Effect of gravomagnetism on the trajectory of light ray

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Abstract

It has been shown by various authors that gravo magnetic field can produce lensing effect. The effect of such a gravitational body with magnetic monopole on the trajectory of light ray is discussed in this paper. The light deflection angle has been calculated in the present works, considering upto fourth order terms. Schwarzschild light deflection angle can be obtained from this expression, by setting magnetism equals to zero. However, for a hypothetical massless, magnetic monopole the light deflection angle does not reduce to zero.

1 INTRODUCTION

One of the most important predictions of general relativity is deflection of light ray in presence of gravitational mass. Three factors that effect the trajectory of light ray are gravitational mass, rotation and charge. The first order contribution of mass on the path of light ray was calculated by Einstein himself. After Einstein, higher order contribution of mass towards light deflection angle was calculated by Keeton and Petters [1]. Recently, Iyer and Petters [2] calculated it for strong field and found that under weak field approximation their expression matches with that of Keeton and Petters [1].

The light deflection angle for a rotating mass (Kerr mass) was calculated by Iyer and Hansen [3, 4]. Later, Bozza [5] obtained the lensing formula and calculated all other components related to lensing. Azzami et. al [6, 7] calculated two individual components (parallel and perpendicular to equatorial plane) of light deflection angle in quasi-equatorial regime. Dubey and Sen have used Kerr [8] and Kerr-Newman [9] geometry, to show how gravitational redshifts are affected as photon is emitted from various latitudes. Chakraborty and Sen [10] have recently obtained the light deflection angle for a charged, rotating body in the equatorial plane and showed how deflection angle changes with charge. Chakraborty and Sen [11] obtained the off equatorial light deflection angle for Kerr geometry. Hasse and Pelrick [12] worked on the lensing by Kerr-Newman mass (charged, rotating) using Morse theory and showed that infinite number of images formed by such body. Eiroa et al [13] worked on Reissner-Nordström (charged, static) mass and calculated light deflection angle in both strong and weak deflection limit.

On the other hand some authors have used material medium approach where the gravitational effect on light ray was calculated by assuming some effective refractive index assigned to the medium through which light is propagating. With similar approach Atkinson [14] studied the trajectory of light ray near a very massive, static and spherically symmetric star. Fischback and Freeman [15] calculated the second order contribution to gravitational deflection by a static mass using the same method. Sen [16] used this method to calculate the gravitational deflection of light without any weak field approximation. Similar method was used in earlier past by Balaz [17] to calculate the change in the direction of polarization vector of electromagnetic wave passing close to a rotating body. Recently Roy and Sen [18] calculated the trajectory of a light ray in Kerr field using the same material medium approach.

All the above mentioned works were done by considering three factors (mass, electric charge and rotation), but various authors showed that not only these three factors but also magnetism can influence space-time curvature i.e. path of light ray. In the year of 1963, Newman, Tamburino and Unti [19] first introduced the concept of "generalized Schwarzschild metric". This metric contains one arbitrary parameter in addition to the mass generally known as NUT factor or gravomagnetic mass. In the same year Misner [20] studied the "generalized Schwarzschild metric" and called it as NUT (named after Newman, Tamburino and Unti) space-time. According to him this line element has a Schwarzschild-like singularity, but this singularity is not observed in the curvature tensor i.e there exists no curvature singularity. The presence of cross term "$dtd\varphi$" shows, that this space has a strength of gravomagnetic monopole [21]. Lensing effect of this type of body was studied by Nouri-Zonoz and Lynden-Bell [22]. They showed that the presence of NUT factor change the observed shape, size and orientation of a source, but they did not show the effect of such body on light deflection angle.

On the other hand Kerr-Taub-NUT (KTN) line element represents the solution of Einstein's field equation for a rotating body with non zero magnetic mass [23]. Wei et al. [24] studied numerically the quasi-equatorial lensing by the stationary, axially-symmetric black hole in KTN space-time in the strong field limit. Abdujabbarov et al. [25] studied the electromagnetic fields in the KTN space time and in the surrounding spacetime of slowly rotating magnetized NUT star and obtained analytical solutions of Maxwell equations. Chakraborty and Majumdar [26] derived the exact LenseThirring precession frequencies for Kerr, KTN and Taub-NUT space-times.

