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Relationship of gauge gravitation theory in Riemann-Cartan spacetime and general relativity theory

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Abstract. The simplest variant of gauge gravitation theory in Riemann-Cartan spacetime leading to the solution of the problem of cosmological singularity and dark energy problem is investigated. It is shown that this theory by certain restrictions on indefinite parameters of gravitational Lagrangian in the case of usual gravitating systems leads to Einstein gravitational equations with effective cosmological constant.

Keywords: modified gravity, gravity, dark energy theory

Contents

1 Introduction

The investigation of gauge gravitation theory in Riemann-Cartan spacetime (GTRC), which is a necessary generalization of metric gravitation theory in the framework of gauge approach by including the Lorentz group into the gauge group corresponding to gravitational interaction, shows that GTRC allows to solve some principal problems of general relativity theory (GR) by virtue of the change of gravitational interaction by certain physical conditions in the frame of GTRC in comparison with GR (see for example [1–3]). The change of gravitational interaction is provoked by more complicated structure of physical spacetime, namely by spacetime torsion. In the frame of GTRC the gravitational interaction in the case of usual gravitating matter with positive values of energy density and pressure can be repulsive. The effect of gravitational repulsion appears at extreme conditions when energy density and pressure are extremely high and also in case when energy density is very small and vacuum effect of gravitational repulsion is essential. This allows to solve the problem of cosmological singularity and to explain accelerating cosmological expansion at present epoch without using the notion of dark energy. Given data were obtained by study of isotropic cosmology built in the frame of GTRC based on general expression of gravitational Lagrangian including both a scalar curvature and quadratic in the curvature and torsion invariants with indefinite parameters by certain restrictions on these parameters. A physical cause of a change of gravitational interaction in GTRC is connected with the fact that torsion according to gravitational equations is function of energy density and pressure and together with energy-momentum tensor affects on spacetime metric. The torsion plays the principal role at extreme conditions by formation of limiting energy density for gravitating matter [4] and also leads to formation of effective cosmological constant at asymptotics of cosmological models by virtue of influence on physical spacetime in the vacuum having the structure of Riemann-Cartan continuum with de Sitter metric (but not Minkowski spacetime) [5].

The following question appears: what is possible role of the torsion in astrophysics in the case of usual gravitating systems, for which energy density is much smaller than limiting energy density [1] but greater than average energy density in the Universe at present epoch (or effective cosmological constant). It should be noted that one torsion function at asymptotics of cosmological models has the structure (see below) which can be essential quantitatively in newtonian approximation though the evolution of cosmological models at asymptotics

[1] We don't discuss here massive stars collapsing in GR. In the frame of GTRC the collapse is impossible if limiting energy density exists in the nature.

coincides practically with that of Friedmann cosmological models with cosmological constant. The study of this question is particular case of research of relationship of GTRC and GR.

This paper is devoted to investigation of relationship of GR and the simplest GTRC (minimum GTRC) which allows to build the theory of regular accelerating Universe.

2 Isotropic cosmology and minimum gauge gravitation theory in Riemann-Cartan spacetime

In the beginning lets introduce the base definitions and relations used in this paper. In the framework of GTRC the role of gravitational field variables play the orthonormalized tetrad $h^i{}_\mu$ and the Lorentz connection $A^{ik}{}_\mu$; corresponding field strengths are the torsion tensor $S^i{}_{\mu\nu}$ and the curvature tensor $F^{ik}{}_{\mu\nu}$ defined as

$$S^i{}_{\mu\,\nu} = \partial_{[\nu}\, h^i{}_{\mu]} - h_{k[\mu} A^{ik}{}_{\nu]}\,,$$

$$F^{ik}{}_{\mu\nu} = 2\partial_{[\mu} A^{ik}{}_{\nu]} + 2A^{il}{}_{[\mu} A^k{}_{|l\,|\nu]},$$

where holonomic and anholonomic spacetime coordinates are denoted by means of greek and latin indices respectively [2] . Isotropic cosmology in Riemann-Cartan spacetime investigated in a number of papers (see for example [1–6]) was built by using the gravitational Lagrangian given in the following sufficiently general form

