k}$ is not restricted to be symmetric. Instead, the antisymmetric part of the connection, $S^k{}_{ij} = \Gamma_{[i\,j]}^{\,k}$ (*i.e.*, the torsion tensor), is regarded as a dynamical variable similar to the metric tensor g_{ij} in general relativity. Then, variation of the total action for the gravitational field and matter with respect to the metric tensor gives Einstein-type field equations that relate the curvature to the dynamical energy-momentum tensor $T_{ij} = (2/\sqrt{-g})\delta\mathfrak{L}/\delta g^{ij}$, where $\mathfrak{L}$ is the matter Lagrangian density. On the other hand, variation of the total action with respect to the torsion tensor gives the Cartan equations for the spin tensor of matter [5]:

$$s^{ijk} = \frac{1}{\kappa} S^{[ijk]}\,, \quad \text{where } \kappa = \frac{8\pi G}{c^4}. \tag{1}$$

Thus ECSK theory of gravity extends general relativity to include intrinsic spin of matter, with fermionic fields such as those of quarks and leptons providing natural sources of torsion. Torsion, in turn, modifies the Dirac equation for elementary fermions by adding to it a cubic term in the spinor fields, as observed by Kibble, Hehl and Datta [1][4][5].

It is this nonlinear Hehl-Datta equation that provides the theoretical background for our proposal. The cubic term in this equation corresponds to an axial-axial self-interaction in the matter Lagrangian, which, among other things, generates a spinor-dependent vacuum-energy term in the energy-momentum tensor (see, for example, Ref. [13]). The torsion tensor $S^k{}_{ij}$ appears in the matter Lagrangian via covariant derivative of a Dirac spinor with respect to the affine connection. The spin tensor for the Dirac spinor ψ then turns out to be totally antisymmetric:

$$s^{ijk} = -\frac{i\hbar c}{4}\bar{\psi}\gamma^{[i}\gamma^j\gamma^{k]}\psi, \tag{2}$$

where $\bar{\psi} \equiv \psi^\dagger\gamma^0 := (\psi_1^*,\ \psi_2^*,\ -\psi_3^*,\ -\psi_4^*)$ is the Dirac adjoint of ψ and γ^i are the Dirac matrices: $\gamma^{(i}\gamma^{j)} = 2g^{ij}$. The Cartan equations (1) render the torsion tensor to be quadratic in spinor fields. Substituting it into the Dirac equation in the Riemann-Cartan spacetime with metric signature $(+,\ -,\ -,\ -)$ gives the cubic Hehl-Datta equation [1][4][5]:

$$i\hbar\,\gamma^k\psi_{:k} = mc\,\psi + \frac{3\kappa\hbar^2 c}{8}\left(\bar{\psi}\gamma^5\gamma_k\psi\right)\gamma^5\gamma^k\psi, \tag{3}$$

where the colon denotes a general-relativistic covariant derivative with respect to the Christoffel symbols, and m is the mass of the spinor. The Hehl-Datta equation (3) and its adjoint can be obtained by varying the following action with respect to $\bar{\psi}$ and ψ (respectively), without varying it with respect to the metric tensor or the torsion tensor [13]:

$$\mathcal{I} = \int d^4x\,\sqrt{-g}\left\{-\frac{1}{2\kappa}R + \frac{i\hbar c}{2}(\bar{\psi}\gamma^k\psi_{:k} - \bar{\psi}_{:k}\gamma^k\psi) - mc^2\bar{\psi}\psi - \frac{3\kappa\hbar^2c^2}{16}(\bar{\psi}\gamma^5\gamma_k\psi)(\bar{\psi}\gamma^5\gamma^k\psi)\right\}. \tag{4}$$

The last term in this action corresponds to the effective axial-axial, self-interaction mentioned above:

$$\mathfrak{L}_{\rm AA} = -\sqrt{-g}\;\frac{3\kappa\hbar^2c^2}{16}(\bar{\psi}\gamma^5\gamma_k\psi)(\bar{\psi}\gamma^5\gamma^k\psi). \tag{5}$$

This self-interaction term is not renormalizable. But it is an *effective* Lagrangian density in which only the metric and spinor fields are dynamical variables. The original Lagrangian density for a Dirac field in which the torsion tensor is also a dynamical variable (giving the Hehl-Datta equation), *is* renormalizable, since it is quadratic in spinor fields. But as we will see, renormalization may not be required if ECSK gravity turns out to be what is realized in Nature, because it gives physical justification for the counter terms.

Before proceeding further we note that the above action is not the most general possible action within the present context. In addition to the axial-axial term, an axial-vector and a vector-vector terms can be added to the action, albeit as non-minimal couplings (see, for example, Ref. [15]). However, it has been argued in Ref. [13] that minimal coupling is the most natural coupling of fermions to gravity because non-minimal couplings are sourced by components of the torsion that do not appear naturally in the models of spinning matter. For this reason we will confine our treatment to the minimal coupling of fermions to gravity and the corresponding Hehl-Datta equation, while recognizing that strictly speaking our neglect of non-minimal couplings amounts to an approximation, albeit a rather good approximation, at least as far as electrodynamics is concerned.

A. S-matrix Evaluation of the Charged Fermionic Self-Energy within QED

It is instructive for our purposes to first review in this subsection the standard treatment of the charged fermionic self-energy within QED, ignoring the ECSK gravity. To this end, recall that the canonical evaluation of the electron self-energy was performed by Weisskopf in 1939 [17], which revealed that in general the self-energy of an electron is logarithmically divergent. An S-matrix evaluation for the electromagnetic mass, δm, confirms Weisskopf's result [18] [19]. In the units of $\hbar = c = 1$ and $\alpha = e^2/4\pi$, together with a high-energy cutoff Λ, this electromagnetic mass can be expressed as

$$\delta m = \frac{3\alpha m}{2\pi}\ln\left(\frac{\Lambda}{m}\right) \;\; (\text{for } \Lambda >> m), \tag{6}$$

where "ln" stands for the natural logarithm. Since $m = 1/r$, we can put the above equation in a more familiar form for electromagnetic energy,

$$\delta m = \frac{3\alpha m}{2\pi}\ln\left(\frac{\Lambda}{m}\right) = \frac{3\alpha}{2\pi r}\ln\left(\frac{\Lambda}{m}\right) = \frac{3\,e^2}{8\pi^2\,r}\ln\left(\frac{\Lambda}{m}\right), \tag{7}$$

so that now we can see more easily that δm goes to infinity as $r \to 0$. In other words, m in the last equation is not really the observed mass of the fermion but is more like a so-called bare mass as Milonni suggests [18]. Moreover, we note that the mass in that expression comes in via the fermion propagator which can be off-mass-shell so it could have any value. Weinberg in [19] obtains a result for the complete electromagnetic self-energy function renormalized to order α in his eq. (11.4.14); and for our rest frame scenario we obtain a corresponding result of

$$\delta m_R = \frac{(\ln(16)+1)\,m_B\,e_R^2}{8\pi^2} = \frac{\ln(16\mathrm{e})\,e_R^2}{8\pi^2\,r_B}, \tag{8}$$

where e in ln(16e) is Euler's number. However, in our view the mass in Weinberg's result is still the "bare" mass because we will soon show that the observed mass of a fermion depends on the electromagnetic mass minus a mechanical mass. If the natural log function in eq. (6) is taken to equal unity, then there is only a small difference between Weinberg's result and Weisskopf's result. We will use this result in further calculations as the cutoff has now been removed.

