Spin connection as Lorentz gauge field: propagating torsion

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Abstract

We propose a modified gravitational action containing besides the Einstein-Hilbert term some quadratic contributions resembling the Yang-Mills lagrangian for the spin connections. We outline how a propagating torsion arises and we solve explicitly the linearised equations of motion on a Minkowski background. We identify among torsion components six degrees of freedom: one is carried by a pseudo-scalar particle, five by a tachyon field. By adding spinor fields, we point out how only the pseudo-scalar particle couples directly with fermions and we evaluate the associated coupling constant, which is suppressed by the ratio between fermion and Planck masses.

Keywords: Lorentz gauge theory of Gravity, Torsion, propagative theory

1. Introduction

As pointed out by the Einstein-Cartan theory [1, 2], it is possible to implement a local symmetry in the description of the space-time by using the tetrad formalism. In such a formulation, a local basis of tangent space is introduced and the theory is characterised by a local Lorentz rotation symmetry.

In his seminal paper [3], Utyiama proposed a method to introduce gauge fields associated with Lorentz transformations; he showed that these fields are nothing more than the spin connections ω_μ^{IJ} corresponding to a physical gauge symmetry as the space-time description is unaffected by a Lorentz rotation of

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the tetrads. Indeed, this interpretation of spin connections as gauge fields of the Lorentz group (LGT) is not physically well-grounded in the Einstein-Cartan theory, since in this framework the spin connections do not include propagating degrees of freedom: they depend on tetrad fields and (algebraically), in the theory coupled with fermions, on the spin density too [4–8].

A revised paradigm for this question is given by Poincaré gauge theory of gravity (PGT) [9–14], where vierbein are identified with the gauge fields corresponding to the translational part of the Poincaré group. Hence, the gravitational interaction follows from the local extension of the invariance under Poincaré transformations.

However, also PGT presents some internal difficulties, since the two gauge fields, the spin connections and the tetrads, are not independent from each other. This is due to the fact that an infinitesimal Lorentz rotation cannot be distinguished from an infinitesimal translation [15].

In this work, we show how it is possible, by modifying the Einstein-Cartan [2] action, to obtain intrinsically propagating gauge fields associated with the local Lorentz group. Our purpose is to consider quadratic terms in curvature tensor with strong analogies with the free Yang-Mills terms for the Lorentz group.

From the equations of motion, we deduce that in this theory the torsion tensor is generally non trivial (also in vacuum). We solve in vacuum the equations of motion of the linearised theory on a Minkowskian background. We outline that torsion owns an intrinsically dynamical behaviour and, by dividing into irreducible components, we show that the propagating degrees of freedom are represented by a pseudo-scalar massive field and a tachyonic field. An interesting feature of this analysis is that the fields are massive and their masses are fixed by the coupling constant γ for the Yang-Mills term. Such Yang-Mills term is the only viable quadratic modification providing a non-trivial contribution to the equations of motion, determining the main difference of this theory with the Einstein-Cartan one.

By studying the interaction with spinor fields, it comes out that only the pseudo-scalar field couples with fermions. In this case, the spin connections receive a contribution sourced by the fermion field, but the resulting interaction is very weak, since it is suppressed by the large value of the Planck mass.

The paper is structured as follows. In section 2 we give a brief introduction of PGT. In section 3 we present the new action, we evaluate the corresponding equations of motion and eventually we solve on a Minkowskian background the linearised theory in section 4. Here we find that the spin connections include propagating degree of freedom, solving the non propagating nature of the spin connections in LGT [4–6, 11, 16]. In section 5 we include in this model spinor fields characterizing their contribution to the spin connections; we also estimate the phenomenological impact of this theory by considering its possible contribution on the muon gyro-magnetic moment. Brief conclusions follow in section 6.

