On a new category of physical effects

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A new category of "intrinsic" effects is proposed to be added to the two already known kinematic and dynamical categories. An example of intrinsic effect is predicted, its origin source is established, and a scheme of its experimental detection is proposed. This effect lowers to non-relativistic values the propagation velocity of a plane electromagnetic wave in a vacuum, when a time-independent homogeneous magnetic field is superposed over it. This result, pertaining to the classical Maxwell theory, follows from exact calculations. A critical remark on gravitational waves' detection is given.

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"In 1889 Morley and Miller... reconfigured the old Michelson–Morley apparatus to search for changes in the speed of polarized light caused by a magnetic field."— With this historic reference begins the paper by Frank Nezrick [6]. Morley and Miller, as well as Nezrick and his group at the Fermilab, have considered dynamical mechanisms in their predictions, in the Nezrick case, of the modern quantum field theoretical origin, and the author categorically summarizes that "[the] unquantized Maxwell Equations are linear in the fields giving no interaction between photons."

Below we explicitly show that the linearity of classical Maxwell's equations in vacuum does not hinder from the existence of an interaction between superposed electromagnetic fields in spite of their perfectly exact and simple superposable nature. It is well known that certain properties of these fields are determined by the non-linear electromagnetic energy-momentum tensor T^μ_ν, further taken in the classical vacuum. Our scientific community has a deep-seated tradition to misapprehend as a nonsense the natural situation when the same theory states that a superposition of two or more exact solutions is itself an exact solution, *and* it meanwhile predicts an intrinsic effect which *does not inflict changes* in the component parts of this superposition, while its physical characteristics cannot be reduced to a sum for these component parts. This new classical intrinsic effect follows from such non-linear and bilinear things as the energy density and the Poynting vector whose combination, even *more* non-linear, was already related by several authors to the observable group velocity of electromagnetic field's propagation. They were Pauli [7], p. 115, Eq. (312): $\mathbf{v}^i = 2T^i_0/T^0_0$; Landau and Lifshitz [3], the Problem on p. 69: $\mathbf{v}^i/(1+v^2) = T^i_0/T^0_0$, dealing (from the authors' viewpoint) only with parallelization of the electric and magnetic fields; Penrose and Rindler [8], vol. 1, p.324, and vol. 2, pp. 33 and 257-258, who considered only the pure electric and magnetic type fields (*cf.* [5]), eliminating the alternative 3-field (transformation to single-field frames). The corresponding boosts are $B^{-2}(\mathbf{E}\times\mathbf{B})$ and $E^{-2}(\mathbf{E}\times\mathbf{B})$, respectively. Thus Penrose–Rindler's boosts look as mixtures of the Poynting vector, taken in the non-co-moving frame, but divided by energy density of the electromagnetic field, pertaining to the frame co-moving with this field. The velocity we are speaking here about is in fact related to that which had to be measured by Frank Nezrick, though he didn't mention the above authors. All this now occurs in the non-quantized electromagnetic theory, thus we do not bother about an "interaction between photons." This situation has to reappear also in general relativity and in quantum mechanics (see below), but in special relativity the same type of effect has to be present, easily calculated, and immediately detectable: see below our computation.

Working on this effect in the special-relativistic Minkowskian space-time, we shall use the $(+,-,-,-)$ signature and natural units (so that the velocity is dimensionless and the velocity-of-light constant is $c=1$), the Gaussian units in Maxwell's equations, Greek 4-dimensional indices, and the Cartan formalism of exterior forms (see [2]) as the simplest and most effective way to treat geometric ideas and to interpret the obtained results. The electromagnetic field tensor splits, with the help of monad τ (a unitary time-like vector field, in fact, the 4-velocities of local test observers) and the dual conjugation (or its Hodge-star form), into two 4-dimensional (co)vectors, electric and magnetic, both $\perp \tau$:

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$$\mathbf{E}_\mu = F_{\mu\nu}\tau^\nu \quad \Leftrightarrow \quad \mathbf{E} = *(\tau \wedge *F), \quad \mathbf{B}_\mu = -\overset{*}{F}_{\mu\nu}\tau^\nu \quad \Leftrightarrow \quad \mathbf{B} = *(\tau \wedge F), \tag{1}$$

$$F = \mathbf{E} \wedge \tau + *(\mathbf{B} \wedge \tau), \;\; *F = *(\mathbf{E} \wedge \tau) - (\mathbf{B} \wedge \tau). \tag{2}$$

