The slow roll condition and the amplitude of the primordial spectrum of cosmic fluctuations: Contrasts and similarities of standard account and the "collapse scheme".

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The inflationary paradigm enjoys a very wide acceptance in the cosmological community, due in large part to the fact that it is said to "naturally account" for a nearly scale independent power primordial spectrum of fluctuations which is in very good agreement with the observations. The expected overall scale of the fluctuations in most models, turns out to be too large, because it is inversely proportional to the slow roll parameter, which is expected to be very small. This fact requires the fine tuning of the inflaton potential. In series of recent works it has been argued that the success of the inflationary picture is not fully justified in terms of the rules of quantum theory as applied to the cosmological setting and that an extra element, something akin to a self induced collapse of the wave function is required. There, it was suggested that the incorporation of such collapse in the treatment might avoid the need for fine tuning of the potential that afflicts most inflationary models. In this article we will discuss in detail the manner in which one obtains the estimation of the magnitude of the perturbations in the new scheme and that of the standard accounts, comparing one of the most popular among the later and that corresponding to the new proposal. We will see that the proposal includes a collapse scheme that bypasses the problem, but we will see that the price seems to a be a teleological, and thus unphysical, fine tuning of the characteristics of collapse.

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I. INTRODUCTION

The status of the standard inflationary scenario among cosmologists has been dramatically enhanced by recent advances in the observations, such as Supernova Surveys [1], the studies of large scale structure [2] and those arising from the Wilkinson Microwave Anisotropy Probe (WMAP)[3]. The model was initially proposed by A. Guth in 1981 [4] in order to deal with the shortcomings of the standard Big Bang cosmology: namely the Flatness Problem, the Horizon Problem and the Unwanted Relics Problem. The essential idea is that if the Universe undergoes an early era of accelerated expansion (lasting at least some 80 e-folds or so), it would come out of this period as an essentially flat, homogeneous and isotropic space-time with an extreme dilution of all unwanted relics. A more dramatic consequence [5, 6] is that when considering the quantum aspect of the scalar field driving the inflation (the inflaton field), which is assumed to be in the vacuum state as a result of the same exponential expansion, one finds that it contains "fluctuations" with the appropriate scale-free -Harrizon-Zeldovich spectrum. These vacuum fluctuations, are thus considered responsible for all the structure we observe in the actual universe, and in particular, the observed Cosmic Microwave Background (CMB) anisotropies. The shape of the observed spectrum turns out to be in excellent agreement -when adjusted to take into account well established physics of plasma oscillations and related phenomena- with a very flat primordial spectrum fluctuations. The shape of the spectrum that results from the general picture emerging from these theoretical considerations, is controlled by the so called "slow roll parameter" ϵ (to be precisely defined below), in such a way that the spectrum becomes flatter as that parameter is decreased, on the other hand the overall scale is inversely proportional to the parameter ϵ indicating that it can not be made too small.

The details of the model, such as the form of the potential, the number of fields, etc, continues to be a subject of extensive research, while much less attention has been devoted to the question on how exactly does the universe transit from a homogeneous and isotropic stage to one where the quantum uncertainties, which are after all still homogeneous and isotropic, become actual inhomogeneities.

In this regard, a recent series of works [7, 8] have argued that the standard inflationary paradigm posees an important shortcoming. The point is that the scheme by itself does not provide a fully satisfactory explanation for emergence of the seeds of structure, as there is nothing in the theory that could account for the transmutation of the initial[1] homogeneous and isotropic situation (as described by the background and the quantum state of the fields corresponding to the early universe) into an in-homogeneous and an-isotropic situation corresponding to the latter state representing the actual universe we observe. *Quantum decoherence* has been a constant reference in the attempts to deal with this issue (also referred as the "quantum to classical transition" [9]), but a careful analysis of the resulting accounts indicates that such approaches fail to provide a true resolution [10].

We will not dwell here on these conceptual issues and will only illustrate the problem by quoting from page 476 of Cosmology by S. Weinberg [11]:

> *These are quantum averages, not averages over an ensemble of classical field configurations. ... Just as in the measurement of a spin in the laboratory, some sort of decoherence must set in; the field configurations must become locked into one of an ensemble of classical configurations, ... It is not apparent just how this happens, ...*

Regarding a widespread belief that one can address the issue invoking decoherence, we will again limit ourselves, to just quote from pages 348, 349 of Physical Foundations of Cosmology by V. Mukhanov [12]:

> *How do quantum fluctuations become classical? ... Decoherence is a necessary condition for the emergence of classical inhomogeneities and can easily be justified for amplified cosmological perturbations. However, decoherence is not sufficient ... It can be shown that as a result of unitary evolution we obtain a state which is a superposition of many macroscopically different states, each corresponding to a particular realization of galaxy distribution. Many of these realizations have the same statistical properties. ... Therefore, to pick an observed macroscopic state from the superposition we have to appeal either to Bohr's reduction postulate or to Everett's many-worlds interpretation of quantum mechanics. The first possibility does not look convincing in the cosmological context....*

Thus it is clear that the standard description, within inflationary cosmology, of the origin of the seeds of cosmic structure, is far from providing a fully satisfactory account and faces important conceptual shortcomings.

In order to overcome such shortcoming of the standard inflationary model, the authors in [7] introduce a new ingredient to the inflationary paradigm: the *self induced collapse hypothesis*: a phenomenological model incorporating

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[1] By this we refer here to the state of the universe after just a few e-folds of inflationary expansion, and not to the state that precedes inflationary regime, presumably emerging from the Planck regime, and which is expected to involve all sorts of inhomogeneities and defects, which inflation is assume to erase.

the description of the effects of a dynamical collapse of the wave function of the inflaton on the subsequent cosmological evolution. The idea is inspired by R. Penrose's arguments in the sense that the unification of quantum theory and the theory of gravitation would likely involve modifications in both theories, rather than only the latter as is more frequently assumed. Moreover the idea is that the resulting modifications of the former should involve something akin to a self-induced collapse of the wave-function occurring when the matter fields are in a quantum superposition that would lead to corresponding space-time geometries which are "too different among themselves". This sort of self induced collapse would in fact be occurring in rather common situations, and would ultimately resolve the long standing problem known as the "measurement problem" in quantum mechanics [13, 14]. We will not further discuss these motivations here. In our treatment, which can be seen as an attempt to realize these ideas in the cosmological setting, as a means to resolve the above mentioned shortcoming, the actual formalism must be considered as an effective description of the fundamentally quantum gravitational mechanism, which, in our situation leads to the transition from the symmetric vacuum state to the asymmetrical (the symmetry being homogeneity and isotropy) latter state. At this stage the analysis should be seen as a purely phenomenological scheme, in the sense that it does not attempt to explain such collapse in terms of some specific new physical mechanism, but merely gives a rather general parametrization of such a transition. We will refer to this phenomenological model as the *collapse scheme*. We will not further recapitulate the motivations and discussion of the original proposal and instead refer the reader to the above mentioned works.

The issue that concern us in this paper is the fact that in order to obtain a suitable fluctuation spectrum, the standard inflationary scenarios require the slow roll parameter, to be on the one hand small enough to give a flat spectrum, and on the other hand has to be large enough to ensure that the fluctuations are as "small" as observed. Sometimes this is presented as indicating that the scale of the inflaton potential has to be carefully fine tuned. In some early works on the collapse scheme it was argued [7, 8] that the new approach seemed to offered a possibility to solve that fine-tuning problem. This is the subject that will occupy us here.

In this manuscript, we will review the manner in which one obtains the estimation of the magnitude of the perturbations both in the standard approach and compare it with the corresponding analysis in the collapse scheme. We will see that, in the standard paradigm, one obtains an expression for the "power spectrum" which is proportional to V/ϵ. On the other hand, in the collapse scheme, one can find a very particular set of characteristics for the collapse that would indeed avoid the need for that adjustment. That is, the collapse scheme admits a specific model for the collapse which leads to a fluctuation spectrum with the shape and the amplitude which move together in the appropriate direction as ϵ is decreased: the flatter the spectrum the smaller its amplitude. This is in accord with the earlier suggestions, however as it will be shown here, this is not a generic feature of the collapse scheme, and in most cases one will end up with an amplitude proportional to V/ϵ. At this time we see no natural way by which the mechanism would select the particular characteristics, and the only way of requiring such behavior seems to involve an adjustment of the parameters of the collapse to the details of the reheating, in what can only be described as a teleological arrangement.

The article is organized as follows: In Section II we review the standard description of the inflationary scenario, making special emphasis in the steps where aspects connected with the scale of the resulting spectrum make their appearance, ending with the calculation of the power spectrum of the metric fluctuations. In section III we review the earlier analysis leading to the suggestion that the collapse scheme might naturally resolve the problem. In Section IV we briefly review the quantum mechanical treatment of the field's fluctuations within the collapse scheme and show how the proposal, offers a path that changes the conclusions, a path however that, as indicted above, seems at this point rather unconvincing. In Section V we end with a brief discussion of our conclusions.

Regarding notation we will use signature $(- + ++)$ for the metric and Wald's convention for the Riemann tensor.

II. THE STANDARD INFLATIONARY SCENARIO

This section will briefly review the standard inflationary scenario following closely Chapter 8 of [12]. We elected to focus on this reference because it serves as a pedagogical introduction to the subject and because [6] is considered as one of the standard references on inflationary quantum perturbations. We will pay particular attention to aspects connected to the estimation of the overall scale of the fluctuation spectrum.

In the standard inflationary model the early universe is dominated by a scalar field ϕ with a particular potential $V(\phi)$ called the inflaton. This potential acts as a cosmological constant, which is later "turned off" (when the inflaton reaches the zero of the potential) as a result of the scalar field dynamics, followed by a reheating period "bringing back" the universe to the standard Big Bang cosmological evolutionary path.

The model is characterized by the action of a scalar field coupled to gravity:

$$S[\phi] = \int d^4x\sqrt{-g}\left[\frac{1}{16\pi G}R[g] - \frac{1}{2}\nabla_a\phi\nabla_b\phi g^{ab} - V[\phi]\right] \tag{1}$$

The analysis starts with a background space-time that, as the result of the exponential inflationary expansion, has been driven to a homogeneous and isotropic stage, characterized by the space time geometry described, in accordance with the general inflationary paradigm, by the spatially flat RW cosmology and a background scalar field:

$$ds^2 = a^2(\eta)[-d\eta^2 + \delta_{ij}dx^i dx^j], \qquad \phi_0(\eta), \tag{2}$$

The background metric and the field up to this point represent an homogeneous-isotropic classical *background* (or "expectation value"). Next one considers the quantum aspect of the field $\delta\phi(\mathbf{x},\eta)$, representing the *quantum fluctuations*, and their effect of the space-time geometry δg_{ab}.

The energy momentum tensor involves both the inflaton field and radiation. The first dominating the early inflationary era which ends in a reheating period after which the radiation dominates. The treatment can be done separately for each regime, but one can simplify the treatment by using in general the expressions appropriate for the perfect fluid with energy-momentum tensor $T_{ab} = (\rho + P)U_aU_b + Pg_{ab}$ and when considering the inflaton's contributions using the identification: $\rho = X + V$, $P = X - V$, where $X \equiv -\frac{1}{2}g^{ab}\partial_a\phi\partial_b\phi$ and $U^a = -g^{ab}\partial_b\phi/\sqrt{2X}$.