In this present work, we studied the NUT line element and obtained the light deflection angle for such space-time geometry which is a function of mass and the NUT factor. We also studied the variation of the light deflection angle as a function of NUT factor. For zero NUT factor our result reduces to well known Schwarzschild light deflection angle.

2 NUT LINE ELEMENT AND GEODESIC EQUATIONS

The NUT line element given by Misner as follows [20],

$$ds^2 = f(r)(cdt - 2l\cos\vartheta d\varphi)^2 - f(r)^{-1}dr^2 - (r^2 + l^2)(d\vartheta^2 + \sin^2\vartheta d\varphi^2) \tag{1}$$

where $f(r) = 1 - \frac{2(mr+l^2)}{l^2+r^2}$ and $2l$ is the strength of the gravomagnetic monopole with the dimension of length, generally known as NUT factor or gravomagnetic mass and $m = \frac{GM}{c^2}$, further G, M, c are the gravitational constant, mass of the gravitating body and free space speed of light respectively.

For a static body, every plane passing through the center can be considered as equatorial plane. So we can consider $\vartheta = \frac{\pi}{2}$ and with this assumption the new modified line element is,

$$ds^2 = f(r)c^2dt^2 - f(r)^{-1}dr^2 - (r^2 + l^2)d\varphi^2 \tag{2}$$

Lagrangian(S) of such a system is given by [24, page: 96],

$$2S = g_{\mu\nu}\frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau} \tag{3}$$

Here τ represents the affine parameter. Thus,

$$2S = f(r)c^2\dot{t}^2 - f(r)^{-1}\dot{r}^2 - (r^2 + l^2)\dot{\varphi}^2 \tag{4}$$

where dot (.) indicates differentiation with respect to τ. Let E and L be the angular momentum and energy of the light ray. Then following [1], we can write from equation (4)

$$\frac{\partial S}{\partial (c\dot{t})} = E$$

and

$$-\frac{\partial S}{\partial \dot{\varphi}} = L$$

The above implies,

$$c\dot{t} = \frac{E}{f(r)} \tag{5}$$

$$\dot{\varphi} = \frac{L}{r^2 + l^2} \tag{6}$$

For null geodesic, $ds^2 = 0$, but, $ds^2 = g_{\mu\nu}dx^\mu dx^\nu$. So, $g_{\mu\nu}dx^{mu}dx^\nu = 0$ or,

$$g_{\mu\nu}\frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau} = 0$$

But from equation (4),

$$g_{\mu\nu}\frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau} = 2S$$

So,

$$2S = 0 \tag{7}$$

Therefore, the Lagrangian for null geodesic is zero. Thus using equation (4) we can write,

$$f(r)c^2\dot{t}^2 - f(r)^{-1}\dot{r}^2 - (r^2 + l^2)\dot{\varphi}^2 = 0 \tag{8}$$

Using equation (5) and (6) in equation (8), we can write:

$$\frac{E^2}{f(r)} - \frac{\dot{r}^2}{f(r)} - \frac{L^2}{r^2 + l^2} = 0$$

or,

$$\dot{r}^2 = E^2 - \frac{L^2 f(r)}{r^2 + l^2}$$

or,

$$\dot{r}^2 = E^2 - \frac{L^2(r^2 - 2mr - l^2)}{(r^2 + l^2)^2}$$

It can be shown that the impact parameter $b = \frac{L}{E}$ [27, page: 123]. So the above equation can be written as,

$$\dot{r}^2 = \frac{L^2}{(r^2 + l^2)^2}[\frac{r^4}{b^2} + \frac{l^2}{b^2}(l^2 + 2r^2) - (r^2 - 2mr - l^2)]$$

or,

$$\dot{r} = \frac{L}{r^2 + l^2}\sqrt{\frac{r^4}{b^2} + \frac{l^2}{b^2}(l^2 + 2r^2) - (r^2 - 2mr - l^2)} \tag{9}$$

