

Nonlinear multidimensional gravity and the Australian dipole

K.A. Bronnikov^{a,b,1}, V.N. Melnikov^{a,b,2}, S.G. Rubin^{c,3}, and I.V. Svadkovsky^{c,4}

^a *Center for Gravitation and Fundamental Metrology, VNIIMS, 46 Ozyornaya St., Moscow 119361, Russia*

^b *Institute of Gravitation and Cosmology, PFUR, 6 Miklukho-Maklaya St., Moscow 117198, Russia*

^c *National Research Nuclear University “MEPhI”, Kashirskoe sh. 31, Moscow, 115409 Russia*

The existing observational data on possible variations of fundamental physical constants (FPC) confirm more or less confidently only a variability of the fine structure constant α in space and time. A model construction method is described, where variations of α and other FPCs (including the gravitational constant G) follow from the dynamics of extra space-time dimensions in the framework of curvature-nonlinear multidimensional theories of gravity. An advantage of this method is a unified approach to variations of different FPCs. A particular model explaining the observable variations of α in space and time has been constructed. It comprises a FRW cosmology with accelerated expansion, perturbed due to slightly inhomogeneous initial data.

1 Introduction

The problem of possible variations of the fundamental physical constants (FPC) in time and space is one of the most challenging problems of modern physics, directly related to the central problem of unification of all interactions. It traces back to Dirac’s and Eddington’s famous papers of the 1930s and since then gains much attention in both theoretical and experimental studies.

However, to date, a variability of only one FPC has been revealed by observations more or less confidently, it is the fine structure constant α . The analysis of absorption spectra of various ions in the radiation of distant quasars, performed in the recent years (above all, from the data obtained at the Keck telescope on the Hawaiian islands), has led to a conclusion that α is changing with time, so that in the past it was slightly smaller than now (the relative change $\delta\alpha/\alpha$ is about 10^{-5} [1]). In 2010, an analysis of new data obtained at the VLT (Very Large Telescope), located in Chile, and their comparison with the Keck data led to a conclusion on spatial variations of α , i.e., on its dependence on the direction of observations. The VLT observations in the Southern part of the celestial sphere gave values of the parameter α in the past slightly larger than now. This anisotropy has a dipole nature [2] and has been termed “the Australian dipole” [3]. The dipole axis is located at a declination of $-61 \pm 9^\circ$ and at a right ascension of $17.3 \pm 0,6$ hours. The deflection of α value at an arbitrary point r of space from its modern value α_0 , measured on Earth, is

$$\delta\alpha/\alpha_0 = (1.10 \pm 0.25) \times 10^{-6} r \cos \psi, \quad (1)$$

where ψ is the angle between the direction of observation and the dipole axis, while the distance r is measured in billions of light years. The confidence level of this result (as compared with a “monopole” model where values of α are the same in all directions) has been estimated as 4.1σ .

¹e-mail: kb20@yandex.ru

²e-mail: melnikov@phys.msu.ru

³e-mail: sergeirubin@list.ru

⁴e-mail: igor_svadkovsky@mail.ru

Let us also mention the laboratory experimental data on possible FPC variations in the modern epoch. The tightest constraints on α variations have been obtained by comparison of readings of atomic clocks using optical transitions in Al and Hg ions (without using cesium clocks that have become classic) [4]: $(d\alpha/dt)/\alpha = (-1.6 \pm 2.3) \times 10^{-17}$ per year. This result is of the same order of magnitude as the tightest constraints obtained previously from an isotopic composition analysis of the decay products in the natural nuclear reactor that operated in the Oklo region (Gabon) about 2 billion years ago. Unlike the laboratory data, the Oklo results [5] and, in particular, the tightest constraint [6]

$$d(\ln \alpha)/dt = (-0.4 \pm 0.5) \times 10^{-17}/\text{yr} \quad (2)$$

rely in the assumption that during these 2 billion years the value of α changed uniformly, if changed at all. This assumption looks rather natural but actually follows from nowhere.

Thus in the modern epoch, at least on Earth since the Oklo times, the parameter α did not change more rapidly than by approximately 10^{-17} per year. If, on the other hand, we use the distant quasar data and take a mean value of $d(\ln \alpha)/dt$ for about 10 billion years, we shall obtain a variation rate of about 10^{-15} per year. Therefore one can conclude that at times earlier (maybe much earlier) than 2 billion years ago the value of α changed relatively rapidly but afterwards stopped or almost stopped to change. The task of theory was to explain such a behavior; however, if one takes into account the most recent observations [2, 7], one should add the necessity of explaining the spatial variations of α . Though, one cannot exclude the opportunity that the variations of α are purely spatial in nature whereas the time dependence is related to the finiteness of the velocity of light: being located at a fixed point and at fixed time, we receive signals from distant regions of the Universe emitted at earlier cosmological epochs, and it is therefore impossible to separate spatial and temporal dependences of the parameters.

Let us briefly discuss the theoretical models describing variations of α . Thus, following the pioneering ideas of Dirac and Eddington, Dicke and Peebles [8] in 1962 considered variations of α in cosmological models admitting a variable gravitational interaction intensity. Staniukovich [9] in 1965 discussed different variants of combined FPC variations in connection with Dirac's Large Number Hypothesis. Bekenstein [10] in 1982 described a model of α variations on the basis of the most general assumptions on the electromagnetic interaction: covariance, gauge invariance, causality and invariance with respect to time reversion. This led to a modified Maxwell electrodynamics and provided a certain dynamics of α . There also appeared predictions of possible FPC variations in cosmological models with Kaluza-Klein type extra dimensions, see, e.g., [11, 12].

Since the advent of astronomical evidence on possible time variations of α , there emerged a whole class of new models describing such variations by introducing certain scalar fields. Thus, Sandvik et al. [13] proposed a cosmological extension of Bekenstein's theory [10] with a term of the form $-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}e^{-2\psi}$ in the initial Lagrangian, where the scalar field ψ interacts only with the electromagnetic field $F^{\mu\nu}$. The effect of the field ψ in the dynamics of the expanding Universe was also considered. It was shown that in this model α remained almost constant in the radiation-dominated epoch, slightly increased in the matter-dominated epoch and approaches a constant value at times when the Universe expansion accelerates due to the presence of a positive cosmological constant.

