Self-accelerating Universe in modified gravity with dynamical torsion

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Abstract

We consider a model belonging to the class of Poincaré gauge gravities. The model is free of ghosts and gradient instabilities about Minkowski and torsionless Einstein backgrounds. We find that at zero cosmological constant, the model admits a self-accelerating solution with non-Riemannian connection. Small value of the effective cosmological constant is obtained at the expense of the hierarchy between the dimensionless couplings.

1 Introduction

The possibility of modifying General Relativity at large distances is both theoretically exciting and potentially interesting for cosmology. In particular, IR-modified gravity may serve as an explanation of the accelerated late-time expansion of the Universe, alternative to the cosmological constant and the dark energy. However, it often happens that self-accelerating solutions in IR-modified gravities are plagued by instabilities. A famous example is the DGP model [1], which admits both Minkowski and self-accelerating backgrounds [2, 3]. The latter, however, has ghost instability [4–8].

There are various approaches to modifying gravity in IR, see a comprehensive review [9]. These include theories with extra dimensions [1, 10–13] (for a review see Ref. [14]), $f(R)$ and scalar-tensor gravities (for reviews see Refs. [15] and [16]), theories with massive gravitons [17–20] (for a review see Ref. [21]) and theories with explicitly broken Lorentz invariance [22–26] (for a review see Ref. [27]).

Yet another class of theories which are self-consistent on the Minkowski background [28–31] and may serve as candidates for the consistent infrared modification of gravity are Poincaré gauge gravities. These theories treat the vierbein and connection as independent variables, and the Lagrangians include bilinear terms in the full curvature tensor, as well as bilinear terms in the torsion. The spectrum of such theories about the Minkowski background contains massless degrees of freedom corresponding to the standard graviton, as well as massive degrees of freedom related to the propagating torsion [30]; the gravitational interaction in these theories contains both Newtonian and Yukawa terms [29].

We consider one theory from the class described in [28–31], which was studied previously in Refs. [32–34]. It was shown that the spectrum of small perturbations about the Minkowski background is free of ghosts and tachyons [30, 33]. Remarkably, ghosts, gradient instabilities and tachyons are absent in de Sitter and anti-de Sitter spaces and in arbitrary torsionless Einstein backgrounds of sufficiently small curvature [32, 33]. The gravitational interaction is mediated by both massless and massive spin-2 fields, with relative strength being a free parameter [33]. Thus, the model is indeed an example of the infrared modification of gravity.

It is natural to ask whether the model admits self-accelerating cosmological solution(s). Other models belonging to the class of Poincaré gauge gravities have been studied from this viewpoint by A. Minkevich and collaborators [35, 36] (see also [37] and references therein) who have found self-accelerating cosmological solutions in certain models from the general class of Refs. [28–31]. Here we show that the model studied in Refs. [32–34] is not an exception: it does admit a self-accelerating solution with the de Sitter metric, even though there is no explicit cosmological constant term in the action. The self-acceleration is due to the non-metric connection; we find that the connection coefficients are consistent with the de Sitter metric provided that they are independent of time and uniquely determined by the parameters of the theory.

The key issue is then the stability of perturbations about this solution. We plan to address this issue in future publications.

The paper is organized as follows. We present the Lagrangian and remind the earlier results in Section 2. In Section 3 we study spatially flat homogenous and isotropic cosmology and find that the model admits a self-accelerating solution with de-Sitter metric and non-zero torsion. In Section 4 we demonstrate that the effective dark energy density can be made small by an appropriate choice of parameters.

2 The Model

We make use of the tetrad formalism. The vierbein and connection are considered as independent fields. Following the notations of Refs. [32–34], we denote the vierbein by e^i_μ and connection by $A_{ij\mu} = -A_{ji\mu}$, where $\mu = (0,1,2,3)$ are the space-time indices, and $i,j = (0,1,2,3)$ are the tangent space indices. The latter are raised and lowered using the Minkowski metric η_{ij}, so we do not distinguish upper and lower tangent space indices in what follows, if this does not lead to an ambiguity. The signature of metric is $(-,+,+,+)$.

