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Space-time variation of the electron-to-proton mass ratio in a Weyl model

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ABSTRACT

Aims. We consider a phenomenological model where the effective fermion masses depend on the local value of Weyl tensor as a possible explanation for the recent data indicating a space-time variation of the electron-to-proton mass ratio ($\Delta\mu/\mu$) within the Milky Way. We also contrast the required value of the model's parameters with the bounds obtained for the same quantity from modern tests on the violation of the Weak Equivalence Principle (WEP).
Methods. We obtain the theoretical expression for the variation of $\Delta\mu/\mu$ and for the violation of the WEP as a function of the model parameters. We perform a least square minimization in order to obtain constraints on the model parameters from bounds on the WEP.
Results. The bounds obtained on the model parameters from the variation of $\Delta\mu/\mu$ are inconsistent with the bounds obtained from constraints on the violation of the WEP.
Conclusions. The variation of nucleon and electron masses through the Weyl tensor is not a viable model.

Key words. fundamental constants, equivalence principle

1. Introduction

The search for space-time dependence of fundamental constants plays a fundamental role in the continuous efforts to put in firmer empirical grounds our current physical theories and, at the same time, explore the possibilities of exotic physics that might become manifest trough small deviations.

The experimental research can be grouped into astronomical and local methods. The latter ones include geophysical methods such as the natural nuclear reactor that operated about 1.8×10^9 years ago in Oklo, Gabon (Damour & Dyson 1996; Petrov et al. 2006; Gould et al. 2006), the analysis of natural long-lived β decayers in geological minerals and meteorites (Olive et al. 2004) and laboratory measurements such as comparisons of rates between clocks with different atomic numbers (Prestage et al. 1995; Sortais et al. 2001; Marion et al. 2003; Bize et al. 2003; Fischer et al. 2004; Peik et al. 2004). The astronomical methods are based mainly on the analysis of high-redshift quasar absorption systems. Most of the reported data are, as expected, consistent with null variation of fundamental constants. Nevertheless, there are reports of intriguing results, for instance Webb et al. (1999) and Murphy et al. (2003) have reported observations made with the Keck telescope which suggest a smaller value of the fine structure constant (α) at high redshift as compared with its local value. However, an independent analysis performed with VLT/UVES data gave null results (Srianand et al. 2004). Furthermore, a recent analysis using VLT/UVES data suggests also a variation in α but in the opposite sense, that is, α appears to be larger in the past (Webb et al. 2010). The discrepancy between Keck/HIRES and VLT/UVES is yet to be resolved. In particular, the two studies rely on data from different telescopes observing different hemispheres. It was pointed out that the Keck/Hires and VLT/UVES observations can be made consistent in the case where the fine structure constant is spatially varying (Webb et al. 2010).

Focussing on a different quantity, observations of molecular hydrogen in quasar absorption systems can be used to set constraints on the electron-to-proton mass ratio $\mu \equiv m_e/m_p$ at high redshift (King et al. 2008; Thompson et al. 2009; Malec et al. 2010), and surprisingly a recent analysis of ammonia spectra in the Milky Way indicates a spatial variation of μ (Molaro et al. 2009a; Levshakov et al. 2010a,b). The study, comparing the spectral lines of the ammonia inversion transition and rotational transitions of other molecules with different sensitivities to the parameter $\Delta\mu/\mu$, finds a statistically significant velocity offset that when interpreted in terms of a variation in μ gives $\Delta\mu/\mu = (2.2 \pm 0.7) \times 10^{-8}$. This will be the focus of the present paper. If we assume that the latter is not the result of some fluke and systematic experimental error, and thus take the result quite seriously, we are naturally led to the following question: What would be the simplest modification of our present physical theories that might account for such phenomena? One of the simplest possibilities one can think of is that the effective value of the coupling constants changes with space-time location. In this sense we note that, within the context of theories that are at the fundamental level background independent, the study of possible space-time dependence of fundamental constants is often considered as equivalent to the search for the existence of dynamical fields which couple to the gauge fields and/or to ordinary matter in ways that mimic the ordinary coupling constants.