2. Poicaré Gauge theory of Gravity

The strong analogy between gauge theories and the space-time description in terms of tetrads suggests the proposal to connect the torsion tensor with the Poincaré gauge symmetry of the space-time [4, 5, 11, 17]. Such a Poincaré gauge theory (PGT) [4, 7–17, 22, 23] can be resumed by tracing two different points of view: the gauge approach and the geometrical one [6, 22, 23].

2.1. Gauge approach

If we perform a global Poincaré transformation in the space-time

$$x^\mu \to x'^\mu = x^\mu + \widetilde{\epsilon}^\mu_{\ \nu} x^\nu + \widetilde{\epsilon^\mu}, \tag{1}$$

$\widetilde{\epsilon}^\mu_{\ \nu}$ and $\widetilde{\epsilon^\mu}$ being infinitesimal parameters characterizing space-time rotations and translations, respectively, the fermion field correspondingly experiences a transformation under its Poincaré group representation:

$$\psi(x) \to \psi'(x) = \left(1 + \frac{1}{2}\widetilde{\epsilon}^{\mu\nu} M_{\mu\nu} + \widetilde{\epsilon}^\mu P_\mu\right) \psi(x), \tag{2}$$

where $M_{\mu\nu}$ and P_μ are the generators of the Poicaré algebra. As far as the transformation is global, assuming that the matter lagrangian depends only on the spinor field and its derivatives, $L = L(\psi, \partial\psi)$, one can find the usual conserved currents (energy-momentum tensor and angular momentum tensor) and Noether charges.

If we consider (1) as a local transformation, some connections have to be implemented on the space-time in order to restore the spinor Lagrangian invariance. The associated gauge fields are f^I_μ and $A_\mu^{\ IJ}$ and the associated generators P_μ and Σ_{IJ}. The covariant derivative is defined as

$$D_I\psi = f^\mu_I D_\mu \psi = f^\mu_I \left(\partial_\mu + A_\mu\right)\psi = f^\mu_I \left(\partial_\mu + \frac{1}{2} A_\mu^{\ IJ} \Sigma_{IJ}\right)\psi. \tag{3}$$

As usual the commutators of covariant derivatives is proportional to the field strengths [24, 25], *i.e.*

$$f^K_\mu f^L_\nu \left[D_K, D_L\right]\psi = \frac{1}{2} F_{\mu\nu}^{\ \ \ IJ} \Sigma_{IJ}\psi - F_{\mu\nu}^{\ \ \ I} D_I \psi, \tag{4}$$

where $F_{\mu\nu}^{\ \ \ IJ}$ and $F_{\mu\nu}^{\ \ \ I}$ are the Lorentz and the translation field strengths, respectively.

In [11] it is shown the inadequacy of Special Relativity to describe the behaviour of matter fields under local Poicaré transformations. Matter fields are characterised by the *rigidity condition*, in other words the global Poicaré transformations preserve the distances between events and the relative orientation of neighbouring matter fields (as a rigid body). It must be equivalent if we compare field amplitudes at nearby points and then we transform the result with a global Poincaré transformation, or whether we compare the two transformed amplitudes at the corresponding points. If not so, it would be possible to distinguish experimentally between two reference frames. Nonetheless, this is just the case for local transformations. It means that special relativity is not the adequate framework where we can demand such a rigidity condition in the framework of a gauge theory [11]. For this reason, the space-time needs two compensating field to restore this invariance. The surprising feature of such emerging fields is their geometrical nature.

