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Universe in a black hole with spin and torsion

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The conservation law for the angular momentum in curved spacetime requires that the antisymmetric part of the affine connection (the torsion tensor) is a variable in the principle of least action. The coupling between spin and torsion generates gravitational repulsion in fermionic matter at extremely high densities and avoids the formation of singularities in black holes. We show that every black hole in the presence of torsion forms a nonsingular, closed, nearly flat, homogeneous, and isotropic universe on the other side of its event horizon. Quantum particle production in such a universe can generate a period of exponential expansion which creates an enormous amount of matter in that universe. Accordingly, our Universe may have originated from the interior of a black hole existing in another universe.

I. INTRODUCTION

Relativistic quantum mechanics predicts that elementary particles that are fermions, described by the Dirac equation, have the intrinsic angular momentum (spin). The conservation law for the total angular momentum (orbital plus intrinsic) which admits the exchange between its orbital and intrinsic components (spin-orbit interaction) in curved spacetime requires an asymmetric affine connection [1]. The Einstein-Cartan or Einstein-Cartan-Sciama-Kibble (ECSK) theory of gravity naturally extends the metric general relativity by removing its constraint of the symmetry of the connection [2]. The antisymmetric part of the affine connection, the torsion tensor, becomes a dynamical variable related to the spin density of matter [2–6]. Since Dirac fields couple to the connection, the spin of fermions acts like a source of torsion. The ECSK theory also passes all tests of general relativity, because even at nuclear densities the contribution from torsion to the Einstein equations is negligibly small and both theories give indistinguishable predictions at these densities.

At extremely high densities existing in black holes and in the very early Universe, the minimal spinor-torsion coupling manifests itself as gravitational repulsion, which avoids the formation of singularities from fermionic matter [7–11]. Accordingly, the singular big bang is replaced by a nonsingular big bounce, before which the Universe was contracting [12, 13]. In addition to eliminating the initial singularity, this scenario solves the flatness and horizon problems in cosmology [10, 13, 14]. Cosmic inflation also solves the flatness and horizon problems but it requires additional matter fields with specific conditions on their form and it does not address the big-bang singularity [15]. Torsion therefore provides the simplest and most natural mechanism that solves these three major problems of the standard big-bang cosmology. The ECSK theory may also solve the problem of divergent integrals in quantum field theory by providing fermions with spatial extension and thus introducing an effective ultraviolet cutoff for their propagators [9].

The contraction of the Universe before the big bounce must be caused by a physical initial condition. Such a contraction can correspond to gravitational collapse of matter inside a newly formed black hole existing in another universe [13, 16]. When the event horizon forms, the interior of a black hole becomes a new, closed universe, which can be thought of as a three-dimensional analogue of the two-dimensional surface of a sphere [13]. Quantum effects in an extremely strong, gravitational field in a collapsing black hole cause an intense particle production which creates an enormous amount of mass inside the black hole [17] without changing the total energy (matter plus gravitational field) of the new universe [18]. The universe in a black hole contracts until it reaches a minimum radius and maximum density, undergoes a bounce, and then expands. Such an expansion is not visible from the outside of the black hole because of an infinite redshift at its event horizon. As the universe in a black hole expands to infinity, the boundary of the black hole becomes an Einstein-Rosen bridge connecting this universe with the outer universe in which the black hole exists. Every astrophysical black hole may thus be an Einstein-Rosen bridge (wormhole) to a new universe on the other side of its event horizon. Accordingly, our Universe may be the interior of a black hole existing in another universe and a part of a multiverse. This scenario also explains the arrow of time and the black hole information paradox [13, 18].

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II. EINSTEIN-CARTAN-SCIAMA-KIBBLE THEORY OF GRAVITY

The affine connection Γ_{ij}^{k} in the ECSK theory of gravity is not symmetric and has the antisymmetric part, the torsion tensor $S_{ij}^{k} = (\Gamma_{ij}^{k} - \Gamma_{ji}^{k})/2$ [2–6, 8, 9]. We use the notation of [1, 6]. The curvature tensor is given by $R_{mjk}^{i} = \partial_{j}\Gamma_{mk}^{i} - \partial_{k}\Gamma_{mj}^{i} + \Gamma_{ij}^{l}\Gamma_{mk}^{l} - \Gamma_{ik}^{l}\Gamma_{mj}^{l}$ and its contraction gives the Ricci tensor $R_{ik} = R_{ijk}^{j}$. The metricity condition $g_{ij;k} = 0$, where semicolon denotes the covariant derivative with respect to Γ_{ij}^{k} , gives $\Gamma_{ij}^{k} = \{_{ij}^{k}\} + C_{ijk}^{k}$, where $\{_{ij}^{k}\} = (1/2)g^{km}(g_{mi,j}+g_{mj,i}-g_{ij,m})$ are the Christoffel symbols of the metric tensor g_{ik} and $C_{ijk}^{i} = S_{ijk}^{i} + 2S_{(jk)}^{i}$ is the contortion tensor. The curvature tensor can be decomposed as $R_{klm}^{i} = P_{klm}^{i} + C_{km:l}^{i} - C_{kl:m}^{i} + C_{km}^{j}C_{ij}^{i} - C_{kl}^{j}C_{ijm}^{i}$, where P_{klm}^{i} is the Riemann tensor (the curvature tensor constructed from the Levi-Civita connection $\{_{ij}^{k}\}$) and colon denotes the covariant derivative with respect to $\{_{ij}^{k}\}$.