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There have been several proposals along those lines with various different motivations. Some of them arise from proposals for basic theories that arise in the search for unification of the four fundamental laws of physics such as string-derived field theories (Wu & Wang 1986; Maeda 1988; Barr & Mohapatra 1988; Damour & Polyakov 1994; Damour et al. 2002a,b), related brane-world theories (Youm 2001a,b; Palma et al. 2003; Brax et al. 2003), and Kaluza-Klein theories (Kaluza 1921; Klein 1926; Weinberg 1983; Gleiser & Taylor 1985; Overduin & Wesson 1997). Other proposals emerge as low energy limit phenomenological models where a scalar field ϕ couples to the Maxwell tensor $F_{\mu\nu}$ and are characterized by Lagrangian density terms such as $-B_F(\phi)F_{\mu\nu}F^{\mu\nu}/4$ (Bekenstein 1982; Barrow et al. 2002; Olive & Pospelov 2002) and/or to the matter fields Ψ_i as $m_i B_i(\phi)\Psi_i\bar{\Psi}_i$ (Olive & Pospelov 2002; Khoury & Weltman 2004; Brax et al. 2004; Mota & Shaw 2007; Olive & Pospelov 2008; Dent et al. 2007; Wetterich 2003). It is quite clear, for instance, that the first case would result in something like an effective fine structure constant given by $1/e^2_{effective} = 1/e^2 + B_F(\phi)$ and/or similarly the effective masses of the elementary particles given by $m_i^{eff} = m_i + B_i(\phi)$. Then, if the field took space-time dependent values, a feature that often requires the new field to be quite light so its value is not too rigidly tied to the minima of any self-interaction potential, then the effective fine structure constant and/or effective masses might look space-time dependent. This part of the story is quite clear, however, one can not focus on just this aspect of the theory when considering it. In fact, it is often the case that the most important bounds on the theory do not arise from the direct search for this dependence but from the effects of the direct exchange of quanta of this putative field would have on the behavior of ordinary matter. The fact that the scalar field must be light, as we have just described, indicates that this quanta exchange would not be drastically suppressed by a large mass in its propagator (Sudarsky 1992). This generically leads to modification of the free fall and very often to signals that would mimic violations of the weak equivalence principle (WEP), something on which there exceedingly good bounds.

This fact, together with the lack of economy that is implied by the introduction of additional new fields into the theory (besides the ever increasing plethora of yet to be observed scalar fields: Higgs, Inflaton, quintessence, etc.), leads us to look for simpler possibilities to explain this observation. The basic idea is that rather than considering new fields which play the role of the changing part of the fundamental constants, nonstandard aspects of ordinary well known fields might play that role. The long range fields in nature are the electromagnetic and gravitational ones. The use of the former in the desired context does not seem as a promising possibility because, for one it is very well understood and tested over very wide class of regimes, even at the quantum level (*i.e.*, QED), and its enormous strength implies that any small modification would have very noticeable effects. The latter, on the other hand, is not yet so well understood (particularly its quantum aspects), and secondly, it seems conceivable that an exotic type of coupling to matter has not been perceived. Considerations along these lines have led to proposals where the the curvature of space-time might affect the propagation of matter fields in rather unusual ways (Corichi & Sudarsky 2005), which might be viewed as violating of the strict letter of the equivalence principle [1]. In this manuscript we well explore this issue and show that, despite this early optimistic assessment, the size of the model parameters needed to explain the variation of μ in the Milky Way can be ruled out by the bounds on the same parameters emerging from experimental tests of the WEP.

The paper is organized as follows. In section 2 we discuss the astronomical data that suggests a variation of $\Delta\mu/\mu$ in the Milky Way. In Section 3 we describe the theoretical model we want to consider. Section 4 is devoted to determine the value of a combination of the free parameters of the model as implied by the astronomical data discussed in section 2. In section 5 we obtain bounds on another combination of the free parameters of the model using the latest tests of the WEP. We end with a brief discussion and some conclusive remarks in section 6.

2. Data discussion

Astronomical spectroscopy can prove physical constants which describe atomic and molecular discrete spectra. It is the case of the electron-to-proton mass ratio, $\mu = m_p/m_e$, which we shall analyze in order to tests Weyl models. Recently, Levshakov et al. (2010a,b) reported new bounds on the electron-to-proton mass, obtained through the ammonia method. Previous bounds with the same method were obtained by Molaro et al. (2009b). It consists in comparing radial velocities of two different molecular transitions in order to study relative shifts. In particular, the authors used precise molecular lines observed in Milky Way cold dark clouds to compare the apparent radial velocity for the NH_3 inversion transition, V_{inv}, with the apparent radial velocity, V_{rot}, for rotational transitions in HC_3N and N_2H^+, arising from the same molecular cloud. The method provides a relation between the shift radial velocity and the relative variation of μ:

$$\frac{\Delta\mu}{\mu} = 0.289\frac{V_{rot} - V_{inv}}{c} \equiv 0.289\frac{\Delta V}{c}. \qquad (1)$$