Einstein's equations for the background are written as $G^{(0)}_{00} = 8\pi GT^{(0)}_{00} = 8\pi Ga^2\rho$ and $G^{(0)}_{ii} = 8\pi GT^{(0)}_{ii} = 8\pi Ga^2P$ and yield Friedmann's equations:

$$3\mathcal{H}^2 = 8\pi Ga^2\rho, \qquad -2\mathcal{H}' - \mathcal{H}^2 = 8\pi Ga^2P \tag{3}$$

where $\mathcal{H} \equiv a'(\eta)/a(\eta)$; the prime denotes derivative with respect to the conformal time η; ρ is the overall energy density while P is the overall pressure.

Friedmann's equations (3) can be combined to yield a useful expression for $\rho + P$:

$$\mathcal{H}^2 - \mathcal{H}' = 4\pi Ga^2(\rho + P) \tag{4}$$

As indicated, in the general setting these densities and pressures include the contributions of both the inflaton field and of other forms of matter and radiation that might be present.

Thus for instance, during the early inflationary phase, the only contributions to ρ and P are those corresponding to the inflaton:

$$\rho = \frac{(\phi_0')^2}{2a^2} + V(\phi_0), \qquad P = \frac{(\phi_0')^2}{2a^2} - V(\phi_0), \qquad U^a = \frac{1}{a(\eta)}(\frac{\partial}{\partial\eta})^a. \tag{5}$$

The evolution equation for ϕ_0 is:

$$\phi_0'' + 2\mathcal{H}\phi_0' + a^2\partial_\phi V = 0 \tag{6}$$

The equations above lead to the standard inflationary regime, which written using conformal time, is characterized by a scale factor $a(\eta) \approx -1/[H_I(1-\epsilon)\eta]$, with $H_I^2 \approx 8\pi G/(3V)$; $\epsilon \equiv 1 - \mathcal{H}'/\mathcal{H}^2$ the slow-roll parameter (characterized by $\epsilon \ll 1$ during inflation) and with the scalar field ϕ_0 in the slow roll regime so $\phi_0' = -(a^3/3a')\partial_\phi V$. This era is supposed to end while giving rise to a "reheating period" whereby the universe is repopulated with ordinary matter fields, and then, to a standard hot big bang cosmology leading up to the present cosmological time.

The normalization of the scale factor will be set so $a = 1$ at the "present cosmological time". The inflationary regime would end at $\eta = \eta_r$, a value which is negative and very small in absolute terms ($\eta_r \approx -10^{-22}$), that is, the conformal time during the inflationary era is in the range $-\infty < \eta < \eta_r$, thus $\eta = 0$ is a particular value of the conformal time that does not correspond to the inflationary period, in fact it belongs to the radiation dominated epoch. The scale factor evaluated at the end of the inflationary regime would be denoted as $a_r \equiv a(\eta_r)$.

Next one considers Einstein's equations to first order in the perturbations. According to the usual approach, at this point it is argued that these quantum fluctuations will result in the space-time metric developing anisotropies and inhomogeneities. It is customary to decompose the metric fluctuations in terms of its scalar, vector, and tensor components. In the case of the Einstein-inflaton system, we need to concern ourselves only with scalar and tensor perturbations. In fact the latter will be ignored by simplicity, therefore we can write the perturbed metric (in the conformal gauge) as:

$$ds^2 = a(\eta)^2[-(1+2\Phi)d\eta^2 + (1-2\Psi)\delta_{ij}dx^i dx^j] \tag{7}$$

where Φ and Ψ are functions of the space-time coordinates η, x^i, with the former referred to as the Newtonian potential.

The equations $\delta G^0_0 = 8\pi G \delta T^0_0$ and $\delta G^0_i = 8\pi G\delta T^0_i$, are given respectively by

$$\nabla^2\Psi - 3\mathcal{H}(\mathcal{H}\Phi + \Psi') = -4\pi G a^2 \delta T^0_0 \tag{8}$$

$$\partial_i(\mathcal{H}\Phi + \Psi') = -4\pi G a^2 \delta T^0_i \tag{9}$$

On the other hand, equation $\delta G^i_j = 8\pi G\delta T^i_j$ is:

$$[\Psi'' + \mathcal{H}(2\Psi + \Phi)' + (2\mathcal{H}' + \mathcal{H}^2)\Phi + \tfrac{1}{2}\nabla^2(\Phi - \Psi)]\delta^i_j - \tfrac{1}{2}\partial^i\partial_j(\Phi - \Psi) = 4\pi G a^2 \delta T^i_j \tag{10}$$

For the situations of interest in this work, we can write generically $\delta T^0_0 = -\delta\rho$, $\delta T^0_i = (\rho+P)U^0\delta U_i$ and $\delta T^i_j = \delta P \delta^i_j$.

It is easy to see that consideration of (10) for the case $i \neq j$, together with appropriate boundary conditions (more easily seen in the Fourier transformed version) leads to $\Psi = \Phi$. From now on we will use this result.

For generic hydrodynamical matter, the pressure is a function of both the energy density ρ and the "entropy per baryon" S, and hence:

$$\delta P = c_s^2 \delta\rho + \tau\delta S \tag{11}$$

with $c_s^2 \equiv (\partial P/\partial\rho)_S$ the adiabatic speed of sound and $\tau \equiv (\partial P/\partial S)_\rho$. The expressions (8), (10) and (11) can be combined to yield the following equation of motion for Ψ

$$\Psi'' - c_s^2\nabla^2\Psi + 3\mathcal{H}(1 + c_s^2)\Psi' + [2\mathcal{H}' + \mathcal{H}^2(1 + 3c_s)]\Psi = 4\pi G\tau\delta S \tag{12}$$

It will be convenient to recast (12) in a slightly different form, with the introduction of the new variable:

$$u \equiv \frac{\Psi}{4\pi G\sqrt{\rho + p}} \tag{13}$$

Using the continuity equation $\rho' = -3\mathcal{H}(\rho+P)$ and the definition of c_s^2 one finds that $1+c_s^2 = -(\rho'+P')/[3\mathcal{H}(\rho+P)]$. Substituting this last expression in (12) and changing the variable Ψ for u, the evolution equation for u can be written in the form:

$$u'' - c_s^2\nabla^2 u - \frac{\theta''}{\theta}u = \mathcal{N} \tag{14}$$

where $\theta \equiv \mathcal{H}/a^2(\rho + P)^{\frac{1}{2}}$ and $\mathcal{N} = a^2\tau\delta S/(\rho + P)^{\frac{1}{2}}$. The motion equation (14) or equivalently (12) is very useful because it allow us to determine the evolution of perturbations in a hydrodynamical universe, and are the main result of section 5 in [6].

For the inflationary era, there is an equation similar to (12), which can be obtained by following the corresponding steps that lead us to the later. That is, from the general expression for the energy-momentum tensor for the inflaton. $T^a_b = g^{ac}\partial_c\phi\partial_b\phi + \delta^a_b(-\frac{1}{2}g^{cd}\partial_c\phi\partial_d\phi - V(\phi))$ one obtains the components of the linear perturbations:

$$\delta T^0_0 = a^{-2}[\phi_0'^2\Phi - \phi_0'\delta\phi' - \partial_\phi V a^2\delta\phi], \qquad \delta T^0_i = \partial_i(-a^{-2}\phi_0'\delta\phi) \tag{15a}$$

$$\delta T^i_j = a^{-2}[\phi'_0\delta\phi' - \phi'^2_0\Phi - \partial_\phi V a^2\delta\phi]\delta^i_j \tag{15b}$$

Thus, Einstein's equations for the metric perturbations (8), (9), (10) take the form:

$$\nabla^2\Psi - 3\mathcal{H}(\mathcal{H}\Phi + \Psi') = 4\pi G[-\phi'^2_0\Phi + \phi'_0\delta\phi' + \partial_\phi V a^2\delta\phi] \tag{16}$$

$$\partial_i(\mathcal{H}\Phi + \Psi') = 4\pi G\partial_i(\phi'_0\delta\phi) \tag{17}$$

$$\begin{aligned}[\Psi'' + \mathcal{H}(2\Psi + \Phi)' + (2\mathcal{H}' + \mathcal{H}^2)\Phi + \tfrac{1}{2}\nabla^2(\Phi - \Psi)]\delta^i_j - \tfrac{1}{2}\partial^i\partial_j(\Phi - \Psi) = \\ 4\pi G[\phi'_0\delta\phi' - \phi'^2_0\Phi - \partial_\phi V a^2\delta\phi]\delta^i_j\end{aligned} \tag{18}$$

Subtracting (16) from (18), using: $\Psi = \Phi$, the equation for the background field (6) and (17), one finds the following equation of motion for Ψ during the inflationary regime:

$$\Psi'' - \nabla^2\Psi + 2\left(\mathcal{H} - \frac{\phi''_0}{\phi'_0}\right)\Psi' + 2\left(\mathcal{H}' - \frac{\mathcal{H}\phi''_0}{\phi'_0}\right)\Psi = 0 \tag{19}$$

Thus, by setting

$$c_s^2 = \frac{-1}{3}\left(1 - \frac{2\phi''_0}{\mathcal{H}\phi'_0}\right) \qquad \text{and} \qquad \tau\delta S = \frac{(1 - c_s^2)\nabla^2\Psi}{4\pi G a^2} \tag{20}$$

in (12), one obtains (19). Equivalently, substituting (20) in (14) one obtains

$$u'' - \nabla^2 u - \frac{\theta''}{\theta}u = 0 \tag{21}$$

Thus we have an equation for the Newtonian potential, (12) (or equivalently (14)) which is valid in the inflationary or matter dominated regimes, as long as in the case of the former one uses the expressions for the "adiabatic sound speed" and the "perturbations to the entropy" as given by (20). One should note however, for the case of the scalar field, the thermodynamical notions such as "equation of state", "entropy per baryon" or "sound speed" must be interpreted with care, as in that case, we are not really dealing with hydrodynamical matter (see [15] for a detailed discussion).

We will now proceed to the treatment of the quantum part of the model, by focusing on the quantum "fluctuations" of the inflaton. The proceeding discussion will follow the analysis presented in [12].

We note that by taking into account the equation of motion for the background field (6) and (5), one can rewrite Einstein's equations (16) and (17) as:

$$\nabla^2\Psi - 3\mathcal{H}(\Psi' + \mathcal{H}\Psi) = 4\pi G a^2(\rho + P)\left[\left(\frac{\delta\phi}{\phi'_0}\right)' - \Psi - 2\mathcal{H}\frac{\delta\phi}{\phi'_0}\right] \tag{22}$$

$$\Psi' + \mathcal{H}\Psi = 4\pi G a^2(\rho + P)\left(\frac{\delta\phi}{\phi'_0}\right) \tag{23}$$

Equation (23), together with (4), can be used to obtain:

$$\left(a^2\frac{\Psi}{\mathcal{H}}\right)' = \frac{4\pi G a^4(\rho + P)}{\mathcal{H}^2}\left(\mathcal{H}\frac{\delta\phi}{\phi'_0} + \Psi\right) \tag{24}$$

while (22), together with (3), (4) and (23), can be used to obtain:

$$\nabla^2\Psi = \frac{4\pi G a^2(\rho+P)}{\mathcal{H}}\left(\mathcal{H}\frac{\delta\phi}{\phi_0'}+\Psi\right)'. \tag{25}$$

Next one notes that (24) and (25), can be expressed in a convenient way in terms of the variable u (introduced in (13)) and a new variable v:

$$v \equiv a\left(\delta\phi + \frac{\phi_0'}{\mathcal{H}}\Psi\right) \tag{26}$$

whereby they take the simple form:

$$\nabla^2 u = z\left(\frac{v}{z}\right)', \qquad v = \theta\left(\frac{u}{\theta}\right)' \tag{27}$$

with $z \equiv \theta^{-1} \equiv a^2(\rho+P)^{1/2}\mathcal{H}^{-1}$. The next step in the usual approach is to write the quantum theory for these field variables. This is done writing the second order perturbation expansion for the action of the gravitational and scalar fields about the background. Dropping total derivative terms, the result reduces to a simple expression containing only the variable v.

$$S \equiv \int d\eta d^3x\mathcal{L} = \frac{1}{2}\int d\eta d^3x\left(v'^2 + v\nabla^2 v + \frac{z''}{z}v^2\right) \tag{28}$$

The details of these calculations can be seen in section 10 of [6]. This action leads to the following simple equation of motion for v,

$$v'' - \nabla^2 v - \frac{z''}{z}v = 0 \tag{29}$$

indicating (see page 341 of [12]) that the quantization of cosmological perturbations with the action (28) is formally equivalent to the quantization of a "free scalar field" v with time-dependent "mass" $m^2 = -z''/z$ in Minkowski space.