We know from experiments that the radius of an electron is likely to be less than 10^{-22}m [20]. Substituting that bound for r_B gives

$$\delta m_R = \frac{\ln(16\mathrm{e})\,e_R^2}{8\pi^2\,(10^{-22}\mathrm{m})} \gtrapprox 5.051\times 10^3\,\mathrm{GeV}, \tag{9}$$

which, although finite, is still a very large electromagnetic energy contribution, with the actual value likely to be even greater. Thus according to this estimate the electromagnetic mass is going to be very large near Planck length and will have to be compensated for in order to recover the observed rest mass of a charged fermion. And the compensation will have to be negative mass-energy relative to the positive electromagnetic energy:

$$m_{obs} = -X + \delta m_R = -X + \frac{\ln(16\mathrm{e})\,\alpha}{2\pi r_B} \longrightarrow -\frac{X}{r} + \frac{\ln(16\mathrm{e})\,\alpha}{2\pi r_B}. \tag{10}$$

We suspect that the unknown variable X might be related to the Hehl-Datta self-interaction term (5) because that term varies as $1/r_x$. Our goal then is to investigate this possibility.

To that end, note that the full second order S-matrix calculation within QED worked out by Milonni [18] gives

$$S_{fi}^{(2)}(E) = -i(2\pi)^4\delta^4(p_f - p_i)\sqrt{\frac{m^2}{E_i E_f}}\frac{1}{r^3}\bar{u}(p_f, s_f)\Sigma(p_i)u(p_i, s_i), \tag{11}$$

$$\text{where} \quad \Sigma(p_i) = -ie^2 \int \frac{d^4k}{(2\pi)^4}\frac{g_{\mu\nu}}{k^2 + i\epsilon}\gamma^\mu \frac{1}{\not{p}_i - \not{k} - m + i\epsilon}\gamma^\nu. \tag{12}$$

Milonni's evaluation of $\Sigma(p_i)$ produces the result for δm noted in our eq. (6). This suggests that a convenient path to evaluate the charged fermion self-energy is in the rest frame so that only time remains a variable parameter apart from r. Because $E_i = E_f = E = m$, $p_f = p_i$ and $s_f = s_i$ in the rest frame, eq. (11) can be further simplified to (cf. Appendix B):

$$S_{fi}^{(2)}(E) = -it\left\{\frac{3\alpha}{2\pi\, r}\ln\left(\frac{\Lambda}{m}\right)\right\}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}. \tag{13}$$

However, we will replace the term in brackets with our renormalized term derived above from Weinberg's result.

$$S_{fi}^{(2)}(E) = -it\left\{\frac{\ln(16\mathrm{e})\alpha}{2\pi\, r}\right\}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}. \tag{14}$$

B. S-matrix Evaluation for the Charged Fermionic Self-Energy within Einstein-Cartan Gravity

Building on the results of the previous subsection, we now reevaluate the S-matrix process including the contribution from the ECSK gravity. For the ECSK theory the interaction Hamiltonian density, analogous to that used in (11), is

$$\mathrm{h}_I(x) = -\frac{3\kappa}{16}(\bar{\psi}(x)\gamma^5\gamma_k\psi(x))(\bar{\psi}(x)\gamma^5\gamma^k\psi(x)), \tag{15}$$

where we have set $\hbar = c = 1$ for the time being. For our purposes it is sufficient to evaluate the S-matrix in a rest frame, in which the above interaction Hamiltonian reduces to a single term of the Hehl-Datta spin density squared factor taken from eq.(5), giving

$$\mathrm{h}_I(t) = -\frac{3\kappa}{16}(\bar{\psi}(t)\gamma^5\gamma_0\psi(t))(\bar{\psi}(t)\gamma^5\gamma^0\psi(t)). \tag{16}$$

Furthermore, as shown in Appendix B, the first order S-matrix term in the rest frame is now simplified to

$$S_{fi}^{(1)}(E1) = it\frac{3\kappa}{16\, r^3}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}. \tag{17}$$

On the other hand, for $m\bar{\psi}\psi$ in the rest frame we have,

$$S_{fi}^{(1)}(E2) = -im\int d^4x\, \langle f \mid \bar{\psi}(t)\psi(t) \mid i\rangle = -i\frac{m}{r^3}(2\pi)^4\delta^4(m, 0)\,\bar{u}^f(m)u^i(m) = -i\, mt\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}. \tag{18}$$

Noting that the transition probability corresponding to this expression is equal to one since the mass remains invariant over time, we assume that the sum of the two self-interaction S-Matrix terms will also have a transition probability of one so that

$$|S^{(2)}_{fi}(E) + S^{(1)}_{fi}(E1)|^2 = |S^{(1)}_{fi}(E2)|^2 \longrightarrow \frac{\ln(16\mathrm{e})\alpha}{2\pi\, r} - \frac{3\kappa}{16\, r^3} = m. \tag{19}$$

Reinstating $\hbar$ and c we thus arrive at

$$\frac{\ln(16\mathrm{e})\alpha\hbar c}{2\pi r} - \frac{3\kappa(\hbar c)^2}{16\, r^3} = mc^2. \tag{20}$$

An identical result can be obtained also for the anti-fermion spinor, $v(m)$, so that the equation remains the same for both fermion and anti-fermion. In what follows we will use this S-matrix result for our numerical approximations. It turns out to be compatible with the semi-classical result obtained below in eq. (24). From these equivalent results, after solving for r, it is evident that the complete process is finite, without divergences. This suggests that the above is the correct S-matrix solution for the fermion self-energy problem, with all other orders of the S-matrix expansion vanishing because the *genuine* fermionic self-energy must naturally be evaluated only in the rest frame, with all other contributions summing to zero. Any higher loop corrections in squared charge will be automatically compensated by higher loop corrections from squared spin.

C. Semi-classical Evaluation of Charged Fermionic Self-Energy within Einstein-Cartan Gravity

In this subsection we present semi-classical results within the ECSK theory for comparison with those derived above using the S-matrix approach. To this end, we begin by requiring the action (4) to be invariant under local phase transformations. Then $\psi_{:k}$ transforms to $\psi_{:k} - iqA_k\psi/\hbar$ for a charge q and gauge field A_k, and eq. (3) generalizes to

$$i\hbar\,\gamma^k\psi_{:k} + q\,\gamma^k A_k\,\psi = mc\,\psi + \frac{3\kappa\hbar^2 c}{8}\left(\bar{\psi}\gamma^5\gamma_k\psi\right)\gamma^5\gamma^k\psi\,. \tag{21}$$

In the rest frame of the particle and anti-particle, with the metric signature $(+,\,-,\,-,\,-)$, this equation simplifies to

$$i\hbar\,\gamma^0\frac{\partial\psi}{\partial t} + qcA_0\,\gamma^0\psi = mc^2\,\psi + \frac{3\kappa\hbar^2 c^2}{8}\left(\bar{\psi}\gamma^5\gamma_0\psi\right)\gamma^5\gamma^0\psi\,, \tag{22}$$

which can be further simplified (see Appendix C) to arrive at our semi-classical equation, for any electroweak fermion of charge q and mass m and its anti-particle in the Riemann-Cartan spacetime in this semi-classical theory:

$$\frac{q^2}{4\pi\varepsilon_0\, r} - \frac{3\pi G\hbar^2}{c^2\, r^3} = mc^2 \tag{23}$$

where r is the radial distance from q and this equation represents both particle and anti-particle. Replacing q with electron or positron charge and simplifying gives us,

$$\frac{\alpha\hbar c}{r} - \frac{3\kappa(\hbar c)^2}{8\, r^3} = mc^2. \tag{24}$$

When we compare this result with the result of the previous subsection we can see that with our QFT results we are on the right track using bare mass for m.

