

Testing Superstring Theories with Gravitational Waves

Ruth Durrer^a and Jasper Hasenkamp^b

a Department of Theoretical Physics and Center for Astroparticle Physics, University of Geneva, Geneva, Switzerland and CEA, SPHT, URA 2306, F-91191 Gif-sur-Yvette, France. (email:Ruth.Durrer@unige.ch)

b II. Institute for Theoretical Physics, University of Hamburg, Hamburg, Germany. (email:Jasper.Hasenkamp@desy.de)

We provide a simple transfer function that determines the effect of an early matter-dominated era on the gravitational wave background and show that a large class of compactifications of superstring theory might be tested by observations of the gravitational wave background from inflation. For large enough reheating temperatures $\gtrsim 10^9$ GeV the test applies to all models containing at least one scalar with mass $\lesssim 10^{12}$ GeV that acquires a large initial oscillation amplitude after inflation and has only gravitational interaction strength, i.e., a field with the typical properties of a modulus.

I. INTRODUCTION

Our description of the forces of Nature consisting of the standard model of particle physics and general relativity as the theory of gravity is astonishingly successful. However, the theory is incomplete. On large scales gravitational dynamics requires the introduction of 'invisible' components like dark matter and dark energy. The standard model of particle physics is lacking neutrino masses and contains many unexplained parameters. The most serious problem, however, is that we have no quantum theory of gravity. Classical general relativity has singularities. It breaks down when the curvature becomes too large. The most developed attempt for a quantum theory of gravity is string theory which may turn out to be the fundamental theory of Nature.

String theory has the weakness that it is very difficult to test experimentally. This is not only due to the fact that 'stringy' effects become relevant only at very high energy, but is also a consequence of the landscape [1]. It has turned out that string models are extremely versatile and able to predict more or less everything. However, if we want to take some version of string theory seriously as a physical theory of Nature, it needs to be falsifiable. In this paper we propose a test for the existence of at least one scalar field in the model that affects the cosmic evolution after inflation. The well-known moduli problem [2] predicts a phase of matter dominated expansion some time after inflation. This matter has either to be diluted by a subsequent phase of 'thermal inflation' [3] or, more naturally, the moduli have to decay. In this paper we concentrate on this second possibility which does not require any additional ingredients. Qualitatively, however, our discussion also holds for thermal inflation [4].

Moduli fields describe the configuration of the curled up extra dimensions of the string compactification. They have gravitational coupling strength only which makes them long-lived. They must be stabilized in order for the measured masses and couplings of standard model particles to have well-defined constant values as observed. Therefore, they must have finite mass. In superstring models their mass m_ϕ is typically of the order

of the gravitino mass $m_{3/2}$, so that their decay width is $\Gamma \sim m_\phi^3/m_{\text{pl}}^2 \sim m_{3/2}^3/m_{\text{pl}}^2$, where m_{pl} denotes the Planck mass. If they are displaced from the origin after inflation, they perform oscillations and the Universe becomes matter dominated soon after.

If the moduli were cosmologically stable they would overclose the Universe and if they would decay after $t_{\text{BBN}} \sim 0.1$ s the success of big bang nucleosynthesis (BBN) were spoiled, because neutrinos would not have had enough time to thermalize [5]. Altogether, the moduli must decay before BBN.

We show that such a matter dominated phase before BBN leaves a detectable imprint on the gravitational wave background from inflation: it significantly reduces the amplitude of the gravitational wave background at frequencies accessible to ground- or space-based detectors, compared to those probed by observations of the cosmic microwave background (CMB). Such a spectrum could be ruled out if a gravitational wave background from inflation were detected not only by future CMB experiments, but also by gravitational wave experiments at higher frequencies with an unsuppressed amplitude. In this paper we calculate this signature in detail.

To circumvent the proposed test the modulus would have to decay before $t \sim 10^{-22}$ s $\simeq \Gamma^{-1}$ which corresponds to $m_\phi = (m_{\text{pl}}^2 \Gamma)^{1/3} \sim 10^{12}$ GeV.

A particularly interesting situation arises for superstring models with stabilized moduli that have at least one long lived modulus or modulus-like field whose mass is less than, or of the order of, the gravitino mass [6, 7]. As mentioned above, the proposed test applies up to scalar masses of 10^{12} GeV. Even though the tested mass range is well below the Planck scale, this would be a crucial test, because to our knowledge there is no other test for so massive, shortlived particles which are way beyond being testable in colliders, also in high precision tests. In this case the gravitino masses up to the same order of magnitude, 10^{12} GeV, are probed. As far as we know, no other possibility has been proposed to probe so high supersymmetry breaking scales, albeit indirectly.

