Vacuum torsion and regular accelerating Universe without dark matter

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Abstract

The simplest gauge gravitation theory in Riemann-Cartan space-time leading to the solution of the problem of cosmological singularity and dark energy problem is investigated with purpose to solve the dark matter problem. It is shown that the interaction of the vacuum torsion with proper angular momentums of gravitating objects can lead to appearance at astrophysical scale (galaxies, galactic clusters) additional force of gravitational attraction, which in the frame of standard theory is associated with dark matter.

Keywords: Riemann-Cartan space-time continuum, regular accelerating Universe, dark energy, dark matter
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1. Introduction

The application of the local gauge invariance principle, which is one of the most important principles of modern theory of fundamental physical interactions, generally speaking leads to generalization of metric gravitation theory (MGT). MGT can be introduced in the frame of gauge approach by considering the 4-translations group as gauge group [1, 2]. By including the Lorentz group into gauge group corresponding to gravitational interaction we obtain gravitation theory in Riemann-Cartan space-time (GTRC) [3, 4, 5] [1]. It should be noted that GTRC but not MGT corresponds to supergravity theory, because the supergravity gauge group includes the Lorentz group. If GTRC is natural generalization of MGT, the question arose about the possibility of the solution of principal problems of general relativity theory (GR) on the base of GTRC, at first of all the problem of the beginning of the Universe in time and problem of dark components of the Universe - dark energy and dark matter. In the frame of GTRC based on sufficiently general expression of gravitational Lagrangian $\mathcal{L}_g$ including both a scalar curvature and terms quadratic in the curvature and torsion tensors with indefinite parameters it was shown that by certain restrictions on these parameters GTRC allows to solve the problem of the beginning of the Universe in time (problem of cosmological singularity) and to explain accelerating cosmological expansion at present epoch without using the notion of dark energy. It is because GTRC leads to the change of gravitational interaction by certain conditions in comparison with GR: gravitational interaction has repulsive character at extreme conditions (extremely high energy densities and pressures) where limiting energy (mass) density appears [12] and in situation when

energy density is very small and the vacuum gravitational repulsion effect is essential [13, 14]. These physical results were obtained on the base of investigations of isotropic cosmology built in the frame of GTRC (see e.g. [12, 13, 14, 15, 16, 17]). The physical significance of space-time torsion in dynamics of gravitating systems was found out. In particular, the principal role of torsion is connected with the fact that physical space-time in the vacuum has the structure of Riemann-Cartan continuum with de Sitter metric (but not Minkowski space-time) [14].

The present letter is devoted to investigation of dark matter problem on the base of GTRC: is it possible to explain phenomena associated in the frame of standard theory with dark matter as a result of the change of gravitational interaction at astrophysical scale in comparison with GR? Our consideration will be realized in the frame of so-called minimum GTRC - the simplest GTRC, which allows to solve the problem of cosmological singularity and dark energy problem.

2. Minimum gauge gravitation theory in Riemann-Cartan space-time and vacuum torsion

The minimum GTRC [17, 18] was introduced as a result of investigations of GTRC based on general expression of gravitational Lagrangian $\mathcal{L}_g$ including both a scalar curvature F, six invariants quadratic in the curvature tensor $F^{ik}_{\ \mu\nu}$ and three invariants quadratic in the torsion tensor $S^i_{\ \mu\nu}$ with indefinite parameters [2]. Restrictions on indefinite parameters of $\mathcal{L}_g$ were obtained, by which the physical consequences of GTRC are the most satisfactory. As a result the following expression of $\mathcal{L}_g$ of

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[1]Now there are many works dedicated to investigations of GTRC, which is known in literature as Poincaré gauge theory of gravity (see e.g. [6, 7, 8, 9, 10, 11] and Refs herein).

