

Particle physics models of inflation and curvaton scenarios

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Abstract

We review the particle theory origin of inflation and curvaton mechanisms for generating large scale structures and the observed temperature anisotropy in the cosmic microwave background (CMB) radiation. Since inflaton or curvaton energy density creates all matter, it is important to understand the process of reheating and preheating into the relevant degrees of freedom required for the success of Big Bang Nucleosynthesis. We discuss two distinct classes of models, one where inflaton and curvaton belong to the hidden sector, which are coupled to the Standard Model gauge sector very weakly. There is another class of models of inflaton and curvaton, which are embedded within Minimal Supersymmetric Standard Model (MSSM) gauge group and beyond, and whose origins lie within *gauge invariant* combinations of supersymmetric quarks and leptons. Their masses and couplings are all well motivated from low energy physics, therefore such models provide us with a unique opportunity that they can be verified/falsified by the CMB data and also by the future collider and non-collider based experiments. We then briefly discuss stringy origin of inflation, alternative cosmological scenarios, and bouncing universes.

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I. INTRODUCTION

The paradigm of primordial inflation has met with glorious successes over the past three decades since its conception [1–6] (for some excellent reviews on inflation, see [7–10]). In the most general scenario, inflation occurs because a slowly rolling scalar field, the inflaton, dynamically gives rise to an epoch of accelerated expansion dominated by a false vacuum (for a review on inflaton models, see [10]). During inflation, quantum fluctuations imprinted on space-time are stretched outside the Hubble patch. These primordial fluctuations eventually re-enter the Hubble patch, whence their form can be extracted by observing the perturbations in the Cosmic Microwave Background (CMB) radiation. Slow-roll inflationary scenarios generically predict almost Gaussian *adiabatic* perturbations with a *nearly* flat spectrum (for a review on generating quantum fluctuations during inflation, see [11]), which have met with an unprecedented success with the latest observations, see the recent data from WMAP [12] ¹. Future CMB experiments such as PLANCK ² will improve the current data, and also provide useful constraints on the scale of inflation in terms of primordial gravity waves [13–16], departure from random Gaussian fluctuations [17–22], isocurvature perturbations [23, 24], etc.

The end of inflation can be considered as a paradigm for the origin of matter, since all matter arises from the vacuum energy stored in the inflaton field. However the present models do not give clear predictions as to what sort of matter there is to be found in the early universe. Theoretical and observational successes of the Big Bang Nucleosynthesis (BBN) have constrained the degrees of freedom around the temperature of $T \geq 1 - 5$ MeV, which contains the Standard Model (SM) quarks and leptons and three generations of neutrinos [25–28]. The present observational uncertainties allow *only* one extra species of relativistic particle at the time of BBN [27, 29]. From the current observations we also know that SM baryons constitute about 4.6% of the total energy density, almost 23% of the total energy density is in non-luminous, non-baryonic dark matter, and the rest of the energy density is in the form of dark energy [12].

Besides the cosmological issues, one of the theoretical challenges is to understand why the mass scale of the SM, of $\mathcal{O}(100)$ GeV, is much lower than the scale of gravity, $M_{\text{P}} =$

¹ See <http://map.gsfc.nasa.gov/>.

² See <http://www.sciops.esa.int/index.php?project=PLANCK&page=index>

$(8\pi G_N)^{-1/2} = 2.436 \times 10^{18}$ GeV. Unfortunately, the SM masses are not protected from the quantum corrections, which is known as the *hierarchy problem*. The most popular remedy is the supersymmetry (SUSY) (for a review, see [30–34]), which is believed to be broken at a scale $\sim \mathcal{O}(100 - 1000)$ GeV. The SUSY is presumably broken first at high scales in some hidden sector then transmitted to the minimal SUSY extension of the SM, known as the MSSM, by gravitational [30, 31] or gauge interactions [35, 36]. The Large Hadronic Collider (LHC)³ at CERN has a potential to discover the MSSM particles. Theoretical estimations of radiative corrections of gauge couplings also suggest that MSSM enables grand unification (GUT) of the gauge interactions at scales $M_{GUT} \sim 10^{16}$ GeV (for a GUT review, see [37, 38]). When SUSY is broken locally, like any other gauge symmetry, an intimate connection with gravity emerges, known as the supergravity (SUGRA), which is valid below the Planck scale [30]. Furthermore, the unification of gravity with the other gauge interactions seems to require viewing fundamental particles as, instead, excitations of extended objects in the framework of string theory [39–41]. Therefore, it is important to ask whether beyond the SM physics can provide all the right ingredients for inflation to occur or not.

Since the origin of baryons and dark matter bring inflation closer to the particle physics, the models of inflation must rely on an effective field theory treatment, where the inflaton belongs to the *hidden sector*, or the *observable sector*. In the former case, the coupling between the inflaton and the (MS)SM sector is either through gravitational strength, or via small unknown Yukawa coupling. Unfortunately, in the case of a hidden sector inflation the particle origin, mass, and couplings are largely unconstrained and at times unmotivated from theoretical point of view. Therefore, the inflationary predictions from such hidden sector models are also highly model dependent.

String theory lends strong support to hidden sector models of inflation. There are plenty of absolute gauge singlet moduli, which mainly arise in the gravitational sector upon compactifications [42–44]. There are many attempts to embed inflation within string theory, for a review, see [8, 45–53]. The most exciting phenomenological revelation from string theory is that it can stabilize all the moduli [42], and also the volume modulus [54], with large and small positive cosmological constants, which has lead us to believe in a stringy landscape [44, 55–57]. At low energies, the number of vacua could be humongous,

³ <http://lhc.web.cern.ch/lhc/>

$10^{500} - 10^{1000}$ [44, 56, 57], and one in 10^{10} could be SM like [58–60]. False vacuum inflation in such a landscape is generic with all possible scales of inflation down to the current cosmological constant [54]. However, our patch of the universe must have had a graceful exit from inflation at least before BBN. Exiting from such eternally inflating regime and exciting the SM degrees of freedom pose a new challenge for string theory.

In order to seek an observable sector origin of inflation, it is important to ask whether inflation can happen within the GUT theory [61–84]. Invariably all of the models of inflation require an *absolute gauge singlet* inflaton couplings to the GUT/MSSM fields to drive the first phase of inflation, or to prepare the initial conditions for inflation. There is also an interesting proposal to realize inflation within the SM, with a non-trivial Higgs coupling to the Ricci scalar [85]. The advantage is that inflation occurs within an observable sector physics, therefore the origin of matter creation is ascertained. But this idea does not rely on SUSY at all, and assumes the SM to be valid all the way up to the Planck scale.

In a recent development, it has been shown that within MSSM parameters allow a unique possibility to realize inflation with the help of *gauge invariant flat directions* [86–89]. In MSSM there are many scalars, which span into a moduli space of *gauge invariant F*- and *D*-flat directions made up of squarks and sleptons (SUSY partners of quarks and leptons) (for a review, see [90, 91]), which carry the SM charges, i.e. baryon and/or lepton number. These inflatons have an *enhanced symmetry point* near the origin (at a VEV defined by zero). Away from the origin the inflatons break wholly or partly the SM gauge symmetry depending on the flat direction. But such a spontaneous breaking of charge and color in the early universe is not considered to be dangerous, provided they all settle down to their minimum before the electroweak phase transition. Note that in all these cases inflation occurs within an observable sector, where their mass and couplings are all well motivated from low energy physics.

In any inflationary scenario, it is important to understand the mechanisms of how to excite the SM quarks and leptons, known as the reheating [92–95] and preheating [96–118], and how to thermalize the universe with the MS(SM) degrees of freedom [119–123]. In this regard the observable sector models of inflation have an advantage, since the inflaton couplings to the matter fields are all known.

There is yet another paradigm for generating the amplitude for the CMB perturbations, known as the curvaton scenario [124–129]. The curvaton is a light scalar field, which obtains

its quantum fluctuations induced by the vacuum energy of the inflaton potential. However, the curvaton being light does not decay as rapidly as the inflaton, albeit its slow dynamics leads to its late decay. The relative perturbations between the fields give rise to entropy perturbations, which feed the curvature perturbations. Once the curvaton decays, it converts all its entropy perturbations into the adiabatic and nearly scale invariant perturbations. One advantage of the curvaton scenario is that it is possible to generate significant non-Gaussianity [124, 129, 130], however the present non-Gaussianity bounds are also tied to the residual isocurvature perturbations [131]. The challenges for the curvaton paradigm are the same as that of the inflaton, if the curvaton decays, then it must excite the (MS)SM degrees of freedom [132–134]. There are also alternative mechanisms to understand the temperature anisotropy in the CMB data *without invoking inflation*, such as in the case of a bouncing cosmology [135–140], we will briefly discuss some of these scenarios.

The main goal of this review is to address the origin of inflation and curvaton, where they explain the large scale structures, and also the microphysical origin of the inflaton and curvaton. This can be achieved provided they belong to a well-motivated sector of particle physics. Our aim is to review such models in details and some of their cosmological consequences.

The review is organized as follows. In section (Sec.) II, we recapitulate some basic inflationary cosmology, in particular quantum fluctuations during inflation, multi-field perturbations, curvaton scenario, non-Gaussianity, challenges and requirements for a successful inflation. In Sec. III, we present some background material for particle physics tools required for inflation and curvaton, we particularly focus on MSSM, renormalization group equations, moduli space of flat directions within MSSM, properties of flat directions and their cosmological consequences, SUGRA and their role in building inflationary models, and a brief discussion on cosmic strings and grand unified theories (GUT). In Sec. IV, we discuss models of inflation, particularly highlighting the connections with particle theory, we discuss non-SUSY models of inflation, SM Higgs inflation and implications for collider and non-collider experiments, SUSY F - and D -term inflation models, and embedding inflation within SUSY GUTs. In Sec. V, we discuss various attempts to gauge the inflaton with the SM charges, we discuss MSSM inflation models where we recognize the inflaton candidates. We also consider thermal production of dark matter in conjunction with inflationary parameter space. In Sec. VI, we discuss inflaton decay, reheating and thermalization. We

focus on gauge singlet inflaton couplings to the SM and the MSSM. We discuss thermalization in perturbative decay of inflaton, basics of preheating and its applications to SUSY inflationary models. In Sec. VII, we discuss models of curvaton where the curvaton is a MSSM flat direction. We then single out curvaton candidates, and discuss predictions for non-Gaussianity. In Sec. VIII, we discuss inflationary models within a string theory setup. We briefly describe alternative mechanisms for generating primordial perturbations in the context of a non-singular bouncing cosmology, and discuss various challenges they face.

II. INFLATION

A. Slow-roll inflation

A completely flat potential would render inflation future eternal (but not past [4, 6, 141–144]), provided the energy density stored in the flat direction dominates. The inflaton direction is however not completely flat but has a potential $V(\phi)$ with some slope. An inflationary phase is obtained when the expansion rate evolution satisfies $\ddot{a} > 0$. Slow-roll inflation assumes that the potential dominates over the kinetic energy of the inflaton $\dot{\phi}^2 \ll V(\phi)$, and $\ddot{\phi} \ll V'(\phi)$, therefore the Friedmann and the Klein-Gordon equations can be approximated as:

$$H^2 \approx \frac{1}{3M_{\text{P}}^2} V(\phi), \quad (1)$$

$$3H\dot{\phi} \approx -V'(\phi), \quad (2)$$

where prime denotes derivative with respect to ϕ . The slow-roll conditions are give by:

$$\epsilon(\phi) \equiv \frac{M_{\text{P}}^2}{2} \left(\frac{V'}{V} \right)^2 \ll 1, \quad (3)$$

$$|\eta(\phi)| \equiv M_{\text{P}}^2 \left| \frac{V''}{V} \right| \ll 1. \quad (4)$$

Note that ϵ is positive by definition. These conditions are necessary but not sufficient for inflation. They only constrain the shape of the potential but not the velocity of the field $\dot{\phi}$. Therefore, a tacit assumption behind the success of the slow-roll conditions is that the inflaton field should not have a large initial velocity.

