Torsion or not torsion, that is the question

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Abstract

A hypothesis of general relativity is that spacetime torsion vanishes identically. This assumption has no empirical support; in fact, a nonvanishing torsion is compatible with all the experimental tests of general relativity. The first part of this essay studies the framework that is suitable to test the vanishing-torsion hypothesis, and an interesting relation with the gravitational degrees of freedom is suggested. In the second part, some original empirical tests are proposed based on the observation that torsion induces new interactions between different spin-polarized particles.

Any pseudo-Riemannian manifold admits a metric-compatible derivative operator with nonvanishing torsion. By definition, the failure of such an operator to commute, when applied to a scalar field, is related to the torsion tensor field $T_{ab}{}^{c}$. In addition, geometrically, torsion measures the failure of infinitesimal parallelograms to close [1].

In general relativity (GR) it is assumed, without empirical support, that torsion vanishes identically. Of course, one may claim that the experimental success of GR justifies the vanishing-torsion hypothesis. However, as it is argued below, all GR tests are compatible with a nonvanishing torsion, and, as a basic assumption of the theory, it is paramount to experimentally test it. The main goal of this essay is to suggest alternative tests of the vanishing-torsion hypothesis, but before embarking on such a task, it is useful to specify the theoretical framework.

It is important to start by narrowing down the action that is used to empirically test the vanishing-torsion hypothesis. The experimental validation of GR allows one to conclude that the dominant term of the gravitational part of this action must be the Einstein–Hilbert action with the torsion-full derivative operator ∇_a; this is known as the Einstein–Cartan theory [2]. Moreover, if torsion is present, the gravitational degrees of freedom have to be described with the Palatini formalism [3]. This is because, in the standard approach, the derivative operator is completely determined by the metric, and torsion, which is a property of such an operator, is supposed to be metric independent.

Since torsion is a property of the derivative operator, it can only couple to matter through such an operator. It turns out that, with this restriction, spinors are the only known particles that couple to torsion [4]. Therefore, in this essay, Dirac spinors are the only matter fields under consideration. Now, if the tetrad e^a_μ and spin connection $\omega_{a\mu\nu}$ are chosen as the independent dynamical variables to describe gravity (the conventions of Ref. [5] are used), then, the total action, for a spinor field Ψ of mass M, takes the form

$$\begin{aligned} S &= \int d^4xe\left[R(e,\omega)+16\pi G\left(\frac{i}{2}e^a_\mu\bar{\Psi}\gamma^\mu\nabla_a\Psi-\frac{i}{2}e^a_\mu(\nabla_a\bar{\Psi})\gamma^\mu\Psi-M\bar{\Psi}\Psi\right)\right] \\ &= \int d^4xe\left[R(e,\omega)+16\pi G\left(\frac{i}{2}e^a_\mu\bar{\Psi}\gamma^\mu\partial_a\Psi-\frac{i}{2}e^a_\mu(\partial_a\bar{\Psi})\gamma^\mu\Psi-\frac{1}{4}e^a_\mu\omega_{a\nu\rho}\epsilon^{\mu\nu\rho}{}_{\sigma}\bar{\Psi}\gamma_5\gamma^\sigma\Psi-M\bar{\Psi}\Psi\right)\right], \end{aligned} \tag{1}$$

where e is determinant of the tetrad components, R is the curvature scalar associated with ∇_a, and G is Newton's constant. Also, for the second identity the explicit form of the covariant derivative acting on spinors is used [6], and $\epsilon_{\mu\nu\rho\sigma}$ are the components of the volume form in the dual tetrad basis, γ^μ are the Dirac matrices, $\gamma_5=i\gamma^0\gamma^1\gamma^2\gamma^3$, and $\bar{\Psi}=\Psi^\dagger\gamma^0$. Note that the action (1) has no free parameters and that the chiral current associated with Ψ, which is given by $\bar{\Psi}\gamma_5\gamma^\mu\Psi$, plays an important role. In addition, the spinor action explicitly depends on the derivative operator through $\omega_{a\mu\nu}$. This last remark is relevant since the standard and the Palatini approaches are inequivalent in such cases [7]. Furthermore, bear in mind that spinors, which are fundamentally quantum-mechanical objects, are introduced through a semi-classical framework. Thus, for practical purposes, such particles behave as extended objects which can be spatially overlapped.

The variation of the action (1) with respect to the tetrad gives an equation whose symmetric part reduces to Einstein equations and the antisymmetric part reflects the freedom to perform local Lorentz transformations (in the presence of torsion, the Ricci tensor is not necessarily symmetric). On the other hand, the spin-connection variation yields

$$0=2\nabla_a(e^{[a}_\mu e^{c]}_\nu)-4\delta^\rho_{[\mu}e^{[a}_{\nu]}e^{c]\sigma}\omega_{a\rho\sigma}-e^a_\mu e^b_\nu T_{ab}{}^{c}+4\pi G e^c_\rho\epsilon^\rho{}_{\mu\nu\sigma}j_5^\sigma, \tag{2}$$

where j_5^μ is the total chiral current. Naturally, equation (2) does not give a unique solution for $\omega_{a\mu\nu}$ and $T_{ab}{}^{c}$. However, for consistency, the solution is chosen to be $\omega_{a\mu\nu}=e_{\mu b}\nabla_a e^b_\nu$ since this relation is used to obtain the equations of motion from the action. This restricts torsion to satisfy

$$T_{ab}{}^{c}=4\pi G e^d_\mu\epsilon^c{}_{abd}j_5^\mu. \tag{3}$$

Observe that torsion is linked to j_5^μ through an algebraic equation, which implies that torsion does not propagate. In addition, j_5^μ has contributions of all the spinor fields under consideration. Moreover, since the chiral current is generated by spin-polarized spinors, to look for torsion, the experiments need to be done inside a spin-polarized media. This justifies the previous claim that all GR tests are compatible with a nonvanishing torsion since, clearly, these tests are done in situations where, at least on average, $j_5^\mu = 0$.