[2]The definitions and notations of our previous papers (see e.g. [13, 17, 18]) are used below. With the purpose to make quantitative estimations the light velocity c is conserved in formulas.

minimum GTRC was found:

$$\mathcal{L}_g = f_0 F + f_5 F^{\mu\nu}F_{\nu\mu} + f_6 F^2$$
$$+ S^{\alpha\mu\nu}(a_1 S_{\alpha\mu\nu} + a_2 S_{\nu\mu\alpha}) + a_3 S^\alpha_{\ \mu\alpha}S_\beta^{\ \mu\beta}, \tag{1}$$

where $f_0 = \frac{c^4}{16\pi G}$ (G is Newton's gravitational constant) and parameters f_5, f_6 and a_k ($k = 1, 2, 3$) are expressed in terms of parameters of isotropic cosmology: $b = a_2 - a_1$, $\alpha = \frac{f_5 + 3f_6}{3f_0^2} > 0$ and $\omega = \frac{f_5}{f_5 + 3f_6}$ [12, 18]:

$$a_1 = b, \qquad a_2 = 2b, \qquad a_3 = -\frac{4}{3}b,$$
$$f_5 = 3f_0^2\alpha\omega, \qquad f_6 = f_0^2\alpha(1 - \omega). \tag{2}$$

The correspondence principle with GR will be fulfilled if parameters b and ω satisfy the following restrictions: $0 < 1 - \frac{b}{f_0} \ll 1$, $0 < \omega \ll 1$ and the value of α^{-1} corresponds to some high energy density (see below).

Gravitational equations of minimum GTRC have the following form:

$$\nabla_\nu U_i^{\ \mu\nu} + 2S^k_{\ i\nu}U_k^{\ \mu\nu} + 2(f_0 + 2f_6 F)F^\mu_{\ i}$$
$$+2f_5(F_{ki}F^{\mu k} + F^\mu_{\ kim}F^{mk}) - h_i^{\ \mu}(f_0 F + f_5 F^{\mu\nu}F_{\nu\mu} + f_6 F^2$$
$$+ S^{\alpha\mu\nu}\left(a_1 S_{\alpha\mu\nu} + a_2 S_{\nu\mu\alpha}\right) + a_3 S^\alpha_{\ \mu\alpha}S_\beta^{\ \mu\beta}) = -T_i^{\ \mu}, \tag{3}$$

$$4\nabla_\nu[(f_0/2 + f_6 F)h_{[i}^{\ \nu}h_{k]}^{\ \mu} +$$
$$+ f_5 F^{[\mu}_{\ [k}h_{i]}^{\ \nu]}] + U_{[ik]}^{\ \ \mu} = -J_{[ik]}^{\ \ \mu}, \tag{4}$$

where $U_i^{\ \mu\nu} = 2(a_1 S_i^{\ \mu\nu} - a_2 S^{[\mu\nu]}_{\ \ \ i} - a_3 S_\alpha^{\ \alpha[\mu}h_i^{\ \nu]})$, ∇_ν denotes the covariant operator having the structure of the covariant derivative defined in the case of tensor holonomic indices by means of Christoffel coefficients and in the case of tetrad tensor indices by means of anholonomic Lorentz connection.

As it was shown in [18], if $0 < \omega \ll 1$ in the case of spinless matter with energy densities, which are much less than limiting energy density, equations (3)-(4) lead to equations for metric in the form of Einstein gravitational equations with effective cosmological constant

$$G^\mu_{\ \lambda} = -\frac{1}{2b}\left[T_\lambda^{\ \mu} + \delta^\mu_\lambda \frac{(1 - \frac{b}{f_0})^2}{12\alpha}\right], \tag{5}$$

where $G^\mu_{\ \lambda}$ is Einstein tensor. The influence of torsion appears in eq. (5) via formation of effective cosmological constant and the change of gravitational constant. We see that the correspondence principle with GR will be fulfilled if parameters b satisfies the condition $0 < 1 - \frac{b}{f_0} \ll 1$ and the value of α^{-1} corresponds to some high energy density, by which effective cosmological constant ensures observed accelerating cosmological expansion.