The action of the model is [34]:

$$S = \int d^4x \; eL \; , \quad L = \frac{3}{2}(\tilde{\alpha}F - \alpha R) + c_3 F^{ij}F_{ij} + c_4 F^{ij}F_{ji} + c_5 F^2 + c_6(\epsilon\cdot F)^2 \; , \tag{1}$$

where $e \equiv det(e^i_\mu)$; F_{ijkl} is the curvature tensor constructed with the connection $A_{ij\mu}$,

$$F_{ijkl} = e^\mu_k e^\nu_l(\partial_\mu A_{ij\nu} - \partial_\nu A_{ij\mu} + A_{im\mu}A_{mj\nu} - A_{jm\mu}A_{mi\nu}) \; ;$$

$$F_{ij} = \eta^{kl}F_{ikjl} \; , \quad F = \eta^{ij}F_{ij} \; , \quad \epsilon\cdot F \equiv \epsilon^{ijkl}F_{ijkl} \; ;$$

ϵ_{ijkl} is the Levi-Civita symbol defined in such a way that $\epsilon^{0123} = -\epsilon_{0123} = 1$; R_{ijkl} is the Riemanian curvature tensor,

$$R_{ijkl} = e^\mu_k e^\nu_l(\partial_\mu \omega_{ij\nu} - \partial_\nu \omega_{ij\mu} + \omega_{im\mu}\omega_{mj\nu} - \omega_{jm\mu}\omega_{mi\nu}) \; ;$$

$$R_{ij} = \eta^{kl}R_{ikjl} \; , \quad R = \eta^{ij}R_{ij} \; ,$$

where $\omega_{ij\mu}$ is the Riemanian spin-connection. It is expressed in terms of the vierbein as follows:

$$\omega_{ij\mu} \equiv \omega_{ijk}e^k_\mu = \frac{1}{2}(C_{ijk} - C_{jik} - C_{kij})e^k_\mu \; ,$$

where

$$C_{ijk} = e^\mu_j e^\nu_k(\partial_\mu e_{i\nu} - \partial_\nu e_{i\mu}) \; .$$

The constants $\alpha, \tilde{\alpha}, c_3, c_4, c_5, c_6$ are the parameters. We impose the following conditions,

$$c_3 + c_4 = -3c_5 \; , \tag{2a}$$

$$\alpha < 0, \quad \tilde{\alpha} > 0, \quad c_5 < 0, \quad c_6 > 0 \; , \tag{2b}$$

in order not to have the pathological degrees of freedom in the Minkowski background [30].

It is worth noting that the action (1) is equivalent to the action used in [28–30, 32, 33], which, instead of the explicit αR-term, involves mass terms for torsion. To this end, one decomposes the connection as follows,

$$A_{ij\mu} \equiv A_{ijk} e^k_\mu = \left[\frac{1}{2} \left(T_{ijk} - T_{jik} - T_{kij} \right) + \omega_{ijk} \right] e^k_\mu \ ,$$

where $T_{ijk} = -T_{ikj}$ is the torsion tensor. The latter can be decomposed into its irreducible components under the local $O(1,3)$ group,

$$T_{ijk} = \frac{2}{3}(t_{ijk} - t_{ikj}) + \frac{1}{3}(\eta_{ij} v_k - \eta_{ik} v_j) + \epsilon_{ijkl} a^l \ ,$$

where the field t_{ijk} is symmetric with respect to the interchange of i and j and satisfies the cyclic and trace identities,

$$t_{ijk} + t_{jki} + t_{kij} = 0, \qquad \eta^{ij} t_{ijk} = 0, \qquad \eta^{ik} t_{ijk} = 0 \ .$$

The action is then

$$S = \int d^4x \ eL \ ,$$

where

$$L = \frac{3}{2}(\tilde{\alpha} - \alpha) F + \alpha (t_{ijk} t^{ijk} - v_i v^i + \frac{9}{4} a_i a^i) + c_3 F^{ij} F_{ij} + c_4 F^{ij} F_{ji} + c_5 F^2 + c_6 (\epsilon \cdot F)^2 \ .$$

We will use the action (1) in what follows.

In Ref. [33] it was found that there are three propagating modes at the linear level in the Minkowski background: the massless spin-2 mode, the massive spin-2 mode with mass

$$m^2 = \frac{\tilde{\alpha}(\tilde{\alpha} - \alpha)}{2\alpha c_5} \tag{3}$$

and the massive spin-0 mode with mass

$$m_0^2 = \frac{\tilde{\alpha}}{16 c_6} \ . \tag{4}$$

There are no ghosts or tachyons in the Minkowski background. In the theory equipped with the cosmological constant, the perturbations are healthy in torsionless Einstein backgrounds as well.

The analysis of gravitational interactions between sources in Minkowski background [33] reveals that for small m there is the correspondence,

$$\alpha = -\frac{M_{Pl}^2}{24\pi} \ . \tag{5}$$

We leave other parameters arbitrary for the time being.

