

Observing Geometric Torsion

Stefano Lucat^{*1} and Tomislav Prokopec^{†1}

¹*Institute for Theoretical Physics, Spinoza Institute and EMMEΦ,
Utrecht University, Postbus 80.195, 3508 TD Utrecht, The Netherlands*

Dynamical (propagating) torsion can be observed by using conventional gravitational wave detectors such as LIGO, Virgo, LISA and bar detectors. We discuss specific signatures of different types of torsion, in particular those of vector and mixed symmetric torsion (skew symmetric torsion cannot be detected in this way). These signatures are specific to torsion and therefore they can be unambiguously distinguished from those of gravitational waves.

PACS numbers: 04.30.w,04.50.Kd,04.80.Cc,04.80.Nn

I. INTRODUCTION

Cartan-Einstein (CE) theory is a very old topic (for reviews that are still actual see [1, 2]), and yet as far as we are aware of there has been no proposal to detect (dynamical) torsion *via* instruments such as gravitational wave detectors. In this letter we argue that conventional gravitational wave detectors can be used to detect propagating (dynamical) torsion. The probable reasons why this has not been proposed before are (a) skew symmetric torsion (which is typically what one means by torsion in Cartan-Einstein theory) does not imprint any signal on gravitational wave detectors and (b) torsion is not dynamical in Cartan-Einstein theory.

If Weyl symmetry is realized at the classical level then torsion trace vector couples to scalar fields, implying that scalars source torsion trace [3], modifying thus the original Cartan-Einstein theory. Furthermore, in the original CE theory torsion figures as a constraint (non-dynamical) field, which exists locally where the source is, but does not propagate. However, when matter is quantized, one can show that (when one integrates out matter fields) already at the one-loop level, both torsion trace and skew symmetric torsion become dynamical [2, 4], such that they can propagate through space in form of torsion waves and carry energy and information throughout our universe just as gravitational waves do. We do not know whether this is realized in nature, but if true the possibilities are exciting enough to deserve a closer attention of both theorists and observers.

Torsion has been used in literature for various purposes. For example, torsion was proposed as a way of avoiding the Big Bang singularity [5–7], to drive inflation [8–10] and create perturbations [11] or primordial

* Electronic address: s.lucatu@uu.nl

† Electronic address: t.prokopec@uu.nl

magnetic fields [12], to generate dark matter [13] or dark energy [14, 15]. Apart from being detectable by gravitational wave detectors, torsion might be also detectable at the LHC [16].

II. DETECTION

Just like in general relativity, where gravitational waves induce a change in distance between two test bodies described by the Jacobi equation, in a more general geometric theory that contains geometric torsion the suitably generalized Jacobi equation [3] governs the distance between two test bodies. In Ref. [3] we have shown that the Jacobi equation reads,

$$\nabla_{\dot{\gamma}}\nabla_{\dot{\gamma}}J + 2\nabla_{\dot{\gamma}}T(\dot{\gamma}, J) = R(\dot{\gamma}, J)\dot{\gamma}, \quad (1)$$

where $T(\cdot, \cdot)$ and $R(\cdot, \cdot)$ denote the torsion and curvature tensors, respectively, and $\nabla_{\dot{\gamma}}$ is the covariant derivative in the direction of the tangent vector $\dot{\gamma}$. Usually Jacobi vector fields J are taken to be orthogonal to $\dot{\gamma}$ ($g(J, \dot{\gamma}) = 0$, where $g(\cdot, \cdot)$ is the metric tensor), but that in fact is not necessary. In Appendix A we present details on how torsion enters Eq. (1). Here it is important to keep in mind that - according to the Young classification of tensors - the torsion tensor T can be decomposed into three distinct tensors,

1. torsion trace vector \mathcal{T} ;
2. skew symmetric torsion Σ ;
3. mixed torsion tensor Q .

The precise relation is given by Eq. (14) in Appendix A. When this decomposition and Eqs. (11–12) are used in (1), one gets how different torsion components contribute to the Jacobi field acceleration. In particular, for torsion trace contributes as (16), skew symmetric torsion as (17) and finally mixed torsion as (18). In what follows we analyze how different torsion components influence the distance J between neighboring geodesics. For completeness we first present the well known result for gravitational waves.