As it follows from equations of isotropic cosmology the effective cosmological constant in equations (5) has the vacuum origin. Let us illustrate this fact. Any homogeneous isotropic model (HIM) in Riemann-Cartan space-time is described by three functions of time: the scale factor of Robertson-Walker

metric $R(t)$ and two torsion functions $S_1(t)$ and $S_2(t)$. Cosmological equations generalizing Friedmann cosmological equations of GR take the form [12]

$$\frac{k}{R^2} + (H - 2S_1)^2 - S_2^2 =$$
$$\frac{1}{6f_0 Z}\left[\rho c^2 - 6bS_2^2 + \frac{\alpha}{4}\left(\rho c^2 - 3p - 12bS_2^2\right)^2\right], \tag{6}$$

$$\dot{H} - 2\dot{S}_1 + H(H - 2S_1) =$$
$$-\frac{1}{12f_0 Z}\left[\rho c^2 + 3p - \frac{\alpha}{2}\left(\rho c^2 - 3p - 12bS_2^2\right)^2\right], \tag{7}$$

where $H = \dot{R}/R$ is the Hubble parameter (a dot denotes the differentiation with respect to $x^0 = ct$), $k = +1, 0, -1$ for closed, flat and open models respectively, ρ is mass density, p is pressure and $Z = 1 + \alpha\left(\rho c^2 - 3p - 12bS_2^2\right)$. The torsion functions S_1 and S_2 are

$$S_1 = -\frac{\alpha}{4Z}[\dot{\rho}c^2 - 3\dot{p} + 12f_0\omega HS_2^2 - 12(2b - \omega f_0)S_2\dot{S}_2], \tag{8}$$

$$S_2^2 = \frac{\rho c^2 - 3p}{12b} + \frac{1 - (b/2f_0)(1 + \sqrt{X})}{12b\alpha(1 - \omega/4)}, \tag{9}$$

where

$$X = 1 + \omega(f_0^2/b^2)[1 - (b/f_0) - 2(1 - \omega/4)\alpha(\rho c^2 + 3p)] \geq 0. \tag{10}$$

The dynamics of HIM depends essentially on torsion functions. The presence of $\sqrt{X}$ in (10) leads to appearance of limiting energy density of order $(\omega\alpha)^{-1}$ and secures regular behaviour of HIM at extreme conditions, and the presence of constant term in S_2^2 - vacuum torsion - induces the cosmological constant at asymptotics. By taking into account restrictions for parameters ω and b the expression (9) for S_2^2 at asymptotics takes the form:

$$S_2^2 = \frac{1}{12b}\left[\rho c^2 - 3p + \frac{1 - b/f_0}{\alpha}\right], \tag{11}$$

and as a result cosmological equations (6)-(7) at asymptotics are:

$$\frac{k}{R^2} + H^2 = \frac{1}{6b}\left[\rho c^2 + \frac{1}{4\alpha}\left(1 - \frac{b}{f_0}\right)^2\right], \tag{12}$$

$$\dot{H} + H^2 = -\frac{1}{12b}\left[(\rho c^2 + 3p) - \frac{1}{2\alpha}\left(1 - \frac{b}{f_0}\right)^2\right]. \tag{13}$$

Unlike standard ΛCDM-model cosmological constant appears in (12)-(13) as a result of solution of gravitational equations for HIM that leads to the change of gravitational interaction when energy density is small and comparable with cosmological constant - the vacuum gravitational repulsion effect. This effect appears at cosmological scale, and it is negligibly small at astrophysical scale and locally (Solar system).

The influence of gravitating vacuum in GTRC can be manifested not only via accelerating cosmological expansion at

present epoch. The fact of the matter is that the vacuum torsion function $S_2^{2(vac)}$, which according to (11) is

$$S_2^{2(vac)} = \frac{1 - b/f_0}{12b\alpha},\qquad(14)$$

can be essential quantitatively at newtonian approximation. The vacuum torsion function $|S_1|$ and the vacuum value of H are negligibly small in comparison with $|S_2^{(vac)}|$. Owing to this the curvature tensor [13] has the following vacuum components, which can be important at newtonian approximation:

$$F^{12}{}_{12} = F^{13}{}_{13} = F^{23}{}_{23} = -S_2^{2(vac)}.\qquad(15)$$

It is easy to show by using [18] that according to eq. (4) corrections of the torsion components $S_{\alpha\mu\nu}$ ($\alpha, \mu, \nu = 1, 2, 3$, $\alpha \neq \mu$, $\alpha \neq \nu$) and consequently curvature components (15) at asymptotics in the case of matter with spin are not essential at newtonian approximation. It should be noted that though the formula (14) is obtained in cosmological concomitant system of reference, formulas (14) and (15) are valid by transition to other systems of reference because $S_2^2 = -\frac{1}{6}S_{\alpha\mu\nu}S^{\alpha\mu\nu}$ ($\alpha, \mu, \nu = 1, 2, 3$; $\alpha \neq \mu$, $\alpha \neq \nu$).