In the first report (Levshakov et al. 2010b), three radio telescopes were used to obtain the data: 32-m Medicina, 100-m Effelsberg and 45-m Nobeyama. The authors found several problems in treating the data: i) not all the molecular profiles can be described adequately with a single component Gaussian model and ii) molecular cores are not ideal spheres and being observed at higher angular resolutions exhibit frequently complex substructures. The line profiles may be asymmetric due to non-thermal bulk motions. Therefore the authors selected 23 pair molecular lines from the initially 55 molecular pairs observed. The weighed mean and errors as well as the robust M-estimate of the mean reported for the 23 data, for 100-m Effelsberg and 45-m Nobeyama data sets are shown in table 1. In order to check if the weighed mean is a representative value of the mean value of each data set of table 1 we have calculated $\chi^2 = \sum_i p_i(x_i - x_W)^2 / \sum_j p_j$ (x_W is the weighed mean, $p_i = 1/\sigma_i^2$ and σ_i refers to the 1σ error reported in Levshakov et al. (2010b)) and compared it with the expected value of χ^2 for a gaussian distribution. In all cases the obtained value of χ^2 is large compared with the expected value of χ^2 for a normal (gaussian) distribution. Therefore, the distribution of the data does not seem to be normal (or gaussian). The authors of Levshakov et al. (2010b) have also calculated the robust M-estimate of the mean. From table 1 it follows that there is a significative difference between the Nobeyama

[1] The proposals are motivated by searches for possible granular structure of space-time which might conflict with the ultra-locality that is implicitly assumed in the latter principle.

weighed mean and robust M-estimator, while for the complete data set and Effelsberg data set both estimators are consistent. On the other hand, although there is a good agreement between the robust M-estimate of the three data sets, the Nobeyama data set has a larger systematic error due to the lower accuracy at the rest frequencies of the N_2^+H transition. Furthermore, from the 23 sources of the complete data set, 2 are in common for the 3 telescopes, and 2 are in common for Effelsberg and Nobeyama. Levshakov et al. (2010b) treated them as independent *observations* because the data was obtained with different instruments, *so such data points have different systematic errors*; however, these are not independent as measurements of the physical quantity.

In a recent paper, the authors map four molecular cores selected from their previous sample in order to estimate systematic effects in the velocity offset and to check the reproducibility of the velocity offset on the year-to-year time base (Levshakov et al. 2010a). Observations were performed with the 100-m Effelsberg telescope. In two cores the velocity offset can be explained by the observed kinematic structure. In the other two cores, L1498 and L1512 they obtain a statistically significant positive velocity offset which is shown in table 1. The differences between densities in clouds are not significative, thus the Effelsberg robust M-estimate reported in Levshakov et al. (2010b) is statistically more significant than the value reported in Levshakov et al. (2010a). We will consider the value of the Effelsberg robust M-estimate reported in Levshakov et al. (2010b) as the final value of the velocity offset between rotational and inversion transitions. However, it should be noted that taking the other values reported in the paper or taking the value reported in Levshakov et al. (2010a) will not produce a significant change in the conclusions of the theoretical model we are testing in this paper.

Table 1. Results for the relative radial velocity for all the data, only data obtained with Effelsberg radio telescope and only data obtained with Nobeyama radio telescope (Levshakov et al. 2010b). Entry name Effelsberg (2) refers to results reported in Levshakov et al. (2010a). $\langle\Delta W\rangle_W$ refers to the weighed mean and corresponding error, $\langle\Delta V\rangle_M$ refers to the robust M-estimate of the mean. All velocities are in units of m/s.

Data	$\langle\Delta V\rangle_W$	$\langle\Delta V\rangle_M$
all	20.7 ± 3	21.5 ± 2.8
Effelsberg	23.0 ± 3.1	23.2 ± 3.8
Nobeyama	14.0 ± 7.2	22.9 ± 4.2
Effelsberg (2)		26.9 ± 4.2