The ECSK theory of gravity is based on the Lagrangian density of the gravitational field that is proportional to the Ricci curvature scalar $R = R_{ik}g^{ik}$, similarly to the metric general relativity. The field equations are obtained from the total action for the gravitational field and matter, $I = \frac{1}{c} \int (-\frac{1}{2\kappa}R\sqrt{-g} + \mathfrak{L}_m)d^4x$, where \mathfrak{L}_m is the Lagrangian density of matter, $g = \det(g_{ik})$, and $\kappa = 8\pi G/c^4$, with respect to the metric and torsion tensors. Varying the action with respect to the torsion gives the Cartan field equations that relate algebraically the torsion of spacetime to the canonical spin tensor of matter $s^{ijk} = 2(\delta \mathfrak{L}_m/\delta C_{ijk})/\sqrt{-g}$ [2–6, 8, 9]:

$$S_{jik} - S_i g_{jk} + S_k g_{ji} = -\frac{1}{2} \kappa s_{ikj}, \tag{1}$$

where $S_i = S_{ik}^k$. Varying the action with respect to the metric and using (1) gives the Einstein field equations that relate the curvature of spacetime to the canonical energy-momentum tensor of matter σ_i^{j} :

$$R_{ik} - \frac{1}{2}Rg_{ik} = \kappa\sigma_{ki}.$$
(2)

The symmetric, dynamical energy-momentum tensor $T_{ik} = 2(\delta \mathfrak{L}_m/\delta g^{ik})/\sqrt{-g}$ [19] is related to the canonical energymomentum tensor by $T_{ik} = \sigma_{ik} - (1/2)(\nabla_j - 2S^l_{jl})(s_{ik}^{\ j} - s_k^{\ j} + s^j_{\ ik})$, where ∇_k denotes the covariant derivative with respect to Γ_{ij}^k .

The Einstein and Cartan equations can be combined to give [4, 6, 8, 9]

$$G^{ik} = \kappa T^{ik} + \frac{1}{2} \kappa^2 \left(s^{ij}_{\ j} s^{kl}_{\ l} - s^{ij}_{\ l} s^{kl}_{\ j} - s^{ijl} s^{k}_{\ jl} + \frac{1}{2} s^{jli} s^{jl}_{\ jl} + \frac{1}{4} g^{ik} (2s^{\ l}_{\ j} s^{jm}_{\ l} - 2s^{\ l}_{\ j} l s^{jm}_{\ m} + s^{jlm} s_{jlm}) \right), \quad (3)$$

where $G_{ik} = P_{ik} - (1/2)Pg_{ik}$ is the Einstein tensor of general relativity constructed from the contractions of the Riemann tensor, $P_{ik} = P_{ijk}^{j}$ and $P = P_{ik}g^{ik}$. The second term on the right-hand side of (3) is the correction to the curvature of spacetime from the spin. The spin tensor also appears in T_{ik} because \mathfrak{L}_m depends on torsion. The contributions from the spin tensor to the right-hand side of the Einstein equations are significant only at extremely high densities, on the order of the Cartan density [9]. Below this density, the predictions of the ECSK theory do not differ from the predictions of the metric general relativity, and reduce to them in vacuum, where torsion vanishes.

III. SPIN FLUID

Quarks and leptons, that compose all stars, are fermions described in relativistic quantum mechanics by the Dirac equation. Since Dirac fields couple minimally to the torsion tensor, the torsion of spacetime at microscopic scales does not vanish in the presence of fermions [2, 6, 9]. At macroscopic scales, such particles can be averaged and described as a spin fluid [6–8, 11–13]. Even if the spin orientation of particles is random, the terms quadratic in the spin tensor in (3) do not vanish. These terms are significant only at densities of matter much higher than the density of nuclear matter because of the factor κ^2 .