Slow-roll inflation comes to an end when the slow-roll conditions are violated, $\epsilon \sim 1$, and $\eta \sim 1$. However, there are certain models where this need not be true, for instance

in hybrid inflation models [145], where inflation comes to an end via a phase transition, or in oscillatory models of inflation where slow-roll conditions are satisfied only on average [146, 147], or inflation happens on average over every oscillations of a bouncing universe [148], or in fast roll inflation where the slow-roll conditions are never met [149]. The K-inflation where only the kinetic term dominates where there is no potential at all [150]

One of the salient features of the slow-roll inflation is that there exists a late time attractor behavior ⁴. This means that during inflation the evolution of a scalar field at a given field value has to be independent of the initial conditions. Therefore slow-roll inflation should provide an attractor behavior which at late times leads to an identical field evolution in the phase space irrespective of the initial conditions [151]. In fact the slow-roll solution does not give an exact attractor solution to the full equation of motion but is nevertheless a fairly good approximation [151]. A similar statement has been proven for multi-field exponential potentials without slow-roll conditions (i.e. assisted inflation) [152].

The standard definition of the number of e-foldings between time, t , and the end of inflation, t_{end} , is given by

$$N \equiv \ln \frac{a(t_{end})}{a(t)} = \int_t^{t_{end}} H dt \approx \frac{1}{M_{\text{P}}^2} \int_{\phi_{end}}^{\phi} \frac{V}{V'} d\phi, \quad (5)$$

where ϕ_{end} is defined by $\epsilon(\phi_{end}) \sim 1$, provided inflation comes to an end via a violation of the slow-roll conditions. The number of e-foldings can be related to the Hubble crossing mode $k = a_k H_k$ by comparing with the present Hubble length $a_0 H_0$. The final result is [153, 154]

$$N(k) = 62 - \ln \frac{k}{a_0 H_0} - \ln \frac{10^{16} \text{ GeV}}{V_k^{1/4}} + \ln \frac{V_k^{1/4}}{V_{end}^{1/4}} - \frac{1}{3} \ln \frac{V_{end}^{1/4}}{\rho_{rh}^{1/4}}, \quad (6)$$

where the subscripts *end* (*rh*) refer to the end of inflation (end of reheating) ⁵.

A simple generalization of the above formula has been derived in [155], where there are multiple stages of inflation, with potentials V_I, V_i, V_L one after the other separated by the matter epochs, whose expansions are parameterized by $t^{n_1}, t^{n_2}, \dots, t^{n_L}$, with $n_1, n_2, \dots, n_L \leq 1$. We can assume instant reheating after the last phase of inflation, which yields, $\rho_{L,end} =$

⁴ A more rigorous set of parameters are presented describing slow-roll inflation in Eq. (41), which is independent of slow-roll conditions or the number of fields.

⁵ For the particular scale of today's Hubble length, we define: $N_Q \equiv N(k = a_0 H_0)$. The corresponding slow-roll parameters at that scale will be denoted by ϵ_Q, η_Q .

$\rho_{rh,L}$. Concentrating on the present horizon scale $N_Q \equiv N_I(a_0 H_0)$, we obtain [155]

$$\sum_{i=1}^L N_i = 62 + \frac{1}{4} \ln \left(\frac{V_I^2}{V_L M_{\text{P}}^4} \right) + \sum_{i=1}^{L-1} \frac{n_i}{2} \ln \left(\frac{V_{i+1}}{V_i} \right). \quad (7)$$

One can impose new constraints arising from the fact that there should be no reprocessing of modes in between the two phases of inflation on the scales probed by CMB experiments. This requirement constraints [155]

$$\ln \left(\frac{1}{a_{k_c} H_i} \right) > \ln \left(\frac{1}{a_{rh,i} H_{i+1}} \right), \quad i = 2, \dots, L-1 \quad (8)$$

Hence, in general we obtain the following set of constraints [155]:

$$\begin{aligned} N_Q &> 6.9 + \frac{1-n_1}{2} \ln \left(\frac{V_I}{V_{II}} \right), \\ \sum_{j=2}^i N_j &> \frac{1}{2} \ln \left(\frac{V_{II}}{V_{i+1}} \right) - \sum_{j=2}^i \frac{n_j}{2} \ln \left(\frac{V_i}{V_{i+1}} \right), \quad i = 2, \dots, L-1. \end{aligned} \quad (9)$$

In the special case $n_i = 1$, it is easy to see that these constraints are trivially satisfied ($\sum_j N_j > 0$). The details of the thermal history of the universe determine the precise number of e-foldings required to solve the horizon problem, but for most practical purposes (for high scale inflation with large reheating temperature) it is sufficient to assume that $N_Q \approx 50 - 60$, keeping all the uncertainties such as the scale of inflation and the end of inflation within a margin of 10 e-foldings. A significant modification can take place only if there is an epoch of late inflation such as thermal inflation [156, 157], or in theories with a low quantum gravity scale [158–160], or if there are phase transitions during inflation [161–163].

B. Primordial density perturbations

Initially, the theory of cosmological perturbations has been developed in the context of FRW cosmology [164], and for models of inflation in [23, 165–179]. For a complete review on this topic, see [11]. For a real single scalar field there arise only adiabatic density perturbations. In case of several fluctuating fields there will in general also be isocurvature perturbations. Irrespective of the nature of perturbations, any light scalar field obtains quantum fluctuations in a de Sitter space, which has many applications.

1. Fluctuations in de Sitter

By solving the Klein-Gordon equation for a light scalar field in a conformal metric: $ds^2 = g_{\mu\nu}dx^\mu dx^\nu = a^2(\tau, x)(d\tau^2 - dx^2)$, one can find the plane wave solution, $\phi(\mathbf{x}, \tau) = \frac{1}{(2\pi)^{3/2}} \int d^3k (\phi_k(\tau)e^{ik\cdot\mathbf{x}} + \text{h.c.})$, for the mode function: [172, 180–186]:

$$\begin{aligned} \phi_k(\tau) &= \left(\frac{\pi}{4}\right)^{1/2} H|\tau|^{3/2} (c_1 H_\nu^{(1)}(k\tau) + c_2 H_\nu^{(2)}(k\tau)) , \\ \tau &= -H^{-1}e^{-Ht}, \quad \text{and} \quad \nu^2 = \frac{9}{4} - \frac{m^2}{H^2}, \end{aligned} \quad (10)$$

where m is the mass of the scalar field, $H_\nu^{(1)}$ and $H_\nu^{(2)}$ are the Hankel functions and c_1, c_2 are constants. By using a point splitting regularization scheme, it is possible to obtain a Bunch-Davies vacuum for a de Sitter background which actually corresponds to taking $c_1 = 0$, and $c_2 = 1$.

Generically, in a de Sitter phase, the main contribution to the two point correlation function comes from the long wavelength modes; $k|\tau| \ll 1$ or $k \ll H \exp(Ht)$, determined by the Hubble expansion rate [172, 183].

$$\langle \phi^2 \rangle \approx \frac{1}{(2\pi)^3} \int_H^{He^{Ht}} d^3k |\phi_k|^2. \quad (11)$$

The integration yields an indefinite increase in the variance with time

$$\langle \phi^2 \rangle \approx \frac{H^3}{4\pi^2} t. \quad (12)$$

This result can also be obtained by considering the Brownian motion of the scalar field [144, 187–190]. For a massive field with $m \ll H$, and $\nu \neq 3/2$, one does not obtain an indefinite growth of the variance of the long wavelength fluctuations, but [173, 181–183, 191]:

$$\langle \phi^2 \rangle = \frac{3H^4}{8\pi^2 m^2} \left(1 - e^{-(2m^2/3H^2)t}\right). \quad (13)$$

In the limiting case when $m \rightarrow H$, the variance goes as $\langle \phi^2 \rangle \approx H^2$. In the limit $m \gg H$, the variance goes as $\langle \phi^2 \rangle \approx (H^3/12\pi^2 m)$. Only in a massless case $\langle \phi^2 \rangle$ can be treated as a homogeneous background field with a long wavelength mode.

2. Adiabatic perturbations and the Sachs-Wolfe effect

Let us consider small inhomogeneities, $\phi(\mathbf{x}, t) = \phi(t) + \delta\phi(\mathbf{x}, t)$, such that $\delta\phi \ll \phi$. Perturbations in matter densities automatically induce perturbations in the background

metric, but the separation between the background metric and a perturbed one is not unique. One needs to choose a gauge. A simple choice would be to fix the observer to the unperturbed matter particles, where the observer will detect a velocity of matter field falling under gravity; this is known as the Newtonian or the longitudinal gauge because the observer in the Newtonian gravity limit measures the gravitational potential well where matter is falling in and clumping. The induced metric can be written as

$$ds^2 = a^2(\tau) [(1 + 2\Phi)d\tau^2 - (1 - 2\Psi)\delta_{ik}dx^i dx^k] , \quad (14)$$

where Φ has a complete analogue of Newtonian gravitational potential. In the case when the spatial part of the energy momentum tensor is diagonal, i.e. $\delta T_j^i = \delta_j^i$, it follows that $\Phi = \Psi$, [11]. Right at the time of horizon crossing one finds a solution for $\delta\phi$ as

$$\langle |\delta\phi_k|^2 \rangle = \frac{H(t_*)^2}{2k^3} , \quad (15)$$

where t_* denotes the instance of horizon crossing. Correspondingly, we can also define a power spectrum

$$\mathcal{P}_\phi(k) = \frac{k^3}{2\pi^2} \langle |\delta\phi_k|^2 \rangle = \left[\frac{H(t_*)}{2\pi} \right]^2 \equiv \left[\frac{H}{2\pi} \right]^2 \Big|_{k=aH} . \quad (16)$$

Note that the phase of $\delta\phi_k$ can be arbitrary, and therefore, inflation has generated a Gaussian perturbation.

In the limit $k \rightarrow 0$, one can find an exact solution for the long wavelength inhomogeneities $k \ll aH$ [7, 11], which reads:

$$\Phi_k \approx c_1 \left(\frac{1}{a} \int_0^t a dt' \right) + c_2 \frac{H}{a} , \quad (17)$$

$$\frac{\delta\dot{\phi}_k}{\dot{\phi}} = \frac{1}{a} \left(c_1 \int_0^t a dt' - c_2 \right) , \quad (18)$$

where the dot denotes derivative with respect to physical time. The growing solutions are proportional to c_1 , the decaying proportional to c_2 . Concentrating upon the growing solution, it is possible to obtain a leading order term in an expansion with the help of the slow-roll conditions:

$$\Phi_k \approx -c_1 \frac{\dot{H}}{H^2} , \quad (19)$$

$$\frac{\delta\dot{\phi}_k}{\dot{\phi}} \approx \frac{c_1}{H} . \quad (20)$$

Note that at the end of inflation, which is indicated by $\ddot{a} = 0$, or equivalently by $\dot{H} = -H^2$, one obtains a constant Newtonian potential $\Phi_k \approx c_1$. This is perhaps the most significant result for a single field perturbation.

In a long wavelength limit one obtains a constant of motion ζ [11, 169, 192] defined as ⁶ :

$$\zeta = \frac{2}{3} \frac{H^{-1} \dot{\Phi}_k + \Phi_k}{1+w} + \Phi_k, \quad w = \frac{p}{\rho}. \quad (21)$$

This is also known as a comoving curvature perturbation [193] reads in the longitudinal gauge [11] for the slow-roll inflation as

$$\zeta_k = \Phi_k - \frac{H^2}{\dot{H}} \left(H^{-1} \dot{\Phi}_k + \Phi_k \right). \quad (22)$$

For CMB and structure formation we need to know the metric perturbation during the matter dominated era when the metric perturbation is $\Phi(t_f) \approx (3/5)c_1$. Substituting the value of c_1 from Eq. (20), we obtain

$$\Phi_k(t_f) \approx \frac{3}{5} H \frac{\delta\phi_k}{\dot{\phi}} \Big|_{k=aH}. \quad (23)$$

In a similar way it is also possible to show that the comoving curvature perturbations is given by

$$\zeta_k \approx \frac{H}{\dot{\phi}} \delta\phi \Big|_{k=aH}, \quad (24)$$

where $\delta\phi$ denotes the field perturbation on a spatially flat hypersurfaces, because on a comoving hypersurface $\delta\phi = 0$, by definition. Therefore, on flat hypersurfaces

$$\delta\phi_k = \dot{\phi} \delta t, \quad (25)$$

where δt is the time displacement going from flat to comoving hypersurfaces [24, 153]. As a result

$$\zeta_k \equiv H \delta t. \quad (26)$$

Note that during matter dominated era the curvature perturbation and the metric perturbations are related to each other

$$\Phi_k = -\frac{3}{5} \zeta_k. \quad (27)$$

⁶ If the equation of state for matter remains constant, there exists a simple relationship which connects the metric perturbations at two different times: $\Phi_k(t_f) = \frac{1+\frac{2}{3}(1+w(t_f))^{-1}}{1+\frac{2}{3}(1+w(t_i))^{-1}} \Phi_k(t_i)$ [11, 169, 192].