In current earthly measurements (and planned measurements in space) any perturbations of spacetime can be viewed as a small perturbation away from Minkowski metric, $\eta_{\mu\nu} = \text{diag}(-1, 1, 1, 1)$. Throughout this work we assume that both gravitational metric perturbations $h_{\mu\nu}(x) = g_{\mu\nu}(x) - \eta_{\mu\nu}$ and torsion perturbations $T^{\alpha}_{\mu\nu}(x)$ are small such that we can linearize in $h_{\mu\nu}$ and in $T^{\alpha}_{\mu\nu}$. In this linear approximation, to the required accuracy one can set, $\dot{\gamma}^{\mu} = (1, 0, 0, 0)^T$ and $\dot{\gamma}_{\mu} = (-1, 0, 0, 0)$.

Gravitational waves. To detect gravitational waves it is convenient to work in traceless, transverse (TT) gauge, in which $h_{\mu 0} = 0$, $\delta_{ij}h_{ij} = 0$ and $\partial_i h_{ij} = 0$. This is also known as the physical gauge because in this

gauge h_{ij} is (gauge) invariant to linear coordinate shifts ξ^μ , *i.e.* $\mathcal{L}_\xi h_{ij} = 0$, such that in this gauge h_{ij} is a physical (measurable) quantity. From Eqs. (1) and (13) we know that only $\overset{\circ}{R}{}^i{}_{00j}$ components of the Riemann tensor contribute. Next, in TT gauge $\overset{\circ}{R}{}^i{}_{00j} = (1/2)\ddot{h}_{ij}(t, \vec{x})$ and $\dot{\gamma}^\mu \dot{\gamma}^\nu \overset{\circ}{\nabla}_\mu \overset{\circ}{\nabla}_\nu J^i$ can be approximated by $d^2 J^i / dt^2$ (because the Levi-Civita connection contributes at second order), such that we have,

$$\frac{d^2 J^i}{dt^2} = \frac{1}{2} \ddot{h}_{ij}(t, \vec{x}) J^j. \quad (2)$$

Gravitational waves are built from spin two massless gravitons, which come in two polarizations, known as the plus (+) and cross (×) polarization. For example, if the axes are chosen such that a plane gravitational wave is moving in the z -direction, then $h_{zz} = 0$ and,

A) Plus polarization: $h_{xx} = -h_{yy} = h_+ \cos(\omega t - kz)$;

B) Cross polarization: $h_{xy} = h_{yx} = h_\times \cos(\omega t - kz)$;

where $\omega = ck$ ($k = \|\vec{k}\|$) is the frequency of the wave and h_+ and h_\times denote the amplitude of the + and ×-polarized wave, respectively. By making the *Ansatz*, $J^i(t, \vec{x}) = J_{(0)}^i + \Delta J_{(0)}^i \cos(\omega t - kz)$, one can easily show that to leading order in h_+ (h_\times) equation (2) is solved by:

A) Plus polarization: $J^x(t, z) = J_{(0)}^x [1 + (h_+/2) \cos(\omega t - kz)]$, $J^y(t, z) = J_{(0)}^y [1 - (h_+/2) \times \cos(\omega t - kz)]$;

B) Cross polarization: $J^x(t, z) = J_{(0)}^x + (h_\times/2) J_{(0)}^y \cos(\omega t - kz)$, $J^y(t, z) = J_{(0)}^y + (h_\times/2) \times J_{(0)}^x \cos(\omega t - kz)$.

These solutions show that the response displacements $\Delta J_{(0)}^i$ are in phase with the original wave and that A) for the plus polarization the relative displacement: $\Delta L/L = \Delta J_{(0)}^x / J_{(0)}^x = \Delta J_{(0)}^y / J_{(0)}^y = h_+/2$ is given by one half of the wave amplitude, while for B) for the cross polarization the relative displacement: $\Delta L/L = \Delta J_{(0)}^x / J_{(0)}^y = \Delta J_{(0)}^y / J_{(0)}^x = h_\times/2$ is also given by one half of the wave amplitude but with the axes x and y switched (explaining the name cross polarization).