In attempts to explain both temporal and spatial variations of the fine structure constant, quite popular are models assuming the existence of domain walls connected with scalar field dynamics (see, e.g., [14, 15]). Thus, in [15] the initial action contains a dilaton-like scalar field ϕ interacting with the electromagnetic field and having a potential of the form $V(\phi) = \frac{1}{4}\lambda(\phi^2 - \eta^2)^2$. A domain wall is formed due to spontaneous symmetry breakdown. At points separated by the domain wall

the values of α are different, which can explain the observable variations if the wall intersects our Hubble volume.

In [16] it has been shown that in $F(R)$ gravity it is possible to obtain a static solution in the form of an effective (gravitational) domain wall, and that the choice of a logarithmic nonminimal interaction of the electromagnetic field with gravity in the form

$$-\frac{1}{4} \left[1 + \ln \left(\frac{R}{R_0} \right) \right] F_{\mu\nu} F^{\mu\nu}$$

(where R_0 is the modern value of the scalar curvature) makes it possible to describe variations of α , whose value grows as the curvature R decreases.

Olive et al. [17] discuss a model with two domain walls, where the scalar field potential has three minima:

$$V(\phi) = \lambda \left(|\Phi|^2 - \frac{\eta^2}{2} \right)^2 - \sqrt{2}i\epsilon (\Phi^3 - (\Phi^*)^3) + V_0$$

It turns out that such a model much better describes the observational data than a similar one [15] with a single domain wall.

The paper [18] suggests an extension of the previous BSBM (Bekenstein-Sandvik-Barrow-Magueijo) theory ([10, 13]) by introducing a dependence of the coupling constant ω of the scalar field ψ on the field itself, so that the Lagrangian contains the terms $L_\psi = -\frac{\omega(\psi)}{2} \partial_\mu \psi \partial^\mu \psi$ and $L_{\text{em}} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} e^{-2\psi}$. The choice of $\omega(\psi)$ allows for obtaining both growing and falling time dependences of α . This model differs from those with domain walls in that the variations of α are smooth and continuous, and a choice between these models must be easy with future more precise and reliable observational data.

Mariano and Perivolaropoulos [19] have reported on a correlation between the spatial distribution of α values and the dipole anisotropy of the dark energy distribution. In the same paper they have suggested a theoretical model explaining this correlation (named “extended topological quintessence”) which naturally predicts inhomogeneous spherical distributions of both the dark energy and the values of α . The model assumes the existence of a huge global monopole with a size of Hubble order, which nonminimally interacts with the electromagnetic field. There emerge mutually related distributions of different parameters with a dipole anisotropy from the viewpoint of any observer located outside the monopole center. The monopole is formed after a phase transition in a set of three scalar fields with an O(3) symmetric Lagrangian.

In a later paper [20] the same authors support their inferences by the data on one more anisotropy also seeming to exist and to be aligned with other “dipoles”, the so-called Large-Scale Velocity Flows (Dark Flow), i.e., recent indications that there is a large-scale peculiar velocity flow with an amplitude larger than 400 km/s on scales up to $100h^{-1}$ Mpc ($z \leq 0.03$).

It should be noted that all the above approaches, to explain variations of α , introduce scalar fields whose existence and manner of interaction with the electromagnetic field are postulated from the outset and are not explained in any way. In what follows, it will be shown how the scalar fields and their interaction law with electromagnetism naturally follow from nonlinear multidimensional gravity. What is postulated is just the existence of extra dimensions and actually nothing else. An advantage of multidimensional gravity in the treatment of FPC variations is that such variations are explained in a unified way from spatial and temporal variations of the size of extra dimensions [21, 22].

The approach we are using has been formulated in [23], where a methodology was suggested allowing for a transition from multidimensional gravity with higher derivatives to Einstein-Hilbert

gravity with effective scalar fields. Later on this approach was successfully applied for a unified description of the inflationary stage of the Universe and the modern secondary inflation [25] and an explanation of the origin of the Higgs field [26]; a mechanism of cascade reduction of multidimensional space to the observable one was suggested [24, 27]. It has been shown under which conditions the compact extra dimensions become stationary (i.e., have a constant volume), and the cause of their maximum symmetry was found [28].

The present study has been performed in the framework of this approach and is an example of its employment. The paper is organized as follows. Sec. 2 briefly describes the general formalism used. In this framework, in Sec. 3 we build a homogeneous and isotropic cosmological model able to describe the present accelerated Universe along with a time dependence of the fine structure constant α . In Sec. 4, this cosmological model is slightly perturbed on large scale, which enables us to explain spatial variations of α . Sec. 5 is a brief conclusion.

2 Multidimensional gravity and its reduction

Consider a $(D = 4 + d_1)$ -dimensional manifold with the metric

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu + e^{2\beta(x)} b_{ab} dx^a dx^b \quad (3)$$

where the extra-dimensional metric components b_{ab} are independent of x^μ , the observable four space-time coordinates.

The D -dimensional Riemann tensor has the nonzero components

$$\begin{aligned} R^{\mu\nu}{}_{\rho\sigma} &= \bar{R}^{\mu\nu}{}_{\rho\sigma}, \\ R^{\mu a}{}_{\nu a} &= \delta_b^a B_\nu^\mu, \quad B_\nu^\mu := e^{-\beta} \nabla_\nu (e^\beta \beta^\mu), \\ R^{ab}{}_{cd} &= e^{-2\beta} \bar{R}^{ab}{}_{cd} + \delta^{ab}{}_{cd} \beta_\mu \beta^\mu, \end{aligned} \quad (4)$$

where capital Latin indices cover all D coordinates, the bar marks quantities obtained from $g_{\mu\nu}$ and b_{ab} taken separately, $\beta_\mu \equiv \partial_\mu \beta$ and $\delta^{ab}{}_{cd} \equiv \delta_c^a \delta_d^b - \delta_d^a \delta_c^b$. The nonzero components of the Ricci tensor and the scalar curvature are