It is obvious that $\mathbf{E}$ is a polar 4-covector and $\mathbf{B}$, an axial 4-covector, both restricted to the local physical 3-subspace of the τ-reference frame. The deduction details of the above formulae see next to Eqs. (3.1.13), (3.1.16), and (3.1.18) in [4]. In that book, the complete monad theory of physical reference frames is given. Jürgen Ehlers [1] was first who formulated the monad formalism; with it he worked exclusively in the cosmology using reference frames co-moving with matter. The monad belonging to such frames he denoted as u which coincides with the 4-velocity of the filler of cosmological space, so that we use here this notation also for a monad (if any) co-moving with the electromagnetic field. A general monad we denote as τ, mostly since the integral lines of the vector field τ are the physical time (not necessarily time coordinate) lines in a space-time diagram. The second author who independently formulated the monad formalism was Abram L. Zel'manov [9], and he worked with it also in relativistic cosmology, like Ehlers.

A combination of T^α_β with an arbitrary monad τ yields

$$T^\nu_\mu T^\mu_\xi \tau_\nu \tau^\xi = \frac{1}{(8\pi)^2}\left[\left(\mathbf{E}^2+\mathbf{B}^2\right)^2 - 4(\mathbf{E}\times\mathbf{B})^2\right] \equiv \frac{1}{(8\pi)^2}\left[\left(\mathbf{B}^2-\mathbf{E}^2\right)^2 + 4(\mathbf{E}\bullet\mathbf{B})^2\right] = \frac{1}{(16\pi)^2}\left({I_1}^2+{I_2}^2\right) \tag{3}$$

where $I_1 = F_{\mu\nu}F^{\mu\nu}$ and $I_2 = \overset{*}{F}_{\mu\nu}F^{\mu\nu}$ are two invariants on which the simplest classification of electromagnetic fields (see [5]) is based. It is remarkable that these constructions are not only scalars under general transformations of coordinates, but they are also independent of the reference frame choice: the right-hand side does not involve the monad. Considering the propagation of electromagnetic field, we do not include the high-frequency limits related to field's discontinuities (bicharacteristics). From (3) and the Landau and Lifshitz 3-velocity taken as an example, we see that

$$0 \le \frac{|\mathbf{v}|}{1+\mathbf{v}^2} = \frac{1}{2}\sqrt{1-\frac{{I_1}^2+{I_2}^2}{4(\mathbf{E}^2+\mathbf{B}^2)^2}} = \frac{|\mathbf{E}||\mathbf{B}|}{\mathbf{E}^2+\mathbf{B}^2}|\sin\alpha| \le \frac{1}{2}, \tag{4}$$

α being the angle between $\mathbf{E}$ and $\mathbf{B}$ in the strict local Euclidean sense; moreover, the function $|\mathbf{v}|/(1+\mathbf{v}^2)$ is everywhere monotonic. In particular, this means that the propagation of all pure null fields ($|\mathbf{E}| = |\mathbf{B}|$, $\alpha = \pi/2$) occurs with the unit absolute value of the 3-velocity, the velocity of light, and all other electromagnetic fields propagate with sub-luminal velocities which can always be made equal to zero in corresponding co-moving reference frames. These conclusions also hold in general relativity, and they are universally expressed in seemingly "3-dimensional" notations characteristic to the general reference frame theory.

Instead of taking any of the 3-velocities given in [3, 7, 8], we shall now use our general definition ([4], p. 42, Eq. (2.2.11)) of the velocity $\mathbf{v}$ between u-monad and τ-monad frames from the viewpoint of the latter frame:

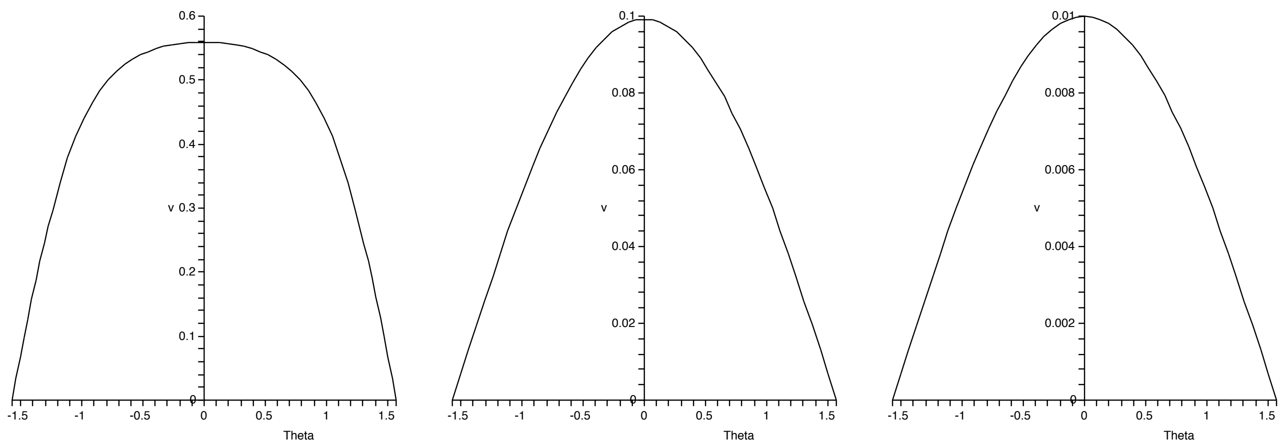
$$u = (\tau \cdot u)(\tau + \mathbf{v}), \quad u \not\perp \mathbf{v} \perp \tau. \tag{5}$$

It is obvious that this $\mathbf{v}$ will be the desired velocity of the electromagnetic field propagation in τ-frame, if the Poynting vector vanishes in u-frame. Thus let us consider a linearly polarized plane monochromatic electromagnetic wave (the situation does not substantially depend on this choice of polarization) as the first component of the superposition with a time-independent homogeneous magnetic field (the second component) in the z-direction of propagation of the wave (a constant electric field yields similar results). In Cartesian coordinates t, x, y, z (the spatial ones forming a right triplet), this superposition reads (since $\mathbf{E}$ and $\mathbf{B}$ are covectors $\perp dt$, the negative signs meaning positivity of the respective vectors' components; to make all expressions more concise, we abbreviate $\omega(t-z)$ as phase of the wave Θ and E/H as A):

$$\mathbf{E} = -E\cos\Theta dx, \quad \mathbf{B} = -Hdz - E\cos\Theta dy \tag{6}$$

where E is the scalar amplitude of both electric and magnetic vectors of the wave, and H is the constant (both in space and time) magnitude of superimposed magnetic field. For two electromagnetic invariants one easily finds that $I_1 := 2(\mathbf{B}^2 - \mathbf{E}^2) = 2H^2 > 0$, $I_2 := -4\mathbf{E} \cdot \mathbf{B} = 0$; consequently, this field belongs to the pure magnetic type (see details of the classification in [5]). This superposition is in fact a specific not precisely monochromatic wave whose behavior can be best understood in the reference frame co-moving with it. We shall find such a frame using the pure-magnetic-type property of this wave's field. First, we write the field $*F$ as a *simple* bivector (see [2], p. 26). Taking for the general frame monad $\tau = dt$, we find from (2) and (6) that

$$\begin{aligned} *F &= \big(E\cos\Theta dz \wedge dy + H\, dz \wedge dt + E\cos\Theta dy \wedge dt\big) \\ &\equiv \underbrace{\big(H dz + E\,\cos\Theta dy\big)}_{P,\text{ a spacelike covector}} \wedge \underbrace{(dt - dz)}_{Q,\text{ a null covector}} = P \wedge Q. \end{aligned} \tag{7}$$



(a) $E/H = 1$ (b) $E/H = 0.1$ (c) $E/H = 0.01$

FIG. 1. Diagrams of the velocity |v| for $-\pi/2 \le \Theta$ =Theta= $\Theta \le \pi/2$.