3. The Weyl Model

The basic idea of the Weyl models involves considering the effect described in the previous section as due to a non-minimal and rather exotic coupling of the matter fields with gravity through the Weyl tensor. At first sight this may seem as an unnatural proposal as gravity is usually neglected in these regimes and, furthermore, one generally does not feel that there might be a fundamental reason to couple gravity with matter fields in exotic ways. However, in contrast to other models (Khoury & Weltman 2004; Brax et al. 2004; Mota & Shaw 2007; Olive & Pospelov 2008; Dent et al. 2007; Wetterich 2003), in this scheme there is no need to invoke new unobserved dynamical fields, or non-dynamical fields that break Poincaré invariance, or other such problems, which in our opinion are ruled out by the analysis of its consequences on virtual particles (Collins et al. 2004; Collins et al. 2009). Moreover, as described in Corichi & Sudarsky (2005); Bonder & Sudarsky (2008); Bonder & Sudarsky (2009); Bonder & Sudarsky (2010), the generic view we adopt in considering these sort of models is that gravity as “the curvature of a manifold” is only an effective description of an emergent manifestation of more fundamental degrees of freedom from an unknown quantum theory, and thus, the unnaturalness of the coupling terms is tied to the need to use the metric description rather that correct language of the quantum gravity theory.

The task is to find a way in which gravity and matter could interact in a phenomenological level causing the described change in the matter's observed mass. We study fermionic matter fields Ψ_i where i labels the field's flavor. The usual mass term in this case is $m_i\bar{\Psi}_i\Psi_i$, therefore, in order to explain the observations we need to replace m_i by some scalar depending on the gravitational environment. This should be implemented by

$$m_i \to m_i[1 + f(\frac{R}{\Lambda^2})], \quad (2)$$

where ξ_i are small phenomenological parameters which may be different for each flavor, f is a function of the curvature tensor and Λ is a energy scale that we set equal to the Planck scale $M_P = 1.22 \times 10^{19}$ GeV. The first scalar function that comes to mind is the Ricci scalar R. Observe that the Ricci tensor at a space-time point x, and thus the Ricci scalar, is completely determined by the matter at the same point x, which implies that coupling $\Psi_i(x)$ with $R(x)$ is a self-coupling that, for phenomenological purposes, is not interesting. Thus, we should build f with what is left when removing from the Riemann tensor the part determined by the Ricci tensor: The Weyl tensor, W_{abcd}. It is trivial to note that it is not possible to construct a Lorentz scalar out of one power of W_{abcd}, thus, the simplest scalar one can write is

$$f = \frac{\zeta_i}{\Lambda^4} W_{abcd}W^{abcd} \equiv W^2. \quad (3)$$

We note that Λ^4 has dimensions 4, ($\hbar$ and c are taken to be 1), and thus ζ_i is dimensionless.

4. Estimates on the Weyl model parameters from electron-to-proton mass ratio

In a model where the interaction between gravity and matter at a phenomenological level is such as described in section 3, the low energy limit of the interaction Lagrangian density can be written as

$$L_{int} = \frac{\xi_e}{\Lambda^4} m_e W^2 \bar{\Psi}_e \Psi_e + \frac{\xi_p}{\Lambda^4} m_p W^2 \bar{\Psi}_p \Psi_p + \frac{\xi_n}{\Lambda^4} m_n W^2 \bar{\Psi}_n \Psi_n, \quad (4)$$

where subscripts e, p, n respectively refer to electrons, protons and neutrons. Thus, the effective masses of the particles can be expressed as

$$m_i^{eff} = m_i \left(1 + \frac{\xi_i}{M_P^4} W^2\right). \quad (5)$$

We define the observable quantity

$$\mu^{eff} \equiv \frac{m_e^{eff}}{m_p^{eff}} = \frac{m_e}{m_p}\left(\frac{1+\frac{\xi_e}{M_P^4}W^2}{1+\frac{\xi_p}{M_P^4}W^2}\right) \simeq \frac{m_e}{m_p}\left(1+\alpha\frac{W^2}{M_P^4}\right), \quad (6)$$

where in the last step we use the fact that ξ_i/M_P^4 are small and we define $\alpha \equiv \xi_e - \xi_p$. Thus, the observed electron-to-proton mass value in cold molecular clouds respect to the same value at Earth is

$$\left(\frac{\Delta\mu}{\mu}\right)^{eff} \equiv \frac{\mu_{cl}^{eff}-\mu_{\oplus}^{eff}}{\mu_{\oplus}^{eff}} = \frac{\alpha}{M_P^4}(W_{cl}^2 - W_{\oplus}^2), \quad (7)$$

where the subscripts cl and $\oplus$ stand respectively for the interstellar clouds and the Earth.