In the matter dominated era the photon sees this potential well created by the primordial fluctuation and the redshift in the emitted photon is given by

$$\frac{\Delta T_k}{T} = -\Phi_k. \quad (28)$$

At the same time, the proper time scale inside the fluctuation becomes slower by an amount $\delta t/t = \Phi_k$. Therefore, for the scale factor $a \propto t^{2/3}$, decoupling occurs earlier with

$$\frac{\delta a}{a} = \frac{2}{3} \frac{\delta t}{t} = \frac{2}{3} \Phi_k. \quad (29)$$

By virtue of $T \propto a^{-1}$ this results in a temperature which is hotter by

$$\frac{\Delta T_k}{T} = -\Phi_k + \frac{2}{3} \Phi_k = -\frac{\Phi_k}{3}. \quad (30)$$

This is known as the Sachs-Wolfe effect [194].

3. Spectrum of adiabatic perturbations

Now, one can immediately calculate the spectrum of the metric perturbations. For a critical density universe

$$\delta_k \equiv \left. \frac{\delta \rho}{\rho} \right|_k = -\frac{2}{3} \left(\frac{k}{aH} \right)^2 \Phi_k, \quad (31)$$

where $\nabla^2 \rightarrow -k^2$, in the Fourier domain. Therefore, with the help of Eqs. (16,23), one obtains

$$\delta_k^2 \equiv \frac{4}{9} \mathcal{P}_\Phi(k) = \frac{4}{9} \frac{9}{25} \left(\frac{H}{\dot{\phi}} \right)^2 \left(\frac{H}{2\pi} \right)^2, \quad (32)$$

where the right hand side can be evaluated at the time of horizon exit $k = aH$. In fact the above expression can also be expressed in terms of curvature perturbations [24, 153]

$$\delta_k = \frac{2}{5} \left(\frac{k}{aH} \right)^2 \zeta_k, \quad (33)$$

and following Eq. (22), we obtain $\delta_k^2 = (4/25) \mathcal{P}_\zeta(k) = (4/25)(H/\dot{\phi})^2(H/2\pi)^2$, exactly the same expression as in Eq. (32). With the help of the slow-roll equation $3H\dot{\phi} = -V'$, and the critical density formula $3H^2 M_{\text{p}}^2 = V$, one obtains

$$\delta_k^2 \approx \frac{1}{75\pi^2 M_{\text{p}}^6} \frac{V^3}{V'^2} = \frac{1}{150\pi^2 M_{\text{p}}^4} \frac{V}{\epsilon}, \quad \text{and} \quad \mathcal{P}_\zeta(k) = \frac{1}{24\pi^2 M_{\text{p}}^4} \frac{V}{\epsilon}, \quad (34)$$

where we have used the slow-roll parameter $\epsilon \equiv (M_{\text{p}}^2/2)(V'/V)^2$. The COBE satellite measured the CMB anisotropy and fixes the normalization of $\mathcal{P}_\zeta(k)$ on a very large scales. For

a critical density universe, if we assume that the primordial spectrum can be approximated by a power law (ignoring the gravitational waves and the k -dependence of the power n_s) [12]

$$\mathcal{P}_\zeta(k) \simeq (2.445 \pm 0.096) \times 10^{-9} \left(\frac{k}{k_0} \right)^{n_s-1}, \quad (35)$$

where n_s is called the spectral index (or spectral tilt), the reference scale is: $k_0 = 7.5a_0H_0 \sim 0.002 \text{ Mpc}^{-1}$, and the error bar on the normalization is given at 1σ . The spectral index $n(k)$ is defined as

$$n(k) - 1 \equiv \frac{d \ln \mathcal{P}_\zeta}{d \ln k}. \quad (36)$$

This definition is equivalent to the power law behavior if $n(k)$ is close to a constant quantity over a range of k of interest. One particular value of interest is $n_s \equiv n(k_0)$. If $n_s = 1$, the spectrum is flat and known as Harrison-Zeldovich spectrum [195, 196]. For $n_s \neq 1$, the spectrum is tilted, and $n_s > 1$ ($n_s < 1$) is known as a blue (red) spectrum. In the slow-roll approximation, this tilt can be expressed in terms of the slow-roll parameters and at first order ⁷

$$n_s - 1 = -6\epsilon + 2\eta + \mathcal{O}(\epsilon^2, \eta^2, \epsilon\eta, \xi^2). \quad (37)$$

The running of these parameters are given by [151]

$$\frac{d\epsilon}{d \ln k} = 2\epsilon\eta - 4\epsilon^2, \quad \frac{d\eta}{d \ln k} = -2\epsilon\eta + \xi^2, \quad \frac{d\xi^2}{d \ln k} = -2\epsilon\xi^2 + \eta\xi^2 + \sigma^3, \quad (38)$$

where

$$\xi^2 \equiv M_{\text{P}}^4 \frac{V'(d^3V/d\phi^3)}{V^2}, \quad \sigma^3 \equiv M_{\text{P}}^6 \frac{V'^2(d^4V/d\phi^4)}{V^3}. \quad (39)$$

Slow-roll inflation requires that $\epsilon \ll 1, |\eta| \ll 1$, and therefore naturally predicts small variation in the spectral index within $\Delta \ln k \approx 1$ [197]

$$\frac{dn(k)}{d \ln k} = -16\epsilon\eta + 24\epsilon^2 + 2\xi^2. \quad (40)$$

Independently of slow-roll considerations or of the number of fields involved in the dynamics of inflation, a new set of parameters, known as the Hubble-flow parameters, were discussed

⁷ At second order,

$$n_s - 1 = 2 \left[-3\epsilon + \eta - \left(\frac{5}{3} + 12C \right) \epsilon^2 + (8C - 1)\epsilon\eta + \frac{1}{3}\eta^2 - \left(C - \frac{1}{3} \right) \xi^2 \right],$$

where C is a numerical constant $C = \ln 2 + \gamma_E - 2 \simeq -0.7296$.

in [198, 199]⁸:

$$\epsilon_0 \equiv H, \quad \epsilon_{n+1} \equiv \frac{\ln |\epsilon_n|}{N}. \quad (41)$$

It gives for the slow-roll parameter $\epsilon_1 = -\dot{H}/H^2$, and inflation takes place only when $\ddot{a} > 0$ which is equivalent to $\epsilon_1 < 1$. Slow-roll inflation takes place when $\forall n, \epsilon_n \ll 1$. In the slow-roll limit, these parameters can be related to the slow-roll parameters,

$$\epsilon_1 \simeq \epsilon + \mathcal{O}(\epsilon^2, \eta^2, \xi^2), \quad \epsilon_2 \simeq 4\epsilon - 2\eta + \mathcal{O}(\epsilon^2, \eta^2, \xi^2). \quad (42)$$

Transient violation of slow-roll conditions were studied in hybrid inflation [201], for computation of the power spectrum, see [202, 203]. Models of inflation with large η were also considered in [149].

4. Gravitational waves

Gravitational waves are linearized tensor perturbations of the metric and do not couple to the energy momentum tensor. Therefore, they do not give rise a gravitational instability, but carry the underlying geometric structure of the space-time. The first calculation of the gravitational wave production was made in [13], and the topic has been considered by many authors [14–16]. For reviews on gravitational waves, see [11, 204].

The gravitational wave perturbations are described by a line element $ds^2 + \delta ds^2$, where

$$ds^2 = a^2(\tau)(d\tau^2 - dx^i dx_i), \quad \delta ds^2 = -a^2(\tau)h_{ij}dx^i dx^j. \quad (43)$$

The gauge invariant and conformally invariant 3-tensor h_{ij} is symmetric, traceless $\delta^{ij}h_{ij} = 0$, and divergenceless $\nabla_i h_{ij} = 0$ (∇_i is a covariant derivative). Massless spin 2 gravitons have two degrees of freedom and as a result are also transverse. This means that in a Fourier domain the gravitational wave has a form

$$h_{ij} = h_+ e_{ij}^+ + h_\times e_{ij}^\times. \quad (44)$$

For the Einstein gravity, the gravitational wave equation of motion follows that of a massless Klein Gordon equation [11]. Especially, for a flat universe

$$\ddot{h}_j^i + 3H\dot{h}_j^i + \left(\frac{k^2}{a^2}\right)h_j^i = 0, \quad (45)$$

⁸ It is possible to extend the calculation of metric perturbation beyond the slow-roll approximations based on a formalism similar to that developed in Refs. [174, 177, 179, 200].

As any massless field, the gravitational waves also feel the quantum fluctuations in an expanding background. The spectrum mimics that of Eq. (16)

$$\mathcal{P}_{\text{grav}}(k) = \frac{2}{M_{\text{P}}^2} \left(\frac{H}{2\pi} \right)^2 \Big|_{k=aH}. \quad (46)$$

Note that the spectrum has a Planck mass suppression, which suggests that the amplitude of the gravitational waves is smaller compared to that of the adiabatic perturbations. Therefore it is usually assumed that their contribution to the CMB anisotropy is small. The corresponding spectral index can be expanded in terms of the slow-roll parameters at first order as ⁹

$$r \equiv \frac{\mathcal{P}_{\text{grav}}}{\mathcal{P}_{\zeta}} = 16\epsilon, \quad \text{and} \quad n_t = \frac{d \ln \mathcal{P}_{\text{grav}}(k)}{d \ln k} \simeq -2\epsilon, \quad (47)$$

Note that the tensor spectral index is negative. It is expected that PLANCK could detect gravity waves if $r \gtrsim 0.1$, however the spectral index will be hard to measure in forthcoming experiments. The primordial gravity waves can be generated for large field value inflationary models. Using the definition of the number of e-foldings it is possible to derive the range of $\Delta\phi$ (see for instance [205–207])

$$16\epsilon = r < 0.003 \left(\frac{50}{N} \right)^2 \left(\frac{\Delta\phi}{M_{\text{P}}} \right). \quad (48)$$

C. Multi-field perturbations

In multi-field inflation models contributions to the density perturbations come from all the fields. However unlike in a single scalar case, in the multi-field case there might not be a unique late time trajectory corresponding to all the fields. In a very few cases it is possible to obtain a late time attractor behavior of all the fields; an example is assisted inflation [152]. If there is no late time attractor then different trajectories inherit difference in perturbations, known as entropy perturbations, which opens new set of constraints which we will discuss below [10, 178, 208–210].

⁹ At second order, $r \simeq 16\epsilon \left[1 + \frac{2}{3}(3C - 1)(2\epsilon - \eta) \right]$, where C is a numerical constant $C = \ln 2 + \gamma_E - 2 \simeq -0.7296$.

1. Adiabatic and isocurvature conditions

There are only two kinds of perturbations that can be generated. The first one is the adiabatic perturbation discussed previously; it is a perturbation along the late time classical trajectories of the scalar fields during inflation. When the primordial perturbations enter our horizon they perturb the matter density with a generic *adiabatic condition*, which is satisfied when the density contrast of the individual species is related to the total density contrast δ_k [23, 24]

$$\frac{1}{3}\delta_{kb} = \frac{1}{3}\delta_{kc} = \frac{1}{4}\delta_{k\nu} = \frac{1}{4}\delta_{k\gamma} = \frac{1}{4}\delta_k, \quad (49)$$

where b stands for baryons, c for cold dark matter, γ for photons and ν for neutrinos.

The other type is the isocurvature perturbation. During inflation this can be viewed as a perturbation orthogonal to the unique late time classical trajectory. Therefore, if there were N fluctuating scalar fields during inflation, there would be $N - 1$ degrees of freedom which would contribute to the isocurvature perturbation [211–214].

The *isocurvature condition* is known as $\delta\rho = 0$: the sum total of all the energy contrasts must be zero. The most general density perturbations is then given by a linear combination of an adiabatic and an isocurvature density perturbations.

2. Adiabatic perturbations due to multi-field

In a comoving gauge, see Eq. (22), $\zeta = -H\delta\phi/\dot{\phi}$ holds good even for multi-field inflation models, provided we identify each field component of ϕ along the slow-roll direction. There also exists a relationship between the comoving curvature perturbations and the number of e-foldings, N , given by [208, 215]

$$\zeta = \delta N = \frac{\partial N}{\partial\phi_a}\delta\phi_a, \quad (50)$$

where N is measured by a comoving observer while passing from flat hypersurface (which defines $\delta\phi$) to the comoving hypersurface (which determines ζ). The repeated indices are summed over and the subscript a denotes a component of the inflaton. A more intuitive discussion has been given in [10, 153, 205].

