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# Local Conformal Symmetry: the Missing Symmetry Component for Space and Time

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## Abstract

Local conformal symmetry is usually considered to be an approximate symmetry of nature, which is explicitly and badly broken. Arguments are brought forward here why it has to be turned into an exact symmetry that is spontaneously broken. As in the B.E.H. mechanism in Yang-Mills theories, we then will have a mechanism for disclosing the small-distance structure of the gravitational force. The symmetry could be as fundamental as Lorentz invariance, and guide us towards a complete understanding of physics at the ultra short distance scale.

All important physical systems have a built-in scale in them, and for that reason, conformal transformations may appear to be useless as a symmetry group; at best, this symmetry is badly broken. The topic of this short note, however, is that one could try to reason differently.

Why have symmetries always been so instrumental for understanding nature? The answer is this: if we know the laws of nature in one particular domain, the laws in other domains can be obtained by applying a symmetry transformation. For instance, translation symmetry tells us that the laws are the same ones everywhere in space and in time. Rotational symmetry tell us that they are the same in all directions. Very important are also the Lorentz transformations: replacing the Galileï transformations, they tell us how a moving particle behaves if we know what that particle does when at rest. In theories without Lorentz invariance, moving particles are altogether different from stationary ones. Discrete symmetries such as isospin tell us how one particle (such as a neutron) behaves if we understand the behaviour of another (i.c., the proton).

Why then is physics still so difficult? Well, we still do not know what happens at higher energies even if we do understand the laws at low energies, or more to the point: small time and distance scales seem not to be related to large time and distance scales. Now, we argue, this is because we fail to understand the symmetry of the scale transformations. This symmetry, of which the local form will be local conformal symmetry, *if exact*, should fulfil our needs. Since the world *appears* not to be scale invariant, this symmetry must be spontaneously broken. This means that the symmetry must be associated with further field transformations, leaving the vacuum not invariant. It is this implementation of the symmetry that we should attempt to construct from the evidence we have.

In conclusion of the above, there must be a component in the space-time symmetry group (the Poincaré group) that both Lorentz and Einstein missed. Lorentz derived the invariance group named after him as a property of electro-magnetism alone. Now *that* system, described by Maxwell's equations, does have conformal symmetry, generated by the transformations $x^\mu \rightarrow l\,x^\mu\,/\,x^2$, with arbitrary constant $l$, to be added to the other generators of the Poincaré group. What this means is that, if we only use light rays, and nothing else, to measure things, then absolute sizes and time spans cannot be observed, only relative ones.[1]

Suppose Einstein had used just that piece of information to build his special theory of relativity. Can we compare observers with scaled observers? Should we not have ended up with the conformal extension of the Poincaré group? Let us have a look at the elementary principles of relativity. Now however, we decide *only* to use light rays for measuring things, and suppose we would wish to set up a theory for gravity. How would that work?

To be sure, we promise to put the scales of things back in the world of physics in the end, by saying that conformal symmetry will be *spontaneously* broken, but we haven't reached that point yet; we first wish to describe the world in the symmetric picture. This is a picture of the world that may be well hidden from our eyes today, in the same way that $SU(2)$ symmetry in the weak interactions has long been hidden from us, before the B.E.H. mechanism was understood.

Back to conformal gravity[2], Einstein could not have started with the elementary line

[1] The reader might be worried about invoking conformal symmetry: did that not transform straight lines into circles, and shouldn't we be able to distinguish straight lines from circles using light? The answer is yes, but here we use *four* dimensional conformal symmetry, which does transform light cones into light cones.

element $\mathrm{d}s$ , but he would have to consider only the light-like geodesics, described by

$$\mathrm{d}s^2 = g_{\mu\nu}\mathrm{d}x^\mu\mathrm{d}x^\nu = 0 \ . \quad \text{lightelement} \tag{1}$$

This means that, at every space-time point $x$ separately, all relative values of the metric tensor components $g_{\mu\nu}(x)$ could be used, but not the common factor, so we would have to consider a 'pseudo' tensor $g_{\mu\nu}$ that is defined apart from this factor:

$$g_{\mu\nu}(\vec x, t) \ \cong \ \Omega^2(\vec x, t) g_{\mu\nu}(\vec x, t) \ , \quad \text{confmetric} \tag{2}$$

an identity equation to be regarded as a one-dimensional local gauge transformation.[2]

If we look at just two of the four coordinates $x^\mu$ , light rays are invariant under

$$\begin{aligned} x + c\,t \quad &\rightarrow \quad \gamma_1(x + c\,t) \ , \\ x - c\,t \quad &\rightarrow \quad \gamma_2(x - c\,t) \ . \end{aligned} \quad \text{confLorentz} \tag{3}$$

Normally, one puts $\gamma_1\gamma_2 = 1$ , but in the case of conformal symmetry, one simply drops this constraint. The importance of this could be that it enables us to consider the small distance limit in the coordinates $x^+ \ = \ x + c\,t$ and $x^- \ = \ x - c\,t$ separately, and this possibility might simplify our attempts to understand what will happen at very tiny distance scales, as was explained.