We note that the variable v that in this approach is subjected to the canonical quantization procedure, is a combination of the scalar field and metric perturbations.

The canonical momentum π conjugated to v is $\pi \equiv \frac{\partial\mathcal{L}}{\partial v'} = v'$ and the standard "equal time" commutation relations among the corresponding operators $\hat{v}$ y $\hat{\pi}$ are: $[\hat{v}(\eta,x),\hat{v}(\eta,y)] = [\hat{\pi}(\eta,x),\hat{\pi}(\eta,y)] = 0, \quad [\hat{v}(\eta,x),\hat{\pi}(\eta,y)] = i\delta(x-y)$. The general solution for the operator version of the equation of motion (29), (as appropriate when viewing the quantum theory in the Heissenberg picture) can be written as:

$$\hat{v}(\eta,x) = \frac{1}{\sqrt{2}}\int\frac{d^3k}{(2\pi)^{3/2}}[v_k^\star(\eta)e^{ikx}\hat{a}_k^- + v_k(\eta)e^{-ikx}\hat{a}_k^+] \tag{30}$$

Where the temporal mode functions $v_k(\eta)$ satisfy:

$$v_k'' + \omega_k^2(\eta)v_k = 0, \qquad \omega_k^2 \equiv k^2 - z''/z \tag{31}$$

The normalization condition for the mode functions $v_k(\eta)$ are chosen so that: $v_k'v_k^\star - v_k v_k^{\star\prime} = 2i$, which leads to the standard commutation relations for the annihilation and creation operators $\hat{a}_k^-$ y $\hat{a}_k^+$:

$$[\hat{a}_k^-,\hat{a}_{k'}^-] = [\hat{a}_k^+,\hat{a}_{k'}^+] = 0, \qquad [\hat{a}_k^-,\hat{a}_{k'}^+] = \delta(k-k') \tag{32}$$

The construction is fully defined by the selection of specific mode functions specified at an "extremely early" time (η_i) by the initial conditions: $v_k(\eta_i) = w_k^{-1/2}$, $v_k'(\eta_i) = iw_k^{1/2}$ leading to a construction of the quantum theory where the vacuum state is the so called Bunch-Davies vacuum [16], which is supposed to describe the state of the quantum field, well into the inflationary regime.

Next one proceeds to evaluate the two point correlation function, and then interprets it as the power spectrum of the gravitational potential. In other words, the analysis is based on the identification:

$$\langle 0|\hat{\Psi}(\eta,x)\hat{\Psi}(\eta,y)|0\rangle \equiv \overline{\Psi(\eta,x)\Psi(\eta,y)} \tag{33}$$

from which one obtains the "prediction" for the primordial spectrum of cosmic structure. The problems related to the justification of such identification are closely connected with the conceptual problem we mentioned in the introduction, and as indicated will not be further discussed here. We will instead focus on obtaining the estimation of the overall scale of the power spectrum in the standard approach:

From (13), (27) and (30) we obtain an expression for the operator $\hat{\Psi}$:

$$\hat{\Psi}(\eta,x) = \frac{4\pi G(\rho+P)^{1/2}}{\sqrt{2}} \int \frac{d^3k}{(2\pi)^{3/2}} [u_k^\star(\eta) e^{ikx} \hat{a}_k^- + u_k(\eta) e^{-ikx} \hat{a}_k^+] \tag{34}$$

where the mode functions $u_k(\eta)$ satisfy:

$$u_k'' + \left(k^2 - \frac{\theta''}{\theta}\right) u_k = 0, \tag{35}$$

the Fourier's version of (21).

The vacuum expectation value for the operators $\hat{\Psi}(\eta,x)\hat{\Psi}(\eta,y)$ is then:

$$\langle 0|\hat{\Psi}(\eta,x)\hat{\Psi}(\eta,y)|0\rangle = \int \frac{dk}{k} 4G^2(\rho+P)|u_k|^2 k^3 \frac{\sin kr}{kr} \tag{36}$$

where $r \equiv |x-y|$. Given that the definition of the power spectrum of a gaussian random field is :

$$\overline{\Psi(x,\eta)\Psi(y,\eta)} = \int \frac{dk}{k} \mathcal{P}_\Psi(k,\eta) \frac{\sin kr}{kr} \tag{37}$$

one can read the power spectrum of the metric perturbations directly from (36):

$$\mathcal{P}_\Psi(k,\eta) = 4G^2(\rho+P)|u_k(\eta)|^2 k^3 \tag{38}$$

The reminder of the calculation can be summarized as follows: given the initial conditions $v_k(\eta_i)$ and $v_k'(\eta_i)$ set by the "Bunch Davis vacuum" one obtains the initial conditions $u_k(\eta_i)$ and $u_k'(\eta_i)$ from (27). Given those, one solves (35) and uses the resulting function $u_k(\eta)$ in the expression (38) of the power spectrum. The equation for $u_k(\eta)$ (35) is usually solved by considering an early situation where the physical scale of the mode is "well inside of the Hubble radius" $k \gg aH$ (short-wavelength) and then connecting the solution to that corresponding to the latter regime where the physical scale of the mode is "outside of the Hubble radius" $k \ll aH$ (long-wavelength). Let us review this series of steps.

The initial condition for $u_k(\eta_i)$ and $u_k'(\eta_i)$ obtained from (27) are: $u_k(\eta_i) = -ik^{-3/2}$, $u_k'(\eta_i) = k^{-1/2}$ where it was used the short-wavelength approximation because all modes fall in that regime at an early enough time. The solution of the equation for $u_k(\eta)$ ((35)) in this regime ($k^2 \gg \mathcal{H}^2 \gg \theta''/\theta$), during the inflationary era, given the above initial conditions is:

$$u_k(\eta) = \frac{-i}{k^{3/2}} e^{ik(\eta-\eta_i)} \tag{39}$$

In the regime of the long-wavelength approximation $k^2 \ll \theta''/\theta \ll \mathcal{H}^2$, and for all eras, (35) takes the form:

$$u_k''(\eta) - \frac{\theta''(\eta)}{\theta(\eta)} u_k(\eta) = 0 \tag{40}$$

It is worthwhile emphasizing that (40) can be obtained from (14) assuming the adiabaticity of the perturbations $\delta S = 0$, in the long-wavelength approximation ($c_s^2 k^2 \ll \theta''/\theta$). That is, under these last two assumptions (40) is valid even after inflation ends, because (14) was obtained independently from any assumption limiting us to particular cosmological epoch. Thus, the solutions to (40) can be used either for the inflationary or the radiation dominated epochs.

The general solution for (40) is:

$$u_k(\eta) = \alpha_k \theta(\eta) + \beta_k \theta(\eta) \int_{\eta_0}^{\eta} \frac{d\tilde{\eta}}{\theta^2(\tilde{\eta})} \tag{41}$$

where α_k and β_k are constants of integration that will be determined later on and η_0 is fixed value of the conformal time. Note that in evaluating the integral in the second term of (41), the contribution from the lower integration limit at η_0, is $C\theta$ where C is a constant. This contribution can thus, be absorbed in the first term, and therefore will be ignored from now on.

Next, making use of (4) one finds a new expression for θ:

$$\theta \equiv \frac{\mathcal{H}}{a^2(\rho + P)^{\frac{1}{2}}} = \frac{\sqrt{4\pi G}}{a}\left(1 - \frac{\mathcal{H}'}{\mathcal{H}^2}\right)^{-\frac{1}{2}} \tag{42}$$

integrating by parts in (41), one obtains:

$$u_k(\eta) = \alpha_k \theta(\eta) + \frac{\beta_k \theta(\eta)}{4\pi G}\left(\frac{a^2(\eta)}{\mathcal{H}(\eta)} - \int^{\eta} a^2(\tilde{\eta}) d\tilde{\eta}\right) \tag{43}$$

The constants of integration α_k and β_k are determined by joining solutions (43) and (39) (requiring continuity of the functions and their first derivatives), at the time of transition from the regime of validity of the short-wavelength approximation to that of the long-wavelength approximation, which is called "the horizon crossing", that is, at $aH = k$. This calculation relies on the fact that for perturbations of interest (those which we observe in the CMB and in the large scale structure), "the horizon crossing" occurred (according to the standard view) when the universe was still in the inflationary phase.

Therefore, one can use the slow roll conditions: $3\mathcal{H}\phi_0' = -a^2 \partial_\phi V$ and $\mathcal{H}^2 = a^2 V/3M_{pl}^2$ in the evaluation of the integral in (43), thus obtaining:

$$u_k(\eta) = \frac{\alpha_k}{M_{pl}\sqrt{2}a}\left(1 - \frac{\mathcal{H}'}{\mathcal{H}^2}\right)^{-\frac{1}{2}} + \beta_k M_{pl} \frac{\sqrt{2}a}{\mathcal{H}}\left(1 - \frac{\mathcal{H}^2}{\mathcal{H}'}\right)^{\frac{1}{2}} = \frac{\alpha_k}{M_{pl}\sqrt{2\epsilon}a} + \beta_k M_{pl}^2 \sqrt{\frac{6\epsilon}{V}} \tag{44}$$

where one has introduced the slow-roll parameter $\epsilon \equiv \frac{1}{2} M_{pl}^2 (\partial_\phi V)^2/V^2 = 1 - \mathcal{H}'/\mathcal{H}^2$ which as its name indicates is assumed to be small $\epsilon \ll 1$, and where we have neglected terms of order $\mathcal{O}(\epsilon^2)$ in the evaluation of the integral in (43). Note that the approximation for $u_k(\eta)$ given by (44) is only valid during inflation.

