Fine structure constant variation or space-time anisotropy?

Zhe Chang,* Sai Wang,† and Xin Li‡

Institute of High Energy Physics

Theoretical Physics Center for Science Facilities

Chinese Academy of Sciences,

100049 Beijing, China

Recent observations on quasar absorption spectra supply evidences for variation of fine structure constant α. In this paper, we propose another interpretation of the observational data on quasar absorption spectra: a scenario with space-time inhomogeneity and anisotropy. Maybe the space-time is characterized by Finsler geometry instead of Riemann one. Finsler geometry admits less symmetries than Riemann geometry does. We investigate the Finslerian geodesic equations in Randers space-time (a special Finsler space-time). It is found that the cosmological redshift in this space-time is deviated from the one in general relativity. The modification term to redshift could be generally revealed as a monopole plus dipole function about space-time locations and directions. We suggest that this modification corresponds to the observed spatial monopole and Australian dipole in quasar absorption spectra.

PACS numbers: 98.80.Es, 98.80.Jk, 02.40.-k

I. INTRODUCTION

It has been widely accepted that Standard cosmological model (ΛCDM model) [1] is the paradigm of the modern cosmology. This model makes several observable predictions which have withstood large quantities of tests by cosmological observations during the last two decades. Until now, almost all observations, such as the cosmic microwave background (CMB) anisotropy [2], the cosmological accelerating expansion [3, 4] and the large scale structure(LSS) [5], agree with the predictions of ΛCDM model. Thus, this model is indeed great successful. Despite of these, it still faces several cosmological large scale anomalies (see review in Ref. [6]), such as large scale velocity flows [7], alignment of low multipoles in the CMB [8–10], large scale alignment in the quasar optical polarization [11] and the preferred axis of Hubble diagram [6, 12]. These anomalies imply that there may exist inhomogeneous and anisotropic at large scale which are led by certain common preferred direction in space-time [6]. All of these are beyond the ΛCDM model and may lead to new physics.

It is well known that there are in principle no any variation of the fundamental physical constants, such as fine structure constant $\alpha = e^2/\hbar c$ (e is the unit electric charge, $\hbar$ is the reducible Planck constant and c is the speed of light in vacuum), in ΛCDM model which is based on the cosmological principle [13] and Einstein's general relativity [14]. However, there have been several propositions for the existence of possible variation of α currently. Since Dirac [15] postulated in 1937 the universal gravitational constant G was not a constant possibly, great interests have been stimulated in studies on the variation of fundamental physical constants including fine structure constant α. Kinds of experiments and observations [16, 17] have been employed in searching for possible α variation. Meanwhile, quantities of theories or models (see review and details in Ref.[17]) have been proposed and studied, which suggest various possibilities for the α variation. There are several reasons for possible variation of α (see review and details in Ref.[17, 18]), such as the existence of extra dimensions, quantum gravity, nonuniqueness of vacuum state, spontaneous symmetry breaking in very early universe and *et al.*. Thus, it is interesting and meaningful to study possible α variation.

To search for the variation of α, there are three classes of experiments: atomic methods, nuclear methods, and gravitational methods (for review and details see, for example, Ref.[16]). Almost all publications presented just constraints or upper limitations on the variation of α [16, 19]. However, recent astrophysical observations provided some evidences [20–22] for the α variation. The many-multiplet (MM) method [23–25] was employed to analyze the data of quasar absorption spectra [23, 26]. The MM method is mainly based on comparing different transitions in different mulitiplets or atoms from cosmic and laboratorial spectra respectively [19, 21]. In addition, the quasar absorption spectra encode information about the atomic energy levels at the positions and time of emissions [16], so that analysis of them will reveal information about α around distant quasars. One has to determine the α-dependence of the atomic spectra in order to observe the variation of α around the distant quasars. In the condition that α has a small shift ($\delta\alpha/\alpha = \frac{\alpha-\alpha_0}{\alpha_0} \ll 1$), the energy levels within one fine-structure multiplet could be described as [16, 20]

$$\omega = \omega_0 + qx \equiv \omega_0 + q \cdot \frac{2\delta\alpha}{\alpha} \ . \tag{1}$$

Throughout of the paper, ω and α also represent respectively for the atomic energy levels and fine structure constant at the positions and time of emissions from quasars with redshift $z = \lambda_{obs}/\lambda_{lab} - 1$.