$$\begin{aligned} R_\mu^\nu &= \bar{R}_\mu^\nu + d_1 B_\mu^\nu, \\ R_a^b &= e^{-2\beta} \bar{R}_a^b + \delta_a^b [\square \beta + d_1 (\partial\beta)^2], \\ R &= \bar{R}[g] + e^{-2\beta} \bar{R}[b] + 2d_1 \square \beta + d_1 (d_1 + 1) (\partial\beta)^2, \end{aligned} \quad (5)$$

where $(\partial\beta)^2 \equiv \beta_\mu \beta^\mu$, $\square = \nabla^\mu \nabla_\mu$ is the d'Alembert operator while $\bar{R}[g]$ and $\bar{R}[b]$ are the Ricci scalars corresponding to $g_{\mu\nu}$ and b_{ab} , respectively. Let us also present, using similar notations, the expressions for two more curvature invariants, the Ricci tensor squared and the Kretschmann scalar $\mathcal{K} = R^{ABCD} R_{ABCD}$:

$$\begin{aligned} R_{AB} R^{AB} &= \bar{R}_{\mu\nu} \bar{R}^{\mu\nu} + 2d_1 \bar{R}_{\mu\nu} B^{\mu\nu} + d_1^2 B_{\mu\nu} B^{\mu\nu} + e^{-4\beta} \bar{R}_{ab} \bar{R}^{ab} \\ &\quad + 2e^{-2\beta} \bar{R}[b] [\square \beta + d_1 (\partial\beta)^2] + d_1 [\square \beta + d_1 (\partial\beta)^2]^2, \end{aligned} \quad (6)$$

$$\mathcal{K} = \bar{\mathcal{K}}[g] + 4d_1 B_{\mu\nu} B^{\mu\nu} + e^{-4\beta} \bar{\mathcal{K}}[b] + 4e^{-2\beta} \bar{R}[b] (\partial\beta)^2 + 2d_1 (d_1 - 1) [(\partial\beta)^2]^2. \quad (7)$$

Suppose now that b_{ab} describes a compact d_1 -dimensional space of nonzero constant curvature, i.e., a sphere ($K = 1$) or a compact d_1 -dimensional hyperbolic space ($K = -1$) with a fixed curvature radius r_0 normalized to the D -dimensional analogue m_D of the Planck mass, i.e., $r_0 =$

$1/m_D$ (we use the natural units, with the speed of light c and Planck's constant \hbar equal to unity). We have

$$\begin{aligned}\overline{R}^{ab}{}_{cd} &= K m_D^2 \delta^{ab}{}_{cd}, \\ \overline{R}_a^b &= K m_D^2 (d_1 - 1) \delta_a^b, \\ \overline{R}[b] &= K m_D^2 d_1 (d_1 - 1) = R_b.\end{aligned}\tag{8}$$

The scale factor $b(x) \equiv e^\beta$ in (3) is thus kept dimensionless; R_b has the meaning of a characteristic curvature scale of the extra dimensions.

Consider, in the above geometry, a sufficiently general curvature-nonlinear theory of gravity with the action

$$\begin{aligned}S &= \frac{1}{2} m_D^{D-2} \int \sqrt{{}^D g} d^D x (L_g + L_m), \\ L_g &= F(R) + c_1 R^{AB} R_{AB} + c_2 \mathcal{K},\end{aligned}\tag{9}$$

where $F(R)$ is an arbitrary smooth function, c_1 and c_2 are constants, L_m is a matter Lagrangian and ${}^D g = |\det(g_{MN})|$.

The extra coordinates are easily integrated out, reducing the action to four dimensions:

$$S = \frac{1}{2} \mathcal{V}[d_1] m_D^2 \int \sqrt{{}^4 g} d^4 x e^{d_1 \beta} [L_g + L_m],\tag{10}$$

where ${}^4 g = |\det(g_{\mu\nu})|$ and $\mathcal{V}[d_1]$ is the volume of a compact d_1 -dimensional space of unit curvature.

Eq. (10) describes a curvature-nonlinear theory with non-minimal coupling between the effective scalar field β and the curvature. Let us simplify it in the following way (putting, for convenience, $m_D = 1$, so that all quantities are now expressed in (D -dimensional) Planck units:

(a) Express everything in terms of 4D variables and $\beta(x)$; we have, in particular,

$$R = R_4 + \phi + f_1, \quad R_4 = \overline{R}[g], \quad f_1 = 2d_1 \square \beta + d_1(d_1 + 1)(\partial\beta)^2,\tag{11}$$

where we have introduced the effective scalar field

$$\phi(x) = R_b e^{-2\beta(x)} = K d_1 (d_1 - 1) e^{-2\beta(x)}\tag{12}$$

The sign of ϕ coincides with $k = \pm 1$, the sign of curvature in the d_1 extra dimensions.

(b) Suppose that all quantities are slowly varying, i.e., consider each derivative ∂_μ (including those in the definition of \overline{R}) as an expression containing a small parameter ε ; neglect all quantities of orders higher than $O(\varepsilon^2)$ (see [23, 24]).

(c) Perform a conformal mapping leading to the Einstein conformal frame, where the 4-curvature appears to be minimally coupled to the scalar ϕ .

In the decomposition (11), both terms f_1 and R_4 are regarded small in our approach, which actually means that all quantities, including the 4D curvature, are small as compared with the D -dimensional Planck scale. The only term which is not small is ϕ , and we can use a Taylor decomposition of the function $F(R) = F(\phi + R_4 + f_1)$:

$$F(R) = F(\phi + R_4 + f_1) \simeq F(\phi) + F'(\phi) \cdot (R_4 + f_1) + \dots,\tag{13}$$

with $F'(\phi) \equiv dF/d\phi$. Substituting this, and the corresponding decompositions of the expressions (6) and (7), into Eq. (10), we obtain, up to $O(\varepsilon^2)$, the following effective gravitational Lagrangian L_g in Eq. (10):

$$L_g = F'(\phi)R_4 + F(\phi) + F'(\phi)f_1 + c_*\phi^2 + 2c_1\phi\Box\beta + 2(c_1d_1 + 2c_2)\phi(\partial\beta)^2 \quad (14)$$

with $c_* = c_1/d_1 + 2c_2/[d_1(d_1 - 1)]$.