Torsion trace. The contributions to the Jacobi field acceleration from the torsion trace vector is given by (16). Analogous to the contributions of Christoffel connection, the terms in the second line of (16) contribute at second order and thus can be neglected (this is because \dot{J}^α is already of the first order in torsion field), such that we have,

$$\ddot{J}^0 = 0, \quad \ddot{J}^i = J^0 \dot{\mathcal{T}}^i + J^j \partial_j \mathcal{T}^i. \quad (3)$$

Now from $g(J, \dot{\gamma}) = \dot{\gamma} \cdot J = 0$ and $\dot{\gamma}^\mu = \delta^\mu_0$ it follows that (to this order) $J^0 = 0$ (which is consistent with the first equation in (3)) and the second equation in (3) simplifies to,

$$\ddot{J}^i = J^j \partial_j \mathcal{T}^i. \quad (4)$$

To facilitate comparison with gravitational waves, we assume that \mathcal{T}^i can be written as a plane wave moving in the z -direction,

$$\mathcal{T}^i = \mathcal{T}_{(0)}^i \cos(\omega t - kz). \quad (5)$$

It is reasonable to assume that torsion wave constitutes a wave of an almost massless field, in which case $\omega \approx ck$ and the waves are to good approximation transverse, meaning that $\mathcal{T}^i = (\mathcal{T}^x, \mathcal{T}^y, \mathcal{T}^z)$ with $|\mathcal{T}^z| \ll |\mathcal{T}^{x,y}|$ in Eq. (5). [17] Inserting the *Ansatz*,

$$J^i(t, z) = J_{(0)}^i + \Delta J_{(0)}^i \sin(\omega t - kz), \quad (6)$$

into the Jacobi equation (4) and making use of (5) results in,

$$\Delta J_{(0)}^i = -\frac{c^2 k}{\omega^2} \mathcal{T}_{(0)}^i J_{(0)}^z \approx -\frac{c}{\omega} \mathcal{T}_{(0)}^i J_{(0)}^z. \quad (7)$$

Let us now pause to discuss this result. Notice first that the phase of the response (7) is shifted by $\pi/2$ with respect to the phase of the original wave (this is to be contrasted with no phase shift in the case of gravitational waves). This phase shift may be difficult to observe, especially because there may be other sources of phase shift (such as dispersivity of the medium through which the waves propagate and the massive nature of the torsion trace). However, if we have some confidence that gravitational wave and torsion wave come from the same source (which can be established by having a directional information) and that dispersive effects of the propagating medium are negligible, then this phase shift might be observable. A second difference is geometric: while gravitational waves induce a response in the relative length along the same transverse direction (for plus polarization) or along the opposite, but still transverse, direction (for cross polarization), torsion trace induces a response along the transverse direction which is proportional to the longitudinal direction of the instrument. Finally third (and probably the most important) difference between the gravitational waves and torsion trace vector signature is in that the relative displacement (7) is inversely proportional to the frequency/wave vector (while no frequency dependence is present in the gravitational wave response). This difference may be crucial when distinguishing torsion wave signatures from those of gravitational waves.

Skew symmetric torsion. From Eq. (17) it follows that, $\ddot{J}^i = 0$, which immediately implies that skew symmetric torsion cannot be observed by gravitational wave detectors [18].

Torsion with mixed symmetry. From Eq. (18) we see that,

$$\ddot{J}^i = -2\dot{Q}^i{}_{0j} J^j, \quad (8)$$

where we have assumed that $J^0 = 0$. Since we do not know any mechanism by which dynamical Q can be generated, [19] we cannot be sure how the wave equation for Q looks like. Therefore the analysis presented in

what follows represents an educated guess. It is reasonable to assume that – just as any massless waves – the Q waves are transverse, motivating the following *Ansatz*,

$$Q^i{}_{0j} = Q_{ij(0)} \cos(\omega t - kz), \quad \delta_{ij} Q_{ij(0)} = 0, \quad \omega = ck \quad (9)$$

where $Q_{ij(0)}$ is a real traceless 3×3 matrix. It is convenient to decompose $Q_{ij(0)}$ into its symmetric and traceless ($S^Q_{ij(0)}$) and antisymmetric ($A^Q_{ij(0)}$) parts, $Q_{ij(0)} = S^Q_{ij(0)} + A^Q_{ij(0)}$. Inserting this into (8) and assuming the form for J^i as in (7) results in,

$$\Delta J^i{}_{(0)} = -\frac{2c}{\omega} \left[S^Q_{ij(0)} + A^Q_{ij(0)} \right] J^j{}_{(0)}. \quad (10)$$