The Bianchi identities, $R^{i}_{n[jk;l]} = 2R^{i}_{nm[j}S^{m}_{kl]}$ and $R^{m}_{[jkl]} = -2S^{m}_{[jk;l]} + 4S^{m}_{n[j}S^{n}_{kl]}$, together with the Einstein and Cartan field equations give the conservation laws for the canonical energy-momentum and spin tensors: $T^{ij}_{;j} = C_{jk}^{\ i}T^{jk} + (1/2)s_{klj}R^{klji}$ and $s^{\ k}_{ij} - 2S_ks_{ij}^{\ k} = T_{ij} - T_{ji}$ [2–6, 8]. Using the Papapetrou method of multipole expansion for these laws [20] leads to the formulas for the macroscopic canonical energy-momentum and spin tensor of a spin fluid in the point-particle approximation [6, 9]. The canonical energy-momentum tensor of a spin fluid is given by

$$\sigma_{ij} = c\Pi_i u_j - p(g_{ij} - u_i u_j),\tag{4}$$

and its spin tensor by

$$s_{ij}^{\ \ k} = s_{ij} u^k, \ s_{ij} u^j = 0,$$
 (5)

where Π_i is the four-momentum density of the fluid, u^i its four-velocity, s_{ij} its spin density, and p its pressure. Substituting the macroscopic tensors to (3) gives [8, 11–13]

$$G^{ij} = \kappa \left(\epsilon - \frac{1}{4}\kappa s^2\right) u^i u^j - \kappa \left(p - \frac{1}{4}\kappa s^2\right) (g^{ij} - u^i u^j) - \frac{1}{2}\kappa (\delta^l_k + u_k u^l) (s^{ki} u^j + s^{kj} u^i)_{:l},\tag{6}$$

where $\epsilon = c \prod_i u^i$ is the rest energy density of the fluid,

$$s^2 = \frac{1}{2}s_{ij}s^{ij} > 0 \tag{7}$$

is the square of the spin density.

If the spin orientation of particles in a spin fluid is random then the last term on the right-hand side of (6) vanishes after averaging. Thus the Einstein-Cartan equations for such a spin fluid are equivalent to the Einstein equations for a perfect fluid with the effective energy density $\epsilon - \kappa s^2/4$ and the effective pressure $p - \kappa s^2/4$ [8, 11–13]. The square of the spin density for a fluid consisting of fermions with no spin polarization is given by [11, 21]

$$s^2 = \frac{1}{8} (\hbar c n_{\rm f})^2.$$
 (8)

where $n_{\rm f}$ is the number density of fermions. The effective energy density and pressure of a spin fluid are thus given by

$$\tilde{\epsilon} = \epsilon - \alpha n_{\rm f}^2, \quad \tilde{p} = p - \alpha n_{\rm f}^2,$$
(9)

where $\alpha = \kappa (\hbar c)^2 / 32$.

IV. FRIEDMANN EQUATIONS WITH TORSION

A closed, homogeneous, and isotropic universe is described by the Friedmann-Lemaître-Robertson-Walker (FLRW) metric which, in the isotropic spherical coordinates, is given by $ds^2 = c^2 dt^2 - a^2(t)(1 + kr^2/4)^{-2}(dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2)$, where a(t) is the scale factor and k = 1 [5, 19]. In the comoving frame of reference, in which the four-velocity u^i of the cosmological spin fluid satisfies $u^0 = 1$ and $u^{\alpha} = 0$ (α denotes spatial indices), the Einstein field equations (6) for this metric become the Friedmann equations (the cosmological constant is negligible in the early Universe) [11–13]:

$$\frac{\dot{a}^2}{c^2} + k = \frac{1}{3}\kappa(\epsilon - \alpha n_{\rm f}^2)a^2,\tag{10}$$

$$\frac{\dot{a}^2 + 2a\ddot{a}}{c^2} + k = -\kappa(p - \alpha n_{\rm f}^2)a^2,\tag{11}$$

where dot denotes the differentiation with respect to the cosmic time t. Differentiating (10) multiplied by a with respect to t and combining it with (11) multiplied by \dot{a} give a conservation law

$$\frac{d}{dt}\left((\epsilon - \alpha n_{\rm f}^2)a^3\right) + (p - \alpha n_{\rm f}^2)\frac{d}{dt}(a^3) = 0,\tag{12}$$

The spin fluid in the early Universe is formed by an ultrarelativistic matter in kinetic equilibrium, for which $\epsilon(T) = h_{\star}T^4$, $p(T) = \frac{\epsilon(T)}{3}$ and $n(T) = h_nT^3$, where T is the temperature of the Universe and ζ is the Riemann zeta function, and we defined $h_{\star} = \frac{\pi^2}{30}g_{\star}k_{\rm B}^4/(\hbar c)^3$ and $h_n = \frac{\zeta(3)}{\pi^2}g_nk_{\rm B}^3/(\hbar c)^3$ [22]. The effective numbers of thermal degrees of freedom are $g_{\star}(T) = g_{\rm b}(T) + \frac{7}{8}g_{\rm f}(T)$ and $g_n(T) = g_{\rm b}(T) + \frac{3}{4}g_{\rm f}(T)$, where $g_{\rm b} = \sum_i g_i$ is summed over relativistic fermions, and g_i is the number of the spin states for each particle species *i*. Accordingly, $n_{\rm f}(T) = h_{\rm nf}T^3$ and $h_{n\rm f} = \frac{\zeta(3)}{\pi^2}\frac{3}{4}g_{\rm f}(T)(k_{\rm B}T)^3/(\hbar c)^3$. Substituting these functions to (12) gives