The action (10) with (14) is typical of a scalar-tensor theory (STT) of gravity in a Jordan frame. To study the dynamics of the system, it is helpful to pass on to the Einstein frame. Applying the conformal mapping

$$g_{\mu\nu} \mapsto \tilde{g}_{\mu\nu} = |f(\phi)|g_{\mu\nu}, \quad f(\phi) = e^{d_1\beta}F'(\phi), \quad (15)$$

after a lengthy calculation, we obtain the action in the Einstein frame as

$$S = \frac{1}{2}\mathcal{V}[d_1] \int \sqrt{\tilde{g}}(\text{sign } F')L, \quad (16)$$

$$L = \tilde{R}_4 + K_E(\phi)(\partial\phi)^2 - 2V_E(\phi) + \tilde{L}_m, \quad (16)$$

$$\tilde{L}_m = (\text{sign } F')\frac{e^{-d_1\beta}}{F'(\phi)^2}L_m; \quad (17)$$

$$K_E(\phi) = \frac{1}{4\phi^2} \left[6\phi^2 \left(\frac{F''}{F'} \right)^2 - 2d_1\phi \frac{F''}{F'} + \frac{1}{2}d_1(d_1+2) + \frac{4(c_1 + c_2)\phi}{F'} \right], \quad (18)$$

$$-2V_E(\phi) = (\text{sign } F')\frac{e^{-d_1\beta}}{F'(\phi)^2}[F(\phi) + c_*\phi^2], \quad (19)$$

where the tilde marks quantities obtained from or with $\tilde{g}_{\mu\nu}$; the indices are raised and lowered with $\tilde{g}_{\mu\nu}$; everywhere $F = F(\phi)$ and $F' = dF/d\phi$; e^β is expressed in terms of ϕ using (12).

Let us consider the electromagnetic field $F_{\mu\nu}$ as matter in the initial Lagrangian, putting

$$L_m = \alpha_1^{-1}F_{\mu\nu}F^{\mu\nu}, \quad (20)$$

where α_1 is a constant. After reduction to four dimensions this expression acquires the factor $e^{d_1\beta}$ arising from the metric determinant: $\sqrt{D}g = \sqrt{4}g e^{d_1\beta}$. In the subsequent transition to the Einstein picture the expression $\sqrt{4}g F_{\mu\nu}F^{\mu\nu}$ remains the same (it is the well-known conformal invariance of the electromagnetic field), hence the Lagrangian (17) takes the form

$$\tilde{L}_m = \alpha_1^{-1}e^{d_1\beta}F_{\mu\nu}F^{\mu\nu}, \quad (21)$$

and for the effective fine structure constant α we obtain

$$\frac{\alpha}{\alpha_0} = e^{d_1(\beta_0 - \beta)}, \quad (22)$$

where α_0 and β_0 are values of the respective quantities at a fixed space-time point, for instance, where and when the observation is taking place.

3 The cosmological model

Depending on the choice of $F(R)$, the parameter c_1 and c_2 and the matter Lagrangian in the action (9), the theory under consideration can lead to a great variety of cosmological models. Some of them were discussed in [23], mostly those related to minima of the effective potential (19) at nonzero values of ϕ . Such minima correspond to stationary states of the scalar ϕ and consequently of the scale factor $b = e^\beta$ of the extra dimensions. If the minimum value of the potential is positive, it can play the role of a cosmological constant that launches an accelerated expansion of the Universe.

Here, we would like to focus on another minimum of the potential V_{Ein} , existing for generic choices of the function $F(R)$ with $F' > 0$ and located at the point $\phi = 0$. The asymptotic $\phi \rightarrow 0$ corresponds to growing rather than stabilized extra dimensions: $b = e^\beta \sim 1/\sqrt{|\phi|} \rightarrow \infty$. A model with such an asymptotic growth at late times may still be of interest if the growth is sufficiently slow and the size b does not reach detectable values by now. Let us recall that the admissible range of such growth comprises as many as 16 orders of magnitudes if the D -dimensional Planck length $1/m_D$ coincides with the 4D one, i.e., about 10^{-33} cm: the upper bound corresponds to lengths about 10^{-17} cm or energies of the order of a few TeV. This estimate certainly changes if there is no such coincidence.

One should note that small values of ϕ to be considered here are still very large as compared to 4D quantities, and so our general assumptions are well justified. Indeed, according to (12),

$$|\phi| = \frac{d_1(d_1 - 1)}{b^2},$$

where $b \lesssim 10^{16}$, hence $|\phi| \gtrsim d_1^2 \cdot 10^{-32}$, while the quantity \tilde{R}_4 , if identified with the curvature of the modern Universe, is of the order 10^{-122} in Planck units (that is, close to the Hubble parameter squared, or (the Hubble time)⁻², see also Eq. (34) below.

Let us check whether it is possible to describe the modern state of the Universe by an asymptotic form of the solution for $\phi \rightarrow 0$ as a spatially flat cosmology with the 4D Einstein-frame metric

$$d\tilde{s}_4^2 = dt^2 - a^2(t)d\vec{x}^2, \quad (23)$$

where $a(t)$ is the Einstein-frame scale factor. At small ϕ , assuming a smooth function $F(\phi)$, we can use its Taylor decomposition⁵

$$F(\phi) = -2\Lambda_D + F_1\phi + F_2\phi^2 + \dots, \quad (24)$$

where Λ_D is the initial cosmological constant. For simplicity, we suppose in this paper⁶ $F_1 = 0$, $F_2 = 1$. Then, substituting $F' = 2\phi$ and $F'' = 2$, we obtain for the kinetic and potential terms in the Lagrangian (16) in the first approximation in ϕ :

$$\begin{aligned} K_{\text{E}} &\approx K_0/(2\phi^2), & K_0 &= \frac{1}{2} \left[\frac{1}{2}d_1^2 - d_1 + 6 + 2(c_1 + c_2) \right]; \\ V_{\text{E}} &\approx V_0 e^{-2\bar{d}\beta}, & V_0 &= \frac{\Lambda_D}{4d_1^2(d_1 - 1)^2}, & 2\bar{d} &= d_1 - 4. \end{aligned} \quad (25)$$

It is clear that this model can work only if $d_1 > 4$. In terms of β instead of ϕ , the Lagrangian takes the form

$$L = \tilde{R}_4 + 2K_0(\partial\beta)^2 - 2V_0 e^{-2\bar{d}\beta} + \tilde{L}_m, \quad (26)$$

⁵We assume for certainty $\phi > 0$, or, which is the same according to (12), $K = +1$, but everything can be easily reformulated for $\phi < 0$.