2.2. Geometrical approach

The Cartan space-time [2] is characterised by a connection ${\Gamma_{\mu\nu}}^{\rho}$ compatible with the metric, *i.e.*

$$\nabla_\rho g_{\mu\nu} = \partial_\rho g_{\mu\nu} - {\Gamma_{\rho\mu}}^{\sigma} g_{\sigma\nu} - {\Gamma_{\rho\nu}}^{\sigma} g_{\mu\sigma} = 0, \tag{5}$$

and a non trivial torsion tensor ${T_{\mu\nu}}^{\rho} = {\Gamma_{\mu\nu}}^{\rho} - {\Gamma_{\nu\mu}}^{\rho}$, such that

$${\Gamma_{\mu\nu}}^{\rho} = {\bar{\Gamma}_{\mu\nu}}^{\rho} - {K_{\mu\nu}}^{\rho}, \tag{6}$$

${\bar{\Gamma}_{\mu\nu}}^{\rho}$ being the Christoffel symbols, while ${K_{\mu\nu}}^{\rho}$ is the contortion tensor [4, 8]. Choosing a set of orthonormal vector fields e^I_μ (the tetrads) as a basis for the local tangent space, one has to define the covariant derivative as follows

$$D_\mu \psi = (\partial_\mu + \omega_\mu)\psi = \left(\partial_\mu + \frac{1}{2}{\omega_\mu}^{IJ}\Sigma_{IJ}\right)\psi. \tag{7}$$

Analogously to (6), the spin connection ${\omega_\mu}^{IJ}$ can be written as a term depending on the tetrads e^I_μ and a torsion contribution: [4, 8]

$$\begin{aligned} &{\omega_\mu}^{IJ} = {\bar{\omega}_\mu}^{IJ} + {K_\mu}^{IJ}, \\ &\qquad\qquad {K_\mu}^{IJ} = e^{I\nu} e^J_\rho {K_{\mu\nu}}^{\rho} = -e^{I\nu} e^J_\rho \left({T_{\mu\nu}}^{\rho} - {T_\nu}^{\rho}{}_{\mu} + {T^\rho}_{\mu\nu}\right). \end{aligned} \tag{8}$$

In this framework the gauge fields e^I_μ represent the map from the coordinate basis to the tetradic one. Moreover, ${\omega_\mu}^{IJ}$ must be connected to the local Lorentz transformation of the vector basis. The torsion dynamic is described by the action

$$\mathcal{L}_T = A\, T_{IJK} T^{IJK} + B\, T_{IJK} T^{JKI} + C\, {T_{IK}}^{K} {T^{IL}}_{L}. \tag{9}$$

A, B and C being some parameters, while the space-time indexes of the torsion tensor have been projected into those of the tangent space via the tetrads, ${T_{IJ}}^{K} = e^\mu_I e^\nu_J e^K_\rho {T_{\mu\nu}}^{\rho}$, and eventually raised or lowered via Minkowsky metric. Indeed, this is the most general Lagrangian generating equations of motion for the torsion with derivatives up to second order only (as shown in [16]).

Comparing the gauge approach with the geometrical one, it is natural to identify the gauge field ${A_\mu}^{IJ}$ with the spin connection ${\omega_\mu}^{IJ}$ and the gauge field corresponding to translations f^I_μ with the component of the tetrad field e^I_μ. By these identifications, the equations of motions for the Einstein-Cartan action induce an algebraic relation between torsion tensor and the spin density [4–6], which results into non-propagating gauge field ${\omega_\mu}^{IJ}$.

3. Lagrangian formulation

The attempt to extend the General Relativity formalism through a change in the Einstein-Hilbert action must be consistent with all the experimental observations constraining the theory. Moreover, by analogy with usual field theories, the equations of motion must contain derivatives up to the second order.
The simplest example of a modifications is the introduction of topological terms, which do not affect the equations of motions.