3. Vacuum torsion and gravitational interaction at astrophysical scale

Lets consider the interaction of proper angular momentums of astrophysical objects (stars in galaxies, galaxies in galactic clusters) with vacuum torsion by using equations of motion of particle with momentum in Riemann-Cartan space-time [19] generalizing Papapetrou's equations for rotating particle in GR [20] [3]. In the case of rotating particle with angular velocity tensor Ω_{ik} corresponding equations of motion, by conserving terms, which are essential at non-relativistic approximation, are:

$$\frac{DP_i}{d\tau} = \frac{1}{2}I\Omega_{mn}F^{mn}{}_{il}v^l \qquad (i, l, m, n = 1, 2, 3),\qquad(16)$$

where $\frac{D}{d\tau}$ denotes riemannian absolute derivative with respect to proper time τ, P_i is generalized momentum, I is inertia momentum and v^l is velocity of particle. In non-relativistic approximation $P_i = mv_i$ (m is particle mass) and $\Omega_{mn} = const$.

We will consider the circular motion of rotating particle in spherically symmetric gravitational field created by mass M in newtonian approximation. By taking into account that $g_{00} = 1 + \frac{2\phi}{c^2}$ (ϕ is newtonian potential), components of angular velocity $\Omega_i = \epsilon_{ikl}\Omega^{kl}$ and relation (15) we obtain in the case of motion in plane XOY (centrum of mass M is in origin of coordinates, vector of orbital angular momentum is directed along the axe OZ) equation of motion in usual form $m\frac{d\mathbf{v}}{dt} = \mathbf{F}$ with the following expression of the force vector:

$$\mathbf{F} = -m\frac{d\phi}{d\mathbf{r}} + I\Omega_3 S_2^{2(vac)}v\frac{\mathbf{r}}{r}.\qquad(17)$$

If $\Omega_3 < 0$ the force (17) includes besides Newtonian term additional force of attraction:

$$F = G\frac{mM}{r^2} + I\Omega S_2^{2(vac)}v,\qquad(18)$$

where $\Omega = |\Omega_3|$. By taking into account that the force (17) is centripetal force we obtain the following dependence of velocity on distance from centrum and parameters of particle and gravitational field:

$$v = \frac{I}{2m}\Omega S_2^{2(vac)}r + \left[(\frac{I}{2m}\Omega S_2^{2(vac)}r)^2 + \frac{GM}{r}\right]^{\frac{1}{2}}.\qquad(19)$$

We see that interaction of proper angular momentum with vacuum torsion leads to terms growing with distance in expression of velocity (19). This allows to explain the behaviour of rotational curves in galaxies. By given parameters of particle and gravitational field the velocity (19) depends on value of parameter $x = 1 - \frac{b}{f_0}$. By taking into account that average mass density in the Universe at present epoch $\rho_1 = \frac{x^2}{4c^2\alpha}$ is of order $10^{-26}\frac{\text{kg}}{\text{m}^3}$, we obtain that at the first approximation

$$S_2^{2(vac)} = \frac{16\pi G}{3c^2 x}\rho_1 \sim \frac{10^{-52}}{x}(\text{m}^{-2}).\qquad(20)$$