Recently, Webb *et al.* [20] analyzed the data of quasar absorption spectra from Keck-Hires Telescope (Keck) with the MM method. They claimed that α would be smaller in large scales

$$\frac{\delta\alpha}{\alpha}(z) = (-0.543 \pm 0.116) \times 10^{-5} \ . \tag{2}$$

This is a spatial monopole function about redshift z. Most recently, they [21, 22] claimed that evidence of a spatial dipole, named as "Australian Dipole", of α-variation was found also by quasar absorption spectra from Keck, Very Large Telescope (VLT) and both combined. They found that α would take a spatial dipole variation with an increase in the northern hemisphere and a decrease in the southern hemisphere. The spatial dipole of α variation could be revealed as

$$\frac{\delta\alpha}{\alpha}(\cos\Theta, z) = (1.10 \pm 0.25) \times 10^{-6} \ , \tag{3}$$

where Θ is the angle between the quasar sightline and the best-fit dipole position [22]. The spatial monopole and dipole of α variation were found to take the same order of magnitude $\sim 10^{-6}$.

It is well known that Glashow and Cohen proposed very special relativity (VSR) [27] in which the Lorentz group is replaced by its subgroup. There exists certain preferred direction [28–31] in VSR, which leads to Lorentz violation. It has also been clear that the preferred direction could give rise to certain anisotropy of the speed of light in the vacuum [32, 33]. Thus the fine structure constant α would vary with directions and show certain anisotropy in space, since it is inverse ratio in the speed of light. In addition, the line element of VSR has been proved to be a Finslerian line element [28–31]. So that Finsler space-time could bring about new insights on the variation of α. Finsler space-time admits less Killing vectors (equivalently less symmetries) than Riemann space-time does [34]. This means there exist certain preferred directions which lead to inhomogeneities and anisotropy in Finsler space-time. Of course, such inhomogeneities and anisotropy would lead to the variation of the speed of light which make the α vary with locations and directions in space. These are the effects of Lorentz violation which make the Finsler space-time different from the ΛCDM model. The attribute of monopole of α variation implies that the universe is inhomogeneous at large scale and the existence of spatial dipole implies that there may exist certain anisotropy at large scale. As mentioned above, this kind of anisotropy may be also the reason why other large scale cosmological anomalies emerge [6]. The inhomogeneities and anisotropy of this kind signal certain non-trivial cosmic topology [9]. Then there may exist a special kind of space-time structure at large scale other than the usual Friedmann-Robertson-Walker (FRW) structure in ΛCDM model. Maybe Finsler space-time a reasonable candidate for new physics correspond to the α variation claimed. These are new results in the work and may potentially lead to new physics.

In Einstein's general relativity, gravity is connected with curvature in Riemann geometry. In the same way, one could discuss gravity based on Finsler geometry [35, 36]. Gravity in Finsler space-time has been studied for a long time [37–40]. An incomplete *list* of works in this field includes: a specified Finsler structure makes the modified Newton's gravity [41] equivalent to

Milgrom's Modified Newtonian Dynamics (MOND) [42]; a Finlerian gravity model accounts to the accelerated expanding universe without invoking dark energy hypothesis [43]; Randers space [44] accounts for the anomalous acceleration [45] in solar system observed by Pioneer 10 and 11 spacecrafts; Finsler space-time leads modification to the gravitational deflection of light [46] corresponding to observations on Bullet Cluster [47]; Finslerian kinematics is in good agreement with secular trend of the Astronomical Unit and secular eccentricity variation of the Moon's orbit [48]; the Finslerian extension of Schwarzchild metric asymptotically approaches Bogoslovsky locally anisotropic space-time instead of Minkowski space-time [49].

In this paper, we suggest an inhomogeneous and anisotropic space-time could describe well the astronomical observations on quasar absorption spectra. The rest of the paper is arranged as follows. In the section II, we discuss the space-time inhomogeneity and anisotropy in the framework of Finsler geometry. A uniform formula for redshift is presented in the section III. The observed data showed monopole and the Australian Dipole is fitted in the new scenario. We give conclusions and remarks in the section IV.