We thus see that the response to a passing Q wave is much richer than that of a gravitational wave. To get a better feeling on what (10) tells us, let us assume that Q is an almost massless field, in which case we have approximately, $Q_{zi(0)} = 0 = Q_{iz(0)}$ and the only non-vanishing components in (10) are $S^Q_{xx(0)} = -S^Q_{yy(0)}$, $S^Q_{xy(0)}$ and $A^Q_{xy(0)}$, implying that S^Q resembles gravitational wave, and A^Q has no gravitational analogue. However, there are important differences: (A) a phase shift of $\pi/2$ characterizes the Q wave response (10) and (B) the response to a Q wave is inversely proportional to frequency.

III. SUMMARY AND DISCUSSION

In this letter we show that dynamical (propagating) torsion can be observed by conventional gravitational wave detectors. More precisely, torsion trace and torsion of mixed symmetry can be observed, while skew symmetric torsion cannot. Roughly speaking, the relative amplitude change due to passage of a torsion wave is, $\Delta L/L \sim T/k \sim cT/\omega$, where T is the torsion wave amplitude and ω and k denote its frequency and wave number, respectively (more precise results are given in Eqs. (7) and (10)). This is to be contrasted with the response to gravitational waves of amplitude h , which is of the form, $\Delta L/L \sim h$. This difference suggests that torsion waves of larger wave length (smaller frequency) will be easier observed, and hence instruments in space such as LISA, pulsar timing arrays and measurements of cosmic microwave background are generally more sensitive to torsion waves. Furthermore, torsion waves induce a response that is delayed by a quarter period, while no such phase shift is present for gravitational waves. Finally, torsion polarization differs from that of gravitational waves, see Eqs. (7) and (10).

In this letter we do not discuss production of torsion waves. Since we assume torsion to be dynamical, the usual suspects – inflation, preheating, phase transitions and violent astrophysical events (such as black hole collisions, collisions of other compact stellar objects, supernovae, etc.) – can be invoked to be responsible for production. Because the response is more sensitive at low frequencies, one does not need to produce in inflation

a (nearly) scale invariant spectrum to make it observable, implying that a blue spectrum of torsion waves from inflation could be also observable.

Acknowledgements. We acknowledge financial support from an NWO-Graduate Program grant and by the D-ITP consortium, a program of the Netherlands Organization for Scientific Research (NWO) that is funded by the Dutch Ministry of Education, Culture and Science (OCW).

Appendix A

In this appendix we present some details on how to analyze the Jacobi equation (1). Jacobi field J represents a space-time vector field that can be used to characterize the distance between neighboring geodesics in a congruence of geodesics and it is therefore useful in determining how gravitational wave detectors respond to a passing (gravitational or torsion) wave.

In a space-time with geometric torsion, metric compatibility condition, $\nabla_\mu g_{\alpha\beta} = 0$ implies that the connection associated with the covariant derivative ∇ can be written as,

$$\Gamma^\alpha_{\mu\nu} = \overset{\circ}{\Gamma}^\alpha_{\mu\nu} + K^\alpha_{\mu\nu} \quad (11)$$

where $\overset{\circ}{\Gamma}^\alpha_{\mu\nu}$ is the Christoffel connection (Levi-Civita symbol) and $K^\alpha_{\mu\nu}$ denotes the contorsion tensor defined as,

$$K^\alpha_{\mu\nu} = T^\alpha_{\mu\nu} + T_{\mu\nu}{}^\alpha + T_{\nu\mu}{}^\alpha \quad (12)$$