⁶The theory is insensitive to multiplying the action by a constant, and we use this freedom to fix $F_2 = 1$.

Neglecting the gravitational influence of the electromagnetic field (that is, considering only vacuum models), one can write down the independent components of the Einstein and scalar field equations with the unknowns $\beta(t)$ and $a(t)$ in the form

$$3\frac{\dot{a}^2}{a^2} = K_0\dot{\beta}^2 + V_0 e^{-2\bar{d}\beta}, \quad (27)$$

$$\ddot{\beta} + 3\frac{\dot{a}}{a}\dot{\beta} = \frac{V_0\bar{d}}{K_0} e^{-2\bar{d}\beta}. \quad (28)$$

These equations, corresponding to a scalar field with an exponential potential, can be solved exactly but the solution looks rather involved, and for our purpose more preferable is the comparatively simple approximate solution that can be obtained in the slow-rolling approximation; the latter should be acceptable at late times. Let us suppose that

$$|\ddot{\beta}| \ll 3\frac{\dot{a}}{a}\dot{\beta}, \quad K_0\dot{\beta}^2 \ll V_0 e^{-2\bar{d}\beta}, \quad (29)$$

and neglect the corresponding terms in Eqs. (27) and (28). Then, expressing the quantity \dot{a}/a from (27) and substituting it into (28), we obtain

$$\dot{\beta} = \frac{\bar{d}\sqrt{V_0}}{K_0\sqrt{3}} e^{-\bar{d}\beta}, \quad (30)$$

whence

$$e^{\bar{d}\beta} = \frac{\bar{d}^2}{K_0} \sqrt{\frac{V_0}{3}} (t + t_1), \quad (31)$$

where t_1 is an integration constant. For the scale factor $a(t)$ we have

$$\frac{\dot{a}}{a} = \frac{p}{t + t_1} \quad \Rightarrow \quad a = a_1(t + t_1)^p, \quad a_1 = \text{const}, \quad p = \frac{K_0}{\bar{d}^2}. \quad (32)$$

Substituting the solution to the slow-rolling conditions (29), we make sure that they hold as long as $p \gg 1$, or in terms of the input parameters of the theory,

$$p = \frac{d_1^2 - 2d_1 + 12 + 4(c_1 + c_2)}{(d_1 - 4)^2} \gg 1. \quad (33)$$

We will assume that this condition holds.

A further interpretation of the results depends on which conformal frame is regarded physical (observational) [29, 30], and this in turn depends on the manner in which fermions appear in the (so far unknown) underlying unification theory involving all interactions.

Let us adopt the simplest hypothesis that the observational picture coincides with the Einstein picture and make some estimates. Thus, the inverse of the modern value of the Hubble parameter (the Hubble time) is estimated as

$$t_H = 1/H_0 = a_0/\dot{a}_0 \approx 4, 4 \times 10^{17} \text{ s} \approx 8 \times 10^{60} t_{\text{pl}}, \quad (34)$$

where t_{pl} is the Planck time and the index “0” marks quantities belonging to the present time, which is a usual notation in cosmology. From (32) it follows that $H_0 = p/(t_0 + t_1)$, whence

$$t_* := t_0 + t_1 = pt_H \gg t_H. \quad (35)$$

With $p \gg 1$, the model satisfies the observational constraints on the factor w in the effective equation of state $p = w\rho$ of dark energy that causes the accelerated expansion of the Universe: at $w = \text{const}$ we have $a \sim t^{2/(3+3w)}$, consequently, $w = -1 + 2/(3p)$ is a number close to -1 .

Furthermore, the ‘‘internal’’ scale factor $b(t) = e^\beta$ grows much slower than $a(t)$:

$$b(t) = b_0 \left(\frac{t + t_1}{t_*} \right)^{1/\bar{d}}, \quad b_0 = \left(\frac{1}{H_0} \sqrt{\frac{V_0}{3}} \right)^{1/\bar{d}}. \quad (36)$$

Using the expression for V_0 from (25), one can estimate the initial parameter Λ_D , connecting it with the size of the extra factor space b_0 : in Planck units,

$$\Lambda_D = 12H_0^2 d_1^2 (d_1 - 1)^2 b_0^{d_1 - 4} \approx \frac{3}{16} d_1^2 (d_1 - 1)^2 b_0^{d_1 - 4} \times 10^{-120}. \quad (37)$$

As already mentioned, the ‘‘internal’’ scale factor $b = e^\beta$ should be in the range $1 \ll b_0 \lesssim 10^{16}$ in Planck units. The estimate (37) shows that the present model makes much easier the well-known ‘‘cosmological constant problem’’ (the difficulty of explaining why in standard cosmology $\Lambda_{\text{standard}} \sim 10^{-122}$ in Planck units). For instance, if (in the admissible range) $b_0 = 10^{15}$ and $d_1 = 12$, it follows $\Lambda_D = 3267$ without any indication of fine tuning.

Let us estimate the possible range of the parameters c_1 and c_2 in the action (9). The present model describes only the modern stage of the Universe evolution, but it should admit an improvement after which it will account for other stages, including the early inflation. Then one should require that the curvature-nonlinear terms in the initial Lagrangian should not violate our slow-change approximation, see Sec. 2 This leads to the condition $c_{1,2} \ll 10^{11}$. Indeed, during inflation, the Hubble parameter is $H \sim 10^{-6}$ in Planck units, while the scalar curvature at inflation, when the 4D geometry is approximately de Sitter, is estimated as $R \simeq 12H^2 \sim 10^{-11}$. Assuming that the Ricci and Riemann tensor components have the same order of magnitude, we find that the condition $R \gg c_1 R^{AB} R_{AB}$, used above in the framework of the slow-change approximation, will be violated if c_1 is too large. The upper bound of the parameter c_2 is obtained in a similar way.