From (41) we find $u_k'(\eta)$:

$$u_k'(\eta) = \frac{d}{d\eta}\left(\alpha_k \theta(\eta) + \beta_k \theta(\eta) \int^{\eta} \frac{d\tilde{\eta}}{\theta^2(\tilde{\eta})}\right) = \left(\alpha_k + \beta_k \int^{\eta} \frac{d\tilde{\eta}}{\theta^2}\right)\theta' + \frac{\beta_k}{\theta} \tag{45}$$

The quantities θ and θ' are obtained directly from consideration of (42) during the inflationary regime. Neglecting terms of order $\mathcal{O}(\epsilon^2)$ one finds:

$$\begin{aligned} u_k'(\eta) &= -\alpha_k \left(2 - \frac{\mathcal{H}^2}{\mathcal{H}'}\right) \frac{\mathcal{H}}{aM_{pl}\sqrt{2}} \left(1 - \frac{\mathcal{H}'}{\mathcal{H}^2}\right)^{-\frac{1}{2}} - \beta_k \left(1 - \frac{\mathcal{H}'}{\mathcal{H}^2}\right)^{\frac{3}{2}} \sqrt{2} M_{pl} a \\ &= -\frac{\alpha_k (1+\epsilon)}{M_{pl}^2} \sqrt{\frac{V}{6\epsilon}} - \beta_k \epsilon^{3/2} \sqrt{2} M_{pl} a \end{aligned} \tag{46}$$

The continuity conditions on $u_k(\eta)$ and $u_k'(\eta)$ at the "time of the first horizon crossing" lead to:

$$\frac{\alpha_k}{kM_{pl}^2}\sqrt{\frac{V}{6\epsilon}} + \beta_k M_{pl}^2\sqrt{\frac{6\epsilon}{V}} = -ik^{-3/2}e^{ik(\eta_k-\eta_i)} \tag{47}$$

$$\frac{-\alpha_k}{M_{pl}^2}\sqrt{\frac{V}{6\epsilon}}(1+\epsilon) - \beta_k M_{pl}^2\sqrt{\frac{6}{V}}k\epsilon^{3/2} = k^{-1/2}e^{ik(\eta_k-\eta_i)} \tag{48}$$

where we used the fact that, at "horizon crossing" $a(\eta_k) = kH^{-1} = kM_{pl}\sqrt{3/V}$, (where use has been made of slow-roll condition during inflation $H^2 = V/3M_{pl}^2$). The constants α_k and β_k are then:

$$\alpha_k = k^{-\frac{1}{2}}M_{pl}^2\sqrt{\frac{6\epsilon}{V}}e^{ik(\eta_k-\eta_i)}(-1+i\epsilon), \qquad \beta_k = k^{-\frac{3}{2}}\frac{e^{ik(\eta_k-\eta_i)}}{M_{pl}^2}\sqrt{\frac{V}{6\epsilon}}(1-i(\epsilon+1)) \tag{49}$$

Therefore we have found the expressions for $u_k(\eta)$ in the two regimes:

$$u_k(\eta) = \frac{-i}{k^{3/2}}e^{ik(\eta-\eta_i)} \qquad \text{for} \qquad k^2 \gg \frac{\theta''}{\theta} \tag{50}$$

and

$$u_k(\eta) = \alpha_k\theta(\eta) + \frac{\beta_k\theta(\eta)}{4\pi G}\left(\frac{a^2(\eta)}{\mathcal{H}(\eta)} - \int^{\eta} a^2(\tilde{\eta})d\tilde{\eta}\right) \qquad \text{for} \quad k^2 \ll \frac{\theta''}{\theta} \tag{51}$$

with α_k and β_k given in (49). The final step is to calculate the power spectrum (38) considering that the relevant modes ($u_k(\eta)$) are in the long-wavelength regime during the radiation-dominated epoch ($a(\eta) \propto \eta$). Thus neglecting the first term in (51) (θ is inversely proportional to $a(\eta)$ which is a rapidly increasing function), we can express $u_k(\eta)$ as:

$$u_k(\eta) \approx \frac{2}{3}\frac{\beta_k}{4\pi G(\rho+P)^{\frac{1}{2}}} \tag{52}$$

Substituting (52) into (38) one obtains:

$$\mathcal{P}_\Psi(k,\eta) = 4G^2(\rho+P)|u_k(\eta)|^2k^3 = \frac{|\beta_k|^2k^3}{4\pi^2}\left(\frac{2}{3}\right)^2 \tag{53}$$

And finally using (49) in (53) we obtain the power spectrum for the metric perturbation:

$$\mathcal{P}_\Psi(k,\eta) = \frac{V}{54\pi^2M_{pl}^4\epsilon}(2+2\epsilon+\epsilon^2) \approx \frac{V}{27\pi^2M_{pl}^4\epsilon} \tag{54}$$

where we have assumed smallness of the slow-roll parameter $\epsilon \ll 1$, obtaining a scale-invariant power spectrum compatible with the flat Harrison-Zeldovich spectrum.

As shown in the previous calculations, the fact that $\mathcal{P}_\Psi$ is proportional to $V/M_{pl}^4\epsilon$ can be traced back to the late time behaviour of $u_k(\eta)$, as shown in (52): It is proportional to $|\beta_k|^2$ which is directly proportional to $V/M_{pl}^4\epsilon$.

Thus we can trace the result regarding the amplitude of the spectrum, to the behaviour $u_k(\eta)$ in the long-wavelength approximation during the radiation dominated epoch, considered as the time where the imprint of these inhomogeneities on the CMB occurs, and to the matching conditions for this quantity at the time of "horizon crossing" (characterized in (46), (47), and (48)).

This result is thus, closely connected to the identifications made in the standard approach to the problem of the quantum-to-classical transition, which we regard as very problematic at the conceptual level, as we have already pointed out.

Next we will see how our approach, which is completely different at the fundamental level, can in principle change the result (54), but we will find that the required characterization of the collapse contains elements that seem to make it unphysical.

III. EARLIER ARGUMENTS

In the standard approach one can trace the problem to an enormous amplification of the fluctuation spectrum occurring during the transition from the inflationary regime to the radiation dominated regime. In order to consider the ratio by which $\Psi_k(\eta)$ is amplified during the transition, one focusses attention on ξ the so called "intrinsic curvature perturbation", the quantity that can be regarded in this context as defined by:

$$\xi \equiv \frac{2}{3}\frac{\mathcal{H}^{-1}\Psi' + \Psi}{1+w} + \Psi \tag{55}$$

where $w \equiv P/\rho$. This quantity was first introduced in [17] and has been extensively used [18, 19]. It can be shown, by using the definition of $c_s^2 \equiv P'/\rho'$, the continuity equation $\rho' = -3\mathcal{H}(\rho + P)$ and with the help of (12), that ξ is, in the long wavelength approximation and for "adiabatic perturbations", roughly a "constant quantity", irrespective of the cosmological regime and the nature of the dominant kind of matter. The constancy of this quantity during the transition from the inflationary epoch to the radiation dominated epoch, is used to obtain a relation between the values of the Newtonian potential during the two relevant regimes: $\Psi_k^{inf}(\eta)$ and $\Psi_k^{rad}(\eta)$.

$$\xi^{inf} = \xi^{rad} \qquad \text{implies} \qquad \Psi_k^{inf}\left[\frac{2}{3}\left(\frac{1}{w_{inf}+1}\right) + 1\right] = \frac{3}{2}\Psi_k^{rad} \tag{56}$$

where, in obtaining the right hand side of (56) the use of the equation of state $P = \rho/3$ was made, and the left hand side was obtained using the equation of state $P = w_{inf}\rho$ where $w_{inf} + 1 = \phi_0'^2/a^2\rho$. We should note that the constancy of the Newtonian potential (for scales larger than the Hubble radius) within the inflationary and radiation dominated regimes, was used in the form $\Psi_k' = 0$ in both sides of the equation. Finally by relying on the assumption of validity of the slow-roll approximation during inflation, $\phi_0'^2/a^2 = \frac{2}{3}V\epsilon$, (56) becomes:

$$\Psi_k^{rad} = \frac{2}{3}\frac{\Psi_k^{inf}}{\epsilon}. \tag{57}$$

We note the difference between the power spectra obtained at different cosmological epochs:

$$\mathcal{P}_\Psi^{inf}(k,\eta) = \frac{k^3}{4\pi^2}\epsilon^2|\beta_k|^2 \qquad \mathcal{P}_\Psi^{rad}(k,\eta) = \frac{k^3}{4\pi^2}\frac{4}{9}|\beta_k|^2 \tag{58}$$

and since $|\beta_k|^2 \propto k^{-3}V/M_{pl}^4\epsilon$ last equations yield:

$$\mathcal{P}_\Psi^{inf}(k,\eta) \propto \frac{V\epsilon}{M_{pl}^4} \qquad \mathcal{P}_\Psi^{rad}(k,\eta) \propto \frac{V}{\epsilon M_{pl}^4} \tag{59}$$

One can next consider, the corresponding analysis for the collapse scheme. The fundamental difference is that in this approach, there is more intrinsic freedom in the model that goes beyond the specification of the set of fields and the potential. One must characterize the collapse time and the state after the collapse. That suggests that a specific scheme of the collapse, might prevent the Newtonian potential Ψ_k from getting amplified during the transition from the inflationary to the radiation dominated epoch.

We can look at this possibility by focusing again on the quantity ξ. During the inflationary epoch, the expression (55) for ξ can be rewritten (by using (23) in its semiclassical version along with $\rho + P = \phi_0'^2/a^2$ and $1 + w = 8\pi G\phi_0'^2/3\mathcal{H}^2$) as:

$$\xi_{inf} = \Psi_{inf} + \frac{\mathcal{H}}{\phi_0'}\langle\hat{\delta\phi}(\eta)\rangle_\Theta \tag{60}$$

where the inflaton field $\delta\phi$ has been replaced by the expectation value of the corresponding quantum operator in the state $|\Theta\rangle$ after the collapse. Here of course, we are dealing with a quantum expectation value, and one might worry about the appropriateness of using properties derived for the corresponding quantity in a classical realm. However we know that the the Schrödinger or Heissenberg evolutions imply, for this system that the expectation values follow the same equations of motion as the classical counterparts (Ehrenfest's Theorem). In our case of course the collapse scheme departs from such smooth evolution, but as we must focuss on the late time (i.e. the reheating era) behaviour of the quantities and assuming that the collapse occurs well within the inflationary stage, we would be justified in taking over, for the behaviour of this ξ, the conclusions obtained from the classical equations of motion.

If the post-collapse state $|\Theta\rangle$, is such that $\langle\hat{\delta\phi}(\eta^c_k)\rangle = 0$, given the constancy of ξ (for modes $k\eta \ll 1$) one infers that:

$$\xi_{inf} = \Psi_{inf} \tag{61}$$

During the radiation dominated epoch, the expression for ξ is obtained from (55) by considering $w = \frac{1}{3}$ and the constancy of Ψ in the long-wavelength regime, therefore

$$\xi_{rad} = \frac{3}{2}\Psi_{rad} \tag{62}$$

Using that ξ is a conserved quantity we can obtain a relation between Ψ_{inf} and Ψ_{rad} within the particular collapse scheme ((61) and (62)):

$$\xi^{rad} = \xi^{inf} \qquad \text{implies} \qquad \Psi^{rad}_k = \frac{2}{3}\Psi^{inf}_k \tag{63}$$

Thus, this last result indicates that in the collapse framework, the amplitude of the metric perturbations would only be "amplified" by a factor of $\frac{2}{3}$ during the transition from the inflationary to the radiation dominated epoch, a result which differs drastically from the standard inflationary approach because in that case, the amplification of the metric perturbation was of $1/\epsilon$. In the absence of the huge amplification, the final amplitude would seem to be simply proportional to $V\epsilon/M^4_{pl}$. We will next see how a more detailed and careful analysis invalidates this happy conclusion.

IV. THE COLLAPSE SCHEME FOR THE QUANTUM FLUCTUATIONS IN THE INFLATIONARY SCENARIO

In this section we will review the formalism used in analyzing the collapse process, the full formalism and motivation is exposed in [7] and [8]. Here the working assumptions are the validity of a classical treatment for the space-time according to the semi-classical Einstein's equations: $G_{ab} = 8\pi G\langle T_{ab}\rangle$, and the standard quantum field theoretical treatment of the inflaton's perturbations, with an appropriate modifications to include the *self induced collapse of the wave function*. The latter is assumed to induce a jump in the quantum state for each mode of the scalar field $|0\rangle \to |\Theta\rangle$, and the corresponding changes in the the expectation of the energy momentum tensor, leading to the emergence in the metric perturbations. For more details we refer the reader to [7, 8].