If again one assumes that the perturbations in $\delta\phi_a$ have random phases with an amplitude

$(H/2\pi)^2$, one obtains:

$$\delta_k^2 = \frac{V}{75\pi^2 M_{\text{P}}^2} \frac{\partial N}{\partial \phi_a} \frac{\partial N}{\partial \phi_a}. \quad (51)$$

For a single component $\partial N/\partial \phi \equiv (M_{\text{P}}^{-2}V/V')$, and then Eq. (51) reduces to Eq. (34). By using slow-roll equations we can again define the spectral index

$$n - 1 = -\frac{M_{\text{P}}^2 V_{,a} V_{,a}}{V^2} - \frac{2}{M_{\text{P}}^2 N_{,a} N_{,a}} + 2 \frac{M_{\text{P}}^2 N_{,a} N_{,b} V_{,ab}}{V N_{,c} N_{,c}}, \quad (52)$$

where $V_{,a} \equiv \partial V/\partial \phi_a$, and similarly $N_{,a} \equiv \partial N/\partial \phi_a$. For a single component we recover Eq. (37) from Eq. (52).

3. Isocurvature perturbations and CMB

One may of course simply assume a purely isocurvature initial condition. For any species the entropy perturbation is defined by

$$S_i = \frac{\delta n_i}{n_i} - \frac{\delta n_\gamma}{n_\gamma}, \quad (53)$$

Thus, if initially there is a radiation bath with a common radiation density contrast δ_r , a baryon-density contrast $\delta_b = 3\delta_r/4$, and a CDM density contrast δ_c , then

$$S_c = \delta_c - \frac{3}{4}\delta_r = \frac{\rho_r \delta \rho_c - (3/4)\rho_c \delta \rho_r}{\rho_r \rho_c} = \frac{\rho_r + (3/4)\rho_c}{\rho_r \rho_c} \delta \rho_c \approx \delta_c, \quad (54)$$

where we have used the isocurvature condition $\delta \rho_r + \delta \rho_c = 0$, and the last equality holds in a radiation dominated universe. Similarly the baryon isocurvature is given by: $S_B = \delta_B - (3/4)\delta_r$ and the neutrino (or any other relativistic species) isocurvature component is given by: $S_\nu = (3/4)\delta_\nu - (3/4)\delta_r$.

However a pure isocurvature perturbation gives five times larger contribution to the Sachs-Wolfe effect compared to the adiabatic case [24, 216, 217]. This result can be derived very easily in a matter dominated era with an isocurvature condition $\delta \rho_c = -\delta \rho_r$, which gives a contribution $\zeta_k = (1/3)S_k$. Therefore from Eqs. (27,30), we obtain $\Delta T_k/T = -S_k/15$. There is an additional contribution from radiation because we are in a matter dominated era, see Eq. (54), $S \approx \delta_c \equiv -(3/4)\delta_r$. The sum total isocurvature perturbation $\Delta T_k/T = -S/15 - S/3 = -6S/15$, where S is measured on the last scattering surface. The Sachs-Wolfe effect for isocurvature perturbations fixes the *slope* of the perturbations, rather than the amplitude

[130, 218]. Present CMB data rules out pure isocurvature perturbation spectrum [12, 219–226], although a mixture of adiabatic and isocurvature perturbations remains a possibility.

In the latter case it has been argued that the adiabatic and isocurvature perturbations might naturally turn out to be correlated [209, 227–229]. It is sometimes useful to consider α defined by:

$$\frac{\alpha}{1-\alpha} = \frac{\mathcal{P}_S(k_0)}{\mathcal{P}_\zeta(k_0)}, \quad (55)$$

where $\mathcal{P}_S(k_0)$ is the power spectrum of the entropy perturbation S_c at the pivot scale. This parameter α is constrained by observations, see Sec. II E below.

4. Non-Gaussianity

The inflaton inevitably has to have interactions with other scalars, fermions and gauge fields for a successful reheating. Furthermore, there could be more than one light scalar dynamics involved during and after inflation. The collective dynamics of more than one light field can source non-Gaussianity [17–21, 230–232] (for a review see [22, 205]). The non-Gaussianity can also be generated by invoking initial conditions which depart from Bunch-Davies vacuum [233], non-canonical kinetic term [234, 235], breaking slow-roll conditions abruptly for a brief period [236, 237], multi-field inflationary models [231], curvaton scenarios [124, 129] and large non-Gaussianity during preheating [238–241]. In [242], non-Gaussianity during preheating has been found less significant when studied in the context of δN formalism. The simplest form for the local non-Gaussianity can be written as [151, 178, 208, 243]:

$$\zeta(x) \equiv g(x) + \frac{3}{5}f_{NL} g^2(x) + \frac{9}{25}g_{NL} g^3(x) + \dots, \quad (56)$$

where $g(x)$ is the Gaussian random fluctuations. In general, the non-Gaussianity can be calculated by studying the bispectrum (three point correlator $\langle g_{k_1}, g_{k_2}, g_{k_3} \rangle \neq 0$) and the trispectrum (four point correlator $\langle g_{k_1}, g_{k_2}, g_{k_3}, g_{k_4} \rangle \neq 0$)¹⁰.

¹⁰ Assuming that ζ is dominated by the Gaussian perturbations, g , the power spectrum \mathcal{P}_ζ is determined by, $\langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \rangle = (2\pi)^3 \mathcal{P}_\zeta(k_1) \delta(\mathbf{k}_1 + \mathbf{k}_2)$, the bispectrum \mathcal{B}_ζ is determined by $\langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \zeta_{\mathbf{k}_3} \rangle = (2\pi)^3 \mathcal{B}_\zeta(k_1, k_2, k_3) \delta(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3)$, and the trispectrum \mathcal{T}_ζ is given by $\langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \zeta_{\mathbf{k}_3} \zeta_{\mathbf{k}_4} \rangle = (2\pi)^3 \mathcal{T}_\zeta(k_1, k_2, k_3, k_4) \delta(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3 + \mathbf{k}_4)$, where \mathcal{B}_ζ and \mathcal{T}_ζ can be written as: $\mathcal{B}_\zeta(k_1, k_2, k_3) = (6/5)f_{NL}(\mathcal{P}_\zeta(k_1)\mathcal{P}_\zeta(k_2) + \mathcal{P}_\zeta(k_2)\mathcal{P}_\zeta(k_3) + \mathcal{P}_\zeta(k_3)\mathcal{P}_\zeta(k_1))$, and $\mathcal{T}_\zeta(k_1, k_2, k_3, k_4) =$

The δN formalism developed in Refs. [208, 244, 245] provides a powerful tool to study the non-Gaussianity. It assumes that light fields contribute to the local evolution of the number of e-foldings [246]:

$$\zeta(x, t) = \delta N(\phi_1(x), \phi_2(x), \dots, t) \equiv N(\phi_1(x), \phi_2(x), \dots, t) - N(\phi_1, \phi_2, \dots, t), \quad (57)$$

where $N(x, t)$ is the number of e-foldings of expansion starting from an initial flat slice ending to a slice of uniform density. For instance, upto the first order in field perturbations, $\zeta(x, t) = \sum N_i(t)\delta\phi_i(x)$ leads to $\mathcal{P}_\zeta = (H_k/2\pi)^2 \sum N_i^2(k)$. For a single field [205],

$$\zeta = N'\delta\phi + \frac{1}{2}N''(\delta\phi)^2 = N'\delta\phi + \frac{1}{2}\frac{N''}{N'^2}(N'\delta\phi)^2. \quad (58)$$

where $\nu \equiv \delta/\delta\phi$ and $(3/5)f_{NL} = (1/2)(N''/N'^2)$. Given the fact that $N' = H(t)/\dot{\phi}$, we can evaluate N' , N'' in terms of the slow-roll parameters, which yields [20]

$$\frac{3}{5}f_{NL} = \frac{\eta - 2\epsilon}{2}. \quad (59)$$

The value of f_{NL} in the case of slow-roll inflation is always bounded by the slow-roll parameters. During inflation these parameters $\epsilon, \eta \ll 1$, therefore non-Gaussianity is negligible. Similar conclusion holds for more than one fields during inflation [231, 247].

D. Curvaton and fluctuating inflaton coupling/mass scenarios

The curvaton paradigm involves at least two fields, the inflaton and a light field curvaton, which are not coupled to each other, we will discuss a slightly variant scenario when the fields have coupling. It is essential that (1) the curvature perturbations created by the inflaton are negligible compared to the total curvature perturbations, (2) the curvaton field is very light during inflation therefore, it obtains random fluctuations of order $H_{inf}/2\pi$, and (3) the curvaton oscillations dominates the universe and its decay generates the total curvature perturbations [124, 125, 127–129].

Let us assume a curvaton field, σ , whose equation of motion and the perturbations read as:

$$\ddot{\sigma} + 3H\dot{\sigma} + V_\sigma = 0, \quad \ddot{\delta\sigma}_k + 3H\dot{\delta\sigma}_k + ((k/a)^2 + V_{\sigma\sigma})\delta\sigma_k = 0. \quad (60)$$

$\tau_{NL} (\mathcal{P}_\zeta(k_{13})\mathcal{P}_\zeta(k_3)\mathcal{P}_\zeta(k_4) + 11 \text{ permutations.}) + (54/25)g_{NL} (\mathcal{P}_\zeta(k_2)\mathcal{P}_\zeta(k_3)\mathcal{P}_\zeta(k_4) + 3 \text{ permutations.})$. Here f_{NL} , τ_{NL} and g_{NL} are non-linearity parameters where $\tau_{NL} = (36/25)f_{NL}^2$.

where $V_{\sigma\sigma} \ll H^2$ during inflation, therefore the VEV is $\sigma \approx \sigma_*$ nearly a constant. The perturbations in σ field is given by: $\delta\sigma/\sigma \sim (H_{inf}/2\pi\sigma_*)$ for $H_{inf} \ll \sigma_*$, therefore $\mathcal{P}_{\delta\sigma/\sigma}^{1/2} \sim (H_{inf}/2\pi\sigma_*)$. It is assumed that the curvaton field rolls slowly as the universe becomes radiation dominated after the inflaton decay.

On large scales the curvature perturbations are given by: $\zeta = -H\delta t = -H\delta\rho/\dot{\rho}$, $\zeta_r = (1/4)\delta\rho_r/\rho_r$ and $\zeta_\sigma = (1/3)\delta\rho_\sigma/\rho_\sigma \equiv (1/3)\delta\sigma$, they all evolve independently [246]. The value of ζ_σ has been calculated assuming that the curvaton is oscillating with a pressureless equation of state. During these oscillations the curvaton converts its fluctuations into the curvature perturbations. The total curvature perturbations is given by [124, 129]:

$$\zeta = \frac{4\rho_r\zeta_r + 3\rho_\sigma\zeta_\sigma}{4\rho_r + 3\rho_\sigma}. \quad (61)$$

Since prior to the curvaton oscillations, the curvature perturbations in the universe is dominated by that of the inflaton decay products, i.e. radiation, therefore, $\zeta = \zeta_r$, which simplifies the above expression:

$$\zeta = \frac{\rho_\sigma}{4\rho_r + 3\rho_\sigma}\delta\sigma. \quad (62)$$

If the curvaton energy density dominates over radiation, then $\zeta = (1/3)\delta\sigma$, otherwise, the fraction, $r \sim \rho_\sigma/\rho_r < 1$, would signify the curvaton energy density at the time of decay. In which case $\zeta = (1/4)r\delta\sigma$, and

$$\mathcal{P}_\zeta \approx r^2 \left(\frac{H_{inf}}{2\pi\sigma_*} \right)^2, \quad (63)$$

and the spectral tilt is given by

$$n_s \equiv 1 - 6\epsilon + 2\eta = 1 + \frac{\dot{H}_{inf}}{H_{inf}^2} + \frac{2}{3} \frac{V_{\sigma\sigma}}{H_{inf}^2}. \quad (64)$$

Since \dot{H}_{inf}/H_{inf}^2 , $V_{\sigma\sigma}/H_{inf}^2 \ll 1$, the spectral tilt is fairly close to one. The coherent oscillations of the curvaton generates non-Gaussian perturbations. A small perturbations around the minimum leads to

$$\frac{\delta\rho_\sigma}{\rho_\sigma} = 2\frac{\delta\sigma}{\sigma} + \frac{(\delta\sigma)^2}{\sigma^2}, \quad (65)$$

averaged over many-many oscillations over Hubble period. In some realistic curvaton scenarios, the curvaton may decay *almost instantly* via SM gauge couplings in less than one Hubble time scale. In which case the curvaton oscillations may not last long enough to generate any significant non-Gaussianity.