Here, we consider a Lagrangian containing at most second derivatives of fields, scalar under parity and reducing to General Relativity in some limit. According with this hypotheses, our Lagrangian formulation reads

$$S[e,\omega] = -\frac{1}{2\chi}\int d^4x\, e(R_{\mu\nu}{}^{IJ}\, e^{\mu}_{I}\, e^{\nu}_{J} + \\ + \gamma\, R^{\mu\nu}{}_{IJ}\, R_{\mu\nu}{}^{IJ} + \beta\, \eta^{\mu\nu\rho\sigma}\epsilon_{IJKL} R_{\mu\nu}{}^{IJ}\, R_{\rho\sigma}{}^{KL}), \quad (10)$$

where $R_{\mu\nu}{}^{IJ}$ is the Riemann tensor

$$R_{\mu\nu}{}^{IJ} = \partial_\mu \omega_\nu{}^{IJ} - \partial_\nu \omega_\mu{}^{IJ} + \omega_\mu{}^{IK}\omega_{\nu K}{}^{J} - \omega_\nu{}^{IK}\omega_{\mu K}{}^{J}, \quad (11)$$

while $\chi = 8\pi G$ and γ, by the analogy with Yang-Mills lagrangian, is a positive coupling constant (this requirement ensures that the theory is ghost-free). In what follows, β will play no role since it multiplies a topological term.

In the first order formalism, the spin connections ω^{IJ} and the tetrads e^I fields are treated as independent. The equations of motion follow from the variation of the action (10) with respect to these independent sets of variables, *i.e.*:

$$D_\mu \left[e\, e^{[\mu}_{I}\, e^{\nu]}_{J} + 2\,\gamma\, e\, R^{\mu\nu}{}_{IJ} \right] = 0 \quad (12)$$

$$R^{K}_{\rho} - \frac{1}{2}\left(R + \gamma\, R_{\mu\nu}{}^{IJ}\, R^{\mu\nu}{}_{IJ} \right) e^{K}_{\rho} + 2\,\gamma\, R_{\mu\nu\, IJ}\, R_{\rho}{}^{\nu IJ}\, e^{K\mu} = 0. \quad (13)$$

While Einstein-Cartan theory in vacuum is equivalent to General Relativity [2, 4, 5, 11, 16], the present theory is always characterized by a non trivial torsion tensor due to the second term on the left-hand side of (12). In fact, the first term in (12) is proportional to the torsion [5, 17], while the second corresponds to the equation of motion of a free Yang-Mills theory. Hence, this is a promising scheme for a theory wherein torsion is characterised by independent degrees of freedom.

4. Linearised theory on minkowskian background

The system of equations (12) and (13) is non-linear both in tetrads and in spin connection fields. Let us now investigate the linearised field equations with respect to the torsion field on a Minkowskian background, by fixing the tetrads as follows

$$e^{I}_{\mu}(x) = \delta^{I}_{\mu}. \quad (14)$$

This particular choice provides an identification between space-time and internal indexes and a significant simplification of the dynamical problem: torsion-free spin connections vanish ($\bar{\omega}_\mu^{IJ} = 0$ and $\omega_\mu^{IJ} = K_\mu^{IJ}$) and the Riemann tensor can be written as

$$R_{\mu\nu}{}^{IJ} = 2\,\partial_{[\mu}K_{\nu]}^{IJ} + \circ(K^2). \tag{15}$$

The system of equations (12) and (13) becomes at the linear order

$$D_\mu\left(\delta_I^{[\mu}\delta_J^{\nu]} + 4\,\gamma\,\partial^{[\mu}K^{\nu]}{}_{IJ}\right) = 0 \tag{16}$$

$$R_\rho{}^J = 2\,e_I^\mu\partial_{[\mu}K_{\rho]}{}^{IJ} = 0. \tag{17}$$

To characterize the solution we firstly solve equation (16) and then we check if these solutions also solve the condition (17).
Expanding the covariant derivative in (16), we obtain the equation

$$\Box K_{SIJ} - \partial_S\partial_L K^L{}_{IJ} + \frac{1}{2\gamma}\left[-K_L{}^L{}_{[I}\,\eta_{J]S} + K_{[IJ]S}\right] = 0, \tag{18}$$

where $K_{SIJ} = e_S^\mu\,K_{\mu\,IJ} = \delta_S^\mu\,K_{\mu\,IJ}$. Hence, we have to solve the following system of equations

