Gravity can be neither classical nor quantized

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Abstract

I argue that it is possible for a theory to be neither quantized nor classical. We should therefore give up the assumption that the fundamental theory which describes gravity at shortest distances must either be quantized, or quantization must emerge from a fundamentally classical theory. To illustrate my point I will discuss an example for a theory that is neither classical nor quantized, and argue that it has the potential to resolve the tensions between the quantum field theories of the standard model and general relativity.

To quantize or not to quantize gravity

Gravity stands apart from the other three interactions of the standard model by its refusal to be quantized. To be more precise, quantizing gravity is not the actual problem; gravity can be perturbatively quantized. The problem is that the so quantized theory cannot be used at energies close by and above the Planck energy, and thus cannot be considered a fundamental theory; it is said to be 'non-renormalizable,' meaning it has no predictive power in the extremely high energy regime.

This mismatch between the quantum field theories of the standard model and classical general relativity is more than an aesthetic problem: It signifies a severe shortcoming of our understanding of nature. This shortcoming has drawn a lot of attention because its resolution it is an opportunity to completely overhaul our understanding of space, time and matter. The search for a consistent theory of quantum gravity that could be applied also at Planckian energies, or strong curvature respectively, has thus lead to many proposals. But progress has been

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slow and in the absence of experimental evidence, our reasons for the necessity of quantizing gravity are theoretical:

1. Classical general relativity predicts the formation of singularities under quite general circumstances. Such singularities are unphysical and should not occur in a fundamentally meaningful theory. It is expected that quantum gravity is necessary to prevent the formation of singularities.

2. Applying quantum field theory in a curved background at small curvature leads to the evaporation of black holes, as first shown by Hawking [1]. This black hole evaporation however seems to violate unitary which is incompatible with quantum mechanics. It is widely believed that quantum gravitational effects restore unitarity and information is conserved.

3. All quantum fields carry energy so they all need to couple to the gravitational field, but we do not know a consistent way to couple a quantum field to a classical field. As Hannah and Eppley have argued [2], the attempt to do such a coupling leads either to a violation of the uncertainty principle (and thus would necessitate a change of the quantum theory) or to the possibility of superluminal signaling, which brings more problems than it solves. While Mattingly has argued [3] that Hannah and Eppley's thought experiment can not be carried out in our universe, that does not address the problem of consistency.

These issues have all been extensively studied and discussed in the literature and are familiar ground. The most obvious way to address them seems to be a non-perturbative theory in one or other form, and several attempts to construct one are under way. I will use the opportunity of the essay contest to stray from the well-trodden ground and argue that we should instead reinvestigate the apparent tension between the quantized matter and non-quantized gravity. It is worthwhile for the following to recall the problems with coupling a classical to a quantum field.

The first problem, as illuminated by Hannah and Eppley is that the classical and the quantum fields would have different uncertainty relations, and their coupling would require a modification of the quantum theory. Just coupling them as they are leads to an inconsistent theory. The beauty of Hannah and Eppley's tought argument is its generality, but that is also its shortcoming, because it does not tell us how a suitable modification of quantum theory could allow such a coupling to be consistent.

The second problem is that it is unclear how mathematically the coupling should be realized, as the quantum field is operator-valued and the classical field is a function on space-time. One possible answer to this is that any function can be identified with an operator on the Hilbert space by multiplying it with the identity. However, the associated operators would always be commuting, so they are of limited use to construct a geometrical quantity that can be set equal to the operator of the stress-energy-tensor (SET) of the quantum fields.

Another way to realize the coupling is to construct classical field from the operator of the SET by taking the expecation value. The problem with this approach is that the expectation value may differ before and after measurement, which then conflicts with the local conservation laws of general relativity. Coupling the classical field to the SET's expectation value is thus usually considered valid only in approximation when superpositions carry negligible amounts of energy.

Because of these difficultites to make sense of the theory, leaving gravity classical while the other interactions are quantized is not a very promising option. However, this theoretical assessment should be supported by experimental test; recent proposals for this have been put forward in [4, 5].

How to be neither classical nor quantized

Let us carefully retrace the logic of the arguments in the previous section.

We have experimental evidence that matter is quantized in the energy regimes that we have tested. We cannot leave gravity unquantized if it couples to quantized matter. Thus gravity has to be quantized in the energy regimes we have tested. We can quantize gravity perturbatively. This theory does make sense in the energy regimes that we have tested, but does not make sense in the strong curvature regime. We have no experimental evidence for the existence and properties of singularities or black hole evaporation, or the behavior of matter in the strong curvature regime.

To conclude from the previous paragraph that we need a non-perturbative completion of quantum gravity necessitates a further assumption, that is that the quantization procedure itself is independent of the energy range at which we apply the theory. It is this assumption that I argue should be given up.

We normally think of a theory as either being quantized or classical, but let us entertain the possibility that quantization is energy-dependent. Concretely, consider that Planck's constant $\hbar$ is a field whose value at high energies goes to zero. In four space-time dimensions, Newton's constant is $G = \hbar c/m_{\mathrm{Pl}}^2$, so if we keep

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mass units fix, G will go to zero together with $\hbar$, thereby decoupling gravity. If gravity decouples, there's no reason for singularities to form. If gravity becomes classical, there's no problem with the perturbative expansion. So this possibility seems intriguing, if somewhat vague. I will now make this idea more concrete and then explain how it addresses the previously listed problems with quantizing gravity.