$$\left(\frac{\dot{a}}{a} + \frac{\dot{T}}{T}\right) \left(1 - \frac{3\alpha h_{\rm nf}^2}{2h_\star} T^2\right) = 0,\tag{13}$$

since $g_{\rm b}$ and $g_{\rm f}$ are constant in the range of T where torsion is significant. The relation (13) is satisfied for all values of T if

$$\frac{d}{dt}(aT) = 0. \tag{14}$$

Integrating (14) gives

$$a = \frac{a_{\rm r} T_{\rm r}}{T},\tag{15}$$

where a_r is the scale factor at a reference temperature T_r . The relation (15) between a and T does not depend on the presence of spin [13]. The energy density of particles in kinetic equilibrium scales like $\epsilon \sim a^{-4}$ and their number density scales like $n \sim a^{-3}$. Accordingly, the number of particles $N \propto na^3$ in the Universe does not change if (14) is satisfied.

V. OSCILLATORY UNIVERSE WITHOUT PARTICLE PRODUCTION

When a collapsing star forms an event horizon, the interior of a black hole is causally connected and becomes a new, closed universe [13]. Such a universe is initially inhomogeneous and anisotropic but it becomes more homogeneous and isotropic near a bounce. In a black hole, fermions composing the spin fluid have energies much greater than their rest energies. These particles are thus described by an ultrarelativistic barotropic equation of state, $p = \epsilon/3$, as for radiation. If we assume that the universe begins to contract when $a = a_i$ and $\dot{a} = 0$, where $\alpha n_f^2 \ll \epsilon$, then (10) gives $\frac{1}{2}\kappa\epsilon a_i^2 = 1$. Accordingly, the initial temperature is given by

$$T_i = \left(\frac{3}{\kappa h_\star a_i^2}\right)^{1/4}.\tag{16}$$

For observers outside the black hole, this initial condition occurs in the future infinity because of an infinite redshift at the event horizon. They observe a black hole with mass

$$M_{\rm BH} = \frac{4}{3}\pi a_i^3 \frac{\epsilon}{c^2} = \frac{c^2 a_i}{2G},$$
(17)

which shows that a_i is its Schwarzschild radius. If no particles are produced, then we can use (14). The reference values in (15) can be $a_r = a_i$ and $T_r = T_i$. The dynamics of the new universe is described by (15) and (10) which can be written as

$$\frac{\dot{a}^2}{c^2} + 1 = \frac{1}{3}\kappa (h_\star T^4 - \alpha h_{nf}^2 T^6)a^2$$
(18)

or

$$\frac{\dot{a}^2}{c^2} + 1 = \frac{1}{3}\kappa \Big(h_\star \frac{T_i^4 a_i^4}{a^2} - \alpha h_{nf}^2 \frac{T_i^6 a_i^6}{a^4}\Big).$$
(19)

The Friedmann equations (10) and (11) also give

$$\frac{\ddot{a}}{c^2 a} = -\frac{1}{3}\kappa\epsilon + \frac{2}{3}\kappa\alpha n_{\rm f}^2 = -\frac{\kappa}{3}(h_\star T^4 - 2\alpha h_{\rm nf}^2 T^6).$$
(20)

As the scale factor a decreases, the temperature T increases. The spin-torsion contribution to the energy density is negative and generates gravitational repulsion which becomes stronger as T increases. Since this contribution scales with T faster $(T \sim T^6)$ than $\epsilon \sim T^4$, \dot{a} eventually reaches zero and the universe undergoes a nonsingular bounce. At a bounce, where we can neglect k, (10) gives $\epsilon - \alpha n_f^2 = 0$. Accordingly, the maximum temperature is given by

$$T_{\rm max} = \left(\frac{h_{\star}}{\alpha h_{\rm nf}^2}\right)^{1/2} = \sqrt{\frac{2\pi^5}{15}} \frac{g_{\star}^{1/2}}{\zeta(3)\frac{3}{4}g_{\rm f}} T_{\rm P},\tag{21}$$

where $T_{\rm P}$ is the Planck temperature. The temperature (21) depends only on the numbers of thermal degrees of freedom. The relation (15) determines the minimum scale factor at a bounce of the universe in a black hole:

$$a_{\min} = \frac{a_i T_i}{T_{\max}}.$$
(22)

The universe is accelerating if $\ddot{a} > 0$ which is equivalent, using (20), to $T > T_{\max}/\sqrt{2}$ ($a < \sqrt{2}a_{\min}$). After a bounce, the universe accelerates until $T = T_{\max}/\sqrt{2}$ ($a = \sqrt{2}a_{\min}$). The universe continues to expands with $\ddot{a} < 0$ until it reaches its maximum scale factor (a crunch) at $a = a_i$, after which another contraction begins. This expansion looks like the time reversal of the contraction from $a = a_i$ to $a = a_{\min}$. The universe is therefore oscillatory: its scale factor oscillates between the minimum and maximum scale factors [12]. The period of the oscillation from a bounce to a crunch and back is equal to

$$\Delta t = 2 \int_{a_{\min}}^{a_i} \frac{da}{\dot{a}} = \frac{2}{c} \int_{a_{\min}}^{a_i} \frac{da}{(\kappa \tilde{\epsilon} a^2/3 - 1)^{1/2}},$$
(23)

where $\tilde{\epsilon} = h_{\star}(a_i T_i/a)^4 - \alpha h_{nf}^2 (a_i T_i/a)^6$.