It is also worth noting that without ameliorating the Dirac equation with a cubic term, eq. (23) would reduce for an electron to $\alpha\hbar/r_e = m_e c$, giving $r_e = \alpha\hbar/(m_e c) \sim 10^{-15}$ m, where $\alpha = e^2/(4\pi\varepsilon_0\hbar c)$ is the fine structure constant. This is the classical electron radius. Experimental evidence, however, suggests that electron radius is much smaller [20]. As we shall see, our calculations with the cubic term included predicts the electron radius to be of the order of 10^{-34} m, which is closer to the Planck length. This may turn out to be the correct value of the electron radius.

Needless to say, what we have presented above is a derivation of eq. (23) within a theory that may be viewed as a semi-classical theory of Dirac fields in a Riemann-Cartan spacetime [4][5]. It can be interpreted also as a theory of gravity-induced self-interaction within standard general relativity [1][5]. An equivalent second-quantized generalization of this theory is presented in subsection II A. However, any such generalization must necessarily reproduce the Hehl-Datta equation (3) for single fermions even at reasonably high energies, just as Dirac equation remains valid for single fermions at high energies [16]. It is therefore not unreasonable to use eq. (23) derived above for our numerical estimates since it is compatible with the result derived using the S-matrix formalism.

Finally, it is important to note here that, despite the appearance of four spinors in the interaction term of eq. (4), it describes the self-interaction of a *single* fermion, of range $\sim 10^{-34}$ m, not mutual interactions among the spins of four distinct fermions. That is to say, it does not describe a "spin field" of some sort as a carrier of a new interaction [5]. If, however, one insists on interpreting the interaction term in eq. (4) as describing interactions among four distinct fermions, then the mass of the corresponding exchange boson would have to exceed 10^{15} GeV, which is evidently quite unreasonable. Moreover, as we shall see in the next section, this energy is a fictitious quantity and therefore there is no justification for assuming some kind of a new torsion interaction between different fermions. What is more, as we shall soon see, within our scheme any corrections due to vacuum polarization are automatically compensated for in the production of electroweak mass-energy, dictated by eq. (23) above.

D. Evaluation of Charged Fermionic Self-Energy by Dimensional Analysis

It is instructive to compare the results of the previous two subsections with the evaluation of the charged fermionic self-energy using only dimensional analysis. To this end, we begin with the following physically reasonable assumptions:

1. ECSK theory of gravity is the correct theory of spacetime for addressing the fermionic self-energy problem since it allows the dimension-full gravitational constant, G, to enter elementary particle physics in a natural manner.
2. Since experiments to date indicate that an electron is a point-like particle without any substructure and put an upper bound of 10^{-22} meters on its radius, we assume that the radius of electron is much less than that value.
3. We assume that the radial distance, r, on which the electromagnetic self-interaction depends is the same radial distance on which the self-interaction arising from the ECSK gravity-induced torsion spin density also depends.

Given these assumptions, we ask: What physical mechanism is responsible for the observed rest mass m_x of the elementary charged fermions? To answer this question we express the rest mass energy in CGS units, and assume that it is at least partially[1] electromagnetic in nature, so that it satisfies a relation like

$$m_x c^2 \sim \frac{e^2}{r} + X, \tag{25}$$

where the dimensionality of X is necessarily that of energy. However we already know that the value of $r < 10^{-15}\, m$ produces an energy greater than $m_x c^2$. Therefore X must be negative energy, giving

$$m_x c^2 \sim \frac{e^2}{r} + (-X). \tag{26}$$

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[1] As is well known, assuming that the rest mass energy is entirely electromagnetic in nature leads to the classical radius of the electron, which has been ruled out by experiments. But our assumption of it being at least partially electromagnetic in nature is quite reasonable.

Now, since fermions have spin $\hbar/2$, it is reasonable to assume that it is involved in a mechanical-like energy resulting from the spin interacting with itself analogous to charge, so that we may have a relation like

$$-X \sim -\left(\frac{\hbar}{2}\right)^2 \frac{1}{r}, \tag{27}$$

and we note that spin-squared will be negative. But it is evident from this expression that what we have on its RHS is energy $\times$ mass $\times$ length $= EML$, so we will have to divide out by mass and length to get the dimensions of energy, giving

$$-X \sim -\left(\frac{\hbar}{2r}\right)^2 \frac{1}{M}, \tag{28}$$

which in terms of the gravitational constant $G \sim L^3/MT^2$ can be written as

$$-X \sim -G\left(\frac{\hbar}{2r}\right)^2. \tag{29}$$

The dimensions on the RHS of this expression now give us $E \times L^3/T^2$, so we will have to cancel out L^3/T^2. A natural candidate to accomplish that is the speed of light, c, giving

$$-X \sim -\frac{G}{c^2 r}\left(\frac{\hbar}{2r}\right)^2 = -\frac{G}{r^3}\left(\frac{\hbar}{2c}\right)^2, \tag{30}$$

provided we cancel out the extra L with $1/r$. Thus we now have the dimensions of energy on the RHS, so that we finally have

$$m_x c^2 \sim \frac{e^2}{r_x} - \frac{G}{r_x^3}\left(\frac{\hbar}{2c}\right)^2. \tag{31}$$

This is the basic form of our central equation (apart from some numerical constants) which we have derived also using two other methods in the previous subsections. Solving this equation for r_x with electron mass for m_x gives a value of the order of 10^{-34} m. However, rather surprisingly, we also found a solution for r_x for the classical electron radius.

III. PARTICLE MASSES VIA TORSION ENERGY CONTRIBUTION

For our numerical analysis it is instructive to bring out the physical content of eq. (23) in purely classical terms. To this end, recall that when the self-energy of an electron is attributed solely to its electrostatic content, it is found that that energy is divergent, provided we assume the electron to be a point particle. This energy is called "self-energy", because it arises from the interaction of the charge of the electron with the electrostatic field that it itself is creating. With the help of our result (23) above we can avoid this fundamental difficultly as follows. Multiplying through eq. (23) with $3/4\pi$, it can be written as

$$\left(\frac{q^2}{4\pi\varepsilon_0\, r}\right)\frac{3}{4\pi r^3} - \left(\frac{3\pi G\hbar^2}{c^2\, r^3}\right)\frac{3}{4\pi r^3} = \left(mc^2\right)\frac{3}{4\pi r^3}. \tag{32}$$

With $4\pi r^3/3$ recognized as a volume of a sphere of radius r, it is now easy to recognize each quantity in the parenthesis in this equation as energy, and each term as the corresponding energy density. From that it is easy to make a comparison of the semi-classical result to our S-matrix based result for a fermion:

$$\left(\frac{\ln(16\mathrm{e})\alpha\hbar c}{2\pi r} - \frac{3\pi G\hbar^2}{2\, c^2\, r^3}\right) = mc^2 \longrightarrow \left(\frac{\ln(16\mathrm{e})q^2}{8\pi^2\varepsilon_0 r} - \frac{3\pi G\hbar^2}{2\, c^2\, r^3}\right) = mc^2. \tag{33}$$

Although this comparison is encouraging, it is unfortunate that the first term in eq. (33) diverges as $r \to 0$. But if we set r to Planck length we obtain

$$\frac{\ln(16\mathrm{e})\alpha}{2\pi\, l_P} \approx 3.125 \times 10^{16}\,\mathrm{GeV}, \tag{34}$$

which is close to Planck energy. Although finite, this is still an extremely large energy. *But such a large energy for charged leptons is not realized in Nature.* A natural question then is: Is there a negative mechanical energy that cancels out most of this energy to produce the observed rest mass-energy of leptons? We believe the answer lies in the second term of eq. (33), which – as we saw above – arises from the non-linear amelioration of the Dirac equation within ECSK theory. Indeed, if we again set the Planck length for r in the second term of eq. (33), then we obtain

$$-\,\frac{3\pi\, G\hbar^2}{2\, c^2\,(l_P)^3} \approx -\,5.773 \times 10^{19}\ \mathrm{GeV}. \tag{35}$$

Comparing this value with the electrostatic energy at the Planck length estimated in eq. (34) we see at once that the torsion-induced mechanical energy (35) can indeed counterbalance the huge electrostatic energy. This is a surprising observation, considering the widespread belief that "the numerical differences which arise [between GR and ECSK theories] are normally very small, so that the advantages of including torsion are entirely theoretical" [16].