The analysis here will be based on the same model as that of section II. As in the usual treatment, one splits both, metric and scalar field into a spatially homogeneous "background" part and an inhomogeneous part "fluctuation", i.e., the scalar field is written $\phi = \phi_0 + \delta\phi$, while the perturbed metric can be written as in (7).

The equations governing the background unperturbed Friedmann-Robertson universe and the homogeneous scalar field $\phi_0(\eta)$ are again, the scalar field equation (6) and Friedmann's equations (3) and (4). We will work in the same set up for the values of η and $a(\eta)$ mentioned at the beginning of section II.

Let us begin the quantum theory within the collapse framework by incorporating the fact that $\Psi = \Phi$ in Einstein's equations (16) and (17). By setting the appropriate boundary condition, (17) reduces to

$$\Psi' + \mathcal{H}\Psi = 4\pi G\phi'_0\delta\phi \tag{64}$$

Then by substituting this last expression in the left hand side of (16) and noting that $4\pi G\phi'^2_0\Psi = 4\pi G a^2(\rho+P)\Psi = (\mathcal{H}^2 - \mathcal{H}')\Psi$ (where the last equality was given by (4)) one obtains:

$$\nabla^2\Psi + \mu\Psi = 4\pi G(\omega\delta\phi + \phi'_0\delta\phi') \tag{65}$$

where $\mu \equiv \mathcal{H}^2 - \mathcal{H}'$ and $\omega \equiv 3\mathcal{H}\phi'_0 + a^2\partial_\phi V$, which upon use of the expression for $\partial_\phi V$ from (6), gives $\omega = \mathcal{H}\phi'_0 - \phi''_0$. Finally the slow-rolling approximation $\partial^2\phi/\partial t^2 = 0$ corresponds in these coordinates to the condition $\omega = 0$. Thus (65) becomes:

$$\nabla^2\Psi + \mu\Psi = 4\pi G\phi_0'\delta\phi' \tag{66}$$

On the other hand, the evolution equation for the fluctuation of the field obtained from action (1) is:

$$\delta\phi'' - \nabla^2\delta\phi + 2\mathcal{H}\delta\phi' = 0 \tag{67}$$

Note that here we have neglected terms proportional to $\partial^2_{\phi\phi}V[\phi]$, in accordance with the standard slow-roll approximation.

It is convenient to work with the auxiliary field $y = a\delta\phi$, thus (67) becomes:

$$y'' - \left(\nabla^2 + \frac{a''}{a}\right)y = 0 \tag{68}$$

The next step involves the quantization of the field fluctuation, that is, one writes the field as $\phi = \phi_0 + \delta\phi$, where the background field ϕ_0 is described in a completely "classical"[2] fashion while only the fluctuation $\delta\phi$ is quantized. After the quantization $\hat{\delta\phi} = a^{-1}\hat{y}$ of $\delta\phi$ it is recognized immediately that a quantization $\hat{y}$ of y occurs automatically. The conjugated canonical momentum of y is $\pi = y' - ya'/a$. In order to avoid infrared problems, the collapse picture considers a restriction of the system to a box of side L, where it is imposed periodic boundary conditions. The field and its momentum is thus:

$$\hat{y}(\eta,\mathbf{x}) = \frac{1}{L^3}\sum_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{x}}\hat{y}_k(\eta), \qquad \hat{\pi}(\eta,\mathbf{x}) = \frac{1}{L^3}\sum_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{x}}\hat{\pi}_k(\eta) \tag{69}$$

The wave vectors satisfy $k_iL = 2\pi n_i$ with $i = 1,2,3$ and $\hat{y}_k(\eta) \equiv y_k(\eta)\hat{a}_k + \overline{y}_k(\eta)\hat{a}^\dagger_{-k}$, and $\hat{\pi}_k(\eta) \equiv g_k(\eta)\hat{a}_k + \overline{g}_k(\eta)\hat{a}^\dagger_{-k}$. The functions $y_k(\eta)$ and $g_k(\eta)$ reflect the election of the vacuum state, the so called Bunch-Davies vacuum:

$$y_k^\pm(\eta) = \frac{1}{\sqrt{2k}}\left(1 \pm \frac{i}{\eta k}\right)\exp(\pm ik\eta), \qquad g_k^\pm(\eta) = \pm i\sqrt{\frac{k}{2}}\exp(\pm ik\eta) \tag{70}$$

The vacuum state is defined by the condition $\hat{a}_k|0\rangle = 0$ for all k, and is homogeneous and isotropic at all scales.

The induced collapse operates in close analogy with a "measurement" in the quantum-mechanical sense, but of course without any external apparatus or observer that could be thought to make the measurement. That is, one assumes that at a certain time η_k^c (from now on we will refer to this particular time as the *time of collapse*) the state of the field, which was initially the Bunch Davies vacuum changes spontaneously into another state that could in principle be a non-symmetrical state. The proposal is inspired by Penrose's ideas [13, 14] in which gravity plays a fundamental role on the collapse of the wave-function and it does not require of observers to perform a measurement in order to induce the collapse. The collapse scheme as employed here however, does not propose at this point a concrete physical mechanism behind it, although one envisions a more profound theory presumably derived from quantum gravity will eventually account for it. At this stage one might consider it as a mere parametrization of its characteristics. These ideas and motivations are discussed in great detail in [7] and [8]. In this paper we will only make use of the collapse scheme to calculate the power spectrum of the metric perturbations. Since the collapse acts as a sort of "measurement", it is convenient to decompose the field $\hat{y}_k$ and its conjugated momentum $\hat{\pi}_k$ in their real and imaginary parts which are completely hermitian $\hat{y}_k(\eta) = \hat{y}_k^R(\eta) + i\hat{y}_k^I(\eta)$ and $\hat{\pi}_k(\eta) = \hat{\pi}_k^R(\eta) + i\hat{\pi}_k^I(\eta)$ and thus qualify as reasonable observables.

$$\hat{y}_k^{R,I}(\eta) = \tfrac{1}{\sqrt{2}}\left(y_k(\eta)\hat{a}_k^{R,I} + \overline{y}_k(\eta)\hat{a}_k^{\dagger R,I}\right), \qquad \hat{\pi}_k^{R,I}(\eta) = \tfrac{1}{\sqrt{2}}\left(g_k(\eta)\hat{a}_k^{R,I} + \overline{g}_k(\eta)\hat{a}_k^{\dagger R,I}\right)$$

where $\hat{a}_k^R \equiv \frac{1}{\sqrt{2}}(\hat{a}_k + \hat{a}_{-k}), \qquad \hat{a}_k^I \equiv \frac{-i}{\sqrt{2}}(\hat{a}_k - \hat{a}_{-k})$

[2] What we mean by "classical" is that the background field ϕ_0 is taken as an approximated description of the quantum quantity $\langle\hat{\phi_0}\rangle$.

The commutator of the real and imaginary annihilation and creation operators is: $[\hat{a}_k^R, \hat{a}_{k'}^{R\dagger}] = L^3(\delta_{k,k'} + \delta_{k,-k'})$, $[\hat{a}_k^I, \hat{a}_{k'}^{I\dagger}] = L^3(\delta_{k,k'} - \delta_{k,-k'})$.

Let $|\Xi\rangle$ be any state in the Fock space of $\hat{y}$. So introducing the following quantity: $d_k^{R,I} = \langle \hat{a}_k^{R,I} \rangle_\Xi$, the expectation values of the modes are expressible as

$$\langle \hat{y}_k^{R,I} \rangle_\Xi = \sqrt{2} Re(y_k d_k^{R,I}), \qquad \langle \hat{\pi}_k^{R,I} \rangle_\Xi = \sqrt{2} Re(g_k d_k^{R,I}) \tag{71}$$

For the vacuum state $|0\rangle$ one has, as expected, $d_k^{R,I} = 0$, and thus $\langle \hat{y}_k^{R,I} \rangle_0 = 0$, $\langle \hat{\pi}_k^{R,I} \rangle_0 = 0$. While their corresponding uncertainties are: $(\Delta \hat{y}_k^{R,I})_0^2 = (1/2)|y_k|^2 L^3$, $\qquad (\Delta \hat{\pi}_k^{R,I})_0^2 = (1/2)|g_k|^2 L^3$

Now one needs to further specify the state after the collapse which will be denoted by $|\Theta\rangle$. For our purposes all it is needed to specify is $d_k^{R,I} = \langle \Theta | \hat{a}_k^{R,I} | \Theta \rangle$. In the vacuum state, $\hat{y}_k$ and $\hat{\pi}_k$ are individually distributed according to a Gaussian distribution centered at 0 with spread $(\Delta \hat{y}_k)_0^2$ and $(\Delta \hat{\pi}_k)_0^2$ respectively, and those are taken as a guidance for considering some of the characteristics of the state after the collapse. We refer to any such concrete specification of characteristics as a collapse scheme. Various concrete collapse schemes, have been studied before, particularly in [20]. The point is that one can test a particular collapse scheme and compare it with the observations to discard or accept it.

In the rest of this paper we focus on the scheme as given by (72) and (73) because, as will be shown next, it seems to be the one with the characteristics needed to deal with the amplitude of the spectrum. We note that nothing of this sort can even be contemplated in the usual approach where there is no liberty whatsoever to make additional assumptions at this stage.

$$\langle \hat{y}_k^{R,I}(\eta_k^c) \rangle_\Theta = \lambda_{k,1} x_{k,1}^{R,I} \sqrt{(\Delta \hat{\pi}_k^{R,I})_0^2} = \lambda_{k,1} x_{k,1}^{R,I} |y_k(\eta_k^c)| \sqrt{L^3/2} \tag{72}$$

$$\langle \hat{\pi}_k^{R,I}(\eta_k^c) \rangle_\Theta = \lambda_{k,2} x_{k,2}^{R,I} \sqrt{(\Delta \hat{\pi}_k^{R,I})_0^2} = \lambda_{k,2} x_{k,2}^{R,I} |g_k(\eta_k^c)| \sqrt{L^3/2} \tag{73}$$

where $\lambda_{k,1}$ and $\lambda_{k,2}$ represent real parameters; η_k^c represents the time of collapse for the mode k and $x_{k,1}^{R,I}$; $x_{k,2}^{R,I}$ are selected randomly from within a Gaussian distribution centered at zero with spread one. Here, we must emphasize that our universe corresponds to a single realization of these random variables, and thus each of these quantities has a single specific value.

Now, as indicated at the beginning of this section one relies on a semi-classical description of gravitation in interaction with quantum fields, in terms of the semi-classical Einstein's equation $G_{ab} = 8\pi G \langle \hat{T}_{ab} \rangle$ whereas the other fields are treated in the standard quantum field theory (in curved space-time) fashion. Putting these elements together for the situation at hand, the semi-classical version of the perturbed Einstein's equations (64), (66) valuated at the time of collapse, after a Fourier's decomposition reduce to:

$$\Psi'_k(\eta_k^c) + \mathcal{H} \Psi_k(\eta_k^c) = 4\pi G \frac{\phi'_0}{a} \langle \hat{y}_k(\eta_k^c) \rangle \tag{74}$$

$$-k^2 \Psi_k(\eta_k^c) + \mu \Psi_k(\eta_k^c) = 4\pi G \frac{\phi'_0}{a} \langle \hat{\pi}_k(\eta_k^c) \rangle \tag{75}$$

It is easy to see that before the collapse occurs, the expectation value on the right hand side of the latter expressions is zero, and the space-time is homogeneous and isotropic (at the corresponding scale).

