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Maxwell's equations in four-dimensional notation and the classical magnetic monopoles

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ABSTRACT

It is shown that, on account of the commutability of the order of derivation of the electromagnetic 4-potential, the first pair of Maxwell equations, when written in a four dimensional notation, is incompatible with the existence of the classical magnetic monopole. It is hypothesized that the appearance in the past of magnetic monopoles could be accepted only if, in exceptional occurrences, it came to be locally lacking the space homogeneity and isotropy on which the commutability of space derivatives is based. The situation is more complex for massive cosmological monopoles.

The first pair of Maxwell Equations in the vectorial form is written:

$$\operatorname{div}\mathbf{B} = 0$$

$$\operatorname{rot}\mathbf{E} = -\frac{1}{c}\frac{\partial}{\partial t}\mathbf{B}$$

with $\mathbf{E} = -grad\,\Phi - \frac{1}{c}\frac{\partial \mathbf{A}}{\partial t}, \quad \mathbf{B} = \operatorname{rot}\mathbf{A}$

These equations are contained in the definition of the electromagnetic field tensor:

$F^{\mu\nu} = \frac{\partial A^{\nu}}{\partial x^{\mu}} - \frac{\partial A^{\mu}}{\partial x^{\nu}}, \; A^{\mu} = (\Phi, \mathbf{A})$.

To obtain them in four dimensional notation we have to construct a tensor of third rank which is antisymmetric in all three indices:

$$T^{\mu\nu\rho} \; = \; F^{\mu\nu,\rho} + F^{\nu\rho,\mu} + F^{\rho\mu,\nu}, \quad \rho \neq \mu \neq \nu = (0,1,2,3) \tag{1}$$

It has numerical value zero because adding the mixed derivatives of the four-potential A^{μ} the six terms, two by two, give zero on account of the commutability of the order of derivation.

$T^{\mu\nu\rho}$ has 64 components but only $24 = D_{4,3}$ are non vanishing; moreover as any set of three digits may be disposed in 6 different ways, its free components are only 4.

So it is convenient to construct a four-vector which is dual to our antisymmetric four tensor $T^{\mu\nu\rho}$ having only 4 components. That is:

$$C_{\sigma} = \frac{1}{6}e_{\sigma\rho\mu\nu}T^{\mu\nu\rho} = \frac{1}{6}\left(\frac{\partial}{\partial x^{\rho}}e_{\sigma\rho\mu\nu}\right)F^{\mu\nu} \tag{2}$$

The 4 components of C_{σ} too, have numerical value of zero.

Calling $G^{*}{}_{\sigma\rho}$ the dual tensor of the electromagnetic field tensor, that is:

$$G^{*}{}_{\sigma\rho} = \frac{1}{2}e_{\sigma\rho\mu\nu}F^{\mu\nu} \tag{3}$$

One obtains $C_{\sigma} = \frac{1}{3}G^{*}{}_{\sigma\rho}{}^{,\rho} = 0$; so the first pair of Maxwell equations is given by the vanishing of the 4-divergence of the tensor dual of the electromagnetic field tensor, that is

$$G^{*}{}_{\sigma\rho}{}^{,\rho} = 0 \tag{4}$$

As above specified this 4-divergence is zero on account of the commutability of the mixed derivatives.

The second pair of the Maxwell equations

$$F^{\mu\nu}{}_{,\nu} = -\frac{4\pi}{c}j^{\mu} \tag{5}$$

is obtained, following the procedure given in the text of Landau and Lifshitz § 30 [1], that is applying the principle of least action to a Lagrangian action made of a term containing the motion of the charges, a second term depending on the interaction between the charges and the field, represented by the four-potential A^{μ} and a

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third term containing an invariant expression, quadratic in the fields: $F_{\mu\nu}\,F^{\mu\nu}$.

Remembering that the motion of the charges and the currents are given and they cannot vary, we arrive to Eq. (5) having considered only the electric charges and their motions without magnetic charges, that is without magnetic monopoles.

If we wish to introduce them, we may introduce the magnetic 4-current

$$I^{\mu} = (\mathrm{mc},\mathbf{i}) \tag{6}$$

where m is the density of magnetic charge and **i** the density of magnetic current, all in analogy to the electric current density j^{μ} .

Adding the magnetic 4-current (6) in the term containing the interaction between the field and electric charges, and adding also the pseudoscalar

$$G^{*\mu\nu}F_{\mu\nu} \tag{7}$$

to the scalar invariant of the electromagnetic field and proceeding as in the case where one has only electric charges, the application of the least action principle to the sum of the two variations gives:

$$F_{\mu\nu,\nu} + G^{*}{}_{\mu\nu,\nu} = -\frac{4\pi}{c}(j_{\mu} + I_{\mu}) \tag{8}$$

from which we see that the 4-divergence of the dual tensor of the electromagnetic field tensor is given by the magnetic 4-current.