3 The self-accelerating solution

We consider now spatially flat homogeneous and isotropic cosmology. Homogeneity and isotropy dictate the following most general Ansatz:

$$e_0^{\tilde{0}} = N(t) \ , \quad e_b^{\tilde{a}} = a(t)\delta_b^{\tilde{a}} \ , \quad A_{\tilde{0}\tilde{a}\tilde{b}} = f(t)\delta_{\tilde{a}\tilde{b}} \ , \quad A_{\tilde{a}\tilde{b}\tilde{c}} = g(t)\varepsilon_{\tilde{a}\tilde{b}\tilde{c}} \ , \tag{6}$$

where $a, \tilde{a} = (1, 2, 3)$, tilde denotes tangent space indices, and space-time indices do not have tilde. In other words, $i = (\tilde{0}, \tilde{a})$, $\mu = (0, a)$. Due to the antisymmetry of A_{ijk} with respect to the interchange of the first pair of indices no other components can be non-vanishing.

With the Ansatz (6), we calculate the non-vanishing components of the curvature tensor F_{ijkl},

$$F_{\tilde{0}\tilde{a}\tilde{0}\tilde{b}} = \frac{1}{Na}\partial_0(af)\delta_{ab} \ , \quad F_{\tilde{0}\tilde{a}\tilde{b}\tilde{c}} = -2fg\varepsilon_{abc} \ ,$$

$$F_{\tilde{a}\tilde{b}\tilde{0}\tilde{c}} = \frac{1}{Na}\partial_0(ag)\varepsilon_{abc} \ , \quad F_{\tilde{a}\tilde{b}\tilde{c}\tilde{d}} = (f^2 - g^2)(\delta_{ac}\delta_{bd} - \delta_{ad}\delta_{bc}) \ .$$

The non-vanishing components of F_{ij} are

$$F_{\tilde{0}\tilde{0}} = \frac{3}{Na}\partial_0(af) \ , \quad F_{\tilde{a}\tilde{b}} = \left[2(f^2 - g^2) - \frac{1}{Na}\partial_0(af)\right]\delta_{ab} \ .$$

Note that F_{ij} is symmetric as required by the homogeneity. Finally, we calculate $F = \eta_{ij}F_{ij}$, $\epsilon \cdot F$ and $R \equiv R_{ijkl}\eta^{ik}\eta^{jl}$:

$$F = 6\left[f^2 - g^2 - \frac{1}{Na}\partial_0(af)\right] \ , \quad \epsilon \cdot F = 12\left[-2fg + \frac{1}{Na}\partial_0(ag)\right] \ ,$$
$$R = 6\left[\frac{1}{Na}\partial_0\left(\frac{\dot{a}}{N}\right) + \frac{1}{a^2}\left(\frac{\dot{a}}{N}\right)^2\right] \ .$$

The action (1) in terms of the homogeneous and isotropic fields is

$$\begin{aligned} eL =& 9\tilde{\alpha}\left[Na^3(f^2 - g^2) - a^2\partial_0(af)\right] - 9\alpha\left[a^2\partial_0\left(\frac{\dot{a}}{N}\right) + \frac{a}{N}(\dot{a})^2\right] \\ &- 36c_5a^2(f^2 - g^2)\partial_0(af) + 144c_6\left\{4Na^3f^2g^2 - 4a^2fg\partial_0(ag) + \frac{a}{N}\left[\partial_0(ag)\right]^2\right\} \ , \end{aligned}$$

where $\dot{a} \equiv \partial_0 a$. Note that, due to (2a) and symmetry of F_{ij}, this expression contains only c_5, c_6, α and $\tilde{\alpha}$, but not c_3 and c_4 separately. The terms with $(f^2 - g^2)^2$ and $[\partial_0(af)]^2$ have canceled out also due to (2a). Upon integrating by parts we write the action in the following form:

$$
\begin{aligned}
eL =& 9\tilde{\alpha}\left[(f^2-g^2)Na^3+2a^2\dot{a}f\right]+9\alpha\frac{a}{N}(\dot{a})^2+36c_5g^2a^2\partial_0(af)\\
&+144c_6\left\{4f^2g^2Na^3-4a^2fg\partial_0(ag)+\frac{a}{N}[\partial_0(ag)]^2\right\}\ .
\end{aligned}
\tag{7}
$$