The scale factor depends on the time according to

$$\left(a^2 - \frac{\alpha h_{nf}^2 a_i^2 T_i^2}{h_\star} - \frac{3ka^4}{\kappa h_\star a_i^4 T_i^4}\right)^{-1/2} a^2 da = \left(\frac{\kappa h_\star}{3}\right)^{1/2} (a_i T_i)^2 c dt.$$
(24)

For $a \ll a_i$, the term with k in (24) can be neglected. If we define $x = a/a_{\min}$ and choose t = 0 at $a = a_{\min}$, then integrating (24) gives [13]

$$\frac{t}{\tau} = \int_{1}^{x} \frac{x^2 dx}{(x^2 - 1)^{1/2}} = \frac{x}{2} \sqrt{x^2 - 1} + \frac{1}{2} \ln|x + \sqrt{x^2 - 1}|,$$
(25)

where

$$\tau = \frac{\alpha h_{nf}^2}{c} \left(\frac{3}{\kappa h_\star^3}\right)^{1/2} = \frac{45}{4} \sqrt{\frac{5}{\pi^{13}}} \frac{\zeta^2(3)(\frac{3}{4}g_f)^2}{g_\star^{3/2}} t_P$$
(26)

is the characteristic time of the torsion-dominated phase of the expansion. This time is on the order of the Planck time $t_{\rm P}$ and depends only on the numbers of thermal degrees of freedom.

The Hubble parameter $H = \dot{a}/a$ is determined by

$$H = \pm c \left(\frac{1}{3}\kappa (h_{\star}T^{4} - \alpha h_{nf}^{2}T^{6}) - \frac{k}{a^{2}}\right)^{1/2},$$
(27)

where plus sign corresponds to expansion and minus sign corresponds to contraction. The magnitude of H has a maximum $|H|_{\text{max}} = \sqrt{4/27}\tau^{-1}$ at $T = \sqrt{2/3}T_{\text{max}}$ (at $a = \sqrt{3/2}a_{\min}$). The total density parameter $\Omega = \tilde{\epsilon}/(\rho_c c^2)$, where $\rho_c = 3H^2/(\kappa c^4)$ is the critical density, is determined by (10): $\Omega = 1 + kc^2/\dot{a}^2$, which leads to

$$\Omega = 1 + \frac{3}{\kappa (h_{\star}T^4 - \alpha h_{nf}^2 T^6)a^2} = 1 + \frac{3}{\kappa (h_{\star}T^2 - \alpha h_{nf}^2 T^4)a_i^2 T_i^2}.$$
(28)

The density parameter has a minimum $\Omega_{\min} = 1 + 4c\tau/a_i$ at $T = \sqrt{1/2}T_{\max}$ (at $a = \sqrt{2}a_{\min}$). A closed universe contains $N \approx (\dot{a}/c)^3 = (\Omega - 1)^{-3}$ causally disconnected regions. Their maximum number is thus $N_{\max} \approx (a_i/l_{\rm P})^3$, where $l_{\rm P}$ is the Planck length. As the universe expands from a_{\min} to $\sqrt{2}a_{\min}$, Ω rapidly decreases from infinity to Ω_{\min} which appears to be tuned to 1 to a high precision since $c\tau \ll a_i$ [13]. As the universe expands from $\sqrt{2}a_{\min}$ to a_i , Ω increases to infinity.

If we assume that only the known standard-model particles exist, then $g_b = 28$ and $g_f = 90$ [22]. For these values, we find:

$$T_{\rm max} = 1.15 \times 10^{32} \,\mathrm{K}, \quad \tau = 4.75 \times 10^{-45} \,\mathrm{s}, |H|_{\rm max} = 8.1 \times 10^{43} \,\mathrm{s}^{-1}, \quad \Omega_{\rm min} - 1 = 5.7 \times 10^{-36}, \quad N_{\rm max} \approx 10^{52}.$$
(29)

An extremely small magnitude of $\Omega_{\min} - 1$ solves the flatness problem. An extremely large N solves the horizon problem. A typical stellar black hole has the Schwarzschild radius $a_i = 10^4$ m. For this value, we find:

$$T_i = 1.38 \times 10^{12} \,\mathrm{K}, \quad a_{\min} = 1.19 \times 10^{-16} \,\mathrm{m}.$$
 (30)

The volume of a closed universe is given by $V = 2\pi^2 a^3$ [19]. The conservation law (12) can thus be written in the form of the first law of thermodynamics:

$$dE + pdV = TdS = 12\pi^2 \alpha h_{nf}^2 a^2 T^5 d(aT),$$
(31)

where $E = \epsilon V$ is the thermal energy of the universe and S its entropy. The relation (14) shows that the entropy of a cyclic universe in a black hole without particle production is constant.