The vectorial form of equation (8) is:

$$\operatorname{div}\mathbf{B} = 4\pi\ \mathrm{m}$$

$$\operatorname{rot}\mathbf{E} = -\frac{1}{c}\frac{\partial}{\partial t}\mathbf{B} - 4\frac{\pi}{c}\mathbf{i}$$

$$\operatorname{div}\mathbf{E} = 4\pi\rho$$

$$\operatorname{rot}\mathbf{B} = \frac{1}{c}\frac{\partial}{\partial t}\mathbf{E} + 4\frac{\pi}{c}\mathbf{j}$$

$$G^{*\mu\nu} = \begin{Bmatrix} \mathbf{0} & \mathbf{B}^1 & \mathbf{B}^2 & \mathbf{B}^3 \\ -\mathbf{B}^1 & \mathbf{0} & \mathbf{E}^3 & -\mathbf{E}^2 \\ -\mathbf{B}^2 & -\mathbf{E}^3 & \mathbf{0} & \mathbf{E}^1 \\ -\mathbf{B}^3 & \mathbf{E}^2 & -\mathbf{E}^1 & \mathbf{0} \end{Bmatrix}$$

However because of (4), it results $I^{\mu} = 0$; so monopoles and magnetic currents are not allowed because they contradict the definition of the electromagnetic tensor from whom (4) is derived.

However Dirac [2] in 1931 stated that, in order to allow the quantization of the electric charge, even a single magnetic monopole had to exist in the Universe.

But so far we do not have certain evidences of the existence of monopoles.

From the measurements of the MACRO collaboration [3,4], we know that the flux of magnetic monopoles from outer space is less than 1.4×10^{-16} $\mathrm{cm^{-2}s^{-1}sr^{-1}}$.

In 1962 Cabibbo and Ferrari [5] noted the incompatibility of Eqs. (4) and (5) ((3) and (4) in their paper) and tried to solve it introducing a new potential for the magnetic charges with attached strings. Eventually, making use of the Mandelstam electrodynamics without potentials [6], they built not-commuting functions of the potentials and from them they obtained the Dirac quantization condition. However, so doing, the field is not determined in the strings.

In 1965 Goldhaber [7] showed that the Dirac quantization condition may be found considering the collision of an electron with the field, extended to all space, of a magnetic monopole. Assuming the variation of the electron's angular momentum be a multiple of $\hbar$, he obtained the Dirac quantization condition in a non relativistic way.

In 1966 Schwinger [8] proved the Dirac condition in a relativistic way.

About the Dirac magnetic monopoles we may also remind the opinion of the mathematician Ross [9] who hypothesized they are not observable because they do not satisfy the Orientation Entanglement Relation which is satisfied by all existing particles.

Another point of view about the magnetic monopole is expressed by the French scientist Lochak [10]. According to him the magnetic monopole is not heavy but light, it is not a boson but a fermion, it has not strong, but weak electromagnetic interaction, it is not rare in Nature, but abundant.

It is a magnetically excited neutrino which could be produced by electric discharges.

In addition to the mathematical derivation of his theory Lochak quotes astonishing experiments, made with intense electric discharges, at the Kurchatov Institute of Moscow by Urutskoev [11] as others experiments made at Dubna by the group headed by Vladimir Kuznetsov [12]. In these experiments may have been observed new elements and the variation of the isotopic structure of the element that composed one of the electrodes. They also show tracks of particles lacking electric charges which are assumed to be magnetic monopoles.

Therefore, if in the past magnetic monopoles were observed, that may indicate that something exceptional took place.

Actually the physicist and philosopher Immanuel Kant [13] in his studies of the 1755 earthquake noted "that cannot pass over in silence that on the frightful day of All Saints the magnets in Augsburg threw off their loads and the magnetic needles where thrown into disorder. Boyle already stated that something similar once happened in Naples after an earthquake".

It is also reported [14] the case of an optician in Edo (now Tokyo) who had in his shop's show window a very large magnet and just two hours before the severe Novenmber 11, 1855 earthquake it lost all its attractive strength and dropped the attached objects; after the quake the magnet regained its strength. Other inexplicable electromagnetic phenomena, happened to a "feinmechaniker" (precision mechanic) during the 1976 Friuli earthquake, are reported by professor Helmut Tributsch, a Friuli native-born, in a paper published ny the N.Y.A.S. [15].

An explanation of these mysterious sudden demagnetizations could be a sudden arrival of magnetic monopoles which settled on the objects with the consequence of destroying the alignment of the Weiss domains during the time the monopoles settled on the poles. The alternative hypothesis of an heat shock above the Curie temperature could not explain the recovery process.

Such mysterious events could be explainable if, locally, in exceptional occurrences, failed the space homogeneity and isotropy on which the commutability of space derivation is based; so it could be:

$$\frac{\partial}{\partial x_a}\frac{\partial}{\partial x_b} \neq \frac{\partial}{\partial x_b}\frac{\partial}{\partial x_a}.$$

In such a case Eq. (4) which comes from the commutability of the order of derivations of the electromagnetic 4-potential would cease to be valid and (8) could be accepted.

These considerations are valid for classical monopoles. The situation is more complex for cosmological supermassive magnetic monopoles. In Grand Unified Theories (GUT) the electroweak and strong interactions at the cosmic time of 10^{-34} sec after the Big Bang could have been created monopoles with a mass $> 10^{+16}$ GeV [16–18]. Some authors speak of intermediate Mass Monopoles, produced later, with masses of about 10^{+10} GeV [19]. Some of them could be still around us as relics of the Big Bang.

However I do not believe that GUT and Standard Model are complete theories. The results of the experiments by Urutskoev

[11] and Kuznetsov [12] are not explainable in the context of the standard model. Therefore, in the future, one may expect the development of more complete theories.

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