Now consider the scalar component $R$ of the Ricci curvature. If we subject the metric tensor to the transformation (2), it transforms as

$$R \ \cong \ \Omega^{-2} R - 6\Omega^{-3} D^2 \Omega \ , \quad \text{Rtransf} \tag{4}$$

where $D_\mu$ is the covariant derivative, and this means that we can always choose $\Omega$ in such a way that $R$ is set equal to zero. Ergo: the scalar Ricci curvature is an ill-defined concept when conformal symmetry is employed. According to the Einstein equations, the trace of the energy-momentum tensor is proportional to the scalar Ricci curvature. This means that the trace of the energy-momentum tensor is also ill-defined. To be precise: *by using light rays alone, one cannot detect the scalar component of the energy-momentum tensor.* It is ill-defined. But if you put it equal to zero, then $\Omega$ is fixed up to boundary terms, and then the 9 remaining components of the Ricci curvature and the energy-momentum tensor are fixed (up to boundary effects).

Indeed, in a conformally invariant theory, the notions of energy, momentum, as well as other matter properties will have to be carefully redefined since the conventional definitions become ambiguous. As for space-time curvature, it is well-known that the Weyl curvature is still well-defined. The Weyl curvature has only 10 independent components where the Riemann curvature has 20.

How *does* one measure the scalar component of $T_{\mu\nu}$ in practice? We cannot use light rays to measure its effect on gravity. Normally, one would use light to measure the matter density, and use the equation of state to determine the pressure, but what if we do not

[2]Sometimes this is referred to as the Weyl group, but Weyl introduced the associated vector field, which we avoid here.

know the equation of state, because we do not know the chemical composition of the material? Suppose that a local conformal transformation is possible that affects matter in a complicated way (as we anticipated already in the 3rd paragraph above), so that its effect on gravity takes the form (4)? Of course, for atoms and molecules, this will not be easy, but I am thinking of a description of matter somewhere near the Planck scale, where we are forced to reconsider almost everything anyway. Then, the possibility of such transformations will affect physics in a drastic way, and this is what we should be interested in.

In empty space-time, the Riemann scalar can also be transformed away, and this means that the cosmological constant has no meaning anymore. We do not claim to have come closer to solving any of the usual problems humanity has with the cosmological constant, but these observations might cast a different light on these problems. In general, it sometimes seems that it is the scalar sector of gravity that is the most mysterious to us, and looking at it in a different way might help.

Spontaneously breaking conformal symmetry is easy at first sight. It happens automatically in the Einstein-Hilbert action. One multiplies the metric tensor $g_{\mu\nu}$ with the square of a scalar dilaton field $\phi(\vec{x},t)$ , which takes over the role of our field $\Omega$ . A curious feature of gravity is that the functional integral over the $\phi$ field has to be shifted to a complex contour, such that the vacuum value of the field becomes (see ref [1])

$$\langle\phi\rangle=\pm i\sqrt{\frac{3}{4\pi G}}\ , \qquad \text{phivac} \tag{5}$$

where the sign can be chosen freely. Gauge-fixing the field to have exactly the value (5) reproduces standard Einstein-Hilbert gravity, and our description of the physical world will be as usual.

The power of our considerations comes if we decide te leave our $\phi$ field alone, and use *something else* to fix the conformal gauge, a consideration that was absolutely crucial in understanding how the B.E.H. mechanism turns the electroweak theory into a manageable, that is, renormalizable, theory. Look at (5) as the "unitarity gauge". We get a "renormalizable gauge" if we decide to choose our conformal factor $\Omega(\vec{x},t)$ in such a way that the *amount of activity* in a given space-time volume element is fixed or at least bounded. Only a limited number of interactions take place in a volume element $\Omega^4\sqrt{-g}$ . This may well render all integrals in gravity convergent, since 4-momentum values much greater than one in the Planck scale may never be needed. Thus, local conformal symmetry may assist us to control the small distance structure of our theories.

As stated, employing a symmetry such as exact local conformal symmetry will be absolutely essential. Without this symmetry, one can always imagine collisions of particles at higher and higher energies, where continuously smaller distance scales will play a role. The Lorentz group is not good enough to link these scales to physics that is already known, while conformal symmetry can accomplish this.

There is another bonus connected to the 'activity gauge' proposed here. As we argued a few years ago, modifying the conformal factor can change the space-time of a black hole, while it forms and disintegrates by Hawking radiation, into a topologically trivial one.

Black holes in such a description may turn into quite ordinary soliton field configurations that may end up not being more singular than magnetic monopoles in gauge theories; quite probably no longer exhibiting information loss problems or firewall problems. Black holes still exist, but their description becomes singularity free and horizons are moved out of sight.

Black holes still being black holes raises the question what happens to baryon number non conservation. Fact is, that demanding exact local conformal symmetry requires careful cancellations of anomalies, which may well be impossible in the presence of local or global $U(1)$ gauge symmetries. Indeed, actually realising our scenario in such a way that all anomalies cancel out precisely, leads to interesting and important new constraints. In passing we emphasise that also in the Standard Model itself, anomalies are known to cause the violation of strict baryon number conservation.

Our 'activity gauge' sets $\Omega(\vec{x},t)$ to be big if 'things are going on at $(\vec{x},t)$' and small otherwise. What this means in practice will have to be sorted out; this note was intended to show a path that has to be further explored.

The condition that all anomalies cancel out gives at least as many numerical constraints as there are physical interaction parameters in the quantum field theory model of the elementary particles – including masses and even the cosmological constant. A preliminary investigation gave discouraging results if we try it upon the Standard Model itself. This led to the conclusion that this model will still require considerable extensions, presumably at very high energy scales. If successful, our approach could lead to new theoretical derivations of improved versions of the Standard Model.

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