VI. UNIVERSE WITH PARTICLE PRODUCTION

A closed universe in a black hole without particle production is oscillatory. It does not reach the size and mass of the observed Universe. To identify our Universe with a universe in a black hole, we must include quantum effects in curved spacetime which are responsible for particle production by the gravitational field [17].

An initial, accelerated expansion of a closed universe after the big bounce defines the torsion-dominated era. As the universe expands, the spin-torsion term $\alpha n_{\rm f}^2$ in (10) decreases faster than ϵ . The universe begins to decelerate and enters the radiation-dominated era. Eventually, the matter in the universe becomes nonrelativistic. At the matter-radiation equality, the energy density of nonrelativistic matter exceeds the energy density of radiation and the universe enters the matter-dominated era. The universe cannot expand to infinity unless it enters another phase of acceleration. The simplest source of such an acceleration is a cosmological constant Λ which modifies the Lagrangian density of the gravitational field: $\mathfrak{L}_{\rm g} = -\frac{1}{2\kappa}(R+2\Lambda)\sqrt{-g}$. The cosmological constant does not change the Cartan equations but adds a term to the Einstein equations (2): $R_{ik} - \frac{1}{2}Rg_{ik} = \kappa\sigma_{ki} + \Lambda g_{ik}$. The Friedmann equation (10) in the radiation-dominated era becomes

$$\frac{\dot{a}^2}{c^2} + 1 = \frac{1}{3}\kappa\epsilon a^2 + \frac{1}{3}\Lambda a^2.$$
(32)

In the matter-dominated era, the energy density is dominated by the mass density of nonrelativistic matter ρ : $\epsilon \approx \rho c^2$. The mass of the nonrelativistic matter in the universe is equal to $M_{\text{univ}} = \rho V$. Accordingly, the Friedmann equation (10) in the matter-dominated era is

$$\frac{\dot{a}^2}{c^2} + 1 = \frac{D}{a} + \frac{1}{3}\Lambda a^2,$$
(33)

where $D = 4GM/(3\pi c^2)$.

A closed universe in a black hole expands to infinity if [5]

$$D > \frac{2}{3\sqrt{\Lambda}}.\tag{34}$$

At the matter-radiation equality, where $a = a_{eq}$ and $T = T_{eq}$, we have $(1/3)\kappa\epsilon_{eq}a_{eq}^2 = D/a_{eq}$ or

$$D = \frac{1}{3} \kappa h_{\star} T_{\rm eq}^4 a_{\rm eq}^3. \tag{35}$$

Using (15), this constant can be written as

$$D = \frac{1}{3} \kappa h_{\star} T_{\rm eq} \tilde{a}_{i}^{3} T_{i}^{3} = \frac{1}{3} \kappa h_{\star} T_{\rm eq} \left(\frac{\tilde{a}_{i}}{a_{i}}\right)^{3} (a_{i} T_{i})^{3}, \tag{36}$$

where \tilde{a}_i is the scale factor at $T = T_i$ in the expanding phase. Without particle production, $\tilde{a}_i = a_i$. With particle production, $\tilde{a}_i > a_i$. Since \tilde{a}_i and a_i correspond to the same temperature, the ratio \tilde{a}_i/a_i represents the expansion of the universe due to particle production. The relation (34) shows that

$$\frac{\tilde{a}_i}{a_i} > \left(\frac{2}{\kappa h_\star T_{\rm eq}(a_i T_i)^3 \sqrt{\Lambda}}\right)^{1/3}.$$
(37)

The value of $T_{\rm eq}$ depends on the number ratio of photons to massive particles and on the masses of the particles. In our Universe, $T_{\rm eq} = 8820$ K and $\Lambda/\kappa = 5.24 \times 10^{-10}$ N/m² [22], which gives

$$\frac{\tilde{a}_i}{a_i} > 10^{10}.$$
 (38)

Accordingly, (28) and (29) lead to

$$\tilde{\Omega}_{\min} - 1 = (\Omega_{\min} - 1) \left(\frac{a_i}{\tilde{a}_i}\right)^2 < 10^{-55}.$$
(39)

This value solves both the flatness and horizon problems [13].

If (34) is not satisfied, then the universe in a black hole eventually reaches a crunch at which it stops expanding and starts contracting. The contraction lasts until the universe reaches the next bounce at $T = T_{\text{max}}$ and $a > a_{\text{min}}$.

After this bounce, the next expanding phase begins. The universe is more isotropic and has a larger scale factor at $T = T_{eq}$ than in the previous expanding phase. Accordingly, it has a larger value of D. If this new value satisfies (34), then the universe expands to infinity. If not, the universe has another cycle of contraction and expansion. After a finite number of cycles, D satisfies (34) and the universe expands indefinitely. The last bounce can be regarded as the big bang, or rather the big bounce, of this universe.