where torsion tensor $T^\alpha_{\mu\nu}$ is defined as the antisymmetric part of the connection, $T^\alpha_{\mu\nu} = \Gamma^\alpha_{[\mu\nu]}$. The curvature tensor $R(\cdot, \cdot)$ in (1) can be conveniently written in terms of the Riemann curvature tensor, $\overset{\circ}{R}^\alpha_{\mu\rho\nu} = \partial_\rho \overset{\circ}{\Gamma}^\alpha_{\mu\nu} - \partial_\nu \overset{\circ}{\Gamma}^\alpha_{\mu\rho} + \overset{\circ}{\Gamma}^\alpha_{\sigma\rho} \overset{\circ}{\Gamma}^\sigma_{\mu\nu} - \overset{\circ}{\Gamma}^\alpha_{\sigma\nu} \overset{\circ}{\Gamma}^\sigma_{\mu\rho}$, and the contorsion tensor K as,

$$R^\alpha_{\mu\rho\nu} = \overset{\circ}{R}^\alpha_{\mu\rho\nu} + \overset{\circ}{\nabla}_\rho K^\alpha_{\mu\nu} - \overset{\circ}{\nabla}_\nu K^\alpha_{\mu\rho} + K^\alpha_{\sigma\rho} K^\sigma_{\mu\nu} - K^\alpha_{\sigma\nu} K^\sigma_{\mu\rho}, \quad (13)$$

where $\overset{\circ}{\nabla}$ represents the (general relativistic) covariant derivative taken with respect to the Christoffel connection $\overset{\circ}{\Gamma}$.

A 3-indexed tensor such as the torsion tensor T can be decomposed into its trace part \mathcal{T} , a skew symmetric part Σ (which is antisymmetric in all indices) and a mixed part Q (which is symmetric in the first two indices and antisymmetric in its latter two indices),

$$T^\alpha_{\nu\gamma} = \delta^\alpha_{[\nu} \mathcal{T}_{\gamma]} + \Sigma^\alpha_{\nu\gamma} + Q^\alpha_{\nu\gamma}, \quad (14)$$

where $Q^\alpha_{\alpha\gamma} = 0$, $Q_{[\alpha\nu\gamma]} = 0$. From (12) and (14) it follows that the contorsion tensor can be decomposed as,

$$K_{\alpha\nu\gamma} = 2g_{\gamma[\nu} \mathcal{T}_{\alpha]} + \Sigma_{\alpha\nu\gamma} + Q_{\alpha\nu\gamma}. \quad (15)$$

When (14) and (15) are inserted into (1) one obtains contributions from various components of the torsion tensor to acceleration of the Jacobi field along γ . To leading order in torsion these read,

$$\left(\dot{\gamma}^\mu \dot{\gamma}^\nu \overset{\circ}{\nabla}_\mu \overset{\circ}{\nabla}_\nu J^\alpha\right)_\mathcal{T} = \left(\overset{\circ}{\nabla}_J \mathcal{T}^\alpha\right) + \dot{\gamma}^\alpha \left(\overset{\circ}{\nabla}_J \mathcal{T}_\gamma\right) \quad (16)$$

$$\begin{aligned} & -2\mathcal{T}^\alpha \left(\frac{\overset{\circ}{D}}{D\tau} J_\gamma\right) - \left(\overset{\circ}{\nabla}_\mathcal{T} J^\alpha\right) + \dot{\gamma}^\alpha \mathcal{T}_\rho \left(\frac{\overset{\circ}{D}}{D\tau} J^\rho\right), \\ \left(\dot{\gamma}^\mu \dot{\gamma}^\nu \overset{\circ}{\nabla}_\mu \overset{\circ}{\nabla}_\nu J^\alpha\right)_\Sigma & = 0, \end{aligned} \quad (17)$$

$$\left(\dot{\gamma}^\mu \dot{\gamma}^\nu \overset{\circ}{\nabla}_\mu \overset{\circ}{\nabla}_\nu J^\alpha\right)_Q = -2\dot{\gamma}^\mu \left(\frac{\overset{\circ}{D}}{D\tau} Q^\alpha{}_{\mu\rho}\right) J^\rho, \quad (18)$$

where $J_\gamma = \dot{\gamma}^\mu J_\mu$, $\mathcal{T}_\gamma = \dot{\gamma}^\mu \mathcal{T}_\mu$, $\overset{\circ}{\nabla}_J = J^\mu \overset{\circ}{\nabla}_\mu$ and $\overset{\circ}{D}/D\tau \equiv \dot{\gamma}^\mu \overset{\circ}{\nabla}_\mu$. These results are used in the main text to investigate how one can detect torsion waves induced by the torsion trace \mathcal{T} , skew symmetric torsion Σ or mixed torsion Q .