Inflation does not only solve the horizon and flatness problems, but it also generates a scale invariant spectrum

of scalar and tensor (gravitational-waves) fluctuations, see e.g. [8]. Even though we know that the Universe was radiation dominated at BBN, its evolution history before that is unknown.

In the following we examine the impact of a moduli dominated phase on the inflationary gravitational-wave background and discuss the prospects for observations in the inflationary gravitational-wave background.

II. EVOLUTION OF GRAVITATIONAL WAVES FROM INFLATION

Gravitational waves are the tensor perturbations h_{ij} of the space-time metric,

$$ds^2 = a^2(\eta)(-d\eta^2 + (\delta_{ij} + 2h_{ij})dx^i dx^j), \quad (1)$$

where $a(\eta)$ denotes the cosmic scale factor. The perturbation h_{ij} is traceless, $h_i^i = 0$, and divergence free, $\partial^i h_{ij} = 0$. The conformal time η is defined by $d\eta = dt/a(t)$, where t denotes the physical time. The energy density of gravitational waves is then given by [9, 10]

$$\rho_{\text{gw}}(\mathbf{x}, t) = \frac{\langle \dot{h}_{ij}(\mathbf{x}, t) \dot{h}^{ij}(\mathbf{x}, t) \rangle}{8\pi G a^2}, \quad (2)$$

where G is Newton's constant. In this work an overdot indicates the derivative w.r.t conformal time η . In Fourier space, the evolution of a gravitational-wave mode h in a Friedmann universe (neglecting anisotropic stresses) is determined by [8]

$$\ddot{h} + 2\frac{\dot{a}}{a}\dot{h} + k^2 h = 0. \quad (3)$$

Introducing $x = k\eta$, and assuming power law expansion, $a \propto \eta^q$, this equation has the simple general solution

$$h = \frac{x}{a(\eta)} (c_1 j_{q-1}(x) + c_2 y_{q-1}(x)), \quad (4)$$

where j_n and y_n denote the spherical Bessel functions of order n as defined, e.g., in [11]. One might replace x/a by x^{1-q} and adjust the pre-factors correspondingly. With this replacement it becomes evident that on super-Hubble scales, $x < 1$, the j -mode is constant while the y -mode behaves as x^{-2q+1} . From this general solution together with Eq. (2) one infers that $\rho_{\text{gw}} \propto a^{-4}$ as soon as the wavelength is sub-Hubble, $x > 1$. (On super Hubble scales the 'energy density' of a mode is not a meaningful concept.)

It is reasonable to assume that both modes have similar amplitudes after inflation, where $x \ll 1$ for all modes of interest. If $q > 1/2$, the y -mode is decaying and soon after inflation we may approximate the solution by the j -mode. Note that a constant value of q corresponds to a constant background equation of state with the ratio of pressure P to energy density ρ given by

$$w = P/\rho \quad \text{and} \quad q = 2/(3w + 1).$$

For a non-inflating ($3w + 1 > 0$) universe, $q \geq 1/2$ corresponds to $w \leq 1$ and comprises all cases of interest. During inflation $-1/3 > w \gtrsim -1$ and $q \lesssim -1$.

In standard cosmology, the Universe is radiation dominated after reheating until the time of equality and matter dominated afterwards. Therefore $q = 1$ until equality, where $\rho_{\text{rad}} = \rho_{\text{mat}}$, and $q = 2$ after that. Since the energy density in gravitational waves scales like radiation, its fraction is constant on scales which enter the horizon during the radiation dominated era and scales like $a(\eta_k) \propto \eta_k^2 \propto 1/k^2$ for scales which enter during the matter dominated era. A good approximation to the transfer function $T_{\text{eq}}^2(k)$ which relates the energy density per logarithmic k -interval to the amplitude of the gravitational-wave spectrum after inflation in standard cosmology is given in [12]. With this we obtain (for simplicity we neglect changes in the number of effective degrees of freedom and the minor effect of today's vacuum domination)

$$\Omega_{\text{gw}}(k) \equiv \frac{1}{\rho_c} \frac{d\rho_{\text{gw}}(k)}{d\log(k)} = \Omega_{\text{rad}} \frac{r \Delta_{\mathcal{R}}^2}{12\pi^2} T_{\text{eq}}^2(k), \quad (5)$$

where

$$T_{\text{eq}}^2(k) = (1 + 1.57\eta_{\text{eq}}k + 3.42(\eta_{\text{eq}}k)^2)(\eta_{\text{cmb}}k)^{-2}. \quad (6)$$