$$\begin{cases}\Box\,K_{SIJ} - \partial_S\partial_\mu K^\mu{}_{IJ} + \frac{1}{2\gamma}(-K_L{}^L{}_{[I}\,\eta_{J]S} + K_{[IJ]S}) = 0\\ \partial^S(-K_L{}^L{}_{[I}\,\eta_{J]S} + K_{[IJ]S}) = 0\\ \partial_{[\mu}K_{\rho]}{}^{\mu J} = 0,\end{cases} \tag{19}$$

where the second condition comes from (18) by acting with ∂^S. In order to find the solutions of (19), we now decompose the contortion tensor in its irreducible components.

4.1. Irreducible components of the contortion tensor

Torsion can be decomposed into three irreducible tensors [18–20] (a multidimensional decomposition can be found in [21]). In this paragraph we decompose in a similar manner the contortion tensor and we show how this decomposition clarifies the nature of its propagating degrees of freedom.
We can write the contortion tensor as follows:

$$K_{SIJ} = \Omega_{SIJ} + t_{SIJ} + q_{SIJ}. \tag{20}$$

where we have isolated the totally-antisymmetric (or pseudo-trace) contortion part Ω_{SIJ}, *i.e.*

$$\Omega_{SIJ} = K_{[SIJ]} = -\frac{1}{6}\epsilon_{SIJK}S^K, \tag{21}$$

the axial-vector $S^K = \epsilon^{KSIJ}K_{SIJ}$ having four independent components, and the trace part t_{SIJ}

$$t_{SIJ} = \frac{1}{3}\left(\eta_{SI}K_J - \eta_{SJ}K_I\right), \tag{22}$$

with $K_I = K^S{}_{SI}$ having also four independent components. The last term in (20) tensor is traceless and its totally-antisymmetric part vanishes, thus the following conditions hold

$$q_{SIJ} = q_{S[IJ]} \tag{23}$$
$$q_{[SIJ]} = 0 \tag{24}$$
$$q^S{}_{IS} = 0, \tag{25}$$

from which it follows that there are sixteen independent components in q_{SIJ}, thus reconciling the total number of independent components with that of the contortion tensor, which is twenty-four.

4.2. Dynamic properties of the irreducible tensors

Let us now investigate the implications of (19) for each irreducible tensor in (20). As soon as Ω_{IJK} is concerned, let us note that the condition $\partial^S\Omega_{SIJ} = 0$ implies $\partial_{[\mu}\Omega_{\rho]}{}^{\mu J} = 0$, thus we get from (19)

$$\begin{cases} \Box\,\Omega_{SIJ} - \partial_S\partial^K\Omega_{K\,IJ} + \frac{1}{2\gamma}\Omega_{SIJ} = 0 \\ \partial^S\Omega_{SIJ} = 0. \end{cases} \tag{26}$$

The second condition in (26) rewrites through (21)

$$\epsilon_{SIJK}\partial^{[S}S^{K]} = 0 \tag{27}$$

and if the spacetime manifold is simply-connected it implies that $S^K = \partial^K\Omega$, for some pseudo-scalar field $\Omega(x)$. The first condition becomes

$$\Box\,\Omega + \frac{1}{2\gamma}\Omega = 0, \tag{28}$$

which is the Klein-Gordon equation for a field with mass $m = 1/2\gamma$. Therefore, the totally-antisymmetric component of the contortion tensor carries one degree of freedom in the form of a massive pseudo-scalar field.
Similarly, from the second equation in (19), we get the following condition for t_{SIJ} (22)

$$\partial^S\, t_{SIJ} = \partial_I K_J - \partial_J K_I = 0, \tag{29}$$

the only possible solution in a simply-connected manifold being $K_I = \partial_I\psi$. The other equations become

$$\begin{cases} \Box\,\psi + \frac{1}{2\gamma}\psi = 0 \\ \delta^J_\rho\Box\psi - 2\partial_\rho\partial^J\psi = 0 \end{cases} \tag{30}$$

and they admit only the trivial solution $\psi = 0$ (this can be seen by multiplying the third condition times δ^ρ_J). Finally, the system (19) for the last part of the contortion tensor reduces to

$$\begin{cases} \Box\; q_{SIJ} - \frac{1}{2\gamma}\, q_{SIJ} = 0 \\ \partial^S\, q_{SIJ} = 0 \\ \partial^S\, q_{IJS} = 0. \end{cases} \tag{31}$$