Appendix B

In this appendix we discuss propagation of torsion trace, as it is implied by the effective gravitational action obtained upon integrating out scalar matter [4]. We shall not discuss here skew symmetric torsion, since it is not detectable by standard gravitational wave instruments. The equation of motion for dynamical torsion trace is of the form [4],

$$\theta \nabla^\mu \mathcal{T}_{\mu\nu} + \nabla_\nu (\alpha R + \xi \phi^2) = 0, \quad (19)$$

where θ , α and ξ are (dimensionless) coupling constants, R is the curvature scalar, ϕ is the (dilaton) scalar and $\mathcal{T}_{\mu\nu} = \partial_\mu \mathcal{T}_\nu - \partial_\nu \mathcal{T}_\mu$ is the torsion trace field strength. To study propagation in the late universe one can replace covariant derivatives with partial derivatives and the metric tensor with Minkowski metric $\eta_{\mu\nu}$. The linear form of (21) then becomes,

$$\partial^2 \mathcal{T}_\mu - \left(1 + \frac{6\alpha}{\theta}\right) \partial_\mu (\partial^\alpha \mathcal{T}_\alpha) + \frac{2M_{\text{P}}^2}{\theta} \mathcal{T}_\mu = 0, \quad \partial^2 = \eta^{\mu\nu} \partial_\mu \partial_\nu, \quad (20)$$

where we wrote $\xi \phi^2 = M_{\text{P}}^2$ is the reduces Planck mass squared and we assumed $R \approx 0$. From (20) we see that \mathcal{T}_μ is very massive, unless θ is very large, which is what we ought to assume (in order for \mathcal{T}_μ to be able to propagate to large distances). That also means that one expects typically, $6|\alpha/\theta| \ll 1$. In Ref. [2] it was argued that \mathcal{T} must be heavy in order to prevent matter fields to excessively decay into torsion. In our model however, a large value of θ (needed to make \mathcal{T} light) also implies a weak coupling ($\sim 1/\theta \ll 1$) to matter fields. Eq. (20) resembles that of a massive photon, but with a covariant gauge term added *a la* Stückelberg. To analyze (20)

it is convenient to break \mathcal{T}_μ into transverse and longitudinal components, $\mathcal{T}_\mu = \mathcal{T}_\mu^T + \mathcal{T}_\mu^L$, $\mathcal{T}_\mu^L = (\partial_\mu/\partial^2)(\partial^\alpha\mathcal{T}_\alpha)$ such that (20) becomes,

$$\partial^2\mathcal{T}_\mu^L - \frac{M_{\text{P}}^2}{3\alpha}\mathcal{T}_\mu^L = 0, \quad \partial^2\mathcal{T}_\mu^T + \frac{2M_{\text{P}}^2}{\theta}\mathcal{T}_\mu^T = 0. \quad (21)$$

That means that, even though the longitudinal field propagates, it is very heavy and it does not propagate very far. On the other hand, if $\theta \gg 1$ the transverse field is light and can propagate far. This is what was assumed when we analyzed detection of torsion trace in (3-7). While the analysis presented in this appendix (based on Ref. [4]) illustrates what kind of equation might govern propagation of torsion trace, our analysis of detection is general and thus not limited to the model presented here.