In order to compute the Weyl tensor both, in the cloud and at the laboratory, we consider a sphere of density ρ and radius R, surrounded by vacuum. Outside the sphere, namely at a distance $r \geq R$ from the center of the sphere we get

$$W^2 = \frac{48GM^2}{r^6}, \quad (8)$$

where $M \equiv 4\pi\rho R^3/3$ is the mass of the sphere. Due to the dependence of W^2 on r we realize that we have to be careful while considering the contributions to W^2, for example, contributions from massive bodies near to the laboratory such as a wall may be greater than the contribution of the entire Earth. Therefore, we calculate the contribution of the entire Earth and from walls of 4 m height, 4 m width and 0.5 m depth made of cement and iron located 0.1 m from the experiment (see table 2). The result is that the contribution to W^2 of an iron wall located very close to the experiment is greater than the effect of the Earth or a cement wall. Since we are dealing with a positive detection of $\Delta\mu$, we consider here the greatest contribution, namely that from the iron wall. Given that we have not the exact details of the laboratory where the rest wavelengths are measured, we keep in mind that we are overestimating the value of W^2 and therefore we obtain a lower estimate for α. Even though the wall is not a sphere, in the regime we are working on it is possible to apply the approximation of linearized gravity and therefore, the superposition principle is valid. Thus, the contribution of the wall can be regarded as the sum of the contribution of a great number of spheres. We estimate that differences with the exact calculation may be at most of order 10.

Table 2. Contribution to W^2 in m^{-4} from the Earth and from massive bodies located at $r = 0.1$ m from the experiment. The dimensions of the walls are: 4 m height, 4 m width and 0.5 m depth.

Source	W^2 (m^{-4})
Earth	1.4×10^{-44}
Wall of cement	6.5×10^{-46}
Wall of iron	1.8×10^{-44}

On the other hand, taking for the clouds a mean density $\rho_{cl} = 3 \times 10^5$ GeV cm^{-3}, a mean radius $R_{cl} = 0.052$ Pc (Molaro et al. 2009a) and $r = R/2$ we obtain

$$W_{cl}^2 = \frac{4^6\pi^4 r^4(\rho_{cl}G)^4(3-8\pi R_{cl}^2\rho_{cl}G)^2}{3(3-8\pi r^2\rho_{cl}G(3r^2+R_{cl}^2))^2} \simeq 1.5 \times 10^{-178}\ \mathrm{m}^{-4}. \quad (9)$$

(We also calculated the value of W_{cl}^2 for $r = 0$ and $r = R$ and the results do not differ significantly from the case $r = R/2$). Taking the value of Effelsberg robust mean discussed in section 2 we get

$$\frac{\alpha}{M_P^4} \geq 5 \times 10^{35}\ \mathrm{m}^4, \quad (10)$$

which is the condition that the parameter appearing in equation (7) needs to satisfy in order for the model to explain the observations described in section 2. The next section is dedicated to study if the relation (10) is compatible tests of the WEP.

5. Bounds from Eötvos type experiments

The gravitational potential of an object composed of N atoms with atomic number Z and baryon number B can be written by taking into account that the effective masses are modified in this model according to equation (5):

$$\begin{aligned} V &= NZ\frac{\xi_e}{M_P^4}m_e W^2 + NZ\frac{\xi_p}{M_P^4}m_p W^2 + N(B-Z)\frac{\xi_n}{M_P^4}m_n W^2 \\ &= N\frac{\alpha'}{M_P^4}W^2 m_p \end{aligned} \quad (11)$$

where

$$\alpha' = Z\left(\xi_e\frac{m_e}{m_p} + \xi_p - \xi_n\frac{m_n}{m_p}\right) + B\xi_n\frac{m_n}{m_p}. \quad (12)$$

The force acting on a freely falling body of mass M_b can be obtained from $\boldsymbol{F} = -\boldsymbol{\nabla}V$, thus, the respective acceleration is

$$\boldsymbol{a} = -\frac{N\alpha' m_p \boldsymbol{\nabla}W^2}{M_P^4 M_b}. \quad (13)$$

The differential acceleration of two bodies with different composition but the same number of atoms N is

$$\Delta\boldsymbol{a} = -\frac{\boldsymbol{\nabla}W^2\ m_p(A\,\alpha_1' + B\alpha_2')}{M_P^4}, \quad (14)$$

where we assume that the mass of the body can be expressed as Nm with m the atomic mass of each body and we define

$$\alpha_1' \equiv \xi_e\frac{m_e}{m_p} + \xi_p - \xi_n\frac{m_n}{m_p}, \qquad \alpha_2' \equiv \xi_n\frac{m_n}{m_p}, \quad (15)$$

and also $A \equiv (Z_1/m_1) - (Z_2/m_2)$ and $B \equiv (B_1/m_1) - (B_2/m_2)$, the subscripts indicating which object we are considering. Observe that A and B depend on the materials tested in each experiment.