By using $\zeta = (1/3)\delta\sigma$ and Eq. (56), the non-Gaussianity parameter can be determined by (for a quadratic potential) [129]¹¹:

$$f_{NL} = \frac{5}{4r}. \quad (66)$$

Another interesting proposal is that the perturbations could be generated from the fluctuations of the inflaton coupling to the SM degrees of freedom [248–252]. It has been argued that the coupling strength of the inflaton to ordinary matter or the inflaton mass, instead of being a constant, could depend on the VEV of various fields in the theory. If these fields are light during inflation their quantum fluctuations will lead to spatial fluctuations in the inflaton decay rate. As a consequence, when the inflaton decays, adiabatic density perturbations will be created because fluctuations in the decay rate translate into fluctuations in the reheating temperature.

This can be understood intuitively from the fact that fluctuations in the inflaton decay rate leads to fluctuations in the reheat temperature of the universe, given by $T_{\text{rh}} \sim \lambda\sqrt{m_\phi M_{\text{P}}}$, where m_ϕ is the mass of the inflaton. The fluctuations in the decay rate, $\Gamma \sim m_\phi\lambda^2$ can be translated into fluctuations in the energy density of a thermal bath with $\delta\rho_\gamma/\rho_\gamma = -(2/3)\delta\Gamma/\Gamma$ [249, 250]. The factor 2/3 appears due to red-shift of the modes during the decay of the inflaton whose coherent oscillations still dominates the energy density of the universe. The inflaton decay rate fluctuates if either λ or m_ϕ is a function of a fluctuating light field.

The fluctuation in the decay rate for the various cases is given by:

$$\frac{\delta\Gamma}{\Gamma} = \begin{cases} 2\frac{\delta S}{M} = \frac{H_{inf}}{\pi M}, & \text{direct decay.} \\ 2\frac{\delta S}{S} = \frac{H_{inf}}{\pi S}, & \text{indirect decay.} \\ \frac{\delta S}{S} = \frac{H_{inf}}{2\pi S}, & \text{fluctuating mass.} \end{cases} \quad (67)$$

Various examples do predict non-Gaussianity within a range of $f_{NL} \sim 5$ [249, 250].

¹¹ For a departure from quadratic potential, the form of f_{NL} modifies by Eq. (457) (see Sec. VII C 6).

E. Confrontation to the CMB and other observational data

The CMB data is currently providing (including WMAP, CBI ¹², VSA ¹³, ACBAR ¹⁴, Boomerang ¹⁵) and will provide (including PLANCK) stringent observational data to constrain the power spectrum of density fluctuations. There are other data set which can be used in conjunction; type Ia supernovae (SN), Baryon Acoustic Oscillations (BAO), large scale structures, Lyman- α forest, etc. In this section, we briefly review the current bounds on the amplitude of the power spectrum, spectral index, and tensor to scalar ratio, running of the spectral index, non-Gaussianities, cosmic strings, and isocurvature perturbations.

1. Primordial power spectrum for scalar and tensor

Most of the observational tests of inflation models arise from the 2 point correlation function, related to the power spectrum of the primordial perturbations, both for scalar and tensor perturbations. The most recent update on the WMAP results, combined with SN and BAO data confirmed that so far the minimal 6-parameter Λ CDM model provides a very good fit of the combined observations. It contains the baryon, CDM, and dark energy fractions; $\Omega_b h^2$, $\Omega_c h^2$, Ω_Λ , the spectral index n_s , the optical depth of reionization, τ_{reion} , and the normalization of the power-spectrum $\mathcal{P}_\zeta(k_0)$, with central values and 1σ error bars for the inflation related parameters given by [12]:

$$\Delta_{\mathcal{R}}^2(k_0) = (2.445 \pm 0.096) \times 10^{-9} \text{ at } k_0 = 0.002 \text{Mpc}^{-1}, \quad n_s(k_0) = 0.960 \pm 0.13. \quad (68)$$

In this minimal model, the primordial power spectrum is approximated by the expression of Eq. (35) and the tensor contribution or the k -dependence of the spectral index are neglected ¹⁶. It is important to stress that these central values and error bars vary significantly when other parameters are introduced to fit the data, in part because of degeneracies between

¹² See <http://www.sciops.esa.int/index.php?project=PLANCK&page=index>

¹³ See <http://www.mrao.cam.ac.uk/telescopes/vsa/>

¹⁴ See <http://cosmology.berkeley.edu/group/swlh/acbar/>

¹⁵ See http://www.astro.caltech.edu/lgg/boomerang_front.htm

¹⁶ The confrontation to the data presented here relies on the slow-roll conditions. Alternatively, as proposed by several teams, one can reconstruct the primordial power spectrum [253, 254] as well as the inflationary potential [255, 256] (see also [12] and Refs. therein). These approaches are limited as only a small range in $\phi \in [\phi_Q, \phi_e]$ is observable and therefore accessible to this reconstruction.

parameters (in particular n_s with $\Omega_b h^2$, the optical depth τ , its running, the tensor-to-scalar ratio, r , and the fraction of cosmic strings).

If the tensor-to-scalar ratio r and/or a running α_s are introduced, the best fit and error bars ¹⁷ (at 1σ) [12]

$$\begin{aligned} n_s &= 1.017_{-0.043}^{+0.042}, & \alpha_s &= -0.028 \pm 0.020, \\ n_s &= 0.970 \pm 0.015, & r &< 0.22 \quad (\text{at } 2\sigma), \\ n_s &= 1.089_{-0.068}^{+0.070}, & r &< 0.55 \quad (\text{at } 2\sigma), & \alpha_s &= -0.053 \pm 0.028. \end{aligned} \tag{69}$$

These data therefore suggest that a red spectrum is favored ($n_s = 1$ excluded at 2.5σ from WMAP and at 3.1σ when other data sets are included) if there is no running. Although, the reader and the model builder should keep in mind that a lot more data is necessary before these results can be used as bench-mark points (see for e.g. [257] for a pedagogical presentation about Bayesian model selection).

These various best fit values are consistent with a model that predicts a non-negligible level of tensor or a running of the spectral index. These results are summarized in Fig. 1.

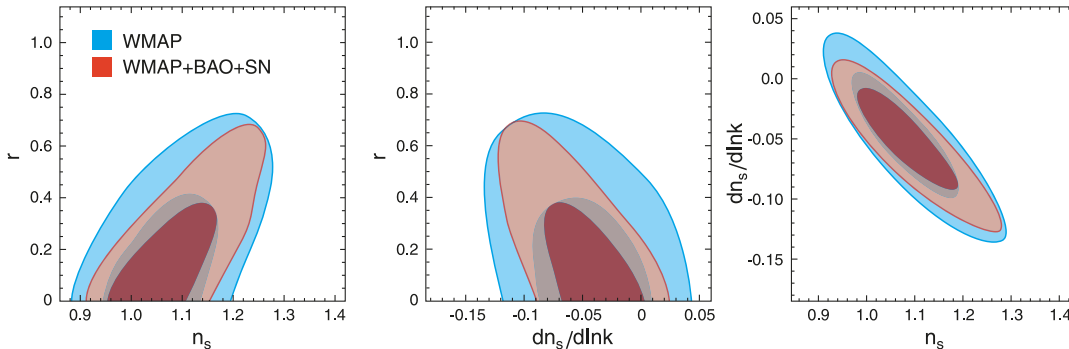


FIG. 1: Two-dimensional likelihood for the spectral index, the running and the ratio tensor/scalar, from the WMAP data only (blue) and the WMAP data combined with the BAO and supernovae data sets (red) at 1σ and 2σ . Figures are taken from [12].

The confrontation of inflationary models to data can also be done by directly constraining the parameters of the potential for each model [202] (see also [203] for a pedagogical review). These methods have the advantage of not relying on a generic parameterization of the power spectrum/potential, or the slow-roll conditions, as some models violate those assumptions

¹⁷ Note that the results vary significantly when WMAP data only or combined observations are used, see Ref. [12] for details.

temporarily [201] or constantly [149]. It is one of the best methods to carry a bayesian analysis for a model selection based on the data. Their disadvantage is that they require to treat each model individually and cannot provide ways to use current constraints to build new models.

2. Cosmic strings and CMB fluctuations

As we shall also see below (at Sec. IV) that several models of inflation can produce cosmic strings (see Sec. III G). This has important consequences when confronting the model to the data as some degeneracy has been observed between the spectral index and the fraction of cosmic strings responsible for the temperature anisotropies at the 10th multipole f_{10} [258]. The 2D likelihood function when f_{10} is included is represented in Fig. 2.

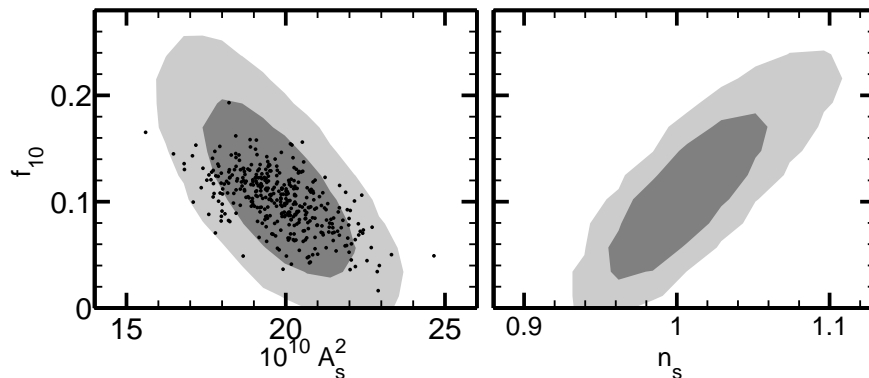


FIG. 2: Two-dimensional likelihood between the fraction of cosmic strings f_{10} and the normalization and tilt of the power spectrum. Figures are taken from [258].

When a certain fraction of cosmic strings is present, the spectral index best fit and error bars - from WMAP 3years data - are shifted and enlarged, and (at 1σ) we read approximately:

$$n_s \simeq 1.01 \pm 0.05 , \quad f_{10} \simeq 0.11 \pm 0.9 . \quad (70)$$

Once other data sets are taken into account (BBN, large scale structures), the current data can only put an upper constraint on the fraction of cosmic strings $f_{10} < 0.11$ at 2σ [258]. This constraint should be improved by the future PLANCK data, both at large multipole and from confrontation to the polarized data, notably the B-modes. Luckily, if cosmic string are present then they should contribute to them [259–261], and their fraction is not degenerate

with the primordial tensor signal from inflation [262].

The current CMB fluctuations generated from strings involve the simplest Nambu-Goto strings, the presence of currents inside cosmic strings can possibly affect their precise signature in CMB [263]. More generally, the properties of cosmic strings arising from non-SUSY theories, from SUSY F - and D -term or $N = 2$ SUSY P-term hybrid inflation, or from brane inflation are different. By adding only 1 parameter to fit the data (f_{10}) and confronting all models to the posterior probabilities might not be the best strategy in such a case (see Sec. III G).

3. Isocurvature perturbations

The isocurvature perturbations measure the deviation from the adiabaticity of the primordial fluctuations, denoted by the quantity S (in the context of cold dark matter in Eq. (54), it was denoted by S_c). The isocurvature perturbations arise if there are light scalar fields fluctuating during inflation, which do not thermalize with the inflaton decay products, i.e. the SM degrees of freedom, after the end of inflation. Usually this deviation is measured by the parameter α related to the ratio between the entropy power spectrum, \mathcal{P}_S , over the curvature perturbation, \mathcal{P}_ζ , via the Eq. (55). There could be some correlations between S and ζ , the parameter:

$$\beta(k_0) \equiv -\frac{\mathcal{P}_{S,\zeta}}{\sqrt{\mathcal{P}_S(k_0)\mathcal{P}_\zeta(k_0)}}, \quad (71)$$

where $\mathcal{P}_{S,\zeta}$ is the cross-correlated power spectrum between S and ζ , distinguish between the totally correlated case ($\beta = -1$) and the totally anti-correlated case ($\beta = 0$). Two parameters α_{-1} (for $\beta = -1$) and α_0 (for $\beta = 0$) are commonly used to describe each case, which are typically encountered in the curvaton scenario and in the axion dark matter scenario, respectively.