From the first equation we see how q_{SIJ} describes the propagation of a tachyon particle. The other conditions can be solved by fixing the frame in which the four-momentum $k_\mu = (0,0,0,1/\sqrt{2\gamma})$ and by requiring q_{SIJ} to vanish when one of the index $S, I, J = 3$ (this can be easly seen in Fourier space). Given these conditions, the total number of independent components within q_{SIJ} is five.

The generality of our solutions can be verified by counting the physical degrees of freedom. Spin connections ${\omega_\mu}^{IJ}$ have 24 components, but the six components ${\omega_0}^{IJ}$ must be removed, because of their non-dynamical character (their time derivatives are not present in the action (10)).
Moreover the condition (17), together with Lorentz invariance, removes twelve additional components, so that the theory is eventually characterised by six physical degrees of freedom only.

It is easy to check that the solutions we have found contains the correct number of physical degrees of freedom: one degree of freedom associated with the pseudo-scalar field Ω and five degrees of freedom corresponding to q_{SIJ}.
Therefore, the contortion tensor solving the equations of motion of the model reads

$$K_{SIJ} = -\frac{1}{6}\epsilon_{SIJK}\,\partial^K\Omega + q_{SIJ}. \tag{32}$$

In the next section we will outline how only the pseudo-scalar field Ω interacts with spinor fields, while the tachyon field decouples (at least classically), thus suggesting that un-physical interactions do not occur.

5. Field equations in presence of spinors

In this section we investigate the role of spinor fields on the curved space-time whose dynamic is described by the action (10).
The tetradic formalism allows a natural implementation of the Dirac algebra on a curved space-time [31–34], so that the internal Lorentz gauge symmetry acts on spinor fields just like Yang-Mills gauge symmetries [3, 17]. The total action can be written as

$$S = S_g[e,\omega] + S_m[e,\psi,D_\mu\psi], \tag{33}$$

where the spinor action reads [5, 24, 25]

$$\begin{aligned} S_m[e,\psi,D_\mu\psi] = \int dx^4\, e\, [\frac{i}{2}\left(\bar\psi\gamma^\mu\partial_\mu\psi - \left(\partial_\mu\bar\psi\right)\gamma^\mu\psi\right) + \\ + \frac{1}{4}\epsilon_{KIJL}\,\omega^{[K\,IJ]}\bar\psi\gamma_5\gamma^L\psi - m\bar\psi\psi]. \end{aligned} \tag{34}$$

It is worth noting that the action above contains an explicit coupling between spinor fields and spin connections, thus spinor enters the I Cartan equation and provides a nonvanishing contribution to torsion. We are going to solve explicitly such an equation in the linearised theory.

5.1. Linearised solution on a minkowskian background

In the presence of spinors equation (18) becomes

$$\Box\omega_{SIJ} - \partial_S\partial_\mu\omega^\mu{}_{IJ} + \frac{1}{2\gamma}\left[-\omega_K{}^K{}_{[I}\,\eta_{J]S} + \omega_{[IJ]S}\right] = J_{SIJ}$$

$$J_{SIJ} = \frac{\chi}{4\gamma}\,\epsilon_{SIJL}\,\bar{\psi}\gamma_5\gamma^L\psi. \quad (35)$$