Lets demonstrate the behaviour of rotational curves numerically by applying obtained relation (19) for hypothetical galaxy similar to Andromeda by choosing particle parameters as for star similar to Solar: $I/m \sim 10^{18}$ m^2, $\Omega \sim 0.5 \cdot 10^{-6}$ s^{-1}; the mass M is taken $M = 2 \cdot 10^{41}$ kg; the parameter $x = 10^{-25}$ and consequently $S_2^{2(vac)} = 10^{-27}(\text{m}^{-2})$ that corresponds to high energy density scale $\alpha^{-1} = 10^7\rho_{nucl}c^2$ (ρ_{nucl} is nuclear mass density). As numerical analysis shows at distances $r < 9$ kpc (1kpc= 0, 3086$\cdot 10^{20}$m) Newtonian term in (19) plays the definitive role, by growth of r from 9 kpc to 25 kpc the velocity v according to Newtonian law decreases from $219 \cdot 10^3$ km/s to $132 \cdot 10^3$ km/s, but according to (19) the velocity v changes only from $256 \cdot 10^3$ km/s to $259 \cdot 10^3$ km/s. By further increase of r essential growth of velocity v takes place according to (19); this effect can be observed in galactic clusters, where we deal with vast space scale of order 10Mpc and more. At the same time this effect can explain why in the frame of standard theory the motion of Dwarf galaxies in galactic clusters is subjected to dark matter, but dark matter does not have an effect on motion in the interior of Dwarf galaxies. If we take into account that linear dimensions of Solar system compose small part of parsec, effects discussed above at galactic scale become negligibly small in Solar system.

Investigation of influence of vacuum torsion on propagation of electromagnetic waves is interesting in connection with observations of gravitational lensing. Such influence is possible if gravitational interaction with electromagnetic field in the frame of GTRC is given by using the minimal coupling (replacing in Lagrangian of electromagnetic field written in Minkowski space-time of partial derivatives via covariant derivatives defined with help of total connection). Because the minimal coupling leads to violation of gauge invariance in the case of electromagnetic field by conservation of electric charge, gravitational interaction in the frame of GTRC generally is specified

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[3]In [19] the curvature tensor was defined with opposite sign and signature (+2) was used.

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similar to GR by using Christoffel coefficients as connection. However, the minimal coupling rather leads in this case to partially gauge theory in terms of [21] that is in agreement with interaction hierarchy and provides an interesting results [10].

4. Conclusion

The physical vacuum in the frame of GTRC is gravitating system possessing the curvature and torsion and having the structure of Riemann-Cartan continuum with de Sitter metric. This conclusion was obtained in the frame of theory based on general expression of gravitational Lagrangian without using any restrictions on indefinite parameters of $\mathcal{L}_g$ [14]. Physical consequences of this result at first were obtained in the frame of isotropic cosmology, where the vacuum gravitational repulsion effect and connected with them accelerating cosmological expansion at present epoch were described. However, investigation of possible role of gravitating vacuum at astrophysical scale is difficult task through complexity of gravitational equations of GTRC. The situation is simplifying, when minimum GTRC was determined. It is because this theory leads to Einstein gravitation equations for metric, which are valid for spinless gravitating systems at wide range of energy density - from extremely high energy densities defined by α^{-1} to energy densities, which are several order greater than average energy density in the Universe at present epoch. This means that Newton's law of gravitational attraction is valid for spinless matter at astrophysical scale in non-relativistic approximation. Corrections connected with spin effects of gravitating matter in the frame of GTRC locally are very small [22]. The possible principal role of gravitating vacuum in astrophysics was determined in this letter. As it is shown rotational curves in galaxies and gravitational phenomena in galactic clusters, for explanation of which the notion of dark matter was introduced in the frame of GR, can be explained in the frame of GTRC as a result of interaction of proper angular momentums of stars and galaxies with torsion of gravitating vacuum [4]. We see that physical phenomena associated in the frame of GR with dark matter as well as dark energy have in the frame of GTRC the vacuum origin. This differs our results from that proposed in the frame of MOND [23] and other alternative theories of gravity, where explanation of discussed phenomena is given by means of modified newtonian gravitational potential. Of course the research of rotational curves for real galaxies and study of motions in galactic clusters suppose considerable information relative to galaxies and galactic clusters. As a result values of parameters b and α can be defined, and only parameter ω, which determines the value of limiting energy density, limiting temperature and depending on them the state and physical processes in the Universe at the beginning of cosmological expansion remains undefined [5].

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[4]Because discussed phenomena quantitatively are essential at newtonian approximation, the search similar to [22] of possible experiments for observations of interaction of proper angular momentums of gyroscopes (similar to Pioneer 10 and Pioneer 11) with vacuum torsion takes on new significance.

[5]In the frame of our classical theory limiting energy density has to be less than the Planckian one. The existence of limiting energy density ensures the

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regular behaviour of all HIM, including inflationary cosmological models [15, 16].