The smallness of the observed variations of α leads to another constraint on c_1 and c_2 : according to (22),

$$\alpha/\alpha_0 = (b/b_0)^{-d_1} = \left(\frac{t + t_1}{t_0 + t_1} \right)^{-2d_1/(d_1-4)} \approx 1 - \frac{2d_1}{d_1 - 4} \frac{t - t_0}{t_*}, \quad (38)$$

so that $\dot{\alpha}/\alpha \sim 10^{-10}/p$ per year. By the empirical data, this quantity cannot be larger than about 10^{-17} per year. A comparison leads to the constraint $p \gtrsim 10^7$. Taking into account the relation (33) between p and the input parameters c_1 and c_2 , we obtain similar bounds on these parameters if the number of extra dimensions d_1 is not too large.

Thus the allowed range of c_1 and c_2 (assuming that they are of the same order of magnitude),

$$10^7 < c_{1,2} \ll 10^{11} \quad (39)$$

is wide enough, which means that any fine tuning is absent.

In the next section we shall see that the inequality $c_{1,2} > 10^7$ is substantially relaxed in the perturbed model.

4 x -dependent perturbations and variations of α

In the previous section we discussed the properties of a homogeneous model which does not contain any spatial variation of α (and any other physical quantity). Let us try to describe variations of α by taking into account spatial perturbations of the scalar field and the metric. We take a metric more general than (23),

$$ds_E^2 = e^{2\gamma} dt^2 - e^{2\lambda} dx^2 - e^{2\eta} (dy^2 + dz^2), \quad (40)$$

where γ, λ, η are functions of x and t . We will not discuss the reasons why the metric perturbation has a distinguished direction, only mentioning a possible weak inhomogeneity at the beginning of the inflationary period and the opportunity of domain walls (mentioned in the introduction) that can be thick on the cosmological scale.

The conditions that the metric (40) only slightly differs from (23) are

$$\gamma = \delta\gamma(x, t), \quad \lambda = \ln a(t) + \delta\lambda(x, t), \quad \eta = \ln a(t) + \delta\eta(x, t),$$

where all “deltas” are assumed to be small. In addition, we replace the effective scalar field $\beta(t)$ with $\beta(t) + \delta\beta(x, t)$. Then the relevant Einstein-scalar equations corresponding to the Lagrangian (26) can be written as follows (preserving only terms linear in the “deltas”):

$$\delta\ddot{\beta} + \frac{3\dot{a}}{a}\delta\dot{\beta} + \dot{\beta}(\delta\dot{\lambda} - \delta\dot{\gamma}) - \frac{1}{a^2}\delta\beta'' + \frac{1}{2K_0}\delta(V_\beta e^{2\gamma}) = 0, \quad (41)$$

$$\frac{\dot{a}}{a}(\delta\dot{\lambda} - \delta\dot{\gamma}) = \delta(V e^{2\gamma}), \quad (42)$$

$$\frac{\dot{a}}{a}\delta\gamma' = K_0\dot{\beta}\delta\beta', \quad (43)$$

where we have chosen the gauge (in other words, the reference frame in perturbed space-time) $\delta\eta \equiv 0$, the dot and the prime stand for $\partial/\partial t$ and $\partial/\partial x$, respectively. We have also denoted $V = V_E = V_0 e^{-2\bar{a}\beta}$ and $V_\beta = dV/d\beta$.

Integration of (43), without loss of generality, leads to

$$\delta\gamma = \frac{K_0}{H}\dot{\beta}\delta\beta, \quad (44)$$

where, as before, $H = \dot{a}/a$. Substituting this $\delta\gamma$ to (41) and taking the difference $\delta\dot{\lambda} - \delta\dot{\gamma}$ from (42), we finally arrive at the following single wave equation for $\delta\beta$:

$$\delta\ddot{\beta} + \frac{3\dot{a}}{a}\delta\dot{\beta} - \frac{1}{a^2}\delta\beta'' + \delta\beta \left[\frac{2\dot{\beta}^2}{H^2} V K_0 + \frac{2\dot{\beta}}{H} V_\beta + \frac{1}{2K_0} V_{\beta\beta} \right] = 0. \quad (45)$$

with an arbitrary constant K_0 and an arbitrary potential $V(\beta)$. In our case, with $V = V_0 e^{-2\bar{a}\beta}$ and K_0 given in (25), we obtain

$$\delta\ddot{\beta} + \frac{3\dot{a}}{a}\delta\dot{\beta} - \frac{1}{a^2}\delta\beta'' + \frac{2V_0 e^{-2\bar{a}\beta}}{p}\delta\beta = 0, \quad (46)$$

while the background quantities $a(t)$ and $\beta(t)$ are determined by the solution (32), (31). It remains to find a solution for $\delta\beta$ which, being added to the background $\beta(t)$, would be able to account for the observed picture of variations of α .

Since the background is x -independent, we can separate the variables and assume

$$\delta\beta = y(t) \sin k(x + x_0)$$

where k has the meaning of a wave number. Then $y(t)$ obeys the equation

$$\ddot{y} + \frac{3p}{t + t_1} \dot{y} + \left[\frac{k^2}{a_1^2(t + t_1)^{2p}} + \frac{6p}{(t + t_1)^2} \right] y = 0. \quad (47)$$

Since the equation (47) has been derived in a certain approximation and describes only a restricted period of time close to the present epoch, it is reasonable to seek the solution in the form of a Taylor series:

$$y(t) = y_0 + y_1(t - t_0) + \frac{1}{2}y_2(t - t_0)^2 + \dots, \quad y_i = \text{const}. \quad (48)$$

Then y_0 and y_1 can be fixed at will as initial conditions, and Eq. (47) leads to expressions of y_2, y_3, \dots in terms of y_0 and y_1 . Even more than that, for a certain neighborhood of $t = t_0$ we can simply suppose $y = y_0 + y_1(t - t_0)$. Actually, this approximation is good enough for $t - t_0 \ll t_* = t_0 - t_1$.