r obtains a local extremum for the closest approach r_0. Therefore, we can write:

$$\dot{r}|_{r=r_0} = 0$$

Thus,from equation (9), we can have

$$\frac{r_0^4}{b^2} + \frac{l^2}{b^2}(l^2 + 2r_0^2) - (r_0^2 - 2mr - l^2) = 0$$

or,

$$\frac{r_0^2}{b^2} = -\frac{l^2}{b^2}(2 + \frac{l^2}{r_0^2}) + (1 - \frac{2m}{r_0} - \frac{l^2}{r_0^2}) \tag{10}$$

Equations (5), (6) and (9) represent the geodesic equations of light ray.

3 LIGHT DEFLECTION ANGLE

The light deflection angle can be in general expressed as [28, page: 188]

$$\alpha = 2\int_{r_0}^{\infty}(\frac{d\varphi}{dr}).dr - \pi \tag{11}$$

Now using equation (6) and (9), we write:

$$\frac{d\varphi}{dr} = \frac{\dot{\varphi}}{\dot{r}} = \frac{1}{\sqrt{\frac{r^4}{b^2} + \frac{l^2}{b^2}(l^2 + 2r^2) - (r^2 - 2mr - l^2)}}$$

or,

$$\frac{\dot{\varphi}}{\dot{r}} = \frac{1}{\sqrt{\frac{r^4}{b^2} + \frac{l^2 r^2}{b^2}(2 + \frac{l^2}{r^2}) - r^2(1 - \frac{2m}{r} - \frac{l^2}{r^2})}} \tag{12}$$

Substituting equation (12) in (11) we have,

$$\alpha = 2\int_{r_0}^{\infty} \frac{dr}{\sqrt{\frac{r^4}{b^2} + \frac{l^2 r^2}{b^2}(2 + \frac{l^2}{r^2}) - r^2(1 - \frac{2m}{r} - \frac{l^2}{r^2})}} - \pi \tag{13}$$

Let us introduce a new variable $x = \frac{r_0}{r}$. So,

$$dx = -\frac{r_0 dr}{r^2}$$

or,

$$\frac{dx}{r_0} = -\frac{dr}{r^2}$$

the limits will change as', when $r \longrightarrow \infty$, then $x \longrightarrow 0$ and when $r \longrightarrow r_0$, then $x \longrightarrow 1$. Using these limits in above equation we have,

$$\alpha = 2\int_{0}^{1} \frac{dx}{r_0\sqrt{\frac{1}{b^2} + \frac{l^2 x^2}{b^2 r_0^2}(2 + \frac{l^2 x^2}{r_0^2}) - \frac{x^2}{r_0^2}(1 - \frac{2mx}{r_0} - \frac{x^2 l^2}{r_0^2})}} - \pi$$

Let us substitute $h = \frac{m}{r_0}$, $n^2 = \frac{l^2}{r_0^2}$ and $\hat{n}^2 = \frac{l^2}{b^2}$. So the above equation can be written as:

$$\alpha = 2\int_{0}^{1} \frac{dx}{\sqrt{\frac{r_0^2}{b^2} + \hat{n}^2 x^2(2 + n^2 x^2) - x^2(1 - 2hx - n^2 x^2)}} - \pi \tag{14}$$

Now after substituting the expression of $\frac{r_0^2}{b^2} = -\frac{l^2}{b^2}(2 + \frac{l^2}{r_0^2}) + (1 - \frac{2m}{r_0} - \frac{l^2}{r_0^2}) = -\hat{n}^2(2 + n^2) + (1 - 2h - n^2)$ from equation (10) in equation (14), we get:

$$\alpha = 2\int_{0}^{1} \frac{dx}{\sqrt{-\hat{n}^2(2 + n^2) + (1 - 2h - n^2) + \hat{n}^2 x^2(2 + n^2 x^2) - x^2(1 - 2hx - n^2 x^2)}} - \pi \tag{15}$$