There are three independent equations of motion that follow from (7). We choose the gauge $N=1$ after varying with respect to N, f and g, divide by a^3 and obtain

$$
\frac{\delta}{\delta N}\quad : \qquad \tilde{\alpha}(f^2-g^2)-\alpha\frac{\dot{a}^2}{a^2}-d_6\frac{[\partial_0(ag)]^2}{a^2}+4d_6f^2g^2=0\ , \tag{8a}
$$

$$
\frac{\delta}{\delta f}\quad : \qquad \tilde{\alpha}(f+\frac{\dot{a}}{a})-d_5g\frac{\partial_0(ag)}{a}+4d_6fg^2=0\ , \tag{8b}
$$

$$
\frac{\delta}{\delta g}\quad : \qquad -\tilde{\alpha}g+d_5g\frac{\partial_0(af)}{a}-d_6\frac{\partial_0[a\partial_0(ag)]}{a^2}+4d_6f^2g=0\ , \tag{8c}
$$

where we have introduced the notations

$$
d_6\equiv 16c_6\ ,\quad d_5\equiv 4c_5+32c_6\ .
$$

We are interested in a self-accelerating solution of this system with

$$
a=e^{\lambda t}\ ,
$$

where $\lambda=$ const. Such a solution necessarily has time-independent f and g. To see this, we note that eq. (8b) can be solved for $\partial_0 g$ as function of f and g. Then eq. (8a) becomes an algebraic equation that determines $f=f(g)$. Hence, we can express $\partial_0 g$, $\partial_0^2 g$ and $\partial_0 f$ as algebraic functions of g. Plugging these into eq. (8c) we obtain an algebraic equation for g with time-independent coefficients. We have checked that g does not drop out of the latter equation (which, in fact, can be cast into the eighth order equation for g^2). The solution to that equation is independent of time, as claimed.

For constant f, g and λ the three equations (8a), (8b) and (8c) become algebraic. Assuming $g\neq 0$ we have,

$$
\tilde{\alpha}(f^2-g^2)-\alpha\lambda^2-d_6g^2\lambda^2+4d_6f^2g^2 \;=\; 0\ , \tag{9a}
$$

$$
\tilde{\alpha}(f+\lambda)-d_5g^2\lambda+4d_6g^2f \;=\; 0\ , \tag{9b}
$$

$$
-\tilde{\alpha}+d_5\lambda f-2d_6\lambda^2+4d_6f^2 \;=\; 0\ . \tag{9c}
$$

A useful consequence of eqs. (9a) - (9c) is

$$\tilde{\alpha}(f^2 - g^2 - \lambda f) - 2\alpha\lambda^2 = 0 \; . \tag{10}$$

Let us solve eq. (9b) for f:

$$f = -\frac{\tilde{\alpha} - d_5 g^2}{\tilde{\alpha} + 4d_6 g^2}\lambda \; . \tag{11}$$

We substitute this into eq. (9a) and solve for λ^2. The result is

$$\lambda^2 = \frac{\tilde{\alpha} g^2(\tilde{\alpha} + 4d_6 g^2)}{(d_5^2 - 4d_6^2)g^4 - [\tilde{\alpha}(2d_5 + d_6) + 4\alpha d_6]g^2 + \tilde{\alpha}(\tilde{\alpha} - \alpha)} \; . \tag{12}$$

Finally, we substitute f from (11) and λ^2 from (12) into (10) and obtain an equation for $x = g^2$:

$$4d_6(d_5^2 - 4d_6^2)x^3 - 4(2\tilde{\alpha}d_6^2 - 4\alpha d_6^2 + \tilde{\alpha}d_5 d_6)x^2 + \tilde{\alpha}(\tilde{\alpha}d_5 - \tilde{\alpha}d_6 + 8\alpha d_6)x - \tilde{\alpha}^2(\tilde{\alpha} - \alpha) = 0 \; .$$

This is cubic in x. The product of the three roots of this equation is

$$\frac{\tilde{\alpha}^2(\tilde{\alpha} - \alpha)}{4d_6(d_5^2 - 4d_6^2)} \; , \tag{13}$$

which is positive provided that we impose the condition $d_5^2 - 4d_6^2 \equiv 4c_5(d_5 + 2d_6) > 0$, i.e., with the restriction (2b),

$$d_5 + 2d_6 \equiv 4c_5 + 64c_6 < 0 \; . \tag{14}$$

The positivity of (13) guarantees that there is one positive root x_1 for g^2. The two other roots are negative. Indeed, the following bilinear combination of the three roots is

$$x_1 x_2 + x_1 x_3 + x_2 x_3 = \frac{\tilde{\alpha}(\tilde{\alpha}d_5 - \tilde{\alpha}d_6 + 8\alpha d_6)}{4d_6(d_5^2 - 4d_6^2)} < 0$$

Furthermore, with our inequality (14), we have,

$$2d_5 + d_6 = 2(d_5 + 2d_6) - 3d_6 \equiv 2(4c_5 + 64c_6) - 48c_6 < 0 \; ,$$

and hence

$$\tilde{\alpha}(2d_5 + d_6) + 4\alpha d_6 \;\; < 0 \; .$$

Using the latter inequality it is straightforward to see that eq. (12) gives positive λ^2 and with positive λ we obtain a negative value for f from (11).