Moving forward to our goal of numerical estimates, let us note that whenever terms quadratic in spin happen to be negligible, then the ECSK theory is observationally indistinguishable from general relativity. Therefore, for post-general-relativistic effects, the density of spin-squared has to be comparable to the density of mass. The corresponding characteristic length scale, say for a nucleon, is referred to as the Cartan or Einstein-Cartan radius [2][16], defined as

$$r_{Cart} \approx (l_P^2\, \lambda_C)^{\frac{1}{3}}, \tag{36}$$

where λ_C is the Compton wavelength of the nucleon. Now it has been noted by Poplawski [6][7][8][9] that quantum field theory based on the Hehl-Datta equation may avert divergent integrals normally encountered in calculating radiative corrections, by self-regulating propagators. Moreover, the multipole expansion applied to Dirac fields within the ECSK theory shows that such fields cannot form singular, point-like configurations because these configurations would violate the conservation law for the spin density, and thus the Bianchi identities. These fields in fact describe non-singular particles whose spatial dimensions are at least of the order of their Cartan radii, defined by the condition

$$\epsilon \sim \kappa s^2, \tag{37}$$

where $\sqrt{s^2} \sim \hbar c\,|\psi|^2$ is the spin density, $\epsilon \sim mc^2|\psi|^2$ is the rest energy density, and $|\psi|^2 \sim 1/r^3$ is the probability density, giving the radius (36). Consequently, at the least the de Broglie energy associated with the Cartan radius of a fermion (which is approximately 10^{-27} m for an electron) may introduce an effective ultraviolet cutoff for it in quantum field theory in the ECKS spacetime. The avoidance of divergences in radiative corrections in quantum fields may thus come from spacetime torsion originating from intrinsic spin. Poplawski and others, however, took ϵ to be the mass-energy density of the fermion to arrive at the Cartan radius (36). But it is easy to work out from the first term of our eq. (33) that at the Cartan radius the electrostatic energy density for an electron is still extremely large:

$$\frac{\ln(16\mathrm{e})\alpha\hbar c}{2\pi(10^{-27}m)^4} \approx 5.051 \times 10^{89}\,\mathrm{GeV\,m^{-3}}. \tag{38}$$

For this reason it is not correct to identify ϵ with the rest mass-energy density, which is $\approx 5.1099 \times 10^{77}$ GeV m^{-3} for an electron at the Cartan radius. The electrostatic energy density of an electron is thus about fourteen orders of magnitude higher. Therefore ϵ is better identified with the electrostatic energy density provided most of it is canceled out.

If in eq. (33) we set the electrostatic energy appearing in its first term to be equal to the spin squared energy induced by the self-interaction appearing in its second term and solve for r, then we obtain

$$\boxed{r_t = \pi\sqrt{\frac{3}{\alpha\,\ln(16\mathrm{e})}}\;l_P}\,, \tag{39}$$

the value of which works out to be

$$r_t \approx 5.300\times 10^{-34}\,\mathrm{m}\,. \tag{40}$$

Thus our r_t is about 33 times larger than the Planck length, and, as we can see, it is a remarkably simple constant in terms of the Planck length, l_P, and the fine structure constant, α (recall that e in ln(16e) is Euler's number). According to eq. (33), r_t is the effective radius at which energy density due to spin density should completely compensate the huge electrostatic energy seen in eq. (34). In our view this is the correct Cartan radius, at least for the charged leptons, and that may still provide a plausible mechanism for averting singularities, since it is still larger than the Planck length. *It is important to note, however, that these huge energy densities never actually occur in Nature, because according to our eq. (33) they are automatically compensated.* The physical mechanism described above is simply to enable extraction of the radius r_t for different charged fermions.

We can now use eq. (33) to solve for r_x for the different charged leptons and anti-leptons which leads us to the following formula for our numerical estimates:

$$\frac{\ln(16\mathrm{e})\alpha\hbar c}{2\pi r_x} - \frac{3\,\pi G\hbar^2}{2c^2\,r_x^3} = +m_x c^2\,. \tag{41}$$

As shown in the Appendix A below, we were able to find solutions for r_x for the charged leptons using arbitrary precision in *Mathematica*. The first in our results listed below is the solution for r_t up to 24 significant figures. Then, using the same precision for comparison, we list the results for r_e for an electron, r_μ for a muon, and r_τ for a tauon, along with the anti-fermions:

$$\begin{aligned}
r_t \quad &= 5.30042667383830634827564\times 10^{-34}\ \mathrm{m} \longrightarrow 0.0\ \mathrm{MeV}\,,\\
r_{e-} = r_{e+} &= 5.30042667383830634910588\times 10^{-34}\ \mathrm{m} \longrightarrow 0.511\ \mathrm{MeV}\,,\\
r_{\mu-} = r_{\mu+} &= 5.30042667383830651994161\times 10^{-34}\ \mathrm{m} \longrightarrow 106\ \mathrm{MeV}\,,\\
r_{\tau-} = r_{\tau+} &= 5.30042667383830923512287\times 10^{-34}\ \mathrm{m} \longrightarrow 1777\ \mathrm{MeV}\,.
\end{aligned}$$

We should note that there is also a positive solution obtained for the reduced Compton wavelength for these fermions in the form of

$$r_x = \frac{\ln(16\mathrm{e})\alpha}{2\pi}\frac{\hbar}{m_x c}. \tag{42}$$

This is so because the spin density squared term becomes very small of order 10^{-39} MeV when r_x is equal to that result so it can be considered effectively as zero. We ignore this solution for fermion radius because experiment has already ruled it out. However, our formulation also reproduces the Compton wavelength for fermions that is useful for other things besides particle radii. Such as the radius of an effective quantum vacuum screening boundary where low energy electrons scatter off each other as if they had this radius.

Evidently, very minute changes in the radii are seen to cause large changes in the observed rest mass-energies of the fermions. But as the differences in the radii go larger, the resultant mass-energies go higher, as one would expect. It seems extraordinary that Nature would subscribe to such tiny differences resulting from a large number of significant figures, but that might explain why the underlying relationship between the observed values of the masses of the elementary particles has remained elusive so far. In addition to the possible reasons for this mentioned above, it is not inconceivable that the difference between the spin energy density and the electrostatic energy density radii with respect to r_t arises due to purely geometrical factors. We also suspect that there may possibly be some kind of symmetry breaking mechanism at work similar to the Higgs mechanism, and this symmetry breaking results in the observed mass-energy generation.