II. SPACE-TIME INHOMOGENEITY AND ANISOTROPY

Finsler geometry [35, 36] has its origination from integrals of the form

$$\int_a^b F\left(x, y\right) d\tau \ , \tag{4}$$

where x and $y \equiv dx/d\tau$ stand respectively for position and velocity under natural coordinate bases. The integrand F is called Finsler structure. Unlike Riemann structure being defined on the manifold M, Finsler structure is defined on the slit tangent bundle $TM \setminus 0$. A Finsler structure of M is a positive-definite function with the property

$$F(x, \lambda y) = \lambda F(x, y) \tag{5}$$

for all $\lambda > 0$. A manifold M associated with a Finsler structure F on $TM \setminus 0$ would be called a Finsler manifold. The Finsler metric tensor is a Hessian matrix, the coefficients of which are defined as [35]

$$g_{\mu\nu} \equiv \frac{\partial}{\partial y^\mu} \frac{\partial}{\partial y^\nu} \left(\frac{1}{2} F^2 \right) . \tag{6}$$

It is also called fundamental tensor and is used to raising and lowering the indices together with its inverse $g^{\mu\nu}$.

The parallel transport has been studied in the framework of Cartan connection [50–52]. The notation of parallel transport in Finsler manifold means that the length $F\left(\frac{dx}{d\tau}\right)$ is constant. The geodesic equation for Finsler manifold is given as [35]

$$\frac{d^2x^\mu}{d\tau^2}+G^\mu=0\ , \tag{7}$$

where

$$G^\mu=\frac{1}{2}g^{\mu\nu}\left(\frac{\partial^2F^2}{\partial x^\lambda\partial y^\nu}y^\lambda-\frac{\partial F^2}{\partial x^\nu}\right) \tag{8}$$

is called geodesic spray coefficient. Obviously, if F is Riemannian metric, then

$$G^\mu=\tilde{\gamma}^\mu_{\nu\lambda}y^\nu y^\lambda\ , \tag{9}$$

where $\tilde{\gamma}^\mu_{\nu\lambda}$ is the Riemannian Christoffel symbol. Since the geodesic equation (7) is directly derived from the integral length of a curve σ

$$L(\sigma)=\int F\left(\frac{dx}{d\tau}\right)d\tau\ , \tag{10}$$

the inner product $\left(\sqrt{g_{\mu\nu}\frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau}}=F\left(\frac{dx}{d\tau}\right)\right)$ of two parallel transported vectors is preserved.

The Randers space is a special kind of Finsler space with Finsler structure F on the slit tangent bundle $TM\backslash 0$ of a manifold M as

$$F(x,y)\equiv\alpha(x,y)+\beta(x,y)\ , \tag{11}$$

where

$$\alpha(x,y)\ \equiv\ \sqrt{\tilde{a}_{\mu\nu}(x)y^\mu y^\nu}\ , \tag{12}$$

$$\beta(x,y)\ \equiv\ \tilde{b}_\mu(x)y^\mu, \tag{13}$$

and $\tilde{a}_{ij}$ is Riemannian metric.

The geodesic spray coefficient G^μ in Randers-Finsler space-time reads [35]

$$G^\mu\ =\ (\tilde{\gamma}^\mu_{\nu\lambda}+l^\mu\tilde{b}_{\nu|\lambda})y^\nu y^\lambda+(\tilde{a}^{\mu\nu}-l^\mu\tilde{b}^\nu)(\tilde{b}_{\nu|\lambda}-\tilde{b}_{\lambda|\nu})\alpha\left(\frac{dx}{d\tau}\right)y^\lambda\ , \tag{14}$$

where $l^\mu\equiv y^\mu/F$, $\tilde{\gamma}^\mu_{\nu\lambda}$ is the Christoffel symbol of Riemannian metric $\tilde{a}$ and $\tilde{b}_{\nu|\lambda}$ denotes the covariant derivative with respect to the Riemannian metric $\tilde{a}$

$$\tilde{b}_{\nu|\lambda}=\frac{\partial\tilde{b}_\nu}{\partial x^\lambda}-\tilde{\gamma}^\mu_{\nu\lambda}\tilde{b}_\mu\ . \tag{15}$$

In the rest of the paper, we just consider the case that β is a closed 1-form. Thus, the geodesic equation of such Randers space-time is given as

$$\frac{d^2x^\mu}{d\tau^2}+(\tilde{\gamma}^\mu_{\nu\lambda}+l^\mu\tilde{b}_{\nu|\lambda})y^\nu y^\lambda=0\ . \tag{16}$$