To summarize, if the condition (14) is satisfied together with the conditions (2b), then there exists the self-accelerating solution with $\lambda > 0$, $f < 0$. The sign of g can be arbitrary, since g is P-odd.

4 The limit of small λ

The solution given in the previous section, although exact and explicit, is fairly complicated. Having in mind the present acceleration of the Universe, we consider the limit of small λ. Let us find out the relevant corner in the parameter space of the action (1). We assume the following power counting:

$$\frac{\tilde{\alpha}}{|\alpha|} = O(\lambda^0)\ ,\ f = O(\lambda^0)\ ,$$

where $|\alpha| \sim M_{Pl}^2$, see eq. (5). We note in passing that this power counting does not exclude the case $\tilde{\alpha} << |\alpha|$ and/or $f^2 << |\alpha|$; it merely means that λ is the smallest parameter in the problem.

We make use of (10) to solve eqs. (9a), (9b) and (9c) for c_5 and c_6:

$$c_6 = \frac{\tilde{\alpha}\lambda(\tilde{\alpha}f + \alpha\lambda)}{16(\lambda^2 - 4f^2)(\tilde{\alpha}f^2 - \tilde{\alpha}\lambda f - 2\alpha\lambda^2)}\ ,$$
$$c_5 = \frac{\tilde{\alpha}[2\tilde{\alpha}f^2 + \lambda f\tilde{\alpha} + \lambda^2(\tilde{\alpha} - 2\alpha)]}{4\lambda(\lambda + 2f)(f^2\tilde{\alpha} - \lambda f\tilde{\alpha} - 2\alpha\lambda^2)}\ .$$

In the small-λ limit, these equations give

$$c_6 = -\frac{\tilde{\alpha}}{64f^3}\lambda\ ,$$
$$c_5 = \frac{\tilde{\alpha}}{4\lambda f}\ ,$$

or

$$\lambda = \left(-\frac{c_6\tilde{\alpha}^2}{c_5^3}\right)^{1/4}\ ,$$
$$f = -\frac{\tilde{\alpha}^{1/2}}{4(-c_5c_6)^{1/4}}\ ,$$

so that the parameter c_6 must be small and c_5 must be large, $c_6 = O(\lambda)$, $c_5 = O(\lambda^{-1})$. Equation (10) then gives

$$g = \pm f + O(\lambda)\ .$$

As we pointed out above, the sign of g can be chosen arbitrarily.

The effective cosmological constant can also be written in terms of the masses (3), (4) of excitations about the Minkowski background:

$$\lambda = m \left(\frac{m}{m_0} \right)^{1/2} \frac{(-\alpha)^{3/4}}{2^{1/4}(\tilde{\alpha} - \alpha)^{3/4}} \ .$$

At $\tilde{\alpha}/|\alpha| = O(\lambda^0)$ this shows that the small value of λ is obtained for small mass m of the spin-2 excitation, and also that large m_0 suppresses λ.

5 Conclusions

To conclude, the model (1) admits the self-accelerating solution,

$$e_0^{\tilde{0}} = 1 \ , \quad e_b^{\tilde{a}} = e^{\lambda t} \delta_b^{\tilde{a}} \ , \quad A_{\tilde{0}\tilde{a}\tilde{b}} = f \delta_{\tilde{a}\tilde{b}} \ , \quad A_{\tilde{a}\tilde{b}\tilde{c}} = g \varepsilon_{\tilde{a}\tilde{b}\tilde{c}} \ ,$$

We have shown that for the most general solution with the de Sitter metric, the functions f and g are necessarily time independent constants. Furthermore, we have established a direct relationship between the dark energy λ and the mass m characteristic of the massive graviton originating from torsion in Minkowski background. The small value of the effective cosmological constant λ is obtained provided that there is a hierarchy between the couplings, $c_6 = O(\lambda)$, $c_5 = O(\lambda^{-1})$. It is worth noting that with this choice of parameters and in Minkowski background, the mass of the spin-2 state (3) is small, $m^2 \sim \lambda f$, while the mass of spin-0 state (4) is large, $m_0^2 \sim f^3/\lambda$. In fact, in the limit of small λ, the scale m_0 may be above the UV cutoff of the effective low energy theory; in that case the scalar degree of freedom is absent in the spectrum about Minkowski background. We emphasize that perturbations about our self-accelerating background may have quite different properties. We plan to address this issue in a forthcoming publication.

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