$$\begin{aligned}\mathcal{L}_{\rm g} = f_0\, F + F^{\alpha\beta\mu\nu}\,(f_1\, F_{\alpha\beta\mu\nu} + f_2\, F_{\alpha\mu\beta\nu} + f_3\, F_{\mu\nu\alpha\beta}) + F^{\mu\nu}\,(f_4\, F_{\mu\nu} + f_5\, F_{\nu\mu}) \\ +f_6\, F^2 + S^{\alpha\mu\nu}\,(a_1\, S_{\alpha\mu\nu} + a_2\, S_{\nu\mu\alpha}) + a_3\, S^\alpha{}_{\mu\alpha} S_\beta{}^{\mu\beta},\end{aligned} \tag{2.1}$$

where $F_{\mu\nu} = F^\alpha{}_{\mu\alpha\nu}$, $F = F^\mu{}_\mu$, f_i $(i = 1, 2, \ldots, 6)$, a_k $(k = 1, 2, 3)$ are indefinite parameters, $f_0 = (16\pi G)^{-1}$, G is Newton's gravitational constant (the light speed in the vacuum $c = 1$). Gravitational equations of PGTG obtained from the action integral $I = \int (\mathcal{L}_g + \mathcal{L}_m)\, hd^4x$, where $h = \det\left(h^i{}_\mu\right)$ and $\mathcal{L}_m$ is the Lagrangian of gravitating matter, contain the system of 16+24 equations corresponding to gravitational variables $h^i{}_\mu$ and $A^{ik}{}_\mu$. By using minimal coupling of gravitational field with matter, the energy-momentum tensor $T_i{}^\mu = -\frac{1}{h}\frac{\delta\mathcal{L}_m}{\delta h^i{}_\mu}$ and spin momentum tensor $J_{[ik]}{}^\mu = -\frac{1}{h}\frac{\delta\mathcal{L}_m}{\delta A^{ik}{}_\mu}$ of gravitating matter manifest as sources of gravitational field in gravitational equations. Gravitational equations are complicated system of differential equations in partial derivatives with indefinite parameters f_i and a_k. Physical consequences depend essentially on restrictions on these parameters. Some of such restrictions were obtained by investigation of isotropic cosmology, notably the solution of cosmological problems mentioned previously was obtained by the following restrictions: $2a_1 + a_2 + 3a_3 = 0$ and $2f_1 - f_2 = 0$. Then cosmological equations and equations for torsion functions include three indefinite parameters: parameter $\alpha = \frac{f}{3f_0^2}$ $(f = f_1 + \frac{f_2}{2} + f_3 + f_4 + f_5 + 3f_6 > 0)$ with inverse dimension of energy density, parameter $b = a_2 - a_1$ with the same dimension as f_0 and dimensionless parameter $\omega = \frac{f_2 + 4f_3 + f_4 + f_5}{f}$. The value of α^{-1} corresponds to the scale of extremely high energy densities, by which the correspondence of GTRC to GR in linear approximation with respect to metric and torsion is violated [1], the parameter b has to satisfy the condition $0 < x = 1 - \frac{b}{f_0} \ll 1$ and for parameter ω we have the following

[2] Like our previous papers we will use notations corresponding to the following relation between holonomic connection $\Gamma^\lambda{}_{\mu\nu}$ and $A^{ik}{}_\mu$: $\Gamma^\lambda{}_{\mu\nu} = h_i{}^\lambda \partial_\nu\, h^i{}_\mu - h_{k\mu} A^{ik}{}_\nu$. Then the tensor $F^\rho{}_{\sigma\mu\nu} = h_i{}^\rho h_{k\sigma} F^{ik}{}_{\mu\nu} = 2\partial_{[\nu}\Gamma^\rho{}_{|\sigma\,|\mu]} + 2\Gamma^\rho{}_{\lambda[\nu}\Gamma^\lambda{}_{|\sigma\,|\mu]}$ has the opposite sign as compared with usually defined curvature tensor and $S^\lambda{}_{\mu\,\nu} = \Gamma^\lambda{}_{[\mu\nu]}$ (cf.[7, 8]). The signature of spacetime metric is (-2).

restriction: $0 < \omega < 2\frac{b}{f_0} \approx 2$, the value of ω is probably small $(0 < \omega \ll 1)$, however, it is not ruled out that $\omega \sim 1$. If parameter ω is small, there is the second scale of extremely high energy densities of order $(\omega\alpha)^{-1}$, which determines the value of limiting energy density.