As a consistency check, let us verify that the tiny length differences seen above vanish, $\Delta r \to 0$, as the corresponding rest mass-energy differences tend to zero: $\Delta E \to 0$. To this end, we recast eq. (41) for arbitrary r_x in a form involving only rest mass-energy on the RHS as:

$$\frac{\ln(16\mathrm{e})\alpha\hbar c}{2\pi\, r_x} - \frac{3\pi\, l_P^2 \hbar c}{2 r_x^3} = m_x c^2. \tag{43}$$

If we now set

$$A \equiv \frac{\ln(16\mathrm{e})\alpha\hbar c}{2\pi} \quad \text{and} \quad B \equiv \frac{3\pi\, l_P^2 \hbar c}{2}\,, \tag{44}$$

then, with $\Delta E = m_x c^2$ and setting $r_x = r_t$ as the cancellation radius for which $\Delta E = 0$, we obtain

$$r_t = \sqrt{B/A}\,. \tag{45}$$

This allows us to derive a general expression for r_x when $\Delta E \neq 0$:

$$\frac{A}{r_x} - \frac{B}{r_x^3} = \Delta E. \tag{46}$$

From this expression it is now easy to see that

$$\lim_{\Delta E \to 0} \left\{ \left(\frac{A}{r_x} - \frac{B}{r_x^3} \right) = \Delta E \right\} = \left(\frac{\sqrt{A}^3}{\sqrt{B}} - \frac{\sqrt{A}^3}{\sqrt{B}} \right) \ln \frac{\sqrt{B/A}}{l_P} = 0 \implies r_x = \sqrt{B/A} = r_t\,, \tag{47}$$

and conversely, using (45),

$$\lim_{r_x \to r_t} \left\{ \left(\frac{A}{r_x} - \frac{B}{r_x^3} \right) = \Delta E \right\} \implies \Delta E = 0\,. \tag{48}$$

Consequently, with $\Delta r \equiv |\, r_t - r_x|$, we see from the above limits that $\Delta r \to 0$ as $\Delta E \to 0$, and vice versa.

As a rough estimate the calculation for the radius r_q of elementary quarks can be performed in a similar manner as that for charged leptons, since at such short distances the strong force reduces to a Coulomb-like force. One must also factor-in the electrostatic energy, so that a relationship like the following must be calculated, say, for the top quark:

$$\frac{2\,\ln(16\mathrm{e})\alpha\hbar c}{9\pi\, r_{qx}} + \frac{\alpha_s \hbar c}{3\, r_{qx}} - \frac{3\pi G\hbar^2}{2c^2\, r_{qx}^3} = m_t c^2\,. \tag{49}$$

Here α_s is the appropriate strong force coupling (we use 0.1). Needless to say, a cancellation radius different from that of the charged leptons should be calculated for comparison, by setting

$$\frac{2\,\ln(16\mathrm{e})\alpha\hbar c}{9\pi\, r_{qt}} + \frac{\alpha_s \hbar c}{3\, r_{qt}} - \frac{3\pi G\hbar^2}{2c^2\, r_{qt}^3} = 0. \tag{50}$$

A calculation of the radius for the top quark based on eq. (49) can be found in the Appendix and is $\approx 1.868 \times 10^{-34}$m. We expect it to be only a very rough estimate of the actual value of the radius. Since only one spin density is involved, the above calculation might be able to approximate the behaviour of the quarks. The calculation of the radii r_{qx} for the up and down quark will probably be problematic, since their masses are not well known.

With regard to neutrinos the story is quite different since they don't have self-energy due to electric or color charge. That means that their rest mass-energy comes entirely from torsion energy due to intrinsic spin. Solving that gives us a radius for electron neutrinos of the order of 10^{-26}m. However, the torsion (spin-squared) self-energy is negative relative to the positive rest mass. We suspect that this means that neutrinos have anti-gravity properties [22]. The anti-gravity effect with normal matter for single neutrinos is practically negligible but the cosmological implications could be large.

IV. POSSIBLE SOLUTION OF THE HIERARCHY PROBLEM

As alluded to in the introduction, the Hierarchy Problem refers to the fact that gravitational interaction is extremely weak compared to the other known interactions in Nature. One way to appreciate this difference is by combining the Newton's gravitational constant G with the reduced Planck's constant $\hbar$ and the speed of light c. The resulting mass scale is the Planck mass, m_P, which some have speculated to be associated with the existence of smallest possible black holes [7]. If we compare the Plank mass with the mass of the top quark (the heaviest known elementary particle),

$$m_P = \sqrt{\frac{\hbar c}{G}} \approx 2.1765 \times 10^{-8} \text{ kg}\,,$$
$$m_t = \frac{173.21 \text{ GeV}}{c^2} \approx 3.1197 \times 10^{-25} \text{ kg}\,,$$

then we see that there is some 17 orders of magnitude difference between them. This illustrates the enormous difference between the Planck scale and the electroweak scale. Many solutions have been proposed to explain this difference, such as supersymmetry and large extra dimensions, but none has been universally accepted, for one reason or another. Furthermore, recent experiments performed with the Large Hadron Collider are gradually ruling out some of these proposals. But regardless of the nature of any specific proposal, it is clear from the above values that predictions of numbers with at least 17 significant figures are necessary to successfully explain the difference between m_P and m_t.

We saw from our numerical demonstration in the previous section that within the ECSK theory minute changes in length can induce sizable changes in the observed masses of elementary particles, and that we do have numbers at our disposal with more than 17 significant figures for producing those masses. Moreover, all length changes occurring in our demonstration are taking place close to the Planck length. Thus, since we are "canceling out" near the Planck length to obtain masses down to the electroweak scale, ours is clearly a possible mechanism for resolving the Hierarchy Problem. We can appreciate this fact by simplifying our central equation by setting $\hbar = c = 1$, which then reduces to

$$\frac{\ln(16\text{e})\alpha}{2\pi\, r_x} - \frac{3\pi G}{2\, r_x^3} = m_x\,. \tag{51}$$

It is now easy to see from this equation that the observed mass-energy only depends on the coupling constants and the radii. Moreover, it is confined entirely within a volume close to the Planck volume, as we saw in our calculations in the previous section. Thus we are led to

$$\text{Planck Scale} \implies \text{Electroweak Scale.}$$

In other words, there is no hierarchy problem in the ECSK theory, because Planck scale physics is producing the electroweak scale physics in the form of the mass-energy of fermions as a byproduct of the very geometry of spacetime.

Within the ECSK theory, which extends general relativity to include spin-induced torsion, gravitational effects near micro scales are not necessarily weak. On the other hand, since torsion is produced in the ECSK theory by the spin density of matter, it is mostly confined to that matter, and thus is a very short range effect, unlike the infinite range effect of Einstein's gravity produced by mass-energy. In fact the torsion field falls off as $1/r^6$, as shown in the calculations of Sect. III, since it is produced by spin density squared, confined to the matter distribution [9].

To compare the strengths of gravitational and torsion effects at various scales, we may define a mass-dependent dimensionless gravitational coupling constant, $Gm^2/(\hbar c)$, and evaluate it for the electron, top quark, and Planck masses:

$$\alpha_{G_e} = \frac{Gm_e^2}{\hbar c} \approx 1.7517 \times 10^{-45},$$
$$\alpha_{G_t} = \frac{Gm_t^2}{\hbar c} \approx 1.1620 \times 10^{-36},$$
$$\alpha_{G_P} = \frac{Gm_P^2}{\hbar c} = 1\,,$$
$$\alpha_e = \frac{e^2}{4\pi\varepsilon_0\hbar c} \simeq 7.2973 \times 10^{-3}.$$

Here α_e is the electromagnetic coupling constant, or the fine structure constant. From these values we see that near the Planck scale the gravitational coupling is very strong compared to the electromagnetic coupling. However, as we noted above and in Sect. III, near the Planck scale torsion effects due to spin density are also very strong, albeit with opposite polarity compared to that of Einstein's gravity, akin to a kind of "anti-gravity" effect of a very short range.