Here $\Delta_{\mathcal{R}}$ is the amplitude of density fluctuations from inflation as measured in the CMB by the WMAP experiment [13], $\Delta_{\mathcal{R}}^2 \simeq 2 \times 10^{-9}$, and $\Omega_{\text{rad}} \simeq 5 \times 10^{-5}$. The ratio r is the tensor to scalar ratio which depends on the inflationary model, η_{eq} and η_{cmb} are the conformal time at matter-radiation equality and at CMB decoupling respectively. This standard spectrum is indicated by the dotted line in Fig. 1. For Eq. (6) a Harrison-Zel'dovich spectrum is assumed. For different primordial spectra with spectral index $n_s \neq 1$ and $n_T \neq 0$ of the primordial scalar and tensor fluctuations from inflation, the result (6) has to be multiplied by $(k/k_c)^{n_T}$ if the amplitude $\Delta_{\mathcal{R}}$ and r (which then are scale dependent) are determined at the pivot scale k_c . Changes in the background by non-standard evolution of the Universe have previously been studied in [14].

We now consider the situation that is likely to occur in superstring models, where soon after inflation moduli come to dominate when $\rho_\phi = \rho_{\text{rad}}$. We denote the corresponding (conformal) time by η_b . We assume that the moduli decay briefly before nucleosynthesis at time η_e . We then compute the final gravitational-wave spectrum by matching the radiation solution ($q = 1$) before η_b to the matter solution ($q = 2$) at η_b and back to the radiation solution at η_e . After η_e the Universe follows the standard evolution, so that the resulting spectrum simply has to be multiplied by the standard transfer function $T_{\text{eq}}^2(k)$. The generic shape of the resulting transfer function T is clear from the general solution: On super-Hubble scales the solution remains constant and $T = 1$. Scales that enter the horizon during the matter dominated phase at $\eta_b < \eta_k = 1/k < \eta_e$ are suppressed by a factor $a(\eta_k)/a(\eta_e) = (k\eta_e)^{-2}$ since $\rho_{\text{gw}} \propto a^{-4}$ while

$\rho_{\text{mat}} \propto a^{-3}$. Scales which have already entered before matter domination are maximally suppressed by a factor $a(\eta_b)/a(\eta_e) = (\eta_b/\eta_e)^2$.

For sufficiently long matter domination, $\eta_e/\eta_b \geq 4$, we find the following simple and accurate analytic approximation to the exact result for the transfer function of an intermediate matter dominated phase:

$$T^2(k; \eta_e, \eta_b) \simeq \frac{1}{\frac{\eta_b^2}{\eta_e^2} \left(\frac{2\pi c}{k\eta_b} - \frac{2\pi}{k\eta_e} + 1 \right)^{-2} + 1}, \quad (7)$$

where the best-fit analysis gives $c = 0.5$. The currently observable gravitational-wave spectrum is then simply

$$\Omega_{\text{gw}}(k) = \Omega_{\text{rad}} \frac{r \Delta_{\mathcal{R}}^2}{12\pi^2} T_{\text{eq}}^2(k) T^2(k; \eta_e, \eta_b), \quad (8)$$

with the fitting formula for $T(k; \eta_e, \eta_b)$ from Eq. (7). The time when matter domination begins, η_b , is determined by the mass of the modulus field. Up to a coupling constant of order one, the mass also determines the time of the modulus decay, η_e , which is the time when matter domination ends. Using the general formula [15] for conformal time,

$$\eta = 1.5 \times 10^5 \text{ s} \left(\frac{100 \text{ GeV}}{T} \right) g_{\text{eff}}^{-1/6}(T), \quad (9)$$

–and assuming a large enough reheating temperature $T_b \sim \sqrt{H_b m_{\text{pl}}}$, $H_b \sim m_\phi$ – we find for a moduli mass of $m_\phi \sim 100 \text{ TeV}$ a value of $\eta_b \sim 10^{-5} \text{ s}$. The resulting gravitational-wave background for $\eta_e \simeq \eta_{\text{BBN}}$ as expected from the usual moduli problem is indicated as the thick solid line in Fig. 1.