The solution of (35) can be written as the homogeneous solution (32) plus $\widetilde{K}_{SIJ}$, satisfying

$$\Box\widetilde{K}_{SIJ} - \partial_S\partial_L\widetilde{K}^L{}_{IJ} + \frac{1}{2\gamma}\widetilde{K}_{SIJ} = J_{SIJ}. \quad (36)$$

Let us write

$$\widetilde{K}_{\mu\,IJ} = \int d^4y\, G_{\mu\nu}{}^{KL}_{IJ}(x-y)J^\nu_{KL}(y), \quad (37)$$

where the kernel $G_{\mu\nu}{}^{KL}_{IJ}$ is defined as the solution of the equation

$$\left(\delta^\nu_\mu\Box - \partial_\mu\partial^\nu + \frac{1}{2\gamma}\delta^\nu_\mu\right)G_{\nu\lambda}{}^{KL}_{IJ} = -\delta^4(x-y)\eta_{\mu\lambda}\delta^K_{[I}\,\delta^L_{J]}. \quad (38)$$

We can rewrite (38) in the Fourier space, so getting

$$\left[\delta^\nu_\mu\left(k^2 - \frac{1}{2\gamma}\right) - k_\mu k^\nu\right]\widetilde{G}_{\nu\lambda}{}^{KL}_{IJ}(k) = \eta_{\mu\lambda}\delta^K_{[I}\,\delta^L_{J]} \quad (39)$$

and by contracting with k^μ we obtain

$$\left[k^\nu\,k^2 - \frac{1}{2\gamma}\,k^\nu - k^2k^\nu\right]\widetilde{G}_{\nu\lambda}{}^{KL}_{IJ}(k) =$$

$$= -\frac{1}{2\gamma}\,k^\nu\,\widetilde{G}_{\nu\lambda}{}^{KL}_{IJ}(k) = \,k^\nu\eta_{\nu\lambda}\delta^K_{[I}\,\delta^L_{J]}. \quad (40)$$

By inserting (40) in (39) one finds

$$\left(k^2 - \frac{1}{2\gamma}\right)\widetilde{G}_{\mu\lambda}{}^{KL}_{IJ}k + 2\gamma\,k_\mu k_\lambda\delta^K_{[I}\,\delta^L_{J]} = \eta_{\mu\lambda}\delta^K_{[I}\,\delta^L_{J]}, \quad (41)$$

which can be easily solved as follows

$$\widetilde{G}_{\mu\lambda}{}^{KL}_{IJ}(k) = \frac{\left(\eta_{\mu\lambda} - \frac{k_\mu k_\lambda}{(2\gamma)^{-1}}\right)}{k^2 - (2\gamma)^{-1} + i\epsilon}\delta^K_{[I}\,\delta^L_{J]}. \quad (42)$$

Hence, the solution of (35) can be written as

$$\widetilde{K}_{\mu\,IJ} = \int d^4y\,\frac{d^4k}{(2\pi)^4}\,\frac{\left(\eta_{\mu\nu} - \frac{k_\mu k_\nu}{(2\gamma)^{-1}}\right)}{k^2 - (2\gamma)^{-1} + i\epsilon}\,e^{ik(x-y)}\,J^\nu_{IJ}(y), \quad (43)$$

which gives the expression of the torsion field sourced by spinors.

5.2. Torsion-spinor coupling

We now consider the interaction between spin connections and spinors at the leading order of a perturbative expansion. Hence, we substitute the vacuum spin connections (32) into the Dirac action (34). Since spinors couples only to the total antisymmetric part of the connection, the tachyon field q_{SIJ} does not interact with spinors (at least at the leading order of the perturbative expansion). So the only contribution is given by the pseudo-scalar field (28) and the spinor lagrangian reads

$$\mathcal{L} = \frac{i\hbar c}{2}\left[\bar{\psi}\gamma^\mu\partial_\mu\psi - \left(\partial_\mu\bar{\psi}\right)\gamma^\mu\psi\right] - \frac{\hbar c}{4}\partial_L\Omega\;\bar{\psi}\gamma^L\gamma^5\psi - \hbar c\, m\bar{\psi}\psi. \tag{44}$$