The expressions (74) and (75) where obtained from Einstein's equations with components $\delta G_0^0 = 8\pi G \delta T_0^0$ and $\delta G_i^0 = 8\pi G \delta T_i^0$. It is a known result, that these particular equations are not actual motion equations but rather constraint equations. The motion equation is the one given by $\delta G_j^i = 8\pi \delta T_j^i$ which, after use of the constraint equation (16) and during the inflationary period, is represented in (19) (or equivalently by (21)).

Therefore, we can use the constraint equations (74) and (75), to obtain the input data for (19), i.e., (74) and (75) allow us to specify $\Psi(\eta_k^c)$ and $\Psi'(\eta_k^c)$ which will serve as the initial conditions for (19). Thus, assuming that the time of collapse η_k^c occurs at the very early stages of the inflationary regime, for which the modes of interest satisfy $k^2 \gg \epsilon \mathcal{H}^2(\eta_k^c) = \mu \Rightarrow |k\eta_k^c| \gg \epsilon$, then (75) yields the initial condition:

$$\Psi(\eta_k^c) = -4\pi G \frac{\phi_0'(\eta_k^c)}{a(\eta_k^c)k^2} \langle \hat{\pi}_k(\eta_k^c) \rangle \tag{76}$$

Using (76), (74) yields the second initial condition:

$$\Psi_k'(\eta_k^c) = 4\pi G \frac{\phi_0'(\eta_k^c)}{a(\eta_k^c)} \left[\langle \hat{y}_k(\eta_k^c) \rangle + \frac{\mathcal{H}_c}{k^2} \langle \hat{\pi}_k(\eta_k^c) \rangle \right] \tag{77}$$

where $\mathcal{H}_c$ denotes $\mathcal{H}$ evaluated at the time of collapse η_k^c.
With the initial conditions at hand, one proceeds to solve (19) or equivalently (21):

$$u_k''(\eta) + \left(k^2 - \frac{\theta''}{\theta} \right) u_k(\eta) = 0$$

During inflation, the quantity θ (defined in (42)) is given by $\theta = 1/\sqrt{2\epsilon} M_{pl} a$. Considering that in the slow-roll approximation $1 \gg \epsilon \approx$ const., one finds:

$$\frac{\theta''}{\theta} = \epsilon \mathcal{H}^2 = \frac{\epsilon}{(1-\epsilon)^2 \eta^2} = \frac{\epsilon + \mathcal{O}(\epsilon^2)}{\eta^2} \approx \frac{\epsilon}{\eta^2} \tag{78}$$

where in the second equality we used the expression for $\mathcal{H}$ during inflation (this was introduced at the beginning of section II). Therefore, (21) takes the form:

$$u_k''(\eta) + \left(k^2 - \frac{\epsilon}{\eta^2} \right) u_k(\eta) = 0 \tag{79}$$

The general solution of (79) is given by:

$$u_k(\eta) = C_1 \sqrt{-\eta} J_\nu(-k\eta) + C_2 \sqrt{-\eta} Y_\nu(-k\eta) \tag{80}$$

where $\nu = \frac{1}{2}\sqrt{1+4\epsilon}$ and J_ν, Y_ν correspond to Bessel's functions of the first and second kind respectively. Using the definition of $u \equiv \Psi/(4\pi G\sqrt{\rho + p})$, we obtain the *exact* expression for the "Newtonian Potential" in the inflationary regime:

$$\Psi_k^{inf}(\eta) = \frac{s}{a} \left(C_1 \sqrt{-\eta} J_\nu(-k\eta) + C_2 \sqrt{-\eta} Y_\nu(-k\eta) \right) \tag{81}$$

where $s \equiv 4\pi G \phi_0'$. After imposing the initial conditions (76) and (77) to the general solution and using the collapse scheme as introduced in (72) and (73), we find C_1 and C_2.

$$\begin{aligned} C_1 = \left(\frac{L}{k} \right)^{\frac{3}{2}} \frac{\pi \sqrt{k}}{4} \Big\{ & \lambda_{k,1}(x_{k,1}^R + i x_{k,1}^I) \sqrt{|z_k|} Y_\nu(|z_k|) + \\ & + \lambda_{k,2}(x_{k,2}^R + i x_{k,2}^I) \left[\left(\frac{1}{1-\epsilon} - \nu - \frac{1}{2} \right) \frac{Y_\nu(|z_k|)}{\sqrt{|z_k|}} + \sqrt{|z_k|} Y_{\nu+1}(|z_k|) \right] \Big\} \end{aligned} \tag{82}$$

$$\begin{aligned} C_2 = -\left(\frac{L}{k} \right)^{\frac{3}{2}} \frac{\pi \sqrt{k}}{4} \Big\{ & \lambda_{k,1}(x_{k,1}^R + i x_{k,1}^I) \sqrt{|z_k|} J_\nu(|z_k|) + \\ & + \lambda_{k,2}(x_{k,2}^R + i x_{k,2}^I) \left[\left(\frac{1}{1-\epsilon} - \nu - \frac{1}{2} \right) \frac{J_\nu(|z_k|)}{\sqrt{|z_k|}} + \sqrt{|z_k|} J_{\nu+1}(|z_k|) \right] \Big\} \end{aligned} \tag{83}$$

where $z_k \equiv k\eta_k^c$ and we used that at the time of collapse $\mathcal{H}_c = -1/(1-\epsilon)\eta_k^c = k/(1-\epsilon)|z_k|$. Since we are assuming that the time of collapse occurs during the early inflationary period, then z_k is in the range $-\infty < z_k \ll k\eta_r$ (we recall that $\eta_r \approx -10^{-22}$, therefore $z_k < 0$).

The result (81) represents the dynamical evolution of $\Psi_k(\eta)$ during the inflationary regime within the collapse framework. In order to obtain a predicted power spectrum and contrast it with the observations, we strictly can not use the $\Psi_k(\eta)$ as given in (81) because it was obtained using $a(\eta)$ in the inflationary epoch. A realistic analysis requires us to focus on $\Psi_k(\eta)$ during the radiation dominated regime. Therefore, the next step is to obtain $\Psi_k(\eta)$ during that epoch.

In order to do this we must again connect the two regimes. This is a point where the analysis necessarily deviates from what is usually done, as here we have, even before inflation ends, actual inhomogeneities and anisotropies in the metric. The so called "reheating regime", where the inflaton field decays into ordinary particles and photons (including presumably dark matter particles) **is a complicated and not fully understood process, quite likely involving huge entropy creation and other complexities. These complications are often ignored in the literature and we will do likewise.** However, what seems rather clear is that the metric perturbations should be considered as evolving continuously during this regime, and to the extent that we ignore reheating period's temporal extent, the matching of the Newtonian potential should be continuous for each mode.

After the inflationary regime has ended, the dynamical evolution of the metric perturbation would be connected by the fluctuations of the radiation energy density, the equations that drive the evolution are naturally, the Einstein Field Equations.

Let us recast (12) which describes such situation:

$$\Psi'' - c_s^2\nabla^2\Psi + 3\mathcal{H}(1+c_s^2)\Psi' + [(2\mathcal{H}' + \mathcal{H}^2(1+3c_s^2)]\Psi = 4\pi G a^2 \tau\delta S$$

We note that, as showed in the beginning of section II, (12) is valid for any cosmological period. In a radiation dominated universe $P = \rho/3$, therefore $c_s^2 = \frac{1}{3}$ and $\tau\delta S = 0$. Given the equation of state, the scale factor can be calculated using the background equations, obtaining $a(\eta) = C_{rad}(\eta - \eta_r) + a_r$ where: $C_{rad}^2 \equiv \frac{8}{3}\pi G\rho a^4$ is a constant; η_r is the conformal time at which the radiation epoch starts; a_r is the value of the scale factor at η_r and $\eta_r < \eta < \eta_{eq}$ with η_{eq} the conformal time at which the universe is populated by matter-radiation equally. According to the comments above, we take the value η_r at which the inflationary regime ends to be the same for which the radiation dominated epoch starts, with a_r representing value of the scale factor at that time.

For the epoch corresponding to a radiation dominated universe, the Fourier transform version of (12) takes the form:

$$\Psi_k'' + \frac{4}{\eta - \eta_r + D_{rad}}\Psi_k' + \frac{1}{3}k^2\Psi_k = 0 \tag{84}$$

where $D_{rad} \equiv a_r/C_{rad}$. The analytical solution to (84) is thus:

$$\begin{aligned}\Psi_k^{rad}(\eta) = \frac{3}{(k\eta - \zeta_k)^2}\Bigg[&C_3\left(\frac{\sqrt{3}}{(k\eta-\zeta_k)}\sin\left(\frac{k\eta-\zeta_k}{\sqrt{3}}\right) - \cos\left(\frac{k\eta-\zeta_k}{\sqrt{3}}\right)\right)\\ &+C_4\left(\frac{\sqrt{3}}{(k\eta-\zeta_k)}\cos\left(\frac{k\eta-\zeta_k}{\sqrt{3}}\right) + \sin\left(\frac{k\eta-\zeta_k}{\sqrt{3}}\right)\right)\Bigg]\end{aligned} \tag{85}$$

where $\zeta_k \equiv k\eta_r - kD_{rad}$. We see that $\Psi_k(\eta)$ as given by (85) contains the denominator $(k\eta - \zeta_k)^{-1}$, and one might worry about it becoming singular, but noting that it arises essentially from $C_{rad}/ka(\eta)$, the fact that $a(\eta) \neq 0$, and moreover is an increasing function there is no possibility of a "blow up" type of behaviour. The constants C_3 and C_4 will be obtained by approximating the continuous change of the equation of state by a sharp jump. Therefore, the matching conditions for Ψ and Ψ' will be derived by rewriting the motion equation for u ((14), assuming adiabatic perturbations) in the following form:

$$\left[\theta^2\left(\frac{u_k}{\theta}\right)'\right]' = -k^2c_s^2\theta u_k \tag{86}$$

The quantity u/θ is continuous because the scale factor a and the energy density ρ are both continuous, the Newtonian Potential Ψ does not jump during the transition. Integrating (86) from $\eta_r - \delta$ to $\eta_r + \delta$, where δ is positive real number and very small in absolute terms, we obtain:

$$\left[\theta_{rad}^2\left(\frac{u_{k,rad}}{\theta_{rad}}\right)'\right]\Big|_{\eta_r+\delta}-\left[\theta_{inf}^2\left(\frac{u_{k,inf}}{\theta_{inf}}\right)'\right]\Big|_{\eta_r-\delta}=-k^2\int_{\eta_r-\delta}^{\eta_r+\delta}c_s^2\theta u_k d\eta \tag{87}$$

Assuming $\delta\to 0$, then the matching conditions given for Ψ_k are:

$$\theta_{inf}^2\left(\frac{u_{k,inf}}{\theta_{inf}}\right)'\Big|_{\eta=\eta_r}=\theta_{rad}^2\left(\frac{u_{k,rad}}{\theta_{rad}}\right)'\Big|_{\eta=\eta_r};\qquad \Psi_k^{inf}(\eta_r)=\Psi_k^{rad}(\eta_r) \tag{88}$$