Any homogeneous isotropic model (HIM) is described by three functions of time: the scale factor of Robertson-Walker metric $R(t)$ and two torsion functions $S_1(t)$ and $S_2(t)$. Cosmological equations generalizing Friedmann cosmological equations of GR take the form

$$\frac{k}{R^2} + (H - 2S_1)^2 - S_2^2 = \frac{1}{6f_0 Z}\left[\rho - 6bS_2^2 + \frac{\alpha}{4}\left(\rho - 3p - 12bS_2^2\right)^2\right], \tag{2.2}$$

$$\dot{H} - 2\dot{S}_1 + H(H - 2S_1) = -\frac{1}{12f_0 Z}\left[\rho + 3p - \frac{\alpha}{2}\left(\rho - 3p - 12bS_2^2\right)^2\right], \tag{2.3}$$

where $H = \dot{R}/R$ is the Hubble parameter (a dot denotes the differentiation with respect to time), $k = +1, 0, -1$ for closed, flat and open models respectively, ρ is energy density, p is pressure and $Z = 1 + \alpha\left(\rho - 3p - 12bS_2^2\right)$. The torsion functions S_1 and S_2 are

$$S_1 = -\frac{\alpha}{4Z}[\dot{\rho} - 3\dot{p} + 12f_0\omega H S_2^2 - 12(2b - \omega f_0)S_2\dot{S}_2]. \tag{2.4}$$

$$S_2^2 = \frac{\rho - 3p}{12b} + \frac{1 - (b/2f_0)(1 + \sqrt{X})}{12b\alpha(1 - \omega/4)}, \tag{2.5}$$

where

$$X = 1 + \omega(f_0^2/b^2)[1 - (b/f_0) - 2(1 - \omega/4)\alpha(\rho + 3p)] \geq 0. \tag{2.6}$$

As consistent with cosmological equations the energy density ρ and pressure p satisfy the equation:

$$\dot{\rho} + 3H\left(\rho + p\right) = 0. \tag{2.7}$$

The torsion function S_2 plays important role at asymptotics when energy density is sufficiently small: $\alpha(\rho + 3p) \ll 1$. Then according to (2.5)-(2.6) if $0 < x = 1 - \frac{b}{f_0} \ll 1$ we have in the lowest approximation with respect to x:

$$S_2^2 = \frac{1}{12b}\left[\rho - 3p + \frac{1 - b/f_0}{\alpha}\right], \tag{2.8}$$

The presence of constant term in (2.8) leads to appearance of effective cosmological constant in cosmological equations, which at asymptotics take the form:

$$\frac{k}{R^2} + H^2 = \frac{1}{6f_0}\left[\rho\frac{f_0}{b} + \frac{1}{4\alpha}\left(1 - \frac{b}{f_0}\right)^2\frac{f_0}{b}\right], \tag{2.9}$$

$$\dot{H} + H^2 = -\frac{1}{12f_0}\left[(\rho + 3p)\frac{f_0}{b} - \frac{1}{2\alpha}\left(1 - \frac{b}{f_0}\right)^2\frac{f_0}{b}\right]. \tag{2.10}$$

In situation when the value of energy density ρ is comparable with effective cosmological constant, equations (2.9)-(2.10) practically coincide with Friedmann cosmological equations with cosmological constant. However, in situation when effective cosmological constant in cosmological equations (2.9)-(2.10) can be neglected in comparison with energy density ρ, the evolution of HIM described by (2.9)-(2.10) weakly differs from that of Friedmann cosmological equations because we have the term $\rho\frac{f_0}{b} \approx \rho(1+x)$ instead ρ in right part of (2.9).