The most recent observations from WMAP 5-years data lead to the values for α_{-1} and α_0 compatible with zero, and respectively, slightly and strongly degenerate with the spectral index [12]. Marginalizing over other parameters, it was found that, at 2σ :

$$\alpha_{-1} < 0.0041, \quad \alpha_0 < 0.072, \quad (72)$$

when the WMAP data were combined with BAO and SN data. These constraints suggest that the deviation from $S = 0$ is smaller than 2.1% and 8.9% respectively at 95% confidence

level.

4. Higher order correlation functions

Higher order correlations, such as bispectrum \mathcal{B}_ζ and trispectrum \mathcal{T}_ζ (defined in Sec. IIC4) can also constrain the inflationary dynamics and the interactions, (see [22] for a review). The amount of non-gaussianities has been recently constrained by the WMAP data [12]. As pointed out in Eq. (59), even a single field inflation model in a slow-roll regime generates small non-Gaussianities at the level, $f_{NL} \sim \epsilon, \eta \sim 10^{-2}$ [20, 151], though the current limits are around four orders of magnitude above this level.

The first constraint on *full* bispectrum was computed for the COBE data [264]

$$-3500 \leq f_{NL} \leq 2000, \quad \text{at } 2\sigma, \quad (73)$$

but due to the computational cost the WMAP employ the “KSW estimator” [265] that combine *squeezed* triangular configurations in the harmonic space to construct an optimal estimator for f_{NL} . For example inflation, curvaton, and preheating induced non-Gaussianity belong to this category, where ζ and ζ^2 are evaluated at the same location in space [12]. In addition, the *equilateral non-linear coupling* parameter $f_{NL\ eq}$ provides a complementary description of the bispectrum, combining triangular configurations in the harmonic space that are equilateral¹⁸.

The constraints from the most recent observations are currently given by WMAP 5-years data [12]¹⁹:

$$-9 < f_{NL} < 115, \quad \text{at } 2\sigma, \quad -151 < f_{NL\ eq} < 253, \quad \text{at } 2\sigma, \quad (74)$$

meaning that there is a hint in favor of a non-vanishing positive f_{NL} . The lower bound on f_{NL} is even raised above zero when the bispectrum maps are less restrictive. This is the origin of the discrepancy between these results and the prior claimed detection of non-Gaussianities from [267] using the WMAP 3-years data. Note that using another statistics

¹⁸ Indeed, models can generate a large amount of non-Gaussianities only in one configuration. Note that several other configurations and statistics to estimate f_{NL} have been proposed and searched for, such as the point-source bispectrum \mathcal{B}_{src} measured in the WMAP 5-years data [12], the flattened triangle configurations from models with departure from the Bunch-Davies initial condition [266].

¹⁹ Note that there is no known constraint on the trispectrum parameter, τ_{NL} . It is expected to be of order $\tau_{NL} \sim (f_{NL})^2$.

(Minkowski functional) has lead so far a negatively preferred value for $f_{NL} \sim -60 \pm 60$. This discrepancy has not yet been fully resolved [12]. The future observations from PLANCK and the galaxy distribution should be able to constrain a deviation up to $|f_{NL}| \gtrsim 5$ [268–271].

F. Dynamical challenges for inflation

Inflation has several dynamical challenges which have been discussed in the literature.

1. Initial conditions for inflation

The question of initial condition is a worrisome factor. universe could have started either cold or hot. Whether universe began hot or cold, once vacuum energy density takes over it would always yield a cold universe. However there are nontrivial initial conditions to be satisfied.

- Homogeneity problem:

In an Einstein gravity inflation does not solve the homogeneity problem, instead inflation requires an initial patch of the universe, r , to be sufficiently homogeneous on scales larger than the Hubble patch, $r \gg H^{-1}$, before inflation could begin see Refs. [272–277]. Initial conditions if set at the Planckian scale do not suffer through this problem as shown by Refs. [4, 143, 278]. Low scale models of inflation require earlier phases of inflation in order to set the initial conditions.

- Chaotic initial conditions:

For sufficiently flat potential the only constraint is given by: $(1/2)\dot{\phi}^2 + (1/2)(\partial_i\phi)^2 + V(\phi) \leq M_{\text{P}}^4$, see Refs. [4, 143, 182, 278]. The initial conditions are set by: $(1/2)\dot{\phi}^2 \sim (1/2)(\partial_i\phi)^2 \sim V(\phi) \sim \mathcal{O}(M_{\text{P}})$. If by any chance $(1/2)\dot{\phi}^2 + (1/2)(\partial_i\phi)^2 \leq V(\phi)$ in a particular domain, the inflation begins and within a Planck time the potential energy density, $V(\phi)$, starts dominating over kinetic term. In domains where $(1/2)\dot{\phi}^2 + (1/2)(\partial_i\phi)^2 > V(\phi)$, inflation does not take place and do not exist classically. The above mentioned conditions are naturally satisfied when $\phi \geq M_{\text{P}}$ for a simple chaotic type potential, $V \sim m^2\phi^2$, where there exists a window, $\mathcal{O}(100 - 10)M_{\text{P}} > \phi > \mathcal{O}(M_{\text{P}})$, where universe enters a process of eternal self-reproduction [143, 278].

In the self-production regime new regions of H_{inf}^{-1} prop up on a timescale of one e-folding with field values $\sim \phi \pm \Delta\phi/2$, where $\Delta\phi \sim H_{inf}/2\pi$. In such regions quantum fluctuations dominate over the classical slow-roll of the field. After few e-foldings, N , these regions are locally homogeneous and grow almost independently. The correlation between the two regions $\langle \phi + \Delta\phi/2, \phi - \Delta\phi/2 \rangle \sim e^{-N}$, die exponentially. Such self-reproduction regions in the inflationary potential can solve the initial homogeneity problem without any trouble. This is also known as eternal inflation ²⁰.

Also note that chaotic initial conditions can be obtained for low scale models of inflation provided the potential is extremely flat. For instance, near the saddle point ϕ_0 of a potential, $V'(\phi_0) = 0$, $V''(\phi_0) = 0$, the quantum fluctuations will dominate over classical motion in a range $\Delta\phi \ll M_P$. Such regions will support self-reproduction of space-time with locally homogeneous regions. However as we shall argue that in order to reach a plateau of such a low scale inflationary potential one requires stochastic jumps of the inflaton field during a prior phase of inflation [89, 279].

Furthermore, if inflation is driven by a collection of scalar fields as in the case of *assisted inflation*, then the initial condition problem for a single field chaotic inflation model can be ameliorated, as one would not require super-Planckian VEVs for the inflatons [280–282].

- Problems with a large VEV:

A natural question arises for many models: can we trust the effective field theory treatment of an inflaton potential when the VEV of the field is super-Planckian. The answer is no, in particular when the inflaton has couplings to the SM or MSSM degrees of freedom. An effective field theory treatment is trustable *only* when the momentum is bounded by the cut-off as well as the total energy density is below the Planckian energy density. For any renormalizable coupling between the inflaton and any matter

²⁰ Note that inflation is not past eternal. The first argument is based on singularity theorems due to Hawking-Penrose [141]. The large stochastic fluctuations cannot take the field forever up the potential. When the field fluctuations become non-linear in one Hubble region, the entire region would collapse to form a blackhole. The second reason is due to [142], where it has been argued that null and time like geodesics are past-incomplete during inflation as long as the averaged expansion rate is such that $H_{av} > 0$ holds along the past directed geodesics. The observers along these geodesics would take finite amount of time to hit the singularity.

field would render super-Planckian VEV dependent mass to the matter field (for a reasonable gauge or Yukawa coupling $\leq \mathcal{O}(1)$). The effective field theory prescription is bound to break down when super-heavy quanta is running in the loops of an inflaton field. For a gauge invariant inflaton, i.e. MSSM inflaton, it is impossible to consider VEVs above the cut-off, as the inflaton has SM gauge interactions. Similarly, embedding inflation in SUGRA or in string theory will always provide inflaton VEV below the Planck scale, this remains true for any realistic potential arising from the low energy effective theory [283–285]. In a limit when the inflaton coupling to matter vanishes, or in a free field theory case, it is possible to obtain VEVs above the cut-off maintaing the effective field theory arguments given in Refs. [4, 143, 278].

- Quantum initial conditions:

If inflation lasts long enough, i.e. $N \sim 60 - 70$ e-foldings, then it is inevitable that the present *observable* mode would originate from sub-Planckian length scales. Potentially quantum gravity corrections at those length scales can leave some imprint in the CMB perturbations, it is known as the trans-Planckian problem for an inflationary cosmology [286–291]. There are two pertinent questions, the first one is related to the choice of Bunch-Davies vacuum as an initial state [180, 292] in order to evolve the quantum perturbations. Second one has to do with an adiabatic evolution of the state throughout the dynamics of inflation. Both the questions have been raised in the literature. It was observed that if either the vacuum or the evolution of a state would violate Lorentz-invariance the corrections to the amplitude of the CMB perturbations would be as large as order one. Typically the corrections will be $\propto (H_{inf}/M_*)$, where M_* is the cut-off above which either the initial state is modified or the evolution. In Ref. [288], the initial state was chosen to be alpha-vacuum (a variant of Bunch-Davies vacuum with large excitations) and the modification to the amplitude of the CMB perturbations were found to be $(H_{inf}/M_*)^2$. It was argued in [293] that as long as the state evolves adiabatically and Lorentz-invariance is maintained, the trans-Planckian corrections would be small of order $\sim (H_{inf}/M_*)^2$, which is a good news for any low scale inflation.

2. *Choice of a vacuum where inflation ends*

In order to realize our observable universe, inflation must come to an end in the right vacuum. The exit must happen such that the relevant degrees of freedom required for the BBN, i.e. the relativistic SM degrees of freedom, and right abundance for the cold dark matter can be excited. There are only few models where inflation can end right in the SM vacuum; the SM Higgs inflation [85], and the MSSM inflation [86]. In many particle physics models of inflation, the existence of a hidden sector coupling to the MSSM or the SM sector is common, see [10]. All these models require extra set of assumptions in order to make sure that the inflaton energy density gets transferred into the MSSM or the SM degrees of freedom. Note that a hidden sector inflaton can excite hidden degrees of freedom as the couplings between the two hidden sectors are not barred by any symmetry. Furthermore, gravity will always couple one such sector to the another. Therefore, it is desirable to end inflation where one can directly excite the SM quarks and leptons ²¹.

In the case of stringy models, there exists no construction where inflation ends right in the MSSM or the SM sector. The problem becomes more challenging with an introduction of a string landscape, since there are nearly 10^{500} to even 10^{1000} vacua [44, 56, 57], with the vast majority of those having large cosmological constants. In such cases exiting inflation from the string landscape and exiting the inflation in our own vacuum becomes even more challenging task [279].

3. *Quantum to classical transition*

The initial sub-Hubble perturbations are quantum in nature. The perturbations are then stretched outside the Hubble patch during inflation, therefore the correlation function, $\langle \delta\phi(x)\delta\phi(x') \rangle$, evolves during inflation. It has been shown that the evolution is similar to that of a squeezed state [294–299], the squeezing happening due to the exponential expansion. For very long wavelength (super Hubble) modes the quantum correlation between the two inflating regimes dies away exponentially by the number of e-foldings of inflation $\sim e^{-N}$.

²¹ Suppose inflation ends in a GUT vacuum, it does not guarantee automatically that the GUT would be broken down to the SM vacuum. There are many ways to break it and one requires special care in realizing such a scenario. See Sec. III F

This lends some support to this idea that two distinct Hubble patches behave for all good purposes classical [143, 166, 278]. But it is still unclear whether the short wavelength modes have any role to play in decohering the long wavelength modes [299]?

The density matrix of the fluctuations within the causal horizon, $\rho[\delta\phi(x), \delta\phi'(y)] = \langle \delta\phi(x) | \rho | \delta\phi'(y) \rangle = \Psi[\delta\phi(x)] \Psi^*[\delta\phi(y)]$, evolves from pure state to a mixed state, $\rho[\delta\phi(x), \delta\phi'(y)] = P[\delta\phi(x)] \delta[\delta\phi(x) - \delta\phi'(y)]$, under the influence of a time dependent interaction Hamiltonian arising from (a) time dependent evolution of the inflaton, and (b) the inflaton interactions to matter. It was pointed out in [299] that short wavelength modes can play a role in decohering the long wavelength modes, once reheating takes place. The thermal bath produced from the inflaton decay can act as an environment for the long wavelength modes when these modes re-enter the Hubble patch after the end of inflation. The decoherence effects during inflation are still an open issue [299–301].