For our demonstration above we have used electrostatic energy density and spin density for matter in a static approximation, for which the field equation within the ECSK theory reduces to $G^{00} = T^{00}$. A numerical estimate for G^{00} from the contributions of the electrostatic energy and spin density parts of T^{00} at our cancellation radius gives

$$G^{00}_{stat} = \frac{8\pi G}{c^4}\frac{\ln(16\mathrm{e})\alpha\hbar c}{2\pi\, r_t^4} \approx +3.644 \times 10^{62}\; m^{-2} \tag{52}$$

$$\text{and} \qquad G^{00}_{spin} = -\frac{8\pi G}{c^4}\frac{3\,\pi G\hbar^2}{2\,c^2 r_t^6} \approx -3.644 \times 10^{62}\; m^{-2}. \tag{53}$$

Evidently, these field strengths at the cancellation radius are quite large even for a single electron. Fortunately they are never realized in Nature, because, as we can see, they cancel each other out to produce $G^{00}_{net} = 0$. On the other hand, if we use only the mass-energy density for electron at the cancellation radius, then we obtain $G^{00}_{mass} \approx 3.0674 \times 10^{43}\,\mathrm{m}^{-2}$, which is again some 19 orders of magnitude off the mark. What is more, the latter field strength does not fall off as fast as that due to the spin-induced torsion field. Thus it is reasonable to conclude that without the cancellation of divergent energies due to the spin self-interaction we have explored here, our universe would be highly improbable.

V. CONCLUDING REMARKS

In this paper we have addressed two longstanding questions in particle physics: (1) Why are the elementary fermionic particles that are so far observed have such low mass-energy compared to the Planck scale? And (2), what mechanical energy may be counterbalancing the divergent electrostatic and strong force energies of point-like charged fermions in the vicinity of the Planck scale? Using a hitherto unrecognized mechanism extracted from the well known Hehl-Datta equation, we have presented numerical estimates suggesting that the torsion contributions within the Einstein-Cartan-Sciama-Kibble extension of general relativity can address both of these questions in conjunction.

The first of these problems, the Hierarchy Problem, can be traced back to the extreme weakness of gravity compared to the other forces, inducing a difference of some 17 orders of magnitude between the electroweak scale and the Planck scale. There have been many attempts to explain this huge difference, but none is simpler than our explanation based on the spin induced torsion contributions within the ECSK theory of gravity. The second problem we addressed here concerns the well known divergences of the electrostatic and strong force self-energies of point-like fermions at short distances. We have demonstrated above, numerically, that torsion contributions within the ECSK theory resolves this difficulty as well, by counterbalancing the divergent electrostatic and strong force energies close to the Planck scale.

It is widely accepted that in the standard model of particle physics charged elementary fermions acquire masses via the Higgs mechanism. Within this mechanism, however, there is no satisfactory explanation for how the different couplings required for the fermions are produced to give the correct values of their masses. While the Higgs mechanism does bestow masses correctly to the heavy gauge bosons and a massless photon, and while our demonstration above does not furnish a fundamental explanation for the fermion masses either, we believe that what we have proposed in this paper is worthy of further research, since our proposal also offers a possible resolution of the Hierarchy Problem.

In Ref. [23] Singh points out that there appears to be a symmetry between small and large masses for spin-torsion coupling and energy-curvature coupling. We have noted that there also appears to be a symmetry in that the energy-curvature coupling is effectively infinite while the spin-torsion coupling is very short-ranged near the Planck length.

Needless to say, the geometrical cancellation mechanism for divergent energies we have proposed here also dispels the need for mass-renormalization, since we have obtained finite solutions for r_x taming the infinities. Thus both classical and quantum electrodynamics appear to be more complete with torsion contributions included.

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Appendix A: Calculations of Fermion Radii using Wolfram Mathematica

In this appendix we explain how we used the arbitrary-precision in *Mathematica* to solve the numerical equations out to 24 significant figures. Each equation displayed below — derived from our central equation (33) — is simplified so that only the numerical factors have to be used, since the dimensional units cancel out, leaving lengths in meters. For decimal factors, the numbers must be padded out to 26 digits with zeros. Then the numerical part of electrostatic energy density is defined as A and the numerical part of spin energy density is defined as B, just as in eq. (44) above. These are then used throughout to perform the calculations. For the values of various physical constants involved in the calculations we have used the 2014 CODATA values, Ref. [24], and values from the Particle Data Group, Ref. [25].

Calculation of the Cancellation Radius for Charged Leptons using Formula (33):

$$\frac{\ln(16\mathrm{e})\alpha\hbar c}{2\pi}\, r_t^2 - \frac{3\,\pi G\hbar^2}{2c^2} = 0 \tag{A1}$$

$A{:=}N[\ln(16\mathrm{e})\,(7.2973525664000000000000000 \times 10^{-3})(1.0545718000000000000000000 \times 10^{-34})$
$\quad \times\,(2.9979245800000000000000000 \times 10^{8})/(2\pi), 26];$
$B{:=}N[(3\pi(6.6740800000000000000000000 \times 10^{-11})(1.0545718001391130000000000 \times 10^{-34})^2)/$
$\quad (2\,(2.9979245800000000000000000 \times 10^{8})^2), 26];$
$N[\mathrm{Solve}[A * r_t^2 - B == 0, r_t], 26]$ //Last
$\{r_t \rightarrow 5.30042667383830634827564 \times 10^{-34}\}$

Calculation of Radius r_e of Electron and Positron

$$\frac{\ln(16\mathrm{e})\alpha\hbar c}{2\pi} - \frac{3\pi G\hbar^2}{2c^2 r_e^2} - m_e c^2 r_e = 0 \tag{A2}$$

$\mathrm{C1}{:=}N[(9.1093835600000000000000000 \times 10^{-31})((2.9979245800000000000000000 \times 10^{8})^2), 26];$
$N\left[\mathrm{Solve}\left[\left(A - B \big/ \left(r_e^2\right)\right) - \mathrm{C1} * r_e == 0, r_e\right], 26\right]$ //Last
$\{r_e \rightarrow 5.30042667383830634910588 \times 10^{-34}\}$

Calculation of Radius r_μ of Muon and Anti-Muon

$$\frac{\ln(16\mathrm{e})\alpha\hbar c}{2\pi} - \frac{3\pi G\hbar^2}{2c^2 r_\mu^2} - m_\mu c^2 r_\mu = 0 \tag{A3}$$

C2:=$N[(1.883531594000000000000000 * 10^{-28})((2.997924580000000000000000 * 10^{8})^{2}), 26]$;
$N\left[\text{Solve}\left[\left(-A + B\,/(r_\mu^2)\right) - \text{C2} * r_\mu == 0, r_\mu\right], 26\right]$ //Last
$\{r_\mu \to 5.300426673838306519941 61 \times 10^{-34}\}$

Calculation of Radius r_τ of Tau and Anti-Tau

$$\frac{\ln(16\mathrm{e})\alpha\hbar c}{2\pi} - \frac{3\,\pi G\hbar^2}{2c^2 r_\tau^2} - m_\tau c^2 r_\tau = 0 \tag{A4}$$

C3:=$N[(3.167470000000000000000000 \times 10^{-27})((2.997924580000000000000000 * 10^{8})^{2}), 26]$;
$N\left[\text{Solve}\left[\left(-A + B\,/\left(r_\tau^2\right)\right) - \text{C3} * r_\tau == 0, r_\tau\right], 26\right]$ //Last
$\{r_\tau \to 5.3004266738383092351228 7\ \times 10^{-34}\}$

Calculation of the Cancellation Radius for (2e/3) Quarks using Formula (50):

$$\frac{4\,\ln(16\mathrm{e})\alpha\hbar c}{9(2\pi)} + \frac{\alpha_s\hbar c}{3} - \frac{3\pi G\hbar^2}{2c^2 r_{qt}^2} = 0. \tag{A5}$$