Limits on violations of the WEP come from Eötvos-Roll-Krotkov-Dicke and Braginsky-Pannov measurements of the differential acceleration of test bodies. The most stringent limits are obtained from measurements of differential acceleration towards the Sun. However, since the force resulting from Weyl models is a short range force, the relevant bounds to test such models are provided by measurements toward the Earth. In this kind of experiments a continuously rotating torsion balance instrument is used to measure the acceleration difference of test bodies with different composition. In table 3, we summarize the current bounds considered in this paper, as well as the composition of the test bodies in each case. All bounds were obtained from an experiment made at the University of Washington Nuclear Physics Laboratory. The authors mentioned two sources for the relevant signals and in our case these would be the sources for $\boldsymbol{\nabla}W^2$, a hillside of 26 m located closed to the laboratory (we estimate 1

m) and a layer of cement blocks added to the wall of the laboratory (Adelberger et al. 1990). We will model these here simply as spherical sources at a given distance. Barring some fortuitous cancelation among the various contributions in the laboratory due to their detailed location and geometry, these simplification cannot amount to more than a change by a geometrical factor of order one, which we will be able to ignore here. Moreover, the expression for ∇W^2 reads

$$\nabla W^2 = -\frac{288}{r^7}\left(\frac{4}{3}\pi G\rho R^3\right)^2, \tag{16}$$

and therefore the contribution from the hillside and cement layer can be estimated to be, respectively,

$$\nabla W_{\mathrm{H}}^2 = 3.8\times 10^{-45}\mathrm{m}^{-5}, \qquad \nabla \mathrm{W}_{\mathrm{L}}^2 = 2.9\times 10^{-49}\mathrm{m}^{-5}. \tag{17}$$

In order to estimate the value of α'_1 and α'_2, we perform a least square minimization using equation (14) and the data from table 3. We obtain for the hillside

$$\frac{\alpha'_1}{M_P^4} = (-1.4 \pm 4.2)\times 10^{16}\ \mathrm{m}^4, \tag{18}$$

$$\frac{\alpha'_2}{M_P^4} = (0.7 \pm 1.6)\times 10^{14}\ \mathrm{m}^4. \tag{19}$$

In addition, we get for the cement layer:

$$\frac{\alpha'_1}{M_P^4} = (-1.9 \pm 5.5)\times 10^{17}\ \mathrm{m}^4, \tag{20}$$

$$\frac{\alpha'_2}{M_P^4} = (0.9 \pm 2.1)\times 10^{16}\ \mathrm{m}^4. \tag{21}$$

The contribution of other massive sources such as other hills closed to the laboratory would give lower values of ∇W^2, resulting in higher values for α'_1. However, we cannot ignore the hill of 26 m and the layer of cement mentioned by the authors, and therefore the limit on α'_1 is not a lower bound in the sense that it has been discussed in section 4 for the astronomical data.

6. Discussion and Conclusions

We have considered a model where a non-minimal coupling of Weyl tensor to matter would result in an effective mass for fermionic fields which would be space-time dependent. The model might be naturally considered as a possible explanation for the recent reported observations of a space-time variation of the electron-to-proton mass ratio in Molaro et al. (2009a); Levshakov et al. (2010a,b). We have developed the model in some detail and extracted the range of values for a combination of the parameters which would be necessary to successfully account for the "exotic" observation. We have also considered the constrains on the model that arise from consideration of precision tests of the WEP and used some of the most modern relevant data to contrast the two results.

In the manuscript at hand and for the model we have considered the two aspects where studied in independent sections. In section 4 we analyzed the variation of the electron-to-proton mass effective ratio between the environments corresponding to the Earth's surface and a molecular cloud and showed that consistency with the reported data requires $\alpha = \xi_e - \xi_p \geq 5\times 10^{35}\mathrm{m}^4\mathrm{M}_{\mathrm{P}}^4$. On the other hand, the result of the statistical analysis performed in section 5 with bounds on the WEP constrain the value of $\alpha'_1 = \xi_e m_e/m_p + \xi_p - \xi_n m_n/m_p$ to be of order $10^{17}\mathrm{m}^4\mathrm{M}_{\mathrm{P}}^4$. Here one might be inclined to note that the two situations are sensitive to sligltly different combination of the fundamental parameters, and is thus conceivable that α might be as large as required to account for the astronomical observations while α'_1 is as small as needed to conform with the laboratory bounds. We view such possibility as very unlikely, as it would imply that the particular choice of materials compared in the the laboratory tests were coincidentally those for which the signal resulting from the generic couplings happened to cancel out almost exactly (at the level of one part in 10^{17}). Moreover, as similar tests with slightly lower precision do exist for other materials, taking this line of reasoning would only lead to a reduction by at most a couple of orders of magnitude in the constraint, something which would still be sufficient to rule the model out.