In ΛCDM model, the cosmological principle indicates that the universe is homogeneous and isotropic at large scale, which leads to the Friedmann-Robertson-Walker (FRW) metric. In the comoving coordinates, the FRW metric takes the form

$$ds^2=dt^2-a^2(t)\left[\frac{dr^2}{1-kr^2}+r^2\left(d\theta^2+\sin^2\theta d\varphi^2\right)\right], \tag{17}$$

where $k=-1,0,+1$ respectively stand for an open, flat, and closed universe, and $a(t)$ is called scale factor. The cosmological redshift $z_R(t)$ is given as

$$1+z_R(t)=\frac{a(t_0)}{a(t)}=\frac{1}{a}\ , \tag{18}$$

which reveals the ratio of expansion undergone by the universe between the time t and the present time t_0.

Unfortunately, the FRW universe does not match with the Keck and VLT observations. The attribute of monopole in α variation implies the universe is not homogenous, and "Australian Dipole "implies the universe is not isotropic. As is mentioned above, Finsler space-time naturally admits less Killing vectors (then less symmetries) than Riemann space-time does. It could be a reasonable framework to corporate with the Keck and VLT observations.

III. UNIFORM REDSHIFT AND AUSTRALIAN DIPOLE

We suppose that the metric of the universe takes the FRW-Randers-Finsler form, in which $\tilde{a}_{\mu\nu}$ is the flat FRW metric and β is a closed 1-form. Then, we find from (16) that

$$0=\frac{d^2x^0}{d\tau^2}+\delta_{ij}\dot{a}a\frac{dx^i}{d\tau}\frac{dx^j}{d\tau}+\frac{dx^0}{d\tau}f\left(x,\frac{dx}{d\tau}\right), \tag{19}$$

$$0=\frac{d^2x^i}{d\tau^2}+2\delta^i_j\frac{\dot{a}}{a}\frac{dx^0}{d\tau}\frac{dx^j}{d\tau}+\frac{dx^i}{d\tau}f\left(x,\frac{dx}{d\tau}\right), \tag{20}$$

where $f\left(x,\frac{dx}{d\tau}\right)\equiv\tilde{b}_{\nu|\lambda}\frac{dx^\nu}{d\tau}\frac{dx^\nu}{d\tau}/F$ and a dot denotes $\frac{d}{dx^0}$. The equation (20) has a solution

$$a^2\frac{dx^i}{d\tau}\propto J_1, \tag{21}$$

where we involve a new quantity J_1. It is defined as $\frac{d\ln J_1}{d\tau}\equiv -f\left(x,\frac{dx}{d\tau}\right)$. By making use of the equation (21), we obtain a solution of (19),

$$a\frac{dx^0}{d\tau}\propto J_1\ . \tag{22}$$

While $\tilde{b}$ vanishes, J_1 reduces to dimensionless constant for photons. The Riemannian norm $\tilde{b}$ is much smaller than 1. Therefore, the energy of the universe is of its Riemannian form $E\simeq\frac{dx^0}{d\tau}$. Then, we find from the solution (22) that the formula of the redshift in FRW-Rander-Finsler space-time is of the form

$$1+z_F(t)=\frac{J_1}{a}\ . \tag{23}$$

In the following, we try to get a formula for J_1 . The derivative of the term $\tilde{b}_\mu\frac{dx^\mu}{d\tau}$ gives

$$\begin{aligned}
\frac{d}{d\tau}\left(\tilde{b}_\mu\frac{dx^\mu}{d\tau}\right) &= \frac{dx^\nu}{d\tau}\frac{\partial}{\partial x^\nu}\left(\tilde{b}_\mu\frac{dx^\mu}{d\tau}\right)=\frac{dx^\nu}{d\tau}\left(\tilde{b}_\mu\frac{dx^\mu}{d\tau}\right)_{|\nu}\\
&= \tilde{b}_{\alpha|\beta}\frac{dx^\alpha}{d\tau}\frac{dx^\beta}{d\tau}+\tilde{b}_\mu\left(\frac{d^2x^\mu}{d\tau^2}+\tilde{\gamma}^\mu_{\nu\lambda}\right)\frac{dx^\nu}{d\tau}\frac{dx^\nu}{d\tau}\\
&= \left(1-\frac{\tilde{b}_\mu}{F}\frac{dx^\mu}{d\tau}\right)\tilde{b}_{\alpha|\beta}\frac{dx^\alpha}{d\tau}\frac{dx^\beta}{d\tau}\ ,
\end{aligned} \tag{24}$$