In an isotropic universe, described by the FLRW metric, an alternating gravitational field causes production of particle-antiparticle pairs [23]. For a strong field in general relativity, the local rate of production of massive particles with spin j = 1 is equal to

$$\frac{1}{\sqrt{-g}}\frac{d}{dt}(\sqrt{-g}n_1) = \frac{c}{288\pi}P^2,$$
(40)

where n_1 is their number density. The local rates of production of massless particles with spin j = 1 and particles with spins j = 0 and j = 1/2 are proportional to P and can be neglected. In the ECSK theory, the torsion tensor may modify (40). We can thus write

$$\frac{1}{\sqrt{-g}}\frac{d}{dt}(\sqrt{-g}n_1) = cK,\tag{41}$$

where K > 0 is a scalar of dimension m⁻⁴, constructed from the curvature, torsion, and metric tensors. For the FLRW metric, we have

$$\frac{1}{a^3}\frac{d}{dt}(a^3n_1) = cK.$$
(42)

Using $n_1(T) = h_{n1}T^3$, where $h_{n1} = \frac{\zeta(3)}{\pi^2}g_{n1}k_{\rm B}^3/(\hbar c)^3$ and $g_{n1} = g_{\rm b1} = \sum_i g_i$ is summed over relativistic, massive bosons with spin j = 1, we find

$$\frac{\dot{a}}{a} + \frac{\dot{T}}{T} = \frac{cK}{3h_{n1}T^3}.$$
 (43)

If we assume that (10) is satisfied, then (11) must be modified. The relation (43) generalizes (14) and describes, together with (18), the dynamics of the universe.

Particle production does not change the total energy and momentum of the matter and gravitational field in the universe. However, it increases the entropy in the universe. Combining (31) and (43) gives

$$\frac{dS}{dt} = \frac{2\alpha ch_{\mathrm{nf}}^2}{h_{\mathrm{n}1}} a^3 T^2 K. \tag{44}$$

The motion of matter through the event horizon is unidirectional and thus it defines the arrow of time in the universe in a black hole. This arrow is also entropic: although black holes are states of maximum entropy in the frame of reference of outside observers, new universes expanding inside black holes continue to increase entropy.

For the FLRW metric, the Weyl tensor vanishes and the two independent invariants of dimension m⁻⁴ constructed from the Riemann and metric tensors are P^2 and $P_{ik}P^{ik}$. Using (6) without the last term on the right-hand side and (9) gives

$$P = -\kappa(\tilde{\epsilon} - 3\tilde{p}) = -2\kappa\alpha n_{\rm f}^2,$$

$$P_{ik}P^{ik} = \kappa^2(\tilde{\epsilon}^2 + 3\tilde{p}^2).$$
(45)

At a bounce, where $\dot{a} = \tilde{\epsilon} = 0$ and $T = T_{\text{max}}$, the rate of production of particles (and thus K) should vanish to avoid $\dot{T} > 0$ in (43). Otherwise, the increase of T would lead to $T > T_{\text{max}}$ and $\dot{a}^2 < 0$ in (18). A scalar, vanishing for $\tilde{\epsilon} = 0$ and constructed from the Riemann and metric tensors, is proportional to $K = P^2 - 3P_{ik}P^{ik}$. Including the torsion tensor can give other forms for K. Ultimately, K should be derived from quantum field theory in the Riemann-Cartan spacetime of the ECSK theory of gravity.

The simplest form of K which vanishes at a bounce and has the same dimension as P^2 is

$$K = \beta(\kappa \tilde{\epsilon})^2,\tag{46}$$

where $\beta > 0$ is a nondimensional constant. Near a bounce, where we can neglect k, (10) can be written as

$$\left(\frac{\dot{a}}{ca}\right)^2 = \frac{1}{3}\kappa\tilde{\epsilon}.\tag{47}$$

Combining (43), (46), and (47) gives

$$\frac{\dot{a}}{a} \left[1 - \frac{3\beta}{c^3 h_{n1} T^3} \left(\frac{\dot{a}}{a} \right)^3 \right] = -\frac{\dot{T}}{T}.$$
(48)

The signs of \dot{a} and \dot{T} must be opposite to avoid an indefinitely long, exponential increase of the scale factor. This condition is satisfied during a contracting phase ($\dot{a} < 0$). During an expanding phase ($\dot{a} > 0$), we must have

$$\frac{3\beta}{c^3 h_{n1}} \left(\frac{\dot{a}}{aT}\right)^3 = \frac{3\beta}{h_{n1}} \left(\frac{\kappa}{3} (h_\star T^2 - \alpha h_{nf}^2 T^4)\right)^{3/2} < 1.$$
(49)

The above function has a maximum at $T = T_{\text{max}}/\sqrt{2}$. We thus find

$$\beta < \beta_{\rm cr} = \frac{\sqrt{6}}{32} \frac{h_{n1} h_{nf}^3 (\hbar c)^3}{h_\star^3} = \frac{15^3 \sqrt{6}}{4\pi^{14}} \frac{\zeta^4 (3) g_{\rm b1} (\frac{3}{4} g_{\rm f})^3}{g_\star^3}.$$
(50)

If we assume that only the known standard-model particles exist, then $g_{b1} = 9$ [22]. For this value, we find

$$\beta_{\rm cr} \approx \frac{1}{929},$$
(51)

which is on the order of $1/(288\pi)$ in (40).