re-arranging equation (15) we have,

$$\alpha = 2\int_{0}^{1} \frac{dx}{\sqrt{(1 - x^2) - 2h(1 - x^3) - n^2(1 - x^4) - 2\hat{n}^2(1 - x^2) - n^2\hat{n}^2(1 - x^4)}} - \pi$$

or,

$$\alpha = 2\int_{0}^{1} \frac{dx}{\sqrt{1 - x^2}\sqrt{1 - 2h\frac{(1-x^2)}{(1-x^3)} - n^2(1 + \hat{n}^2)(1 + x^2) - 2\hat{n}^2}} - \pi$$

or,

$$\alpha = 2\int_{0}^{1} \frac{dx}{\sqrt{1 - x^2}\sqrt{1 - 2h\frac{(1-x^3)}{(1-x^2)}}\sqrt{1 - n^2\frac{(1+\hat{n}^2)(1+x^2)}{1-2h\frac{(1-x^2)}{(1-x^3)}}}\sqrt{1 - \frac{2\hat{n}^2}{[1-2h\frac{(1-x^3)}{(1-x^2)}][1-\frac{n^2(1+\hat{n}^2)(1+x^2)}{1-2h\frac{(1-x^3)}{(1-x^2)}}]}}} - \pi$$

or,

$$\alpha = 2\int_0^1 \frac{dx}{\sqrt{1-x^2}}[1-2h\frac{(1-x^3)}{(1-x^2)}]^{-\frac{1}{2}}[1-n^2(1+\hat{n}^2)(1+x^2)\{1-2h\frac{(1-x^3)}{(1-x^2)}\}^{-1}]^{-\frac{1}{2}}$$
$$[1-2\hat{n}^2\{1-2h\frac{(1-x^3)}{(1-x^2)}\}^{-1}\{1-n^2(1+\hat{n}^2)(1+x^2)(1-2h\frac{1-x^3}{1-x^2})^{-1}\}^{-1}]^{-\frac{1}{2}}-\pi$$

For weak deflection limit $l, m \ll r_o, b$, in other words we can write, $h, n, \hat{n} \ll 1$. So the above equation can be expanded in Taylor series in terms of both h, n and $\hat{n}$. In the following we retain terms upto fourth order only. We further write:

$$\alpha = 2\int_0^1 \frac{dx}{\sqrt{1-x^2}}[1+h\frac{1-x^3}{1-x^2}+\frac{3}{2}(h\frac{1-x^3}{1-x^2})^2$$
$$+\frac{5}{2}(h\frac{1-x^3}{1-x^2})^3+\frac{35}{8}(h\frac{1-x^3}{1-x^2})^4][1+\frac{n^2}{2}(1+x^2)+n^2h(1+x^2)\frac{1-x^3}{1-x^2}$$
$$+2n^2h^2(1+x^2)\frac{(1-x^3)^2}{(1-x^2)^2}+n^2\hat{n}^2(1+x^2)+\frac{3}{8}n^4(1+x^2)^2][1+\hat{n}^2+2h\hat{n}^2\frac{(1-x^3)}{(1-x^2)}$$
$$4\hat{n}^2h^2\frac{(1-x^3)^2}{(1-x^2)^2}+\hat{n}^2n^2(1+x^2)+\frac{3}{2}\hat{n}^4]-\pi$$

Multiplying term by term taking up to fourth order of h, n and $\hat{n}$ we get,

$$\alpha = 2\int_0^1 \frac{dx}{\sqrt{1-x^2}}[1+h\frac{1-x^3}{1-x^2}+\frac{3}{2}h^2\frac{(1-x^3)^2}{(1-x^2)^2}+\frac{5}{2}h^3\frac{(1-x^3)^3}{(1-x^2)^3}$$
$$+\frac{35}{8}h^4\frac{(1-x^3)^4}{(1-x^2)^4}+\frac{n^2}{2}(1+x^2)+\frac{3}{2}n^2h(1+x^2)\frac{1-x^3}{1-x^2}+\frac{15}{4}n^2h^2(1+x^2)\frac{(1-x^3)^2}{(1-x^2)^2}$$
$$+2n^2\hat{n}^2(1+x^2)+\frac{3}{8}n^4(1+x^2)^2+\hat{n}^2+3\hat{n}^2h\frac{(1-x^3)}{(1-x^2)}$$
$$+\frac{15}{2}\hat{n}^2h^2\frac{(1-x^3)^2}{(1-x^2)^2}+\frac{3}{2}\hat{n}^4]-\pi \tag{16}$$