D:=$N[(1/3)(1/10)(1.054571800000000000000000 \times 10^{-34})(2.997924580000000000000000 \times 10^{8}), 26]$;
$N\left[\text{Solve}[((4/9)A + D) * r_{qt}^2 - B == 0, r_{qt}], 26\right]$ //Last
$\{r_{qt} \to 1.8679052501370808562855 9\ \times 10^{-34}\}$

Calculation of Radius r_{tq} of Top Quark:

$$\frac{4\ln(16\mathrm{e})\alpha\hbar c}{9(2\pi)} + \frac{\alpha_s\hbar c}{3} - \frac{3\pi G\hbar^2}{2c^2 r_{tq}^2} - m_t c^2 r_{tq} = 0 \tag{A6}$$

E:=$N[(3.087700000000000000000000 \times 10^{-25})((2.997924580000000000000000 \times 10^{8})^{2}), 26]$;
$N[\text{Solve}[\left((4/9)A + D - B\,/\left(r_{tq}^2\right)\right) - \text{E} * r_{tq} == 0, r_{tq}], 26]$ //Last
$\{r_{tq} \to 1.8679052501370851966124 6\ \times 10^{-34}\}$

Appendix B: Miscellaneous Derivations for S-matrix Evaluation

Since the initial and final momenta are the same in the rest frame, the Dirac delta function can be split into space and time components, and then, together with $E = m = \omega = 2\pi/t$, where ω is angular frequency, it evaluates to

$$\delta^4(p_f - p_i) = \delta^4(m, 0) = \int \frac{d^3\mathbf{r}}{(2\pi)^3}\, e^{i\cdot(\mathbf{p}=0)\cdot\mathbf{r}} \int \frac{dt}{2\pi}\, e^{iEt} = \frac{r^3\, t}{(2\pi)^4}, \tag{B1}$$

where $\mathbf{r}$ is a 3-vector and the time-dependence of the exponential is unity.

The derivation for eq.(13) in the rest frame is as follows. From eqs.(6) and (11) we have

$$
\begin{aligned}
S^{(2)}_{fi}(E) &= -i(2\pi)^4\delta^4(p_f-p_i)\sqrt{\frac{m^2}{E_iE_f}}\frac{1}{r^3}\bar{u}(p_f,s_f)\left(\frac{3m\alpha}{2\pi}\ln\frac{\Lambda}{m}\right)u(p_i,s_i),\\
&= -i(2\pi)^4\delta^4(m,\,0)\frac{1}{r^3}\bar{u}(m)\left(\frac{3m\alpha}{2\pi}\ln\frac{\Lambda}{m}\right)u(m),\\
&= -i(2\pi)^4\frac{r^3\,t}{(2\pi)^4}\frac{1}{r^3}\bar{u}(m)\left(\frac{3m\alpha}{2\pi}\ln\frac{\Lambda}{m}\right)u(m),\\
&= -it\left(\frac{3m\alpha}{2\pi}\ln\frac{\Lambda}{m}\right)\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&0&0\\0&0&0&0\end{pmatrix},\\
&= -it\left(\frac{3\alpha}{2\pi\,r}\ln\frac{\Lambda}{m}\right)\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&0&0\\0&0&0&0\end{pmatrix}.
\end{aligned}
\tag{B2}
$$

For use in deriving eq.(B7) below, we begin with the fermion anticommutator,

$$
\{b_i(p),b^\dagger_j(p')\} = \{d_i(p),d^\dagger_j(p')\} = (2\pi)^3\frac{E}{m}\delta_{ij}\delta^3(\mathbf{p}-\mathbf{p}'),
\tag{B3}
$$

and since there is only one electron and no positron in both the initial and final states for the rest frame and $E_i = E_f = p_i = p_f = m = \omega = 2\pi/t$, we have

$$
\begin{aligned}
\psi(0,t)\mid i\rangle &= \sqrt{\frac{m}{E_ir^3}}\psi(0,t)b^\dagger_i(p_i)\mid 0\rangle\\
&= \sqrt{\frac{1}{r^3}}\int\frac{d^3p}{(2\pi)^3}\sum_{j=1}^{2} b_j(m)u^j(m)e^{-iEt}b^\dagger_i(m)\mid 0\rangle\\
&= \sqrt{\frac{1}{r^3}}\int\frac{d^3p}{(2\pi)^3}\sum_{j=1}^{2} u^j(m)e^{-iEt}\{b_j(m),b^\dagger_i(m)\}\mid 0\rangle\\
&= \sqrt{\frac{1}{r^3}}\,u^i(m)e^{-iEt},
\end{aligned}
$$

and similarly

$$
\begin{aligned}
\langle f\mid\bar{\psi}(0,t) &= \sqrt{\frac{1}{r^3}}\,\bar{u}^f(m)e^{+iEt},\\
\psi(0,t)\mid i\rangle &= \sqrt{\frac{1}{r^3}}\,v^i(m)e^{+iEt},\\
\langle f\mid\bar{\psi}(0,t) &= \sqrt{\frac{1}{r^3}}\,\bar{v}^f(m)e^{-iEt},
\end{aligned}
\tag{B4}
$$

in which the fermion anticommutator eq.(B3) was used for simplification.

The derivation for eq.(17) in the text is as follows. We begin with eq.(16),

$$
\mathrm{h}_I(t) = -\frac{3\kappa}{16}(\bar{\psi}(t)\gamma^5\gamma_0\psi(t))(\bar{\psi}(t)\gamma^5\gamma^0\psi(t)),
\tag{B5}
$$

and substitute it into the first order S-Matrix, remembering that we are working in the rest frame:

$$
S^{(1)}_{fi}(E1) = i\frac{3\kappa}{16}\int d^4x\,\langle f\mid(\bar{\psi}(t)\gamma^5\gamma_0\psi(t))(\bar{\psi}(t)\gamma^5\gamma^0\psi(t))\mid i\,\rangle.
\tag{B6}
$$

Switching to momentum space and using eq.(B4) above, the S-matrix expression works out to give

$$
\begin{aligned}
S^{(1)}_{fi}(E1) &= i\frac{3\kappa}{16\,r^6}\int d^4\,p\;\left(\bar{u}^f(m)e^{+imt}\,\gamma^5\gamma_0\,v^i(m)e^{+imt}\,\bar{v}^f(m)e^{-imt}\,\gamma^5\gamma^0\,u^i(m)e^{-imt}\right)\\
&= i\frac{3\kappa}{16\,r^6}\int d^4\,p\;\bar{u}^f(m)\,\gamma^5\gamma_0\,(-1)\begin{pmatrix}0&0&0&0\\0&0&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix}\gamma^5\gamma^0\,u^i(m)\\
&= i(2\pi)^4\frac{3\kappa}{16\,r^6}\delta^4(m,0)\,(1)\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&0&0\\0&0&0&0\end{pmatrix}\\
&= i\frac{3\kappa\,r^3t}{16\,r^6}\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&0&0\\0&0&0&0\end{pmatrix}\\
&= i\,t\,\frac{3\kappa}{16\,r^3}\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&0&0\\0&0&0&0\end{pmatrix},
\end{aligned}
\tag{B7}
$$

where we have taken $\delta^4(m,0) = r^3t/(2\pi)^4$ as shown above. Similarly, the S-matrix expression for the anti-fermion works out to give

$$
\begin{aligned}
S^{(1)}_{fi}(E3) &= i\frac{3\kappa}{16\,r^6}\int d^4\,p\;\left(\bar{v}^f(m)e^{-imt}\,\gamma^5\gamma_0\,u^i(m)e^{-imt}\,\bar{u}^f(m)e^{+imt}\,\gamma^5\gamma^0\,v^i(m)e^{+imt}\right)\\
&= i\frac{3\kappa}{16\,r^6}\int d^4\,p\;\bar{v}^f(m)\,\gamma^5\gamma_0\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&0&0\\0&0&0&0\end{pmatrix}\gamma^5\gamma^0\,v^i(m)\\
&= i(2\pi)^4\frac{3\kappa}{16\,r^6}\delta^4(m,0)(-1)\begin{pmatrix}0&0&0&0\\0&0&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix}\\
&= -i\,t\,\frac{3\kappa}{16\,r^3}\begin{pmatrix}0&0&0&0\\0&0&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix}.
\end{aligned}
\tag{B8}
$$

From the last two equations it is easy to see that the fermion and anti-fermion S-matrix equations are coupled similar to that in the semi-classical derivation.