We have also indicated in Fig. 1 the results for moduli decaying much earlier, namely at $T_\phi^d \sim 10^6 \text{ GeV}$ (dashed line). Furthermore, for illustration we have indicated the suppression of the gravitational-wave spectrum in standard cosmology from a particle with 30 TeV mass that enters thermal equilibrium after inflation and decays before weakly-interacting massive particles (WIMP) freeze-out (thin solid line). In standard cosmology such signals might not only arise from any extension of the standard model containing long-lived particles like the axino or modulinos, but could even be desired [16].

We compare the spectra with present [17] and future [18] gravitational-wave experiments in Fig. 2; see footnotes and references therein for details on the experiments. It is important that the decay temperature even of a much earlier modulus decay might be inferred from observation. To compare to the signal of an intermediate reheating temperature see [19]. For larger moduli masses the moduli dynamics might depend directly on the reheating temperature. For the simplest potential, $V = m_\phi^2 \phi^2/2$, we estimate $T_R \sim 10^9 \text{ GeV}$ as the minimal reheating temperature corresponding to the sensitivity of WBO for our test to apply.

Interestingly, pulsar timing arrays probe the scale of BBN, where the Universe is surely radiation dominated.

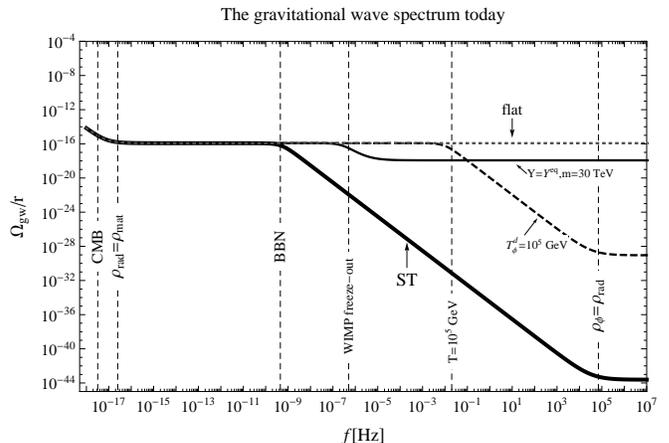


Figure 1: Intensity of inflationary gravitational waves vs. their frequency $f = k/(2\pi)$ observed today. The thick solid line shows the expectation from the usual cosmological moduli problem. The dashed line corresponds to the case of a much earlier modulus decay at $T_\phi^d = 10^6 \text{ GeV}$. For comparison the dotted line shows a perfectly flat spectrum. The thin solid line demonstrates the impact on the spectrum of a particle with 30 TeV mass that enters thermal equilibrium after inflation and decays before WIMP freeze-out. Various frequencies corresponding to important and suggestive scales are highlighted by the vertical dashed lines: CMB indicates the scale of best sensitivity of CMB experiments, and the other frequencies relate to the horizon scale at the indicated event.

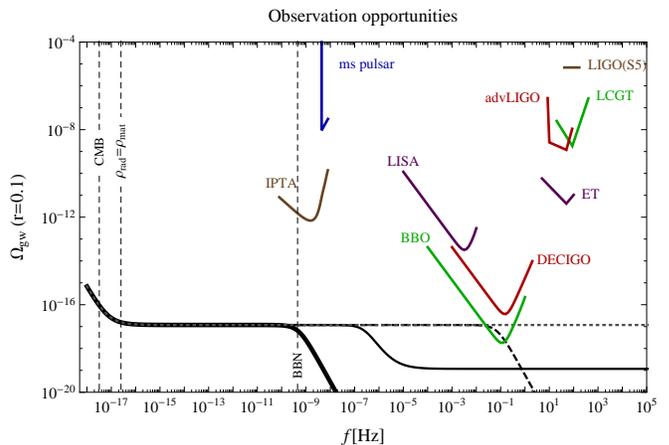


Figure 2: Observation opportunities for the spectra of Fig. 1. In addition to the spectra of Fig. 1, sensitivity curves of existing [17] and future [18] gravitational-wave observatories are plotted.

It would be particularly interesting, if they became sensitive to inflationary gravitational waves. The present CMB limit on r from [20, 21] is $r < 0.2$ on CMB scales.