4. *Inflaton decay, reheating and thermalization*

Reheating takes place due to the perturbative decay of the inflaton [92–95]. After the end of inflation, when $H \leq m_\phi$, the inflaton field oscillates about the minimum of the potential. Averaging over one oscillation results in pressureless equation of state where $\langle p \rangle = \langle \dot{\phi}^2/2 - V(\phi) \rangle$ vanishes [92, 93], so that the energy density starts evolving like matter domination (in a quadratic potential) with $\rho_\phi = \rho_i (a_i/a)^3$ (subscript i denotes the quantities right after the end of inflation) ²². If Γ_ϕ represents the *total* decay width of the inflaton to pairs of fermions. This releases the energy into the thermal bath of relativistic particles when $H(a) = \sqrt{(1/3M_{\text{P}}^2)\rho_i(a_i/a)^3/2} \approx \Gamma_\phi$. The energy density of the thermal bath is determined by the reheat temperature T_R , given by:

$$T_R = \left(\frac{90}{\pi^2 g_*} \right)^{1/4} \sqrt{\Gamma_\phi M_{\text{P}}} = 0.3 \left(\frac{200}{g_*} \right)^{1/4} \sqrt{\Gamma_\phi M_{\text{P}}}, \quad (75)$$

where g_* denotes the effective relativistic degrees of freedom in the plasma. However the inflaton might not decay instantaneously. In such a case there might already exist a thermal plasma of *some* relativistic species at a temperature higher than the reheat temperature

²² For $\lambda\phi^4$ potential the coherent oscillations yield an effective equation of state similar to that of a radiation epoch.

already before the end of reheating [95]. If the inflaton decays with a rate Γ_ϕ , then the instantaneous plasma temperature is found to be [95]:

$$T_{inst}(a) \sim (g_*^{-1/2} H \Gamma_\phi M_{\text{P}}^2)^{1/4}. \quad (76)$$

The temperature of the universe reaches its maximum T_{max} soon after the inflaton field starts oscillating around the minimum. Once the maximum temperature is reached, then $\rho_\psi \sim a^{-3/2}$, and $T \sim a^{-3/8}$ until reheating and thermalization is completely over. Thermalization is achieved when both *kinetic* and *chemical* equilibrium are reached.

Note that the above analysis is solely based on energetic argument. It claims that $T_{\text{R}} \leq \rho_{inf}^{1/4}$, but ignored the microphysical aspects such as what degrees of freedom are excited after the end of inflation. For a successful cosmology one needs to ask how the inflaton energy gets converted into the SM degrees of freedom. This will be discussed in chapters V and VI.

For large reheat temperatures, $T_{\text{R}} \sim 10^9$ GeV, the universe could abundantly create thermal relics of unstable gravitinos with a mass of order 100 – 1000 GeV, which could spoil the success of BBN [302–307]. For extremely low reheat temperatures, i.e. $T_{\text{R}} \sim \mathcal{O}(1-10)$ MeV, it becomes a great challenge to obtain matter-anti-matter asymmetry and the right abundance for the dark matter. Only a few particle physics scenarios can successfully create baryons and dark matter at such a low temperature, see for instance [308–311].

G. Requirements for a successful inflation

The success of inflation is closely tied to the success of BBN [25, 26, 28] as the epoch constraints the number of relativistic degrees of freedom beyond the SM, and the baryonic asymmetry. Furthermore the CMB data and galaxy formation also constraints the abundance of cold dark matter and baryons.

1. Baryons and nucleosynthesis

For a successful BBN, which takes place within the first few hundred seconds, the abundances of light elements ^2H , ^3He , ^4He and ^7Li crucially depends on the baryon-to-photon ratio:

$$\eta \equiv \frac{n_B}{n_\gamma}. \quad (77)$$

All the relevant physical processes take place essentially in the range from a few MeV ~ 0.1 sec down to $60 - 70$ KeV $\sim 10^3$ sec. During this period only photons, e^\pm pairs, and the three neutrino flavors contribute significantly to the energy density. Any additional energy density may be parameterized in terms of the effective number of light neutrino species N_ν , so that

$$g_* = 10.75 + \frac{7}{4}(N_\nu - 3). \quad (78)$$

BBN constrains the number of light neutrino species by $N_\nu \leq 4$ [27, 29]. The four LEP experiments combined give the best fit as $N_\nu = 2.994 \pm 0.12$ [312]. The likelihood analysis which includes all the three elements (D, ${}^4\text{He}$, and ${}^7\text{Li}$) yields the baryon to photon ratio [313]

$$4.7 \times 10^{-10} < \eta < 6.2 \times 10^{-10}, \quad 0.017 < \Omega_b h^2 < 0.023. \quad (79)$$

Despite the uncertainties there appears to be a general concordance between theoretical BBN predictions and observations, which is now being bolstered by the CMB data $\Omega_b h^2 = 0.02229 \pm 0.00073$ [253].

2. Baryogenesis

The baryon asymmetry of the universe (BAU) parameterized as $\eta_B \equiv (n_B - n_{\bar{B}})/s \approx \eta$ is determined to be 0.9×10^{-10} by the recent analysis of WMAP data [12]. As pointed out by Sakharov [314], baryogenesis requires three ingredients: (1) baryon number non-conservation, (2) C and CP violation, and (3) out-of-equilibrium condition.

All these three conditions are believed to be met in the very early universe. Baryogenesis during the electroweak phase transition [315] has been studied widely, see [316]. Another mechanism known as Affleck-Dine baryogenesis, which happens due to the non-trivial dynamics of a light scalar condensate is a natural outcome of inflation [317–319]. It is also possible to convert leptonic asymmetry into baryonic asymmetry, $B = a(B - L)$ [320–323], for a review see [324], where $a = 28/79$ in the case of SM and $a = 8/23$ for the MSSM [325].

A lepton asymmetry can be generated from the out-of-equilibrium decay of the right handed (RH) neutrinos into Higgs bosons and light leptons, provided CP -violating phases exist in the neutrino Yukawa couplings. The created lepton asymmetry will be converted into a baryonic asymmetry via sphaleron processes. This scenario works most comfortably

if $T_R \geq M_1 \geq 10^9$ GeV, where M_1 is the lightest RH neutrino [326]²³. There exist various scenarios of non-thermal leptogenesis [329–333] which can work for $T_R \leq M_N$.

3. Cold dark matter

The WMAP data, galaxy clusters and large scale structure data pin down the dark matter abundance to be : $\Omega_{DM} = 0.22$ [269]. It is important to note that at the end of inflation right abundance of dark matter must be created. There are many well motivated particle physics candidates for cold dark matter [334, 335]. All the plausible candidates arise from beyond the SM physics. Within MSSM the lightest SUSY particle (LSP) is an excellent candidate by virtue of R-parity [30], the non-topological solitons such as Q-balls [90, 91], and Kaluza-Klein dark matter particles in theories with extra dimensions [335] are the most popular ones.

The dark matter particles can be created via thermal scatterings in the case of thermal cold relics, and non-thermally in the process of decay of heavier particles. The thermal relic abundance is easy to calculate, the number density of dark matter X , n_X , gets exponential suppression in comparison with the number density of relativistic degrees of freedom determined by the freeze-out temperature, $T_f < m_X$. The abundance is calculated by solving the Boltzmann equation [95]:

$$\frac{dn_X}{dt} + 3Hn_X = -\langle\sigma v\rangle(n_X^2 - (n_X^{eq})^2), \quad (80)$$

where σ is the total annihilation cross section, v is the velocity and bracket denotes thermally averaged quantities. In the case of heavy X , the cross section can be expanded with respect to the velocity in powers of v^2 , $\langle\sigma v\rangle = a + b\langle v^2\rangle + \mathcal{O}(\langle v^4\rangle) + \dots \approx a + 6b/x$, where $x = m_X/T$ and a, b are expressed in GeV^{-2} . In the regime where the dark matter abundance is frozen-out for $x \gg x_f \equiv m_X/T_f$, the relic density can be expressed in terms of the critical density:

$$\Omega_X h^2 \approx \frac{1.07 \times 10^9}{M_{\text{P}}} \frac{x_f}{\sqrt{g_*}(a + 3b/x_f)} \text{GeV}^{-1}, \quad (81)$$

where g_* is the relativistic degrees of freedom and $x_f \sim 25 - 30$ (in the standard LSP case) are evaluated at the time of freeze-out. An approximate order of magnitude estimation of

²³ Thermal leptogenesis can work below the reheat temperature $T_R < 10^9$ GeV in the case of a resonant leptogenesis [327, 328].

the abundance can be written as:

$$\Omega_X h^2 \approx \frac{3 \times 10^{-27} \text{ cm}^3 \text{ s}^{-1}}{\langle \sigma v \rangle}. \quad (82)$$

III. PARTICLE PHYSICS TOOLS FOR INFLATION

A. Standard Model of particle physics

The Glashow-Weinberg-Salam model of electroweak interactions [336–338], for details see [339], is based on $SU(2)_L \times U(1)_Y$ gauge theory containing three $SU(2)_L$ gauge bosons, W_μ^i , $i = 1, 2, 3$, and one $U(1)_Y$ gauge boson, B_μ , with kinetic energy terms, $\mathcal{L}_{\text{KE}} = -\frac{1}{4}W_{\mu\nu}^i W^{\mu\nu i} - \frac{1}{4}B_{\mu\nu} B^{\mu\nu}$, where $W_{\mu\nu}^i = \partial_\nu W_\mu^i - \partial_\mu W_\nu^i + g\epsilon^{ijk}W_\mu^j W_\nu^k$ and $B_{\mu\nu} = \partial_\nu B_\mu - \partial_\mu B_\nu$. Coupled to the gauge fields is a complex scalar $SU(2)$ doublet, H ,

$$H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix} \quad (83)$$

with a scalar Higgs potential given by

$$V = \mu^2 |H^\dagger H| + \lambda \left(|H^\dagger H| \right)^2, \quad (84)$$

where $\lambda > 0$. For $\mu^2 < 0$, the minimum energy configuration is given by the Higgs VEV, which mediates the symmetry breaking: $SU(2)_L \times U(1)_Y \rightarrow U(1)_{em}$ such that the electromagnetism is unbroken.

$$\langle H \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}. \quad (85)$$

The Higgs VEV, also known as the Higgs mechanism, generates masses for the W and Z gauge bosons

$$\mathcal{L}_s = (D^\mu H)^\dagger (D_\mu H) - V(H), \quad \text{where} \quad D_\mu = \partial_\mu + i\frac{g}{2}\tau \cdot W_\mu + i\frac{g'}{2}B_\mu Y. \quad (86)$$

Since, after symmetry breaking, in the unitary gauge there are no Goldstone bosons left, only the physical Higgs scalar remains in the spectrum. The mass to the gauge boson arises from the scalar kinetic energy term,

$$\frac{1}{2}(0, v) \left(\frac{1}{2}g\tau \cdot W_\mu + \frac{1}{2}g'B_\mu \right)^2 \begin{pmatrix} 0 \\ v \end{pmatrix}. \quad (87)$$

The physical gauge fields are two charged fields, W^\pm , and two neutral gauge bosons, Z and γ . The masses of the gauge bosons are given by:

$$M_W^2 = \frac{1}{4}g^2v^2, \quad M_Z^2 = \frac{1}{4}(g^2 + g'^2)v^2, \quad M_A = 0. \quad (88)$$

Since the massless photon must couple with electromagnetic strength, e , the coupling constants define the weak mixing angle θ_W , $e = g \sin \theta_W$ and $e = g' \cos \theta_W$.

One can also include fermions, let us consider the electron and its neutrino as an example. The fermions in terms of their left- and right-handed projections, $\psi_{L,R} = \frac{1}{2}(1 \mp \gamma_5)\psi$. We need to couple the fermions with the $SU(2)_L$ doublets, therefore:

$$L_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}. \quad (89)$$

The known matter content of the SM has the following charges: $Q_L = (3, 2, 1/6)$, ($B = 1/3, L = 0$), $u_R = (\bar{3}, 1, -2/3)$, ($B = -1/3, L = 0$), $d_R = (\bar{3}, 1, 1/3)$, ($B = -1/3, L = 0$), $L_L = (1, 2, -1/2)$, ($B = 0, L = 1$), $e_R = (1, 1, 1)$, ($B = 0, L = -1$), $H = (1, 2, 1/2)$, ($B = 0, L = 0$), using the notation (n_3, n_2, Y) , with color $SU(3)$, weak $SU(2)$ and hypercharge $U(1)$, respectively. In MSSM, anomaly cancelation will require additional Higgs with an opposite hypercharge, $(1, 2, -1/2)$.