VII. INFLATION AS A SPECIAL CASE

The exponential expansion of space in the early Universe, known as cosmic inflation, also solves the flatness and horizon problems [15]. During inflation, the scale factor increases according to $a \propto e^{Ht}$ with constant values of H and T. In addition, inflation predicts the observed spectrum of primordial density fluctuations [24]. Standard inflation, which assumes the existence of a fundamental scalar field with a specific potential, requires fine tuning and is geodesically incomplete in the past [25].

If β is slightly lesser than $\beta_{\rm cr}$, then at $T = T_{\rm max}/\sqrt{2}$:

$$\frac{3\beta}{c^3h_{n1}} \left(\frac{\dot{a}}{aT}\right)^3 \bigg| \lesssim 1.$$
(52)

At this temperature, (48) therefore gives

$$\frac{\dot{T}}{T} \approx -\frac{2\dot{a}}{a} \tag{53}$$

in a contracting phase and

$$\dot{T} \approx 0$$
 (54)

in an expanding phase. Accordingly, (43) in an expanding phase gives an exponential expansion,

$$\frac{\dot{a}}{a} \approx \frac{c\beta(\kappa\tilde{\epsilon})^2}{3h_{n1}T^3} \approx c \left(\frac{1}{3}\kappa\tilde{\epsilon}\right)^{1/2},\tag{55}$$

at a nearly constant energy density

$$\tilde{\epsilon} \approx \frac{h_{\star}^3}{8\alpha^2 h_{nf}^4}.$$
(56)

As a increases after a bounce, the term on the left-hand side of (52) increases from 0 to nearly 1 and T decreases from $T = T_{\text{max}}$ to $T = T_{\text{max}}/\sqrt{2}$. When $T \approx T_{\text{max}}/\sqrt{2}$, the term in (52) is nearly 1 and the universe begins to expand exponentially at a nearly constant T (inflation). An exponential expansion lasts until the term in (52) decreases significantly below 1 and T continues to decrease. The universe then starts expanding according to (43) and moves from the torsion-dominated era to the radiation-dominated era. In a scenario with one bounce, the relations (38) and (55) constrain the time interval t_{infl} of the exponential expansion: $Ht_{\text{infl}} \gtrsim 23$. This interval depends on $\tau(\beta_{\text{cr}} - \beta)^{-1}$.

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For $\beta = 0$, the universe in a black hole is oscillatory with an infinite number of bounces and crunches (cycles). If $0 < \beta < \beta_{\rm cr}$, then the universe is cyclic with a finite number of cycles. In this case, the scale factor at a crunch is greater than the scale factor at the preceding crunch. As β increases, the number of cycles decreases and the accelerated expansion of the universe in each cycle is closer to exponential. If $\beta \leq \beta_{\rm cr}$, then the universe has only one bounce and expands indefinitely through the torsion-dominated era with a period of an inflationary phase, radiation-dominated era, matter-dominated era, and cosmological-constant era. If $\beta \geq \beta_{\rm cr}$, then an exponential expansion of the universe would last indefinitely (eternal inflation).

VIII. REMARKS

Gravitational repulsion induced by spin and torsion, which becomes significant at extremely high densities, prevents singularities in black holes and at the big bang. Because of this repulsion and particle production, every black hole creates a new universe on the other side of its event horizon. This scenario simultaneously explains the future of black hole interiors and the origin of the Universe [13]. Moreover, it naturally solves the flatness and horizon problems, and can generate an inflationary phase which lasts for a finite amount of time without requiring finely tuned scalar fields, replacing R in the gravitational Lagrangian density by more complicated functions, or adding other hypothetical objects with new free parameters. Our Universe is a stable part of the multiverse and may contain inflating regions of spacetime (new universes) only in black holes.

To complete this scenario, we need to explore the spin-torsion coupling beyond the point-particle approximation of a spin fluid and combine it with the Dirac-spinor description of fermions. We also need to explore how torsion affects particle production by quantum effects in strong gravitational fields. Furthermore, we need to explain the observed matter-antimatter asymmetry in the Universe; this asymmetry may be caused by the nonlinearity of the Dirac equation in the presence of torsion [26]. Finally, we need to derive primordial density fluctuations generated in the early universe formed in a black hole and compare them with the observed spectrum. The nature of dark matter and dark energy still remains unsolved. The presented theory predicts phenomena that are similar to those in some other cosmological scenarios such as inflationary bubbles in the multiverse [15], cyclic universe [27], or holographic world [28]. However, contrary to the other scenarios, this theory adheres to the law of parsimony: *Entia non sunt multiplicanda praeter necessitatem*.

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