We note that these conditions are equivalent to the Deruelle-Mukhanov conditions obtained in [21]. From these conditions, one can easily find the value of the constants C_3 and C_4:

$$\begin{aligned} C_3=&-\left[\left(\tfrac{D_k^2}{3}-3\right)\cos\left(\tfrac{D_k}{\sqrt{3}}\right)-\sqrt{3}D_k\sin\left(\tfrac{D_k}{\sqrt{3}}\right)\right]A(k\eta_r,z_k)\\ &+\left[\tfrac{D_k}{\sqrt{3}}\cos\left(\tfrac{D_k}{\sqrt{3}}\right)+\tfrac{D_k^2}{3}\sin\left(\tfrac{D_k}{\sqrt{3}}\right)\right]\tfrac{\sqrt{3}}{k}B(k\eta_r,z_k) \end{aligned} \tag{89}$$

$$\begin{aligned} C_4=&\left[\left(\tfrac{D_k^2}{3}-3\right)\sin\left(\tfrac{D_k}{\sqrt{3}}\right)+\sqrt{3}D_k\cos\left(\tfrac{D_k}{\sqrt{3}}\right)\right]A(k\eta_r,z_k)\\ &-\left[\tfrac{D_k}{\sqrt{3}}\sin\left(\tfrac{D_k}{\sqrt{3}}\right)-\tfrac{D_k^2}{3}\cos\left(\tfrac{D_k}{\sqrt{3}}\right)\right]\tfrac{\sqrt{3}}{k}B(k\eta_r,z_k) \end{aligned} \tag{90}$$

where

$$\begin{aligned} A(k\eta_r,z_k)\equiv&\frac{-s\pi}{4a}\left(\frac{L}{k}\right)^{\frac{3}{2}}\Bigg\{\lambda_{k,1}(x_{k,1}^R+ix_{k,1}^I)\left(1+\frac{1}{z_k^2}\right)^{\frac{1}{2}}\sqrt{|z_k|}\Big[J_\nu(|z_k|)Y_\nu(-k\eta_r)-\\ &-Y_\nu(|z_k|)J_\nu(-k\eta_r)\Big]+\lambda_{k,2}(x_{k,2}^R+ix_{k,2}^I)\Bigg[-\Bigg(\sqrt{|z_k|}Y_{\nu+1}(|z_k|)\\ &+\left(\frac{1}{1-\epsilon}-\nu-\frac{1}{2}\right)\frac{Y_\nu(|z_k|)}{\sqrt{|z_k|}}\Bigg)J_\nu(-k\eta_r)+\Bigg(\left(\frac{1}{1-\epsilon}-\nu-\frac{1}{2}\right)\frac{J_\nu(|z_k|)}{\sqrt{|z_k|}}+\\ &+\sqrt{|z_k|}J_{\nu+1}(|z_k|)\Bigg)Y_\nu(-k\eta_r)\Bigg]\Bigg\}\sqrt{-k\eta_r} \end{aligned} \tag{91}$$

$$B(k\eta_r,z_k)\equiv\frac{2}{\epsilon}\left[-k\frac{\partial A(-k\eta_r,z_k)}{\partial(-k\eta_r)}+\mathcal{H}(\eta_r)A(-k\eta_r,z_k)\right] \tag{92}$$

and $D_k\equiv kD_{rad}$. The quantity $A(k\eta_r,z_k)$ can be approximated, by considering that if $\epsilon\ll 1$ then $\nu\approx\frac{1}{2}$, thus we have

$$\begin{aligned} A(k\eta_r,z_k)\approx&\frac{-s}{2a}\left(\frac{L}{k}\right)^{\frac{3}{2}}\Bigg[\lambda_{k,1}(x_{k,1}^R+ix_{k,1}^I)\left(1+\frac{1}{z_k^2}\right)^{\frac{1}{2}}\sin\Delta_r+\\ &+\lambda_{k,2}(x_{k,2}^R+ix_{k,2}^I)\left(\cos\Delta_r+\frac{\sin\Delta_r}{z_k}\right)\Bigg] \end{aligned} \tag{93}$$

$$\begin{aligned} B(k\eta_r,z_k)\approx&\frac{s}{2a}\left(\frac{L}{k}\right)^{\frac{3}{2}}\frac{2k}{\epsilon}\Bigg\{\lambda_{k,1}(x_{k,1}^R+ix_{k,1}^I)\left(1+\frac{1}{z_k^2}\right)^{\frac{1}{2}}\left(\cos\Delta_r-\frac{\sin\Delta_r}{k\eta_r}\right)\\ &+\lambda_{k,2}(x_{k,2}^R+ix_{k,2}^I)\left[\cos\Delta_r\left(\frac{1}{k\eta_r}-\frac{1}{z_k}\right)+\sin\Delta_r\left(\frac{1}{k\eta_r z_k}+1\right)\right]\Bigg\} \end{aligned} \tag{94}$$

where $\Delta_r \equiv k\eta_r - z_k$ Despite the apparent complexity for the constants C_3 and C_4, we note that for the scales of interest 10^{-3} Mpc^{-1} $< k <$ 1 Mpc^{-1} and $D_{rad} = a_r/C_{rad} \approx 1.5 \times 10^{-22}$ Mpc, we have $D_k \in [10^{-25}, 10^{-22}]$. Therefore, the approximated expressions for (89) and (90) up to first order in D_k are:

$$C_3 \approx 3A(k\eta_r, z_k) + \frac{D_k}{k} B(k\eta_r, z_k) \tag{95}$$

$$C_4 \approx 0 \tag{96}$$

We should note that the approximation given by the expressions above would correspond, in the standard treatment, to neglecting the "decaying mode". Here we can see clearly how this comes about as a result of the matching conditions and the range of values of the relevant quantities.

With this results we can now write an approximate expression for the Newtonian Potential as:

$$\Psi_k^{rad}(\eta) \approx \left[3A(k\eta_r, z_k) + \frac{D_k}{k} B(k\eta_r, z_k)\right] \frac{3}{(k\eta - \zeta_k)^2} \left[\frac{\sqrt{3}}{(k\eta - \zeta_k)} \sin\left(\frac{k\eta - \zeta_k}{\sqrt{3}}\right) - \right.$$
$$\left. - \cos\left(\frac{k\eta - \zeta_k}{\sqrt{3}}\right)\right] \tag{97}$$

where the constants $A(k\eta_r, z_k)$ and $B(k\eta_r, z_k)$ are given by (93) and (94) respectively.

Next we turn to the observational quantities. The quantity that is measured is $\Delta T(\theta, \varphi)/T$ which is a function of the coordinates on the celestial two-sphere and is expressed as $\sum_{lm} \alpha_{lm} Y_{lm}(\theta, \varphi)$. The angular variations of the temperature are then identified with the corresponding variations in the "Newtonian Potential" Ψ, by the understanding that they are the result of gravitational red-shift in the CMB photon frequency ν so $\delta T/T = \delta\nu/\nu = \delta(\sqrt{g_{00}})/\sqrt{g_{00}} \approx \Psi$

The quantity that is presented as the result of observations is $OB_l = l(l+1)(2l+1)^{-1} \sum_m |\alpha_{lm}^{obs}|^2$. The observations indicate that (ignoring the acoustic oscillations, which is anyway an aspect that is not being considered in this work) the quantity OB_l is essentially independent of l and this is interpreted as a reflection of the "scale invariance" of the primordial spectrum of the fluctuations.

The quantity of observational interest is the "Newtonian potential" on the surface of last scattering: $\Psi(\eta_D, \mathbf{x}_D)$, from where one extracts

$$\alpha_{lm} = \int d^2\Omega \quad \Psi(\eta_D, \mathbf{x}_D) Y_{lm}^\star(\theta, \varphi) \tag{98}$$

with $\mathbf{x}_D = R_D(\sin\theta \sin\varphi, \sin\theta \cos\varphi, \cos\theta)$ and R_D represents the radius of the surface of last scattering. In order to evaluate the expected value for the quantity of interest, we will first use the Fourier's decomposition of the metric's perturbation:

$$\Psi(\eta, \mathbf{x}) = \sum_{\mathbf{k}} \frac{1}{L^3} \Psi_k(\eta) e^{i\mathbf{k}\cdot\mathbf{x}} \tag{99}$$

With the expressions at hand, and after some algebra, one obtains an expression for a_{lm}:

$$\alpha_{lm} = \sum_{\mathbf{k}} \frac{-s}{2a_r(Lk)^{\frac{3}{2}}} \left[3F(k\eta_r, z_k) - \frac{2D_k}{\epsilon} G(k\eta_r, z_k)\right] E(k\eta_D, k\eta_r) 4\pi i^l j_l(kR_D) Y_{lm}^\star(\hat{k}) \tag{100}$$

where

$$F(k\eta_r, z_k) \equiv \lambda_{k,1}(x_{k,1}^R + ix_{k,1}^I)\left(1 + \frac{1}{z_k^2}\right)^{\frac{1}{2}} \sin\Delta_r +$$
$$+\lambda_{k,2}(x_{k,2}^R + ix_{k,2}^I)\left(\cos\Delta_r + \frac{\sin\Delta_r}{z_k}\right) \tag{101}$$

$$G(k\eta_r, z_k) \equiv \lambda_{k,1}(x^R_{k,1} + ix^I_{k,1})\left(1+\frac{1}{z_k^2}\right)^{\frac{1}{2}}\left(\cos\Delta_r - \frac{\sin\Delta_r}{k\eta_r}\right) +$$
$$+\lambda_{k,2}(x^R_{k,2} + ix^I_{k,2})\left[\cos\Delta_r\left(\frac{1}{k\eta_r} - \frac{1}{z_k}\right) + \sin\Delta_r\left(\frac{1}{k\eta_r z_k}+1\right)\right] \tag{102}$$

$$E(k\eta, k\eta_r) \equiv \frac{3}{(k\eta-\zeta_k)^2}\left[\frac{\sqrt{3}}{(k\eta-\zeta_k)}\sin\left(\frac{k\eta-\zeta_k}{\sqrt{3}}\right) - \cos\left(\frac{k\eta-\zeta_k}{\sqrt{3}}\right)\right], \tag{103}$$

$j_l(x)$ is the spherical Bessel function of the first kind, and where $\hat{k}$ indicates the direction of the vector $\mathbf{k}$. The above quantity should be evaluated at the conformal time of decoupling η_D which lies in the matter dominated epoch. Nevertheless, we have used the expression for $\Psi_k(\eta)$ in the radiation dominated era (85), extending if one wants the range of validity for (100) which is from η_r to $\eta_{eq} < \eta_D$. The changes during the brief period from the start of "matter domination" to "decoupling" (where the scale factor changes only by a factor of 10, i.e. $a(\eta_D)/a(\eta_{eq}) \approx 10$), are naturally considered to be irrelevant for the issues concerning us here, and thus the approximated value for α_{lm} obtained using (85) should be a very good approximation for the exact value of α_{lm}.

We note here that the slow roll parameter ϵ has entered in the denominator of one of the terms in (100), and as we will see latter one can trace the unwelcome amplification of the overall scale of the fluctuation spectrum precisely to this factor. The possibility of avoiding this problem in the collapse scheme, which has no counterpart in the ordinary treatments, arises precisely from the liberty to select the details of the collapse so as to ensure the vanishing of the coefficient of the $1/\epsilon$ in (100). We will discuss this possibility shortly.

One should note in passing that it is in (100) that one can find the justification for the reliance on statistical considerations, despite the fact that we are dealing with a single universe (even if we assume that many exist the fact is that we have empirical access only to one). The quantity we are interested on, α_{lm} is, as shown in (100), the result of the combined contributions of the collapse of the wave functions of an ensemble of harmonic oscillators, (one for each one of the quantum field modes $\vec{k}$), with each one contributing with a complex number to the sum, leading to what is in effect a 2-dimensional random walk whose total displacement corresponds to the quantity of actual observational interest. It is clear that, as in the case of any random walk, such quantity can not be evaluated and the only thing that can be done is to evaluate the most likely value for such total displacement, with the expectation that the observed quantity will be close to that value.