III. CONCLUSIONS AND OUTLOOK

We have shown that the gravitational-wave background from inflation is strongly modified by an intermediate matter dominated era. If string compactification leads to an intermediate matter dominated phase, this will be observable in the gravitational-wave background: on CMB scales one will detect the unmodified background from inflation, e.g., with the Planck satellite [22] or with a future CMB polarimeter [23]. However, on higher frequencies like milli-Hz or Hz probed by the gravitational-wave detectors indicated in Fig. 2 no signal will be detected. In other words, if these experiments will detect the signal from the inflationary gravitational-wave background as expected from the CMB, this will rule out all string compactifications that contain at least one scalar with a mass $\lesssim 10^{12}$ GeV—corresponding to the sensitivity of BBO—that acquires a large initial oscillation amplitude after inflation and has only gravitational interaction strength. A correspondingly high supersymmetry breaking scale, for example, of the order of the GUT scale may well render superstring models unobservable.

Even though our derivation presented here is for an intermediate matter dominated phase, the qualitative result remains true also for a phase of thermal inflation. Such a phase would dilute gravitational waves on sub-Hubble scales even more strongly, and would render them undetectable for the experiments indicated in Fig. 2, while not affecting CMB scales. A (possibly) observable gravitational-wave spectrum created after thermal inflation were easily distinguishable from the primordial one by its shape [4].

Furthermore, from our derivation it is clear that for some other, non-inflationary, intermediate epoch starting at time η_b and ending at η_e , with equation of state $P = w\rho$ with $-1/3 < w \leq 1$, the inflationary gravitational-wave spectrum will be suppressed (or enhanced for $w > 1/3$) by a factor $\alpha(k)$ with

$$\Omega_{\text{gw}}^{\text{final}} = \alpha(k) \times \Omega_{\text{gw}}^{\text{std}} \quad \text{with}$$

$$\alpha(k) = \begin{cases} 1 & \text{if } k < \eta_e^{-1} \\ (k\eta_e)^{2(3w-1)/(3w+1)} & \text{if } \eta_e^{-1} < k < \eta_b^{-1} \\ \left(\frac{\eta_e}{\eta_b}\right)^{2(3w-1)/(3w+1)} & \text{if } \eta_e^{-1} < k. \end{cases} \quad (10)$$

Our fitting formula for $T(k)$ reproduces this behavior for an intermediate matter dominated era, $w = 0$. For $w \neq 0$ the exponents ± 2 in the denominator would have to be replaced by $\pm 2(1 - 3w)/(1 + 3w)$.

Of course there is the caveat that (as also many other inflationary models) inflation in models inspired by string theory [24], depending on the compactification and moduli stabilization, may predict only a very low gravitational-wave background that cannot be measured by proposed experiments, neither in the CMB nor directly on smaller scales. In this case, an experiment that would be able to detect the background generated in the intermediate matter dominated era as predicted in [25] should be conceived. Such a background is, however, suppressed with respect to the amplitude from inflation by the ratio of the corresponding Hubble rates $(H_b/H_{\text{inf}})^2$, where H_b denotes the Hubble rate at the beginning of the intermediate matter dominated phase and H_{inf} the one during inflation.

In conclusion, we have shown that combining future CMB polarization measurements with very sensitive gravitational-wave probes such as BBO can provide a crucial test for a large class of string models. Since string theory is our best candidate for a fundamental theory of Nature, and there has not been proposed any experiment to test it, it is of uttermost importance that we realize a BBO-like experiment.

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- [17] Existing measurements are LIGO(S5) (Laser Interferometer Gravitational wave Observatory) [26] and the millisecond (ms) pulsar bound [27]. The Parkes- and European Pulsar Timing Array have recently reported sensitivities [28].
- [18] Future ground-based observatories are advLIGO [29] (we have scaled up the sensitivity curve found in this ref. by roughly a factor of 10, which corresponds to the ratio between the actual LIGO(S5) sensitivity and the one predicted in [29]), LCGT (Large Scale Cryogenic Gravitational wave Telescope) [30] and ET (Einstein Telescope) [31]. The limits shown in Fig. 2 assume two-detector correlations with an observation time $t_{\text{obs}} = 4$ months.
- Proposed future satellite missions are LISA (Laser Interferometer Space Antenna) [32], BBO (Big Bang Observatory) [34] and DECIGO (DECI-hertz interferometer Gravitational wave Observatory) [33]. For the satellite missions we assume $t_{\text{obs}} = 10$ y.
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