The interaction term can be integrated by parts, so getting

$$\mathcal{L} = \frac{i\hbar c}{2}\left[\bar{\psi}\gamma^\mu\partial_\mu\psi - \left(\partial_\mu\bar{\psi}\right)\gamma^\mu\psi\right] + \frac{i\hbar c}{2}m\,\Omega\,(\,\bar{\psi}\gamma^5\psi) - \hbar c\, m\bar{\psi}\psi, \tag{45}$$

where the following relation has been used

$$\partial_L(\,\bar{\psi}\gamma^L\gamma^5\psi) = 2im\bar{\psi}\gamma^5\psi. \tag{46}$$

Let us now redefine Ω as

$$\Omega \to \sqrt{\frac{6}{\chi}}\,\Omega; \tag{47}$$

such that it has the dimensionality of a scalar field (this can be seen from its kinetic term), while the interaction lagrangian with spinors rewrites

$$\mathcal{L}_{int} = i\,g\,\Omega\,\bar{\psi}\gamma^5\psi, \qquad g = \sqrt{\frac{\pi}{3}}\,\frac{m}{M_p}, \tag{48}$$

M_p being Planck mass. Therefore, the coupling constant g between spinors and the pseudo-scalar torsion component depends on the fermion mass. However, in view of the hierarchy between particle and Planck masses the value of g is much smaller than the coupling constants of other interactions.

In order to estimate the possible phenomenological implications of our model, we evaluated the contribution given by the interaction with Ω to the gyromagnetic moment of a lepton, finding a displacement with respect to the standard value [35]

$$\Delta a = -\frac{g^2}{8\pi^2}\,\lambda^2\int_0^1 dx\,\frac{x^3}{(1-x)(1-\lambda^2 x)+\lambda^2 x}, \tag{49}$$

where $\lambda = \frac{m}{M_\Omega}$ and $M_\Omega = (2\gamma)^{-1/2}$ is the pseudo-scalar field mass. The maximum of Δa is reached for $\lambda \to \infty$ and it reads

$$|\Delta a| = \frac{1}{2}\frac{g^2}{8\pi^2}, \tag{50}$$

which is suppressed by the factor g^2. For instance, for a μ particle, $g \approx 10^{-20}$ and the corresponding Δa is several orders of magnitude below the experimental uncertainty [36]. Therefore, we do not expect any sensible deviation to the standard particle physics phenomenology coming from our model.

6. Conclusions

In this work we considered a propagating torsion theory, obtained by adding to the Einstein-Hilbert term a quadratic contribution in the curvature, which resembles a Yang-Mills action for the spin connection. We analyzed classical equations of motion and we solved them on a Minkowski background in the linearised limit. Torsion is generically nonvanishing also in vacuum and it carries five degrees of freedom, described by a pseudo-scalar field and a tachyon particle. The latter does not couple with spinor fields, thus at least classically no unphysical interaction takes place. On the contrary, there is an interaction between the pseudoscalar field and spinors. However, the associated coupling constant is suppressed by the ratio between the mass of the spinor field and Planck mass. Therefore, no sensible deviation to the standard particle physics phenomenology emerges from the present approach to the torsion dynamics.

This may not be the case beyond the linearised limit, when one considers next-to-the-leading order terms in the equations of motion: nontrivial interactions between torsion components are expected to occur and they may spoil the conclusions of the present analysis. In particular, the presence of a tachyon field is extremely dangerous, since it could induce causality violations. Hence, the decoupling of the tachyon has to be regarded as a consistency-check for the viability of the proposed modification of gravity.

It would also be interesting to pursue the quantization of the present model as an effective field theory, using the results and techniques developed for non-Abelian gauge theories.

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