Interestingly, in a theory described by an action similar to (1), but with a vanishing torsion, and which is treated *à la* Palatini, the solution to the equation of motion associated with the spin connection is $\omega_{a\mu\nu} = e_{\mu b}\nabla_a e_\nu^b - 2\pi G e_a^\rho \epsilon_{\rho\mu\nu\sigma} j_5^\sigma$, which differs from the expression used to get the equations of motion in the first place. Also, it can be verified that this last expression, if used in the action, leads to a set of equations of motion that are inequivalent to those obtained with $\omega_{a\mu\nu} = e_{\mu b}\nabla_a e_\nu^b$. In this sense it seems that Dirac spinors cannot be consistently treated within the Palatini framework. In turn, this suggests that the issue of whether torsion vanishes could be closely related to the question of whether gravity is described by the standard or the Palatini approach. Moreover, observe that the chiral current plays an important role in both of these issues. Needless to say that determining the gravitational degrees of freedom is a relevant question since the best available description of matter, *i.e.*, the standard model, is given in terms of an action that does depend on the derivative operator.

To describe the experimental consequences of torsion it is useful to appeal to a test-particle approximation. For that purpose, the test spinor is denoted by ψ and all other spinor fields, which are collectively called source spinors, are assumed to generate the chiral current J_5^μ. The test-spinor equation of motion, assuming it has mass m, takes the form

$$0 = i e_\mu^a \gamma^\mu \partial_a \psi + \frac{i}{2}(\nabla_a e_\mu^a)\gamma^\mu \psi - \frac{1}{4} e_\mu^a \omega_{a\rho\sigma} \epsilon^{\mu\rho\sigma}{}_\nu \gamma_5 \gamma^\nu \psi - m\psi. \tag{4}$$

Note that torsion is present in equation (4) through ∇_a and $\omega_{a\mu\nu}$. Therefore, it is possible to use equation (3) to replace $T_{ab}{}^c$ in equation (4), leading to the well-known nonlinear terms in ψ [8], and additional interactions between the test and source spinors.

Surprisingly, the consequences of the torsion interactions involving test and source spinors have not been extensively explored. For the purpose of characterizing such interactions, it is useful to obtain the corresponding Hamiltonian. As it is customary, this Hamiltonian can be read off from equation (4). For particular experimental situations where it is possible to neglect the torsion self-interactions, the special-relativistic corrections, and the curvature effects, the Hamiltonian, which is obtained using standard methods [9], takes the form

$$H = m + \frac{\vec{p}^2}{2m} - 3\pi G \vec{J}_5 \cdot \vec{\sigma} + \frac{3\pi G}{2m} \vec{\sigma} \cdot \left(\vec{p} J_5^0 + J_5^0 \vec{p}\right), \tag{5}$$

where $\vec{p}$ is the momentum operator, σ^i are the Pauli matrices, and the standard 3-dimensional vectorial notation is utilized. The first two terms in this Hamiltonian are the conventional rest and kinetic energy terms, and the last two terms are due to torsion and are suppressed by G. Moreover, from the explicit form of these torsion interactions, it is evident that, to produce nontrivial contributions, the test spinor has to be in a spin-polarized state and overlapping the source spinors.

The $\vec{J}_5 \cdot \vec{\sigma}$ term in the Hamiltonian (5) can be probed in polarized-neutron transmission experiments through polarized media [10]. Three alternatives look promising in this direction: First, there is an experiment where a polarized neutron beam is sent into a polarized Holmium target [11], which is, in principle, sensitive to torsion effects. However, in this experiment, the torsion signal would behave like the experimental noise, which is discarded. Thus, to test the torsion term, a method to isolate the torsion signal has to be developed. Second, to perform a neutron spin-rotation experiment, as in Ref. [12], but where the target is spin-polarized liquid Helium. Third, there are spin sources that have an extremely high spin-polarization density but are insensitive to magnetic effects [13], and it is conceivable to perform an experiment where a polarized neutron beam passes through such a spin source. Interestingly, one could separate torsion effects from spurious signals by, for example, changing the relative spin orientation, or using that, in contrast to the electromagnetic interaction, this torsion term is momentum independent. In addition, in this case one would probe a neutron-electron coupling where the noise is basically electromagnetic. This is a huge improvement with respect to the other two proposals where the noise is due to the strong interaction.

Interferometry tests are sensitive to the last term in the Hamiltonian (5). In fact, it can be shown that the phase difference produced by such a term is related to the difference of the line integral of J_5^0 along the interferometer arms. Naively, it is tempting to eliminate the noise from spin-spin interactions by surrounding the spin-polarized region as in an Aharonov–Bohm setup. However, J_5^0 is not a potential, thus, no phase difference is produced if J_5^0 vanishes in the integration region. Hence, to look for these effects one must do interferometry within spin-polarized media, which is technologically challenging. In any case, looking for the effects of both unconventional terms in the Hamiltonian (5) may be a valuable tool to separate possible torsion effects from other interactions.

In conclusion, the vanishing-torsion hypothesis of GR has no empirical support and it must be empirically tested. Here some original strategies to test this hypothesis are suggested, whose ultimate sensitivities and consequences are impossible to foresee. Yet, these tests should allow us to quantify the validity of the vanishing-torsion hypothesis, and a potential discovery of a nonvanishing spacetime torsion would have deep conceptual consequences.

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