The dependence of S_2^2 on energy density and pressure is similar to that of energy-momentum tensor in gravitational equations, moreover the constant term in expression (2.8) is much greater than effective cosmological constant in cosmological equations. However, terms in (2.2)-(2.3) depending on S_2^2 are mutually cancelled and influence of torsion function S_2 appears by formation of effective cosmological constant because of terms S_2^4 in cosmological equations [3]. Is it distinctive feature of HIM connected probably with its high symmetry or is it some characteristic property of gravitational equations of GTRC?

We will study this question by using discussed earlier restrictions on indefinite parameters introduced in the frame of isotropic cosmology. The remaining indefinite parameters in gravitational Lagrangian (2.1) can be excluded by using additional physical considerations. So we can use restrictions on indefinite parameters obtained in [7] from analysis of particle content of GTRC in linear approximation and exception of ghosts and tachyons [4]. Restrictions on indefinite parameters obtained in the frame of isotropic cosmology are compatible with the following conditions: $f_1 = f_2 = f_3 = f_4 = 0$ and

$$\begin{gathered} a_1 = f_0(1-x), \qquad a_2 = 2f_0(1-x), \\ a_3 = -\frac{4}{3}f_0(1-x), \qquad f_5 = 3f_0^2\alpha\omega, \\ f_6 = f_0^2\alpha(1-\omega) \qquad (x = 1 - \frac{b}{f_0}). \end{gathered} \tag{2.11}$$

The particle content of GTRC with such restrictions on indefinite parameters includes besides massless graviton massive particles with spin-parity 2^+. Our further consideration will be connected with this GTRC - so-called minimum GTRC.

3 Relationship of minimum GTRC and GR

Gravitational equations of minimum GTRC have the following form:

$$\begin{aligned} &\nabla_\nu U_i{}^{\mu\nu} + 2S^k{}_{i\nu}U_k{}^{\mu\nu} + 2(f_0 + 2f_6\,F)F^\mu{}_i + 2f_5(F_{ki}F^{\mu k} + F^\mu{}_{kim}F^{mk}) - \\ &h_i{}^\mu(f_0F + f_5F^{\mu\nu}F_{\nu\mu} + f_6\,F^2 + S^{\alpha\mu\nu}\,(a_1\,S_{\alpha\mu\nu} + a_2\,S_{\nu\mu\alpha}) + a_3\,S^\alpha{}_{\mu\alpha}S_\beta{}^{\mu\beta}) = -T_i{}^\mu, \end{aligned} \tag{3.1}$$

$$4\nabla_\nu[(f_0/2 + f_6\,F)h_{[i}{}^\nu h_{k]}{}^\mu + +f_5\,F^{[\mu}{}_{[k}h_{i]}{}^{\nu]}] + U_{[ik]}{}^\mu = -J_{[ik]}{}^\mu, \tag{3.2}$$

where $U_i{}^{\mu\nu} = 2(a_1\,S_i{}^{\mu\nu} - a_2\,S^{[\mu\nu]}{}_i - a_3\,S_\alpha{}^{\alpha[\mu}h_i{}^{\nu]})$, ∇_ν denotes the covariant operator having the structure of the covariant derivative defined in the case of tensor holonomic indices by

[3]This means that establishing of correspondence between GTRC and GR in linear approximation with respect to torsion [1, 7] is not sufficient.