In this approximation we obtain the following expression for variations of α :

$$\frac{\alpha}{\alpha_0} \approx 1 - \frac{d_1}{\bar{d}} \frac{t - t_0}{t_*} - d_1 \sin[k(x + x_0)] [y_0 + y_1(t - t_0)] + O(\epsilon^2), \quad (49)$$

where $O(\epsilon^2)$ means $O((t - t_0)^2/t_*^2)$. Assuming that the observer is located at $x = 0$ and requiring $\alpha/\alpha_0 = 1 + O(\epsilon^2)$ at $x = 0$, we obtain the condition

$$y_1 \sin(kx_0) = -1/(\bar{d}t_*). \quad (50)$$

This explains very small, if any, variations of α on Earth at present and since the Oklo times. To account for the ‘‘Australian dipole’’, it is desirable to have an approximately linear dependence on x on the past light cone of the point $t = t_0, x = 0$.

And indeed, a substitution of (50) into (49) at $t - t_0 = -x$ for $x > 0$ gives

$$\alpha/\alpha_0 \approx 1 - d_1 y_0 \sin(kx_0) + d_1 y_0 kx \cos(kx_0) + O(\epsilon^2) \quad (51)$$

at $x \ll t_*$ and $kx \ll 1$. The same result is obtained if we substitute $t - t_0 = x$ for $x < 0$. (In fact, the measurement errors are rather large, and this x dependence should not necessarily be strictly linear.) We are using the conventional normalization $a_0 = 1$.

Evidently, our model, in addition to the input theoretical parameters like d_1, c_1, c_2 , contains the parameters k, x_0, y_0, y_1 , depending on the initial form of the extra space metric. In the framework of chaotic inflation, these parameters vary in different regions of the visible part of the Universe. Their choice enables us to explain the spatial variations of α in agreement with the observations [2]. Actually, there are only two conditions imposed on them: (50) and the relationship identifying (51) with the expression (1) at $r = x$ and $\cos \psi = 1$, i.e., on the dipole axis. We obtain (in Planck units)

$$d_1 y_0 k \cos(kx_0) \approx -2 \times 10^{-66}. \quad (52)$$

The small constant shift of the α value at $x = 0$ against the background does not change the interpretation of the results obtained.

As to the input parameters c_1 and c_2 , they are no longer constrained by the condition of slow variations of α on Earth: this condition is already provided by the equality (50). We must only take care of the validity of the approximation in which the solution is obtained, that is, $p \gg 1$, which holds fairly well if $c_1 + c_2 \gtrsim 100$. Hence the inequality (39) is replaced with a much weaker one:

$$100 \lesssim c_1 \sim c_2 \ll 10^{11}. \quad (53)$$

Fig. 1 presents the distance dependence of $\delta\alpha/\alpha$ (see (49)) for some values of the parameters.

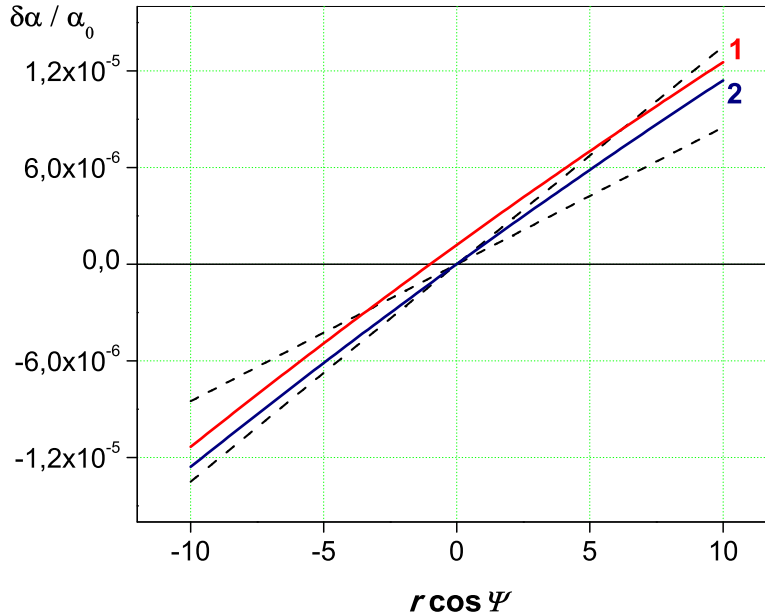


Figure 1: The r dependence of $\delta\alpha/\alpha_0$ (the distance r is measured in billions of light years). The dashed lines correspond to Eq. (1), the solid lines to Eq. (49) at the parameter values $d_1 = 12$, $p = 10^7$, $y_0 = -2 \times 10^{-5}$, $y_1 = -10^{-7}$ (bill. years) $^{-1}$, $k = 0.005$ (billion light years) $^{-1}$. Line 1 corresponds to $x_0 = 1$ billion of light years, line 2 to $x_0 = 0.01$ billion light years.

5 Conclusion

We have studied the possible effect of extra dimensions on large-scale variations of the fine structure constant α in space and time. In the multidimensional paradigm under consideration, the observable values of α and probably other physical quantities, including fundamental constants, depend on the size of the extra factor space. Variations of the dark energy density can be mentioned as an example. Indeed, the space-time variations of the energy density are dominated by those of the potential $V = V_E$ given in (25). The relative variation $\delta V/V = -2\bar{d}\delta\beta$ is of the same order of magnitude as the space-time variations of α according to (22). They are too small to be observed in the near future.

We have focused on the behavior of α because it is the only fundamental constant for which there are more or less reliable data indicating its variations. We are also planning to analyze the behavior of other constants, above all, the gravitational constant and the particle masses.

The agreement with observations is provided in our model by the choice of initial data, which can be interpreted as random values of the extra-dimensional metric at the inflationary stage of the Universe. Thus spatial and temporal variations of α can be manifestations of the multidimensional space-time geometry.

The model described here is very simple, tentative and approximate and works fairly well for times not too far from the present epoch. It does not consistently include other kinds of matter than dark energy (represented by a scalar field of multidimensional origin). However, even such a simple model shows a better agreement with observational data (see Fig. 1) than some other existing models with some *ad hoc* scalar field potentials.