Again integrating the above equation term by term we have,

$$\alpha = 4h+(-4+\frac{15\pi}{4})h^2+(\frac{122}{5}-\frac{15\pi}{4})h^3+(-130+\frac{3465\pi}{64})h^4+\frac{3\pi}{4}n^2+(14-\frac{3\pi}{2})hn^2+(-50+\frac{825\pi}{32})h^2n^2$$
$$\frac{57}{64}n^4\pi+3\pi\hat{n}^2n^2+\pi\hat{n}^2+12\pi\hat{n}^2h+(-20+\frac{75\pi}{4})h^2\hat{n}^2+\frac{3\pi}{2}\hat{n}^4 \tag{17}$$

This is the expression of light deflection angle due to a NUT body. If the NUT factor is set to zero in equation (17), we find the above equation reduces to,

$$\alpha = 4h+(-4+\frac{15\pi}{4})h^2+(\frac{122}{3}-\frac{15\pi}{2})h^3+(-130+\frac{3465\pi}{64})h^4 \tag{18}$$

This is the well known expression of light deflection angle due to Schwarzschild mass. If we set mass is equal to zero in equation (17), the bending angle expression becomes:

$$\alpha = \frac{3\pi}{4}n^2+\frac{57}{64}n^4\pi+3\pi\hat{n}^2n^2+\pi\hat{n}^2+\frac{3\pi}{2}\hat{n}^4 \tag{19}$$

4 DISCUSSION OF RESULTS

The main focus of the paper is to understand how a gravomagnetic monopole influences the geometry of space time curvature. To explain our results more clearly, we plotted bending angle (α) against impact parameter (b) in Fig.1 for three different values of NUT factor ($n = \frac{l}{r_0} = 0, 0.05, 0.08$), where $n = 0$ represents the Schwarzschild body. From the pattern of the graph in Fig.1, it is clear that light deflection angle increases with the increase of NUT factor. When NUT factor is zero, i.e. for Schwarzschild body, light ray gets the minimum deflection.

As already mentioned, the NUT line element represents the exterior of a static body with non zero magnetism. On the other hand Reissner-Nordström line element represents the exterior of a static charged body. Both the bodies are static but have two different types of field. Here, we would like to compare the effect of these two fields on the geometry of space time. To fulfill this objective, we plotted light deflection angle as a function of both NUT factor (calculated in this paper) and static charge (calculated by Eiroa et al [9]) in Fig.2 where both NUT factor and static charge have been taken in the same scale. In Fig.2 we can see that NUT factor and static charge influence the space time geometry in opposite direction. The presence of NUT factor increases the light deflection angle compared to zero field Schwarzschild case. On the other hand, the presence of static charge decreases the light deflection angle with respect to Schwarzschild body.

5 CONCLUSIONS

1. Expression for deflection of light due to a static body with non-zero gravomagnetism has been calculated considering contributions from mass and NUT factor, considering their effects up to fourth order terms.
2. NUT factor has a noticeable effect on the path of the light ray. When compared with Schwarzschild expression for bending, we find that there are some extra terms in the expression for deflection, which occur due to the presence of NUT factor. If we set the NUT factor equal to zero,deflection angle will reduce to that of Schwarzschild deflection angle.
3. Presence of NUT factor increases the deflection angle, on other hand presence of static charge decreases the deflection angle. Thus these two parameters have opposite effect on the space-time geometry.
4. If we set mass is equal to zero, our bending angle does not reduce to zero, i.e. the gravomagnetism itself can influence the path of the light ray.