[4]The strict analysis of particle content has to be connected with consideration of gravitational perturbations above the vacuum spacetime having the structure of Riemann-Cartan continuum with de Sitter metric. However, the deviation of the structure of the vacuum space-time from Minkowski space-time, which is essential at cosmological scale, can be unimportant by local analysis given in [7] because of smallness of values of parameter H and torsion for the vacuum.

means of Christoffel coefficients $\left\{ {\lambda \atop \mu\nu} \right\}$ and in the case of tetrad tensor indices by means of anholonomic Lorentz connection $A^{ik}{}_\nu$ (for example $\nabla_\nu h^i{}_\mu = \partial_\nu h^i{}_\mu - \left\{ {\lambda \atop \mu\nu} \right\} h^i{}_\lambda - A^{ik}{}_\nu h_{k\mu}$). Analytic analysis is possible if $\omega \ll 1$, then according to (2.11) $f_5 \ll f_6$. We will analyze the system of equations (3.1)-(3.2) in the case of spinless matter ($J_{[ik]}{}^\mu = 0$) by neglecting terms with f_5. Then the system of gravitational equations takes the form:

$$\nabla_\nu U_i{}^{\mu\nu} + 2S^k{}_{i\nu} U_k{}^{\mu\nu} + 2(f_0 + 2f_6\, F)F^\mu{}_i - \\ h_i{}^\mu (f_0 F + f_6\, F^2 + S^{\alpha\mu\nu}\,(a_1\, S_{\alpha\mu\nu} + a_2\, S_{\nu\mu\alpha}) + a_3\, S^\alpha{}_{\mu\alpha} S_\beta{}^{\mu\beta}) = -T_i{}^\mu, \tag{3.3}$$

$$4\nabla_\nu[(f_0/2 + f_6\, F) h_{[i}{}^\nu h_{k]}{}^\mu] + U_{[ik]}{}^\mu = 0. \tag{3.4}$$

By taking into account that

$$\Gamma^\lambda{}_{\mu\nu} = \left\{ {\lambda \atop \mu\nu} \right\} + K^\lambda{}_{\mu\nu}, \tag{3.5}$$

where

$$K^\lambda{}_{\mu\nu} = S^\lambda{}_{\mu\nu} + S_{\mu\nu}{}^\lambda + S_{\nu\mu}{}^\lambda, \tag{3.6}$$

we obtain that

$$\nabla_\nu h_i{}^\mu = \partial_\nu\, h_i{}^\mu + \left\{ {\mu \atop \lambda\nu} \right\} h_i{}^\lambda - A^k{}_{i\nu} h_k{}^\mu = -K^\mu{}_{\lambda\nu} h_i{}^\lambda. \tag{3.7}$$

By using (3.7) and the relation $a_2 = 2a_1$ and by multiplying eq. (3.4) with $h^i{}_\sigma h^k{}_\rho$ we transform the equation (3.4) to the following form:

$$2f_0(1 + \frac{2f_6}{f_0} F)(S^\mu{}_{\sigma\rho} + 2S^\nu{}_{\nu[\sigma}\delta^\mu_{\rho]}) - a_2 S^\mu{}_{\sigma\rho} + a_3 S^\nu{}_{\nu[\sigma}\delta^\mu_{\rho]} + 4f_6 \partial_\nu F \delta^\nu_{[\sigma}\delta^\mu_{\rho]} = 0. \tag{3.8}$$

By denoting $Z_1 = 1 + \frac{2f_6}{f_0} F \approx 1 + 2f_0\alpha F$ we write eq. (3.8) in the form[5]:

$$(2f_0 Z_1 - a_2) S^\mu{}_{\sigma\rho} + (4f_0 Z_1 + a_3) S^\nu{}_{\nu[\sigma}\delta^\mu_{\rho]} + 4f_6 \partial_\nu F \delta^\nu_{[\sigma}\delta^\mu_{\rho]} = 0. \tag{3.9}$$

From (3.9) follows that if there is nonvanishing component of torsion with $\mu \neq \rho$ and $\mu \neq \sigma$ we obtain $f_0 Z_1 - \frac{a_2}{2} = 0$ and for minimum GTRC with restrictions (2.11) $Z_1 = \frac{b}{f_0}$. As result the scalar curvature F is constant: $F = -\frac{1 - \frac{b}{f_0}}{2f_0\alpha}$. In the case $\mu = \rho$ and $\mu \neq \sigma$ eq. (3.9) leads to $S^\nu{}_{\nu\sigma} = 0$.