- [1] F. W. Hehl, P. Von Der Heyde, G. D. Kerlick and J. M. Nester, “General Relativity with Spin and Torsion: Foundations and Prospects,” *Rev. Mod. Phys.* **48** (1976) 393. doi:10.1103/RevModPhys.48.393
- [2] I. L. Shapiro, “Physical aspects of the space-time torsion,” *Phys. Rept.* **357** (2002) 113 doi:10.1016/S0370-1573(01)00030-8 [hep-th/0103093].
- [3] S. Lucat and T. Prokopec, “The role of conformal symmetry in gravity and the standard model,” *Class. Quant. Grav.* **33** (2016) no.24, 245002 doi:10.1088/0264-9381/33/24/245002 [arXiv:1606.02677 [hep-th]].
- [4] S. Lucat and T. Prokopec, in progress (2017).
- [5] N. J. Poplawski, “Nonsingular, big-bounce cosmology from spinor-torsion coupling,” *Phys. Rev. D* **85** (2012) 107502 doi:10.1103/PhysRevD.85.107502 [arXiv:1111.4595 [gr-qc]].
- [6] N. J. Poplawski, “Big bounce from spin and torsion,” *Gen. Rel. Grav.* **44** (2012) 1007 doi:10.1007/s10714-011-1323-2 [arXiv:1105.6127 [astro-ph.CO]].
- [7] S. Lucat and T. Prokopec, “Cosmological singularities and bounce in Cartan-Einstein theory,” arXiv:1512.06074 [gr-qc].
- [8] C. G. Boehmer, “On inflation and torsion in cosmology,” *Acta Phys. Polon. B* **36** (2005) 2841 [gr-qc/0506033].
- [9] C. H. Wang and Y. H. Wu, “Inflation in $R + R^{*2}$ Gravity with Torsion,” *Class. Quant. Grav.* **26** (2009) 045016 doi:10.1088/0264-9381/26/4/045016 [arXiv:0807.0069 [gr-qc]].
- [10] N. J. Popawski, “Cosmology with torsion: An alternative to cosmic inflation,” *Phys. Lett. B* **694** (2010) 181 Erratum: [*Phys. Lett. B* **701** (2011) 672] doi:10.1016/j.physletb.2010.09.056, 10.1016/j.physletb.2011.05.047 [arXiv:1007.0587 [astro-ph.CO]].
- [11] Y. P. Wu, “Inflation with teleparallelism: Can torsion generate primordial fluctuations without local Lorentz symmetry?,” *Phys. Lett. B* **762** (2016) 157 doi:10.1016/j.physletb.2016.09.025 [arXiv:1609.04959 [gr-qc]].
- [12] L. C. Garcia de Andrade, “Primordial magnetic fields and dynamos from parity violated torsion,” *Phys. Lett. B* **711** (2012) 143. doi:10.1016/j.physletb.2012.03.075
- [13] A. S. Belyaev, I. L. Shapiro and M. C. Thomas, “Torsion as a Dark Matter Candidate from the Higgs Portal,” arXiv:1611.03651 [hep-ph].

- [14] M. I. Wanas, “Dark Energy: Is It of Torsion Origin?,” arXiv:1006.2154 [gr-qc].
- [15] N. J. Poplawski, “Four-fermion interaction from torsion as dark energy,” *Gen. Rel. Grav.* **44** (2012) 491 doi:10.1007/s10714-011-1288-1 [arXiv:1102.5667 [gr-qc]].
- [16] A. S. Belyaev, I. L. Shapiro and M. A. B. do Vale, “Torsion phenomenology at the LHC,” *Phys. Rev. D* **75** (2007) 034014 doi:10.1103/PhysRevD.75.034014 [hep-ph/0701002].
- [17] From the analysis in Appendix B it follows that one will typically observe \mathcal{T}_μ^T which obeys a massive vector field (Proca) equation (21), from which one can naturally impose a Lorentz condition (obeyed by any massive vector field), $\partial_\mu \mathcal{T}^\mu = 0$, or $\mathcal{T}_0 = -(ck/\omega)\mathcal{T}_z$, where for simplicity we have dropped the superscript T . Therefore, even though Eq. (4) tells us that we cannot directly observe \mathcal{T}_0 , we can imply its value from the Lorentz condition.
- [18] This conclusion could have been reached by a careful look at the derivation of Jacobi equation (1) in Ref [3], where it is derived by making use of the geodesic equation, $\nabla_{\dot{\gamma}}\dot{\gamma} = 0$ and of equation, $\mathcal{L}_{\dot{\gamma}}J = [J, \dot{\gamma}] = 0$. None of these equations contains any dependence on skew symmetric torsion.
- [19] Namely, neither matter (scalars, fermions, vectors) nor gravity couples bilinearly to Q in the Cartan-Einstein theory, implying that no kinetic term for Q can be generated at the one-loop order by integrating out matter or gravitational fields. This is to be contrasted with torsion trace vector and skew symmetric torsion, whose kinetic terms are generated at one-loop order when scalar and fermionic matter is integrated out, respectively, thereby making them dynamical [2, 4].