We thus have found that barring some miraculous cancelation or some unnatural fine tuning of the experimental conditions and/or of the model, the two sets are incompatible, and that a model where the variation of the electron, proton and neutron effective mass is driven by the scalar magnitude of the Weyl tensor can not account for the experimental constraints and the observational data and should be ruled out.

If the observations of Molaro et al. (2009a); Levshakov et al. (2010a,b) were to be further confirmed and the evidence for a change in the value of the electron-to-proton mass ratio became incontrovertible, one would need some different sort of explanation, however, due to the connection between space-time dependency of parameters and the couplings of ordinary matter with dynamical fields, which appears inherent of background independent theories, it seems very unlikely that one might find one such model where the the bounds imposed by tests of the WEP would not be of great relevance and impact. Nonetheless, we should stress our belief that this kind of models should be further explored, not only as potential explanatory grounds for atypical observations, but also as leading to robust constraints on the possible nontrivial couplings of matter and gravitation.

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References

Adelberger, E. G., Stubbs, C. W., Heckel, B. R., et al. 1990, Phys. Rev. D, 42, 3267
Barr, S. M. & Mohapatra, P. K. 1988, Phys. Rev. D, 38, 3011
Barrow, J. D., Sandvik, H. B., & Magueijo, J. 2002, Phys. Rev. D, 65, 063504
Bekenstein, J. D. 1982, Phys. Rev. D, 25, 1527
Bize, S., Diddams, S. A., Tanaka, U., et al. 2003, Phys. Rev. Lett., 90, 150802
Bonder, Y. & Sudarsky, D. 2008, Classical and Quantum Gravity, 25, 105017
Bonder, Y. & Sudarsky, D. 2009, Rep. Math. Phys., 64, 169
Bonder, Y. & Sudarsky, D. 2010, in American Institute of Physics Conference Series, Vol. 1256, American Institute of Physics Conference Series, ed. H. A. Morales-Tecotl, L. A. Urena-Lopez, R. Linares-Romero, & H. H. Garcia-Compean, 157–163
Brax, P., van de Bruck, C., Davis, A., Khoury, J., & Weltman, A. 2004, Phys. Rev. D, 70, 123518

Table 3. Bounds on the equivalence principle considered in this paper.

$\Delta \boldsymbol{a}$ (m^{-1})	Z_1	Z_2	$(B/\mu)_1$	$(B/\mu)_2$	M(g)	Reference
$(2.5 \pm 7.23) \times 10^{-31}$	4	13	0.998648	1.000684	10.0414	Su et al. (1994)
$(5.3 \pm 6.45) \times 10^{-31}$	4	29	0.998648	1.001117	10.0414	Su et al. (1994)
$(0.67 \pm 3.45) \times 10^{-32}$	4	22	0.99868	1.001077	4.84	Schlamminger et al. (2008)
$(2.3 \pm 2.3) \times 10^{-30}$	4	13	0.998648	1.000684	10.03178	Adelberger et al. (1990)
$(0.9 \pm 1.89) \times 10^{-30}$	4	29	0.998648	1.001117	10.03178	Adelberger et al. (1990)
$(1.11 \pm 3.11) \times 10^{-32}$	29	82	1.0011166	1.0001694	9.980	Smith et al. (2000)