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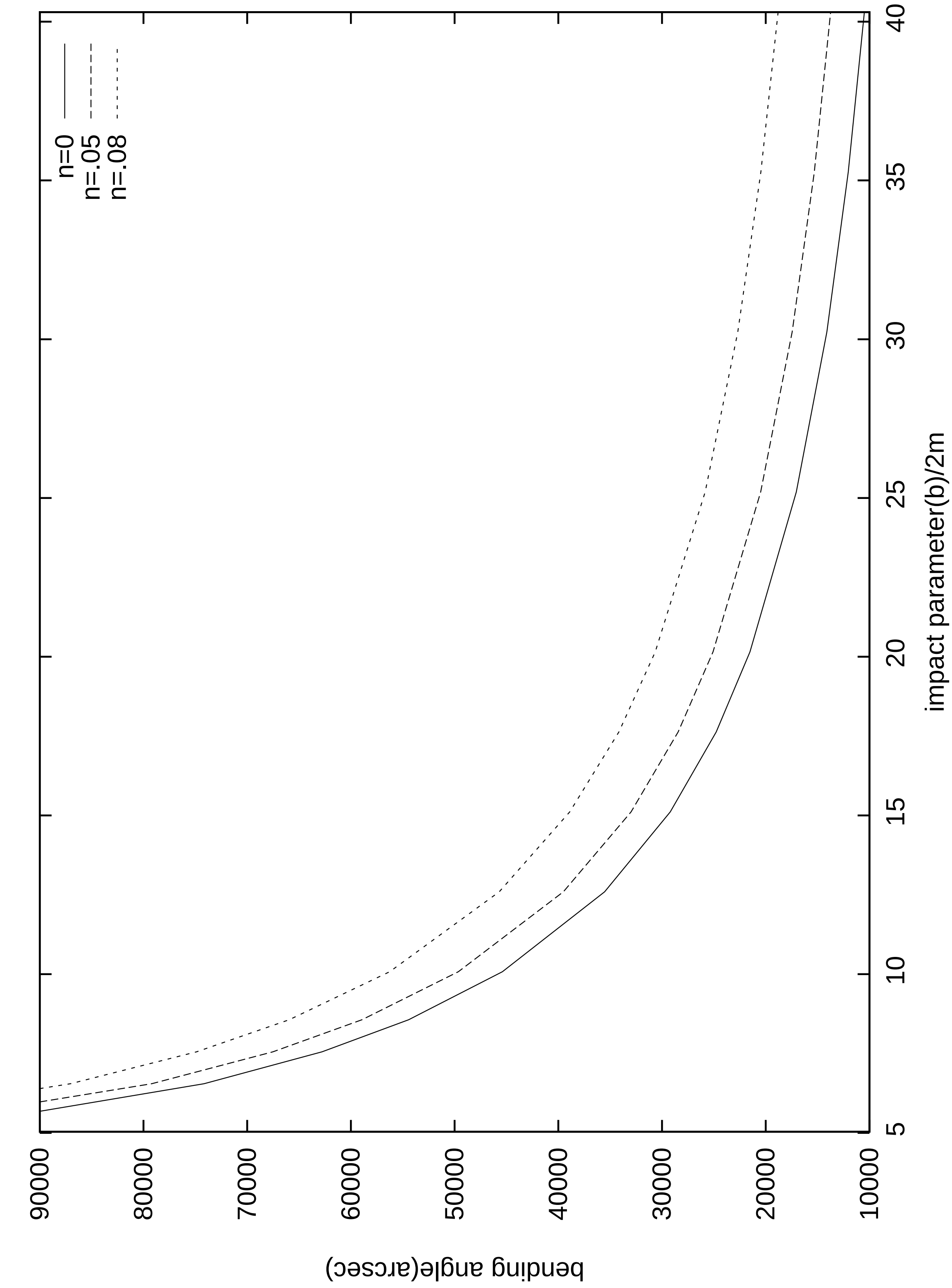

Figure 1: Bending angle (arcsec) as a function of impact parameter $b/2m$ for three values of NUT factor $n = 0, 0.05, 0.08$, where $n = 0$ represents the Schwarzschild body.