where "|" denotes the covariant derivative with respect to the Riemannian metric α. Here, we have used the fact that the term $\tilde{b}_\mu\frac{dx^\mu}{d\tau}$ is a scaler in Riemannian spacetime with metric $\tilde{a}_{\mu\nu}$, to get the second equation of (24). And we have used the geodesic equation (16) to get the last equation of (24). Noticing that F is constant along the geodesic, we find from equation (24) that

$$\frac{d\ln\left(F-\tilde{b}_\mu\frac{dx^\mu}{d\tau}\right)}{d\tau}=-\tilde{b}_{\nu|\lambda}\frac{dx^\nu}{d\tau}\frac{dx^\nu}{d\tau}/F=-f\left(x,\frac{dx}{d\tau}\right). \tag{25}$$

It implies that

$$J_1=1-\tilde{b}_\mu\frac{dx^\mu}{d\tau}, \tag{26}$$

with normalization of τ (F has been normalized).

Combining the equations (18), (23) and (26) together, we obtain the cosmological redshift deviation in Finsler space-time from the one in Riemann space-time:

$$\frac{1+z_R}{1+z_F}\simeq 1+\tilde{b}_\mu\hat{p}^\mu\ , \tag{27}$$

where the over-hat represents that $\hat{p}^\mu$ is a unit four-momentum of light.

It is obvious that the second term $\tilde{b}_\mu\hat{p}^\mu$ on the right of the equation (27) could be rewritten into a monopole plus dipole function about space-time locations and directions. The dipole term comes from the inner product of the space components of $\tilde{b}^\mu$ and $\hat{p}^\mu$, and the monopole one comes from the inner product of the time components of them. We may live in an inhomogeneous and anisotropic universe, but used to calculate all quantities from a homogeneous and isotropic view. This may be the reasons that the observational formulas for the fine structure constant α is varied from point to point in spacetime. Thus, one should try to setup a new scenario with space-time inhomogeneity and anisotropy, and deal with the observational data of quasar absorption spectra from a uniform view of FRW-Randers-Finsler space-time. The relative frequency $\Delta\omega$ (Δ means relative value between two parameters) between two given transitions emitted from quasars should be observed or detected as $(\Delta\omega_0)/(1+z_F)$ around the Earth in the frame of Finsler space-time. However, the observational data show that it takes the form $((\Delta\omega_0)+(\Delta q)x)/(1+z_R)$ in the perspective of Riemann space-time. By using the equations (1)(18)(27), we would obtain the observational formulas of spatial monopole (2) and dipole (3) for α variation. Both the dipole and monopole terms of α-variation appear naturally in the Finsler space-time. In the condition that the dipole and monopole terms in equation (27) take the order of magnitude $\sim 10^{-7}$, the order of magnitude of α variation would appear to be $\sim 10^{-6}$, which is compatible with the formulas from observations.

IV. CONCLUSIONS AND REMARKS

The line element of VSR could be written as [28–31]:

$$ds = \left(\eta_{\mu\nu}dx^\mu dx^\nu\right)^{\frac{1-b}{2}}\left(n_\sigma dx^\sigma\right)^b\ . \tag{28}$$

where the unit vector n_k stands for a preferred direction in the three-dimensional space. The parameter b stands for the level of space anisotropy characterizing the deviation of the metric (28) from Minkowski metric $\eta_{\mu\nu} = diag(+1,-1,-1,-1)$ [29]. In the case that b and $\left(n_\sigma dx^\sigma\right)/\left(\eta_{\mu\nu}dx^\mu dx^\nu\right)^{1/2}$ are small($\ll 1$), the right side of equation (28) could be expanded approximately into a line element of Randers form. From the "ether drift" experiment obtained in 1970 [29, 53, 54], the space anisotropy has an upper limit $b < 5\times 10^{-10}$ around the Earth. However, there could be a little larger space anisotropy at large scale (for instance, Hubble scale). Then there would be no contradiction between our result at large scale and the constraint from the "ether drift" experiment around the Earth.