The starting point is that Planck's constant is a massless scalar field over space time $\hbar(x,t)$, and the equal time commutation relations for all fields, including Planck's constant itself, are proportional then to $\hbar(x,t)$. Since we have no experimental evidence for the variation of Planck's constant, the most conservative assumption is that the $\hbar$-field is presently in its ground state, and difficult to excite with energies that we have access to. This suggests that we think about quantization as the consequence of a spontaneous symmetry breaking, and we have to add a suitable potential for $\hbar$ to the Lagrangian to achieve this. We are presently experiencing $\hbar(x,t)$ as having a non-zero vacuum expectation value that we will denote with $\hbar_0$. This is the measured value of Planck's constant. But at high temperature, presumably close by the Planck energy, the symmetry can be restored, resulting in a classical theory.

Gravity and matter then have a quantized phase and an unquantized phase, and are fundamentally neither quantized nor classical in the same sense that water is fundamentally neither liquid nor solid. Quantization, in this case, is also not emergent from a classical theory because the condition for second quantization does always contain the $\hbar(x,t)$.

A new look at old problems

Let us now come back to the three problems mentioned in the first section that a theory for quantum gravity should address.

First, there is the formation of singularities. We know of two types of singularities that we should worry about, the Big Bang singularity and the singularities inside black holes.

If we move backwards in time towards the early universe, the temperature of matter increases and will eventually exceed the Planck energy. This is the standard scenario in which symmetry restoration takes place [6], so the expectation value of $\hbar$ goes to zero, gravity becomes classical, and matter decouples. If matter decouples, it cannot collapse to a singularity.

Collapse to a black hole is somewhat more complicated because it's not a

priori clear that the temperature of the collapsing matter necessarily increases, but it plausibly does so for the following reason[1]. If matter collapses to a black hole, it does so rapidly and after horizon formation lightcones topple inward, so no heat exchange with the environment can take place and the process is adiabatic. The entropy of the degenerate Fermi gas is proportional to $Tn^{-2/3}$, where T is the temperature and n is the number density. This means that if the number density rises and entropy remains constant, the temperature has to rise [7]. So again, matter decouples and there is nothing left to drive the formation of singularities.

Note that the $\hbar$-field makes a contribution to the source term, necessary for energy conservation.

Second, there is the black hole information loss. It was argued in [8] that the problem is caused by the singularity, not the black hole horizon, and that removing the singularity can resolve the information loss problem. This necessitates the weak interpretation of the Bekenstein-Hawking entropy so that a stable or quasi-stable Planck scale remnant, or a baby-universe, can store a large amount of information. There are some objections to the existence of such remnants, but they rely on the use of effective field theory in strong curvature regimes, the validity of which is questionable [9]. Thus, unitarity in black hole evaporation can be addressed by the first point, avoiding the formation of singularities.

Third, the difficulty of coupling a quantum field to a classical field and the non-renormalizability of perturbatively quantized gravitty. In the here proposed scenario, there is never a classical field coupled to a quantum field. Instead, gravity and matter are of the same type and together either in a quantum phase or a classical phase. In the quantum phase, gravity is quantized perturbatively. It then needs to be shown that the perturbation series cleanly converges for high energy scattering because $\hbar$ is no longer a constant. This is a subtle point and I can here only give a rough argument.

To see how this would work, first note that we can rewrite the equal time commutation relation into a commutation relation for annihilation and creation operators of the fields. The commutator between annihilation and creation operators is then proportional to the Fourier-transform of $\hbar(x,t)$, which I will denote $\tilde{\hbar}$. The same is true for the annihilation and creation operators of $\hbar(x,t)$ (though the prefactors differ for dimensional reasons).

Now consider an arbitrary S-matrix transition amplitude with some interaction vertices. We evaluate it by using the commutation relations repeatedly until annihilation operators are shifted to the very right side, acting on the vacuum,

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which leaves c-numbers, or the Feynman rules respectively. If Planck's constant is a field, then every time we use the commutation relation, we get a power of the $\hbar$-field, and the S-matrix expansion is a series in expectation value of powers of $\tilde{\hbar}$ times the other factors of the transition amplitudes. Then, we use the commutation relations on $\hbar$, or its annihilation and creation operators respetively. Now note that exchanging two of these will only give back one $\tilde{\hbar}$. Thus, we can get rid of the expectation value of powers, so that in the end we will have a series in powers of vacuum expectation values of $\tilde{\hbar}$ (as opposed to a series of expectation values of powers, note the difference).

If we consider the symmetry breaking potential to be induced by quantum corrections at low order, the transition to full symmetry restoration may be at a finite value of energy. In this case then, the quantum corrections which would normally diverge would cleanly go to zero, removing this last problem with the perturbative quantization of gravity.

Summary

I have argued that the fundamental theory can be neither classical nor quantized, but that quantization may be a phase that results from spontaneous symmetry breaking. Needless to say, this proposal is presently very speculative and immature. Some more details can be found in [10], but open questions remain. However, I hope to have convinced the reader that giving up the assumption that a theory is either classical or quantized can be fruitful and offers a new possibility to address the problems with quantum gravity.

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