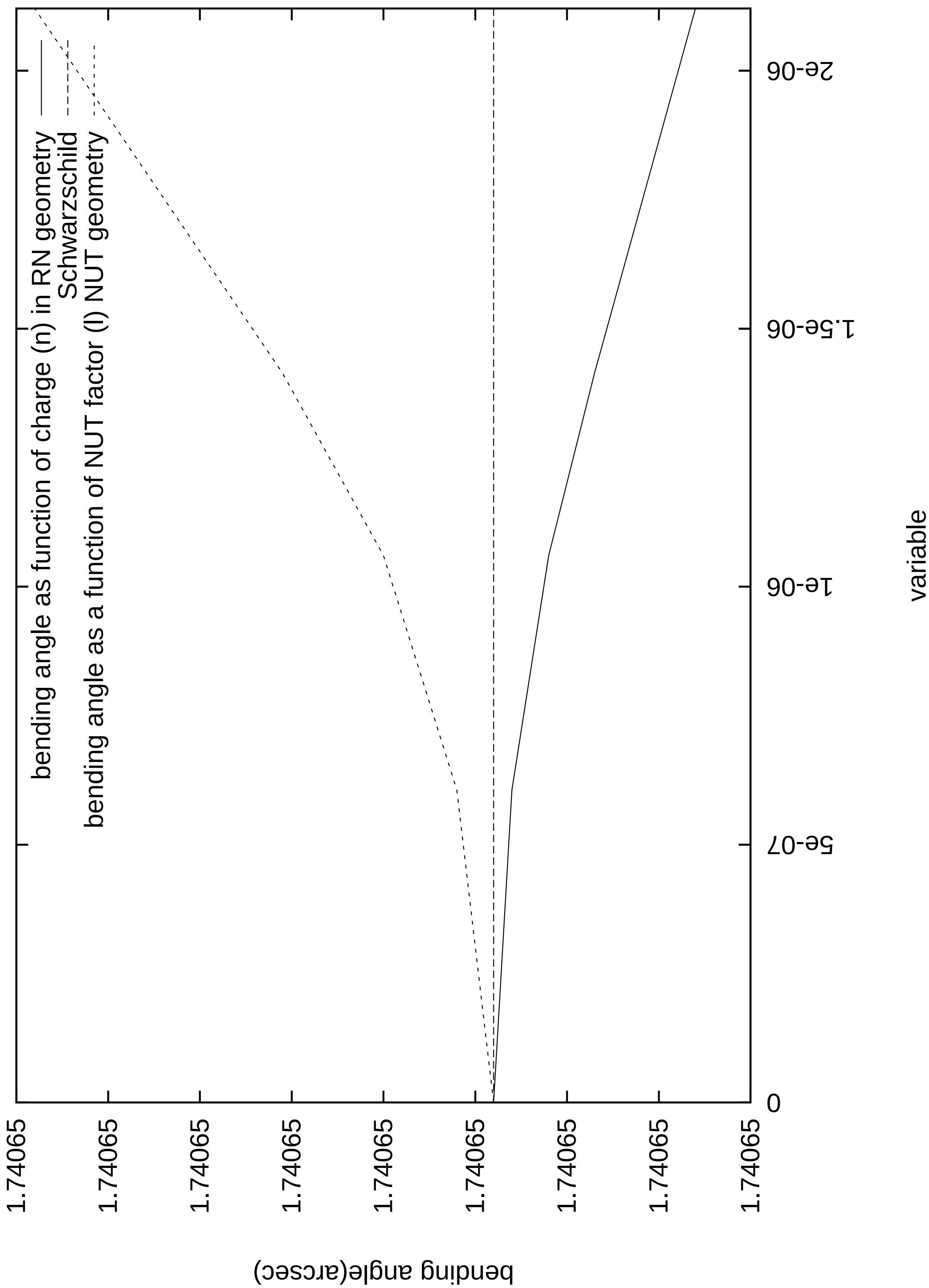

Figure 2: Bending angle (arcsec) as a function of NUT factor (calculated in the present work) and static charge (calculated by Eiroa et al [9]), where both NUT factor and static charge have taken in the same scale.