In this paper, we have proposed that the Finsler space-time with inhomogeneities and anisotropy could account for the observational formulas of the variation of fine structure constant α. Based on the Finslerian geodesic equations in a FRW-Randers-Finsler space-time, a uniform cosmological redshift was obtained, which is deviated from the one in FRW-Riemann space-time. And the deviation could be revealed generally as one monopole plus dipole function about space-time locations and directions. Such a monopole plus dipole function is found to account for the formulas of α variation possibly. Thus, the observations of variation of fine structure constant from quasar absorption spectra could be viewed as a test of space-time inhomogeneity and anisotropy at large scale. As was mentioned above, the standard theories could not reasonably explain any variation of α while Randers-Finsler space-time with inhomogeneities and anisotropy could. This may signal certain hints for new physics.

We would like to thank Prof. C. J. Zhu and C. G. Huang for useful discussions. The work was supported by the NSF of China under Grant No. 10525522, 10875129 and 11075166.

* Electronic address: changz@ihep.ac.cn

† Electronic address: wangsai@ihep.ac.cn

‡ Electronic address: lixin@ihep.ac.cn

[1] S. Dodelson, *Modern Cosmological*, Elsevier(Singapore) Pte Ltd., Singapore, 2008.

[2] E. Komatsu *et al.*, Astrophys. J. Suppl. **192**, 18 (2011), arXiv: 1001.4538 [astro-ph.CO].

[3] R. Kessler *et al.*, arXiv: 0908.4274 [astro-ph.CO].

[4] R. Amanullah *et al.*, Astrophys. J. **716**, 712-738 (2010), arXiv: 1004.1711 [astro-ph.CO].

[5] S. Nesseris and L. Perivolaropoulos, Phys. Rev. D **77**, 023504 (2008), arXiv: 0710.1092.

[6] I. Antoniou and L. Perivolaropoulos, arXiv: 1007.4347[astro-ph.CO].

[7] R. Watkins, H. A. Feldman, and M. J. Hudson, Mon. Not. R. Astron.Soc. **392**, 743 (2009).

[8] C. L. Bennett *et al.*, arXiv: 1001.4758 [astro-ph.CO].

[9] K. Land and J. Magueijo, Phys. Rev. Lett. **95**, 071301 (2005), arXiv: astro-ph/0502237.

[10] M. Tegmark, A. de Oliveira-Costa, and A. Hamilton, Phys. Rev. D **68**, 123523 (2003).

[11] D. Hutsemekers, R. Cabanac, H. Lamy, and D. Sluse, Astron. Astrophys. **441**, 915 (2005).

[12] D. J. Schwarz and B. Weinhorst, Astron. Astrophys. **474**, 717-729 (2007), arXiv:0706.0165 [astro-ph].

[13] S. Weinberg, *Cosmology*, Oxford University Press, New York, 2008.

[14] S. Weinberg, *Gravitation and cosmology*, Wiley, New York, 1972.

[15] P. A. M. Dirac, Nature **139**, 323 (1937).

[16] J. P. Uzan, Rev. Mod. Phys. **75**, 403 (2003), arXiv: hep-ph/0205340.

[17] J. P. Uzan, Review for Living Reviews in Relativity, arXiv: 1009.5514v1 [astro-ph.CO].

[18] J. D. Barrow, Paper for the Proceedings of the Grassmann Bicentennial Conference, 'Grasscosmofun'09', Univ. Szczecin Sept. 14-19 (2009), arXiv: 0912.5510v1 [gr-qc].

[19] V. V. Flambaum, International Journal of Modern Physics A, Vol. **22**, No. 27, 4937-4950 (2007).

[20] M. T. Murphy, V. V. Flambaum, and J. K. Webb, Mon. Not. Roy. Astron. Soc. **345**, 609 (2003), arXiv: astro-ph/0306483.

[21] J. C. Berengut and V. V. Flambaum, J. Phys. Conf. Ser. **264**: 012010 (2011), arXiv: 1009.3693.

[22] J. K. Webb *et al.*, arXiv: 1008.3907.

[23] V. A. Dzuba, V. V. Flambaum, and J. K. Webb, Phys. Rev. Lett. **82**, 888 (1999), arXiv: physics/9802029.