Acknowledgments

The authors wish to thank A. Panov for his interest in our work. The work of S.R. and I.S. was supported in part by the Ministry of Education and Science of the Russian Federation, project 14.A18.21.0789.

References

- [1] J.K. Webb et al., Further evidence for cosmological evolution of the fine structure constant. *Phys. Rev. Lett.* **87**, 091301 (2001).
- [2] J.K. Webb et al., Evidence for spatial variation of the fine structure constant. *Phys. Rev. Lett.* **107**, 191101 (2011); ArXiv: 1008.3907.
- [3] J.C. Berengut and V.V. Flambaum, Astronomical and laboratory searches for space-time variation of fundamental constants. *J. Phys. Conf. Ser.* **264**, 012010 (2011); Arxiv: 1009.3693.
- [4] T. Rosenband et al., Observation of the $^1S_0 \rightarrow ^3P_0$ clock transition in $^{27}\text{Al}^+$. *Phys. Rev. Lett.* **98**, 220801 (2007).
- [5] A.I. Shlyakhter, Direct test of the constancy of fundamental nuclear constants. *Nature* **260**, 340 (1976).
- [6] Y. Fujii et al. The nuclear interaction at Oklo 2 billion years ago. Ed.: *Nucl. Phys.* **B573**. 377 (2000).
- [7] T. Chiba. The constancy of the constants of Nature: Updates. *Prog. Theor. Phys.* **126**, 993–1019 (2011); ArXiv: 1111.0092.
- [8] P.J.E. Peebles and R.H. Dicke. Cosmology and the radioactive decay ages of terrestrial rocks and meteorites, *Phys. Rev.* **128**, 2006 (1962).
- [9] K.P. Staniukovich, *The Gravitational Field and Elementary Particles*, Nauka, Moscow, 1965 (in Russian).
- [10] J.D. Bekenstein, Fine-structure constant: Is it really a constant? *Phys. Rev. D* **25**, 1527 (1982).
- [11] K.A. Bronnikov, V.D. Ivashchuk, V.N. Melnikov, Time variation of the gravitational constant in multidimensional cosmology. *Nuovo Cim. B* **102**, 209-216 (1988).
- [12] Yu.S. Vladimirov and A.A. Peraza, Fundamental constants in the 6-dimensional evolving Universe, *Vestnik Mosk. Univ. Fiz. Astron.* **34** (5), 16–21 (1993).
- [13] H.B. Sandvik, J.D. Barrow, J. Magueijo, a simple cosmology with a varying fine structure constant. *Phys. Rev. Lett.* **88**, 031302 (2002); astro-ph/0107512.
- [14] T. Chiba and M. Yamaguchi, Runaway domain wall and space-time varying α . *JCAP* **1103**, 044 (2011); ArXiv: 1102.0105.
- [15] K.A. Olive, M. Peloso, J.-P. Uzan, The wall of fundamental constants. *Phys. Rev. D* **83**, 043509 (2011); ArXiv: 1011.1504.
- [16] K. Bamba, S. Nojiri, S.D. Odintsov, Domain wall solution in $F(R)$ gravity and variation of the fine structure constant, *Phys. Rev. D* **85**, 044012 (2012); ArXiv: 1107.2538.
- [17] K.A. Olive, M. Peloso, A.J. Peterson, Where are the walls? *Phys. Rev. D* **86**, 043501 (2012); ArXiv: 1204.4391.

- [18] J.D. Barrow and S.Z.W. Lip, A generalized theory of varying alpha. *Phys. Rev. D* **85**, 023514 (2012); ArXiv: 1110.3120.
- [19] A. Mariano and L. Perivolaropoulos, Is there correlation between fine structure and dark energy cosmic dipoles? *Phys. Rev. D* **86**, 083517 (2012); ArXiv: 1206.4055.
- [20] A. Mariano and L. Perivolaropoulos, CMB maximum temperature asymmetry axis: alignment with other cosmic asymmetries. ArXiv: 1211.5915.
- [21] V.N. Melnikov, Multidimensional classical and quantum cosmology and gravitation. Exact solutions and variations of constants. In: *Cosmology and Gravitation*, ed. M. Novello, Editions Frontieres, Singapore, 1994, p. 147.
- [22] V.N. Melnikov, Gravity and cosmology as key problems of the millennium. In: *Albert Einstein Century Int. Conf.*, eds. J.-M. Alimi and A. Fuzfa, AIP Conf. Proc. 861, 2006, p. 109.
- [23] K.A. Bronnikov and S.G. Rubin, Self-stabilization of extra dimensions. *Phys. Rev. D* **73**, 124019 (2006).
- [24] K.A. Bronnikov and S.G. Rubin, *Black Holes, Cosmology and Extra Dimensions* (World Scientific, Singapore, 2012).
- [25] K.A. Bronnikov, S.G. Rubin, and I.V. Svadkovsky, Multidimensional world, inflation and modern acceleration *Phys. Rev. D* **81**, 084010 (2010).
- [26] S.V. Bolokhov, K.A. Bronnikov, and S.G. Rubin, Extra dimensions as a source of the electroweak model, *Phys. Rev. D* **84**, 044015 (2011).
- [27] S.G. Rubin and A.S. Zinger, The Universe formation by a space reduction cascade with random initial parameters, *Gen. Rel. Grav.* **44**, 2283 (2012); ArXiv: 1101.1274.
- [28] A.A. Kirillov, A.A. Korotkevich, and S.G. Rubin, Emergence of symmetries. *Phys. Lett. B* **718**, 237-240 (2012); ArXiv: 1205.1108.
- [29] K.A. Bronnikov and V.N. Melnikov, On Observational Predictions from Multidimensional Gravity, *Gen. Rel. Grav.* **33**, 1549 (2001).
- [30] K.A. Bronnikov and V.N. Melnikov, Conformal frames and D-dimensional gravity, in: *Proc. 18th Course of the School on Cosmology and Gravitation: The Gravitational Constant. Generalized Gravitational Theories and Experiments* (30 April–10 May 2003, Erice), Ed. G.T. Gillies, V.N. Melnikov and V. de Sabbata, (Kluwer, Dordrecht/Boston/London, 2004) pp. 39–64; gr-qc/0310112.