The expected magnitude of the quantity α_{lm}, after taking the continuum limit ($L \to \infty$) is (See [7] for details):

$$|\alpha_{lm}|^2_{M.L.} = \frac{s^2}{2\pi a_r^2}\int \frac{d^3k}{k^3} H(k\eta_r, z_k) E^2(k\eta_D, k\eta_r) j_l^2(kR_D)|Y_{lm}(\hat{k})|^2 \tag{104}$$

where

$$H(k\eta_r, z_k) \equiv 2\lambda_{k,1}^2\left(1+\frac{1}{z_k^2}\right)\left[3\sin\Delta_r - \frac{2D_k}{\epsilon}\left(\cos\Delta_r - \frac{\sin\Delta_r}{k\eta_r}\right)\right]^2 +$$
$$+2\lambda_{k,2}^2\left\{3\left(\cos\Delta_r + \frac{\sin\Delta_r}{z_k}\right) - \frac{2D_k}{\epsilon}\left[\cos\Delta_r\left(\frac{1}{k\eta_r} - \frac{1}{z_k}\right) +\right.\right.$$
$$\left.\left.+\sin\Delta_r\left(\frac{1}{k\eta_r z_k}+1\right)\right]\right\}^2 \tag{105}$$

The expected value for the observed quantity $OB_l = l(l+1)(2l+1)^{-1}\sum_m |\alpha^{obs}_{lm}|^2$ is thus:

$$OB_l = l(l+1)\frac{s^2}{\epsilon^2 \pi a_r^2}\int \frac{dk}{k} H(k\eta_r, z_k) E^2(k\eta_D, k\eta_r) j_l^2(kR_D) \tag{106}$$

The quantity OB_l is related with the amplitude of the Newtonian potential, that is, one can extract an "equivalent power spectrum" for the metric perturbations:

$$\mathcal{P}_{\Psi}^{col}(k,\eta) = \frac{s^2}{8\pi^2 a_r^2} H(k\eta_r, z_k) E^2(k\eta_D, k\eta_r) \tag{107}$$

We note that if $\mathcal{P}_{\Psi}^{col}(k,\eta)$ is independent of k, then the quantity OB_l is independent of l in correspondence with the observations.

It is easy to show, with the help of $a(\eta)$ during the radiation dominated epoch, that the quantity $(k\eta - \zeta_k)/\sqrt{3}$ which appears in (103), is in fact $k/\sqrt{3}\mathcal{H}$. As discussed previously this quantity should be evaluated at the time of decoupling η_D. However for the modes of interest we have the following condition $k/\mathcal{H} \ll 1$. This corresponds to focuss on the so called "scales larger than the Hubble radius". That is, we must consider (103), for scales $k \ll aH$, whereby we find that to a good approximation:

$$E(k\eta, k\eta_r) \approx \frac{1}{3} \tag{108}$$

Considering (108) and recalling the definition of $s = 4\pi G\phi_0'$; the equation of motion for the field in the slow-roll approximation $\phi_0' = -\partial_\phi V a^3/3a'$; Friedmann's equation in the slow-roll regime $3a'^2 = 8\pi G a^4 V(\phi_0)$; the definition of the slow-roll parameter $\epsilon = \frac{1}{2}M_{pl}^2(\partial_\phi V/V)^2$ and the definition of the reduced Planck mass $M_{pl}^2 = 1/(8\pi G)$, then the overall amplitude of the predicted power spectrum is:

$$\mathcal{P}_{\Psi}^{col}(k,\eta) = \frac{1}{432\pi^2}\frac{V\epsilon}{M_{pl}^4} H(k\eta_r, z_k) \tag{109}$$

The quantity $H(k\eta_r, z_k)$ depends on the parameters characterizing the collapse, that is, it depends on $\lambda_{k,1}$, $\lambda_{k,2}$, and z_k. We should note that the liberty to choose those value corresponds to a characterization of some of the details of the mechanism of collapse, and that no analogous freedoms can be identified when one ignores the problems we had mentioned at the beginning, and which motivates the proposals to modify the inflationary paradigm with the "collapse of the wave function" hypothesis. As we indicated one can now assume that details of collapse are such that the undesired terms disappear. For instance, by adjusting the parameters of the collapse to be $\lambda_{k,1} = 0$, $\lambda_{k,2} = 1$ and the time of collapse to satisfy the following equation:

$$\cos\Delta_r\left(\frac{1}{k\eta_r} - \frac{1}{z_k}\right) + \sin\Delta_r\left(\frac{1}{k\eta_r z_k} + 1\right) = 0 \tag{110}$$

the quantity $H(k\eta_r, z_k)$ will turn out to be just:

$$H(k\eta_r, z_k) = 18\left(\cos\Delta_r + \frac{\sin\Delta_r}{z_k}\right)^2 \tag{111}$$

which does not contain the bothersome terms proportional to $1/\epsilon$. The overall amplitude of the metric perturbations will, thus, have the following form:

$$\mathcal{P}_{\Psi}^{\star col}(k,\eta) = \frac{1}{24\pi^2}\frac{V\epsilon}{M_{pl}^4}\left(\cos(k\eta_r - z_k) + \frac{\sin(k\eta_r - z_k)}{z_k}\right)^2 \tag{112}$$

where the $\star$ over the $\mathcal{P}_{\Psi}$ denotes that it is a very specific model for the collapse which leads to the result (112). The amplitude given by (112) has the desired feature of having the factor ϵ in the numerator rather than in the denominator, in contrast with the standard inflationary results. Therefore, as the slow roll parameter ϵ takes smaller values, the amplitude of the fluctuation spectrum becomes smaller, the complete opposite of what happens in the usual approach. This would be a natural resolution of the fine tuning problem affecting most of the inflationary models, if there was a natural way to explain the particular values characterizing this collapse scheme. In principle, and given the fact that we certainly do not know the physics behind the collapse (as indicated it seems likely that it might be connected with aspects of quantum gravity as suggested by R. Penrose), one would not concern oneself with this issue at this time. However there is a very problematic aspect of the choice of parameters we made, that seems to be shared by all the other possible choices compatible with the desirable behavior, namely that it depends on the value of z_k, a quantity that is determined by the stage at which the reheating occurs, and which is presumed to be to the future of the time at which the collapse should occur. In other words there is a teleological aspect in the choice of the collapse parameters and it is very hard to see how could it be part of any sort of reasonable physical process, tied or not, with quantum gravity.

Regarding the "scale invariance" of the spectrum, we note that if $|z_k| \gg |k\eta_r|$, which is not a strong assumption as we have $|\eta_r| \approx 1.157 \times 10^{-22}$ Mpc, which implies $10^{-25} < |k\eta_r| < 10^{-22}$. Then, the scale dependance of the predicted spectrum will be contained only in z_k, leading to the conclusion that there is one simple way to obtain a flat spectrum leading to a matching with the observed shape (but as discussed in [20] this is not the only option), and we will focus on that case here for simplicity, and because we are concerned here with the overall scale of the fluctuation spectrum rather than its shape. The point is that by assuming that the time of collapse of the different modes should depend on the mode's frequency according to $\eta_k^c = z/k$, so that z_k is independent of k (see [7] for a possible physical explanation behind of such pattern), we obtain a form of the predicted spectrum that is in agreement with the so called scale invariant spectrum obtained in ordinary treatments and in the observational studies.

Going back to the overall scale of the fluctuation spectrum we note that for a generic collapse scheme, the term in (105) which contains the factor $1/\epsilon$ becomes dominant and the quantity $H(k\eta_r, z_k)$ can be approximated by

$$
\begin{aligned}
H(k\eta_r, z_k) \approx \frac{8D_k^2}{\epsilon^2}\Bigg\{ & \lambda_{k,1}^2\left(1+\frac{1}{z_k^2}\right)\left(\cos\Delta_r - \frac{\sin\Delta_r}{k\eta_r}\right)^2 + \\
& + \lambda_{k,2}^2\left[\cos\Delta_r\left(\frac{1}{k\eta_r}-\frac{1}{z_k}\right)+\sin\Delta_r\left(\frac{1}{k\eta_r z_k}+1\right)\right]^2\Bigg\}
\end{aligned}
\tag{113}
$$

Inserting this last expression into (109), we obtain the power spectrum of the perturbations for a generic collapse:

$$
\begin{aligned}
\mathcal{P}_\Psi^{col}(k,\eta) = \frac{1}{54\pi^2}\frac{V}{\epsilon M_{pl}^4}\Bigg\{ & D_k^2\lambda_{k,1}^2\left(1+\frac{1}{z_k^2}\right)\left(\cos\Delta_r - \frac{\sin\Delta_r}{k\eta_r}\right)^2 + \\
& + D_k^2\lambda_{k,2}^2\left[\cos\Delta_r\left(\frac{1}{k\eta_r}-\frac{1}{z_k}\right)+\sin\Delta_r\left(\frac{1}{k\eta_r z_k}+1\right)\right]^2\Bigg\}
\end{aligned}
\tag{114}
$$

showing that in general the overall amplitude for power the spectrum would be proportional to $V/\epsilon M_{pl}^4$ as in the standard approach.

V. CONCLUSIONS

In the standard inflationary approach, one finds one must un-naturally constraint in the energy scale of V because of the fact that the prediction for the amplitude of the primordial fluctuation spectrum is $\mathcal{P}_\Psi^{std} \sim V/\epsilon M_{pl}^4$. This is considered as a fine tuning problem as it indicates that in the model one must decrease V as ϵ decreases. That is, as one adjust ϵ to get a flatter spectrum one must adjust the potential to prevent the overall scale of the fluctuations from becoming too large. On the other hand the standard approach suffers from some serious conceptual shortcomings as discussed for instance in [7]. We thus saw as quite hopeful the early indications that the modified approached proposed to deal with the conceptual shortcomings would naturally resolve the more technical, fine tuning problem.

We have seen here that indeed, in the "collapse picture" the prediction for the amplitude of the power spectrum can be $\mathcal{P}_\Psi^{\star col} \sim \epsilon V/M_{pl}^4$ if we choose a very particular characterization of the collapse's parameters. For a generic collapse, however we do obtain the same unwelcome result as in the standard approach. Furthermore, we found that the choice of the parameters that lead to the un-amplified spectrum, seems to involve an undesirable teleological aspect. This is as far as we can see now a serious blow for our early hopes in this particular regard. However, the fact that the new approach involves different aspects as part of explanation of the birth of the primordial inhomogeneities might still lead to unexpected possible approaches to deal with this and other problematic aspects of the inflationary models.

It is worthwhile noting that the collapse model was not originally conceived to deal with the problems such as that of the amplitude of the spectrum, but to deal with the conceptual issues affecting the standard explanation of the origin of cosmic structure. In this sense we should stress that this work not only shows that the ideas tied to the collapse scheme are not mere philosophical in nature, but are susceptible to standard theoretical analysis, and indeed the fact that the collapse model offered, a technical possibility of adjusting the parameters describing the collapse, in such a way to eliminate the fine tuning problem, and despite the fact that such solution seems rather unconvincing (at least in the absence of a causal mechanism that could enforce the condition of (110)), can be considered as illustrative of the potential of new approach in dealing with more specific and less conceptual aspects of inflationary cosmology.

We end our discussion by noting that the inflationary scenario provides an important source of actual observational data -perhaps the only one- about the gravity/quantum interface, and if, as we believe, these can be connected to some aspect of quantum gravity, the careful study of said issues might at long last offer potential clues on a subject generally considered as empirically unreachable.

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