Brax, P., van de Bruck, C., Davis, A.-C., & Rhodes, C. S. 2003, Astrophysics and Space Science, 283, 627
Collins, J., Perez, A., & Sudarsky, D. 2009, in Approaches to Quantum Gravity: Toward a New Understanding of Space, Time and Matter, ed. D. Oriti (Cambridge University Press)
Collins, J., Perez, A., Sudarsky, D., Urrutia, L., & Vucetich, H. 2004, Physical Review Letters, 93, 191301
Corichi, A. & Sudarsky, D. 2005, International Journal of Modern Physics D, 14, 1685
Damour, T. & Dyson, F. 1996, Nuclear Physics B, 480, 37
Damour, T., Piazza, F., & Veneziano, G. 2002a, Phys. Rev. Lett., 89, 081601
Damour, T., Piazza, F., & Veneziano, G. 2002b, Phys. Rev. D, 66, 046007
Damour, T. & Polyakov, A. M. 1994, Nuclear Physics B, 423, 532
Dent, T., Stern, S., & Wetterich, C. 2007, Phys. Rev. D, 76, 063513
Fischer, M., Kolachevsky, N., Zimmermann, M., et al. 2004, Phys. Rev. Lett., 92, 230802
Gleiser, M. & Taylor, J. G. 1985, Phys. Rev. D, 31, 1904
Gould, C. R., Sharapov, E. I., & Lamoreaux, S. K. 2006, Phys. Rev. C, 74, 024607
Kaluza, T. 1921, Sitzungber. Preuss. Akad. Wiss.K, 1, 966
Khoury, J. & Weltman, A. 2004, Physical Review Letters, 93, 171104
King, J. A., Webb, J. K., Murphy, M. T., & Carswell, R. F. 2008, Physical Review Letters, 101, 251304
Klein, O. 1926, Z. Phys., 37, 895
Levshakov, S. A., Lapinov, A. V., Henkel, C., et al. 2010a, ArXiv e-prints
Levshakov, S. A., Molaro, P., Lapinov, A. V., et al. 2010b, Astron.Astrophys., 512, A44+
Maeda, K. 1988, Modern Physics. Letters A, 3, 243
Malec, A. L., Buning, R., Murphy, M. T., et al. 2010, MNRAS, 403, 1541
Marion, H., Pereira Dos Santos, F., Abgrall, M., et al. 2003, Phys. Rev. Lett., 90, 150801
Molaro, P., Levshakov, S. A., & Kozlov, M. G. 2009a, Nuclear Physics B Proceedings Supplements, 194, 287
Molaro, P., Levshakov, S. A., & Kozlov, M. G. 2009b, Nuclear Physics B Proceedings Supplements, 194, 287
Mota, D. F. & Shaw, D. J. 2007, Phys. Rev. D, 75, 063501
Murphy, M. T., Webb, J. K., & Flambaum, V. V. 2003, Mon.Not.R.Astron.Soc., 345, 609
Olive, K. A. & Pospelov, M. 2002, Phys. Rev. D, 65, 085044
Olive, K. A. & Pospelov, M. 2008, Phys. Rev. D, 77, 043524
Olive, K. A., Pospelov, M., Qian, Y. Z., et al. 2004, Phys. Rev. D, 69, 027701
Overduin, J. M. & Wesson, P. S. 1997, Phys.Rep., 283, 303
Palma, G. A., Brax, P., Davis, A. C., & van de Bruck, C. 2003, Phys. Rev. D, 68, 123519
Peik, E., Lipphardt, B., Schnatz, H., et al. 2004, Phys. Rev. Lett., 93, 170801
Petrov, Y. V., Nazarov, A. I., Onegin, M. S., Petrov, V. Y., & Sakhnovsky, E. G. 2006, Phys. Rev. C, 74, 064610
Prestage, J. D., Tjoelker, R. L., & Maleki, L. 1995, Phys. Rev. Lett., 74, 3511
Schlamminger, S., Choi, K., Wagner, T. A., Gundlach, J. H., & Adelberger, E. G. 2008, Physical Review Letters, 100, 041101
Smith, G. L., Hoyle, C. D., Gundlach, J. H., et al. 2000, Phys. Rev. D, 61, 022001
Sortais, Y., Bize, S., Abgrall, M., et al. 2001, Physica Scripta Volume T, 95, 50
Srianand, R., Chand, H., Petitjean, P., & Aracil, B. 2004, Phys. Rev. Lett., 92, 121302
Su, Y., Heckel, B. R., Adelberger, E. G., et al. 1994, Phys. Rev. D, 50, 3614
Sudarsky, D. 1992, Physics Letters B, 281, 98
Thompson, R. I., Bechtold, J., Black, J. H., et al. 2009, ApJ, 703, 1648
Webb, J. K., Flambaum, V. V., Churchill, C. W., Drinkwater, M. J., & Barrow, J. D. 1999, Phys. Rev. Lett., 82, 884
Webb, J. K., King, J. A., Murphy, M. T., et al. 2010, ArXiv e-prints
Weinberg, S. 1983, Physics Letters B, 125, 265
Wetterich, C. 2003, J. Cosmology Astropart. Phys., 10, 2
Wu, Y. & Wang, Z. 1986, Phys. Rev. Lett., 57, 1978
Youm, D. 2001a, Phys. Rev. D, 63, 125011
Youm, D. 2001b, Phys. Rev. D, 64, 085011