Now we will analyze gravitational equation (3.3). By using that $\nabla_\nu h^i{}_\mu = K^\lambda{}_{\mu\nu} h^i{}_\lambda$ and by multiplying eq. (3.3) with $h^i{}_\lambda$ we transform eq. (3.3) to the following form:

$$\nabla_\nu U_\lambda{}^{\mu\nu} - K^\rho{}_{\lambda\nu} U_\rho{}^{\mu\nu} + 2S^\rho{}_{\lambda\nu} U_\rho{}^{\mu\nu} + 2f_0(1 + \frac{2f_6}{f_0}\, F)F^\mu{}_\lambda - \\ \delta^\mu_\lambda (f_0 F + f_6\, F^2 + S^{\alpha\mu\nu}\,(a_1\, S_{\alpha\mu\nu} + a_2\, S_{\nu\mu\alpha}) + a_3\, S^\alpha{}_{\mu\alpha} S_\beta{}^{\mu\beta}) = -T_\lambda{}^\mu. \tag{3.10}$$

From (3.10) we obtain the following expression for scalar curvature:

$$F = \frac{1}{2f_0}[T - 2bS_{\lambda\mu\nu}(S^{\lambda\mu\nu} - 2S^{\mu\nu\lambda})], \tag{3.11}$$

[5]The quantity Z_1 corresponds to Z used earlier in cosmology.

where $T = T_\mu{}^\mu$. The expression (3.11) corresponds to scalar curvature of HIM obtained earlier [1].

By using obtained value of constant scalar curvature $F = -\frac{1-\frac{b}{f_0}}{2f_0\alpha}$ and the formula $S^\nu{}_{\nu\sigma} = 0$ we transform eq. (3.10) to the form:

$$\frac{1}{2b}(\nabla_\nu U_\lambda{}^{\mu\nu} - K^\rho{}_{\lambda\nu}U_\rho{}^{\mu\nu} + 2S^\rho{}_{\lambda\nu}U_\rho{}^{\mu\nu}) + F^\mu{}_\lambda - \frac{1}{2}\delta^\mu_\lambda F -$$
$$\frac{1}{2}\delta^\mu_\lambda S^{\alpha\mu\nu}(S_{\alpha\mu\nu} + 2S_{\nu\mu\alpha}) = -\frac{1}{2b}(T_\lambda{}^\mu + \delta^\mu_\lambda \frac{(1-\frac{b}{f_0})^2}{12\alpha}). \tag{3.12}$$

The tensor $F^\rho{}_{\sigma\mu\nu}$ can be presented in the form of sum of riemannian part depending on Christoffel coefficients and denoting by $R^\rho{}_{\sigma\mu\nu}(\{\})$ and part depending on torsion and denoting by $F^\rho{}_{\sigma\mu\nu}(K)$. As result the tensors $F^\mu{}_\lambda$ and F in (3.12) are divided by the following way: $F^\mu{}_\lambda = R^\mu{}_\lambda + F^\mu{}_\lambda(K)$ and $F = R + F(K)$ and by taking into account $S^\nu{}_{\nu\sigma} = 0$ we have:

$$F^\mu{}_\lambda(K) = -\nabla_\nu K^{\nu\mu}{}_\lambda + K_{\nu\rho\lambda}K^{\rho\mu\nu}, \qquad F(K) = K_{\lambda\mu\nu}K^{\mu\nu\lambda}. \tag{3.13}$$

By using formulas (3.13) and (3.6) we find that all terms with torsion in (3.12) are mutually eliminated and eq. (3.12) takes the form of Einstein gravitational equations with cosmological constant[6]:

$$R^\mu{}_\lambda - \frac{1}{2}\delta^\mu_\lambda R = -\frac{1}{2b}(T_\lambda{}^\mu + \delta^\mu_\lambda \frac{(1-\frac{b}{f_0})^2}{12\alpha}). \tag{3.14}$$

Besides effective cosmological constant the influence of torsion in eq. (3.14) appears via the change of gravitational constant, however, because the value of b is very near to f_0 corresponding consequences are insignificant. Note that analysis leading to eq. (3.14) is not applicable at extreme conditions near limiting energy density, where terms with parameter ω in gravitational equations play principal role.