[24] J. K. Webb *et al.*, Phys. Rev. Lett. **82**, 884-887 (1999), arXiv: astro-ph/9803165.

[25] V. A. Dzuba, V. V. Flambaum, and J. K. Webb, Phys. Rev. A **59**, 230-237 (1999), arXiv:physics/9808021.

[26] D. A. Varshalovich and A. Y. Potekhin, Space Science Review, **74**, 259 (1995).

[27] A. G. Cohen, S. L. Glashow, Phys. Rev. Lett. **97**, 021601 (2006), arXiv: hep-ph/0601236.

[28] G. W. Gibbons, J. Gomis, and C. N. Pope, Phys. Rev. D **76**, 081701 (2007), arXiv: hep-th/0707.2174.

[29] G. Y. Bogoslovsky, arXiv: 0706.2621.

[30] H. F. Goenner and G. Y. Bogoslovsky, Gen. Rel. Grav. **31**, 1383-1394 (1999), arXiv: gr-qc/9701067.

[31] A. P. Kouretsis, M. Stathakopoulos, and P. C. Stavrinos, Phys. Rev. D **79**, 104011 (2009), arXiv: 0810.3267.

[32] S. R. Coleman and S. L. Glashow, Phys. Rev. D **59**, 116008 (1999), arXiv: hep-ph/9812418.

[33] S. R. Coleman and S. L. Glashow, Phys. Lett. B **405**, 249 (1997), arXiv: hep-ph/9703240.

[34] X. Li and Z. Chang, arXiv: 1010.2020 [gr-qc].

[35] D. Bao, S. S. Chern, and Z. Shen, *An Introduction to Riemann–Finsler Geometry*, Graduate Texts in Mathmatics **200**, Springer, New York, 2000.

[36] Z. Shen, *Lectures on Finsler Geometry*, World Scientific, Singapore, 2001.

[37] Y. Takano, Lett. Nuovo Cimento **10**, 747 (1974).

[38] S. Ikeda, Ann. der Phys. **44**, 558 (1987).

[39] R. Tavakol, and N. van den Bergh, Phys. Lett. A **112**, 23 (1985).

[40] G. Yu. Bogoslovsky, Phys. Part. Nucl. **24**, 354 (1993).

[41] Z. Chang and X. Li, Phys.Lett.B **668**, 453 (2008).

[42] M. Milgrom, Astrophys. J. **270**, 365 (1983).

[43] Z. Chang and X. Li, Phys. Lett. B **676**, 173 (2009); X. Li, Z. Chang and M. H. Li, arXiv: 1001.0066; Z. Chang, M. H. Li and X. Li, arXiv: 1009.1509.

[44] X. Li and Z. Chang, Phys. Lett. B **692**, 1 (2010).

[45] J. D. Anderson *et al.*, Phys. Rev. Lett. **81**, 2858 (1998); J. D. Anderson *et al.*, Phys. Rev. Lett. **65**, 082004 (2002); J. D. Anderson *et al.*, Mod. Phys. Lett. A **17**, 875 (2002).

[46] X. Li and Z. Chang, Phys. Rev. D **82**, 124009 (2010).

[47] D. Clowe, S. W. Randall, and M. Markevitch, http://flamingos.astro.ufl.edu/1e0657/index.html; Nucl. Phys. B, Proc. suppl. **173**, 28 (2007).

[48] X. Li and Z. Chang, arXiv: 0911.1890.

[49] Z. K. Silagadze, Acta Phys. Polon. B **42**, 1199-1206 (2011), arXiv:1007.4632.

[50] M. Matsumoto, *Foundations of Finsler Geometry and Special Finsler Spaces*, Kaiseisha Press, Saikawa Shigaken, Japan 1986.

[51] P. L. Antonelli and S. F. Rutz, "Finsler Geometry" Advanced studies in Pure Mathematics 48, Sapporo (2005) p. 210 -In memory of M.Matsumoto.

[52] Z. Szabo, Ann. Glob. Anal. Geom **34**, 381 (2008).

[53] D. C. Champeney, G. R. Isaak, and A. M. Khan, Phys. Lett. **7**, 241-243 (1963).

[54] G. R. Isaak, Phys. Bull. **21**, 255 (1970).