If to P we add lQ (l being an arbitrary function) and use this sum P' instead of the former P, $*F$ does not suffer any change (neither F does). While $P \cdot P < 0$, $P' \cdot P' = 2Hl - H^2 - E^2\cos^2\Theta$. Thus if we choose $l = H + \frac{E^2}{2H}\cos^2\Theta$, the vector P' becomes timelike, $P' \cdot P' = H^2 > 0$, and we can take P'/H as a properly normalized monad $u = \left(1 + \frac{A^2}{2}\cos^2\Theta\right)(dt - dz) + dz + A\cos\Theta dy$. Now (7) reads $*F = Hu \wedge (dt - dz)$, so that in the new frame u the electric field identically vanishes due to (1) rewritten for u, hence we have found one of the field's co-moving frames. In such calculations one has to remember that when only one (here, magnetic) field survives after the reference frame is transformed, there are other possible transformations which already do not change this situation (in fact, all those which involve an additional motion in the direction of this 3-field, even when this motion occurs to be with a non-constant magnitude of the 3-velocity described by strictly local Lorentz transformations, thus working in non-inertial frames). Consequently, there appears a continuum of such single-field (**E**- or **B**-) frames (*cf.* [8]; of course, they work in general as well as in the special relativity), and the search for more elegant frames depends on the individual taste of the researcher. This means that the 3-velocities given in [3, 7, 8] may describe only particular choices of co-moving frames (if they are correct at all).

Let us now calculate 3-velocity $\mathbf{v}$ of the co-moving frame u [group propagation velocity of the electromagnetic field (6)] with respect to the frame $\tau = dt$, using our general definition (5):

$$\mathbf{v} = \frac{A\cos\Theta}{1 + \frac{A^2}{2}\cos^2\Theta}\left(dy - \frac{A}{2}\cos\Theta dz\right), \tag{8}$$

$$|\mathbf{v}| = \frac{A\cos\Theta\sqrt{1 + \frac{A^2}{4}\cos^2\Theta}}{1 + \frac{A^2}{2}\cos^2\Theta}, \tag{9}$$

thus when $E/H \equiv A \le 1$, $|\mathbf{v}| < 1$, and frames co-moving with the superposition of fields are, in principle, realizable. When $H \to 0$, the propagation velocity approaches to that of light, while if $E \ll H$, it becomes as low as one wishes; see Fig. 1 where $|\mathbf{v}|$ is given in natural units.

The most simply realizable experiment for detection of this intrinsic effect can be performed with a large distance of the light propagation in a sufficiently long optic fiber wound upon a bobbin containing a straight concentric conductor with a direct electric current in it to produce a magnetic field along the fiber with the light beam. The dielectric properties of the fiber can be easily filtered out since in this experiment the superimposed constant magnetic field will dominate, and the comparatively weak light beam can be manipulated to get different velocities of its propagation, practically not changing the dielectric properties of the fiber.

Of course, the effects expressed in mutually analogous characteristics (*e.g*, those which are related to a shift of the propagation velocity of electromagnetic field), can be superimposed. However, in so doing, such parallel effects are of entirely different orders of magnitude: the dynamical effects depend on interaction constants (in particular, when there appears a non-linearity in the dynamical field equations, for example, via quantum theoretical corrections like those mentioned in [6]); the kinematic effects (*e.g.*, the 3-velocities composition law) are more universal, but they do not change $c = 1$ composed with any subluminal velocity; the intrinsic effects are significantly stronger than the other ones in view of the stable non-linearity of the expressions which yield them, without any participation of interaction constants. Therefore the intrinsic effects generally are dominant also in other branches of physics. The intrinsic effect related to the group velocity shift has to occur in quantum mechanics (without the second quantization, in the linear equations such as the Schrödinger and Dirac ones) where the 3-velocity should follow from the non-linear probability density flow and probability density itself.

The gravitational deformation in general relativity does in fact belong to the kinematic effects, when it is described without the use of geodesic deviation equation. Thus, for example, the interferometric detection of gravitational waves cannot give a non-zero result, since the scales of all types of equally oriented lengths do change in gravitational fields in the same proportion, and the numbers of light wavelengths fitting along the alternative arms of interferometer cannot suffer changes in a passing gravitational wave. I am regretful not to tell these considerations to Kip S. Thorne more than two decades ago, simply because of a kind of awkward modesty (at a seminar of the Institute for Physical Problems in Moscow in 1970ies during the talk of Herzenstein and Pustovoyt on their proposal of such a detection of gravitational waves, when I had told them this fact, the talk immediately collapsed, and I felt very sorry for it).

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