It should be noted that equations of minimum GTRC (3.1)-(3.2) like GTRC based on gravitational Lagrangian (2.1) have a number of solutions which are unacceptable from physical point of view. In particular, as it was shown in [7] any vacuum solution of GR with vanishing torsion is exact solution of GTRC independently on values of indefinite parameters f_i and a_k while solutions of GTRC far from spatially limited systems have to tend to the vacuum solution with nonvanishing torsion. In connection with this we have to state the criterion [1], which allows to distinguish acceptable solutions from unphysical ones. Such criterion can be based on investigation of solutions at asymptotics: far from spatially limited systems and at asymptotics of cosmological models solutions of GTRC have to tend to the vacuum solution in the form of corresponding Riemann-Cartan continuum.

4 Conclusion

We obtain that minimum GTRC does not lead to essential distinction in behavior of usual astrophysical objects in comparison with GR, if our assumptions ($\omega \ll 1$) are valid. However, consideration of the question about relationship of GTRC and GR is not terminated. Analysis of this question in the case $\omega \sim 1$ is essentially more complicated. If spacetime torsion appears as energy density and pressure, its influence on gravitational interaction in connection with dark matter problem obtains principal interest.

[6] These equations were derived in the case of spinless matter. Investigation of significance of spin effects in the frame of minimum GTRC similar to that in Einstein-Cartan theory [9] is of physical interest.

References

[1] A.V. Minkevich, *Gauge gravitation theory in Riemann-Cartan space-time and gravitational interaction*, *Gravitation and Cosmology* **22** (2016) 148.

[2] A.V. Minkevich, *Towards the theory of regular accelerating Universe in Riemann-Cartan space-time*, *Journal of Modern Physics A* **31**, Nos. 2-3 (2016) 1641011.

[3] A.V. Minkevich, A.S. Garkun and V.I. Kudin, *On some physical aspects of isotropic cosmology in Riemann-Cartan spacetime*, *Journal of Cosmology and Astropart. Physics* (2013) 03 040 [arXiv:1302.2578].

[4] A.V. Minkevich, *Limiting energy density and a regular accelerating Universe in Riemann-Cartan spacetime*, *JETP Letters* **94** (2011) 831.

[5] A.V. Minkevich, *De Sitter spacetime with torsion as physical spacetime in the vacuum and isotropic cosmology*, *Modern Physics Letters A* , **26** (2011) 259 [arXiv:1002.0538].

[6] A.V. Minkevich, A.S. Garkun and V.I. Kudin, *Regular accelerating universe without dark energy in Poincaré gauge theory of gravity*, *Clas. Quantum Grav.* **24** (2007) 5835.

[7] K. Hayashi and T. Shirafuji, *Gravity from Poincaré Gauge Theory of the Fundamental Particles, 1. Linear and quadratic Lagrangians*, *Progress of Theoretical Physics*, **64** (1980), 866 [*Erratum ibid.*, **65** (1981), 2079]; *Gravity from Poincaré Gauge Theory of the Fundamental Particles, 2. Equations of motion for test bodies and various limits*, *Progress of Theoretical Physics*, **64** (1980), 883 [*Erratum ibid.*, **65** (1981), 2079]; *Gravity from Poincaré Gauge Theory of the Fundamental Particles, 3. Weak field approximation*, *Progress of Theoretical Physics*, **64** (1980), 1435 [*Erratum ibid.*, **66** (1981), 741]; *Gravity from Poincaré Gauge Theory of the Fundamental Particles, 4. Mass and energy of particle spectrum*, *Progress of Theoretical Physics*, **64** (1980), 2222.

[8] M. Blagojević, *Gravitation and Gauge Symmetries*, IOP Publishing, Bristol, 2002.

[9] A. Trautman, *The Einstein-Cartan theory*, in *Encyclopedia of Mathematical Physics* **2**, J.-P. Francoise et al. (eds.), Elsevier, Oxford (2006) pg. 189.