Appendix C: Semi-Classical Derivation of Charged Fermion Self-Energy

From eq.(22), in the rest frame, with natural units $\hbar = c = 1$, we have

$$
i\,\gamma^0\frac{\partial\psi}{\partial t} + qA_0\,\gamma^0\psi = m\,\psi + \frac{3\kappa}{8}\left(\bar{\psi}\gamma^5\gamma_0\psi\right)\gamma^5\gamma^0\psi\,,
\tag{C1}
$$

which can be further simplified to

$$i\begin{pmatrix} +\frac{\partial\psi_1}{\partial t} \\ \\ +\frac{\partial\psi_2}{\partial t} \\ \\ -\frac{\partial\psi_3}{\partial t} \\ \\ -\frac{\partial\psi_4}{\partial t}\end{pmatrix} + qA_0\begin{pmatrix} +\psi_1 \\ \\ +\psi_2 \\ \\ -\psi_3 \\ \\ -\psi_4\end{pmatrix} = m\begin{pmatrix} +\psi_1 \\ \\ +\psi_2 \\ \\ +\psi_3 \\ \\ +\psi_4\end{pmatrix} - \frac{3\kappa}{8}\left\{\psi_1^*\psi_3 + \psi_2^*\psi_4 + \psi_1\psi_3^* + \psi_2\psi_4^*\right\}\begin{pmatrix} -\psi_3 \\ \\ -\psi_4 \\ \\ +\psi_1 \\ \\ +\psi_2\end{pmatrix}, \tag{C2}$$

where we have used

$$\gamma^0 = \begin{pmatrix} +1 & 0 & 0 & 0 \\ 0 & +1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1\end{pmatrix} \qquad \text{and} \qquad \gamma^5 = \begin{pmatrix} 0 & 0 & +1 & 0 \\ 0 & 0 & 0 & +1 \\ +1 & 0 & 0 & 0 \\ 0 & +1 & 0 & 0\end{pmatrix}. \tag{C3}$$

If we now represent the particles and anti-particles with two-component spinors ψ_a and ψ_b, respectively [21], where

$$\psi_a := \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} \quad \text{and} \quad \psi_b := \begin{pmatrix} \psi_3 \\ \psi_4 \end{pmatrix} \tag{C4}$$

are the two-component spinors constituting the four-component Dirac spinor, then the above equation can be written as two *coupled* partial differential equations:

$$+i\,\frac{\partial\psi_a}{\partial t} + qA_0\,\psi_a = m\,\psi_a + \frac{3\kappa}{8}\left\{\psi_1^*\psi_3 + \psi_2^*\psi_4 + \psi_1\psi_3^* + \psi_2\psi_4^*\right\}\,\psi_b \tag{C5}$$

$$-i\,\frac{\partial\psi_b}{\partial t} - qA_0\,\psi_b = m\,\psi_b - \frac{3\kappa}{8}\left\{\psi_1^*\psi_3 + \psi_2^*\psi_4 + \psi_1\psi_3^* + \psi_2\psi_4^*\right\}\,\psi_a\,. \tag{C6}$$

Unlike the case in Dirac equation, these equations for the spinors ψ_a and ψ_b are coupled equations even in the rest frame. They decouple in the limit when the torsion-induced axial-axial self-interaction is negligible. On the other hand, at low energies it is reasonable to assume that, in analogy with the Dirac spinors in flat spacetime, the above two-component spinors for free particles decouple in the rest frame, admitting plane wave solutions of the form

$$\psi_a(t) = \sqrt{\frac{1}{V}}\,e^{-iEt}\psi_a(0) \qquad \text{and} \qquad \psi_b(t) = \sqrt{\frac{1}{V}}\,e^{+iEt}\psi_b(0), \tag{C7}$$

where $E = m = \omega = 2\pi/t$ in the rest frame as specified for eq.(B1). We now note that in the rest frame the derivative term in the eqs. (C5) and (C6) vanishes since ψ is constant, because in the rest frame the probability of finding the particle in a given volume at time t is one. Moreover $E = 2\pi/t$ gives

$$\psi_a(t) = \psi_b(t). \tag{C8}$$

Consequently eqs. (C5) and (C6) uncouple and simplify to

$$+qA_0\,\psi_a = m\,\psi_a + \frac{3\kappa}{8}\left\{\psi_1^*\psi_1 + \psi_2^*\psi_2 + \psi_3\psi_3^* + \psi_4\psi_4^*\right\}\,\psi_a\,, \tag{C9}$$

$$-qA_0\,\psi_b = m\,\psi_b - \frac{3\kappa}{8}\left\{\psi_1^*\psi_1 + \psi_2^*\psi_2 + \psi_3\psi_3^* + \psi_4\psi_4^*\right\}\,\psi_b\,. \tag{C10}$$

Substituting the ψ's from eq. (C7) and simplifying reduces eqs. (C9) and (C10) to the following pair of equations:

$$+qA_0\psi_a(0) = m\,\psi_a(0) + \frac{3\kappa}{8}|\psi(0)|^2\,\psi_a(0) \tag{C11}$$

$$\text{and} \quad -qA_0\psi_b(0) = m\,\psi_b(0) - \frac{3\kappa}{8}\,|\psi(0)|^2\,\psi_b(0)\,. \tag{C12}$$

And since $|\psi(0)|^2 = 1/r^3$ in the rest frame, these equations can be further simplified to

$$+\,qA_0 = m + \frac{3\kappa}{8\,r^3}\,, \tag{C13}$$

$$\text{and} \quad -\,qA_0 = m - \frac{3\kappa}{8\,r^3}\,. \tag{C14}$$

Substituting in natural units for the scalar field $A_0 = V = q/(r)$ in the Lorentz gauge (where V is the electric potential), and for $\kappa = 8\pi G$, we finally arrive at our central equations, for any electroweak fermion of charge q and mass m and its anti-particle in the Riemann-Cartan spacetime:

$$\frac{q^2}{r} - \frac{3\pi G}{r^3} = +m \tag{C15}$$

$$\text{and} \quad \frac{q^2}{r} - \frac{3\pi G}{r^3} = -m, \tag{C16}$$

where r is the radial distance from q and the two equations correspond to the particle and anti-particle, respectively. However, since the anti-fermion must satisfy $(\not{p} + m)v(\mathbf{p}) = 0$, the sign on the mass term must be changed giving for both fermion and anti-fermion

$$\frac{q^2}{r} - \frac{3\pi G}{r^3} = m\,. \tag{C17}$$

Finally, replacing q with electron or positron charge and replacing $\hbar$ and c gives us eq. (24) as our central equation for the semi-classical evaluation:

$$\frac{\alpha\hbar c}{r} - \frac{3\kappa(\hbar c)^2}{8\,r^3} = mc^2. \tag{C18}$$

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