Using the hypercharge assignments of the fields, the leptons can be coupled in a gauge invariant manner to the $SU(2)_L \times U(1)_Y$ gauge fields,

$$\mathcal{L}_{lepton} = i\bar{e}_R\gamma^\mu\left(\partial_\mu + i\frac{g'}{2}Y_e B_\mu\right)e_R + i\bar{L}_L\gamma^\mu\left(\partial_\mu + i\frac{g}{2}\tau \cdot W_\mu + i\frac{g'}{2}Y_L B_\mu\right)L_L. \quad (90)$$

All of the known fermions can be accommodated in the Standard Model in an identical manner as was done for the leptons.

A fermion mass term, $\mathcal{L}_{mass} = -m\bar{\psi}\psi = -m(\bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L)$, is not gauge invariant under $SU(2)_L$ and $U(1)_Y$. The gauge invariant Yukawa coupling of the Higgs boson to the up and down quarks is given by, $\mathcal{L}_d \sim -\lambda_d\bar{Q}_L H d_R + h.c.$:

$$-\lambda_d\frac{1}{\sqrt{2}}(\bar{u}_L, \bar{d}_L) \begin{pmatrix} 0 \\ v \end{pmatrix} d_R + h.c. \quad (91)$$

which provides mass term for the down quark if $\lambda_d = m_d\sqrt{2}/v$, while the up quark mass is determined by: $\mathcal{L}_u = -\lambda_u\bar{Q}_L\Phi^c u_R + h.c.$, since fact that $H^c \equiv -i\tau_2 H^*$ (τ is a Pauli matrix)

Similar couplings are used to generate mass terms for the charged leptons, the neutrino has no right handed partner, it remains massless within SM.

In order to obtain neutrino masses, one would have to introduce right handed neutrinos with a Yukawa coupling, i.e. $\mathcal{L} \sim h(LH)\nu_R$. From the VEV of the Higgs the neutrinos will obtain masses $\propto hv$. In order to get neutrino masses in the interesting range $m_\nu \sim 10^{-1}$ eV, for solar and atmospheric neutrino mixing, the Yukawa coupling has to be very tiny, i.e. $h \sim 10^{-12}$. The origin of neutrino masses can also arise from higher dimensional lepton number violating operator [340–343]:

$$\mathcal{L} \sim \frac{\infty}{\mathcal{M}}(\mathcal{L}\mathcal{H})(\mathcal{L}\mathcal{H}). \quad (92)$$

When the Higgs gets a VEV, these gives rise to Majorana masses for the neutrinos of order

$$m_\nu \sim \frac{v^2}{M}. \quad (93)$$

In order to get neutrino masses in the interesting range we require $M \sim 10^{14}$ GeV, remarkably close to the GUT scale.

B. Radiative corrections in an effective field theory

An effective field theory is a powerful tool to study the loop corrections of the scalars coupled to gravity [283, 285, 344–349]. The most general action for N dimensionless scalar fields, θ^i , and the metric $g_{\mu\nu}$ can be written in terms of derivative expansion, with terms involving up to two derivatives, see [283, 285, 346, 347]:

$$\begin{aligned} -\frac{\mathcal{L}_{\text{eff}}}{\sqrt{-g}} = & v^4 V(\theta) + \frac{M_{\text{P}}^2}{2} g^{\mu\nu} [W(\theta) R_{\mu\nu} + G_{ij}(\theta) \partial_\mu \theta^i \partial_\nu \theta^j] \\ & + A(\theta)(\partial\theta)^4 + B(\theta) R^2 + C(\theta) R(\partial\theta)^2 + \frac{E(\theta)}{M^2} (\partial\theta)^6 + \frac{F(\theta)}{M^2} R^3 + \dots, \end{aligned} \quad (94)$$

where $M \ll M_{\text{P}}$ is regarded as the lightest mass scale of particles which would be integrated out, the coefficient functions, $V(\theta)$, $G_{ij}(\theta)$, $A(\theta)$, $B(\theta)$, $C(\theta)$, $E(\theta)$, $F(\theta)$ are dimensionless, and R^3 collectively represents all possible independent invariants constructed from three Riemann tensors, or two Riemann tensors and two of its covariant derivatives; $R(\partial\theta)^2$ denotes all possible invariants involving one power of the Riemann tensor and two derivatives acting on θ^i .

An effective action for a scalar field expanded about a classical solution $\theta^i(x) = v^i(x) + \phi^i(x)/M_{\text{P}}$, $g_{\mu\nu}(x) = \hat{g}_{\mu\nu}(x) + h_{\mu\nu}(x)/M_{\text{P}}$ can be given by [285]:

$$\mathcal{L}_{\text{eff}} = \hat{\mathcal{L}}_{\text{eff}} + M^2 M_{\text{P}}^2 \sum_n \frac{c_n}{M^{d_n}} \mathcal{O}_n \left(\frac{\phi}{M_{\text{P}}}, \frac{h_{\mu\nu}}{M_{\text{P}}} \right) \quad (95)$$

where $\hat{\mathcal{L}}_{\text{eff}}$ is the classical Lagrangian density evaluated at the background configuration, ϕ and $h_{\mu\nu}$ are the small perturbations around the background scalar field and the metric. The sum over n runs over the labels for a complete set of interactions, \mathcal{O}_n , each of which involves $N_n = N_n^{(\phi)} + N_n^{(h)} \geq 2$ powers of the fields ϕ^i (for N fields) and $h_{\mu\nu}$ (with $N_n \neq 1$). The parameter d_n counts the number of derivatives appearing in \mathcal{O}_n , and c_n/M^{d_n} is a dimensionless quantity, where $M \ll M_{\text{P}}$ denotes the scale at which heavy degrees of freedom have been integrated out. The overall prefactor, $M^2 M_{\text{P}}^2$, is to keep the kinetic terms canonical for individual fields.

In terms of dimensionless couplings, λ_n , the scalar part of the potential can be expanded as ²⁴:

$$V(\phi) = v^4 \left[\lambda_0 + \lambda_2 \left(\frac{\phi}{M_{\text{P}}} \right)^2 + \lambda_4 \left(\frac{\phi}{M_{\text{P}}} \right)^4 + \dots \right], \quad (96)$$

Note that the natural scale for the scalar masses under the above assumptions is $m \simeq v^2/M_{\text{P}}$. The quartic coupling constant, $\lambda_4(v/M_{\text{P}})^4$, is similarly Planck suppressed. For the purpose of inflation, the scale of the scalar potential will be governed by $V \sim v^4 \ll M^4$. Such small masses and couplings are needed to keep the successes of inflation.

In general, in order to study the evolution of couplings, c_n , and how the two scales M and M_{P} appear in a given problem, one has to study the 1– particle irreducible (1PI) graphs perturbatively in the interaction part of the Lagrangian density. To this end, one can obtain the leading order expression for the amplitude of such graphs, by assuming that E denotes the largest of physical scales that appear explicitly in the propagators or vertices of the calculation. One can neglect any other smaller scales compared with E when estimating the size of a particular Feynman graph [283, 285, 347].

$$\mathcal{A}_\epsilon(E) \simeq E^2 M_{\text{P}}^2 \left(\frac{1}{M_{\text{P}}} \right)^\epsilon \left(\frac{E}{4\pi M_{\text{P}}} \right)^{2L} \prod_n \left[c_n \left(\frac{E}{M} \right)^{d_n-2} \right]^{V_n}, \quad (97)$$

²⁴ Similar potential can also be derived in the context of supergravity (SUGRA), see Refs. [161, 350].

where ϵ denotes the number of external lines, L loops and V_n vertices with d_n derivatives. The factor $E/4\pi M_{\text{P}} \ll 1$ ensures that successive insertions of interactions to be smaller than preceding ones.

One particular application is integrating out a particle of mass $m \gg M$. In the inflationary context, $M \sim H_{\text{inf}} \sim v^2/M_{\text{P}}$ for the above potential Eq. (96), and $\lambda_n \sim \mathcal{O}(1)$. Expanding the above amplitude Eq. (97) for arbitrary derivatives and comparing the coefficients with Eq. (95), one finds $v^4/(E^2 M_{\text{P}}^2) \simeq v^4/(m^2 M_{\text{P}}^2) \simeq H_{\text{inf}}^2/m^2 \ll 1$. In addition, the generic loop factor $(m/4\pi M_{\text{P}})^2$, the $d_n \geq 4$ interactions are suppressed by at least two powers of m^2/M_{P}^2 , and λ_n are additionally suppressed by powers of H_{inf}^2/m^2 . Only the $d_n = 2$ interactions remain unsuppressed beyond the basic loop factor if $\lambda_n \lesssim \mathcal{O}(1)$. On the other hand, if there are interactions in the scalar potential that are unsuppressed by powers of M_{P} , such as if $\lambda_n \simeq (M_{\text{P}}/v)^{N_n} \lambda_n$, i.e. higher powers of ϕ and $h_{\mu\nu}$, then loops involving these interactions can modify the inflaton potential [285, 346]²⁵.

One such known example is in the case of Higgs inflation [85], where the presence of $\xi H^\dagger H R$ (interaction vertex involves $d_n = 2$ derivatives) term helps flattening the Higgs potential²⁶. A successful inflation requires $\xi \sim 5 \times 10^4 \sqrt{\lambda}$, where λ is the Higgs self-coupling. The unitarity bound on interactions such as Higgs-Higgs scattering, graviton-Higgs scattering (through graviton exchange) leads to $E < E_{\text{max}} \approx M_{\text{P}}/\xi$, which constraints the validity range for an effective field theory treatment for the Higgs inflation to be within a narrow range $\sqrt{\lambda} \ll H_{\text{inf}}/M \ll 1$, where $H_{\text{inf}} \sim \sqrt{\lambda M_{\text{P}}}/\xi$. Any Higgs coupling to the matter field will induce corrections to the Higgs potential and alter the predictions for inflation, see [285].

In other extreme limit, when a light particle is running in the loop, i.e. $m \ll H_{\text{inf}}$ and $m \ll \dot{\phi}/\phi$, the above analysis when applied to 2-point correlations, yields a well known result, $\langle \phi^2 \rangle \sim H_{\text{inf}}^2/4\pi^2$ [285].

The one-loop effective potential, which can be generated when the heavy fields are explicitly integrated out, serves as an useful tool to lift a generic flat direction, which is helpful

²⁵ There are two types of operators; (i) *relevant operators*, which are proportional to positive powers of M , and (ii) *marginal operators*, which grow logarithmically with M , have been considered in studying the trans-Planckian physics, see for instance [289, 351].

²⁶ In an Einstein frame the Higgs potential with canonical kinetic terms is given by Eq. (215), see section IV D.

to obtain inflationary potential. A simple calculation of the virtual effects of the heavy scalars and fermions on the light scalar potential, which can be the inflaton, can be obtained by matching the one-loop corrected effective potential for the full theory. This gives the following result following Coleman-Weinberg [352]²⁷:

$$\begin{aligned} V_{eff}(\phi) &= V_{inf}(\phi) + \Delta V \\ \Delta V &= \frac{1}{64\pi^2} \sum_i (-)^{F_i} M_i(\phi)^4 \ln \frac{M_i(\phi)^2}{\Lambda(\phi)^2}, \end{aligned} \quad (98)$$

where V_{inf} is now the renormalized potential, $\Lambda(\phi)$ is the renormalization mass scale. The sum extends over all helicity states i , F_i is the fermion number, and $M(\phi)$ is the mass of the i -th state.

As an example, let us consider a chiral Lagrangian for N fermions invariant under Z_N symmetry [353]. In general, Z_N symmetry can be defined for an Abelian and non-Abelian gauge group, which keeps the action, S , invariant under the symmetry transformation: $S \longrightarrow e^{2\pi ij/N} \times S$, where $j = 1, 2, \dots, N$. The relevant Lagrangian is:

$$\mathcal{L} = (1/2)\partial_\mu\phi\partial^\mu\phi + \sum_{j=0}^{N-1} \bar{\psi}_j i\gamma^\mu \partial_\mu \psi_j + [m_0 + \epsilon e^{i(\phi/f + 2\pi j/N)}] \bar{\psi}_{jL} \psi_{jR} + h.c., \quad (99)$$

where $\psi_{(R, L)} = (1 \pm \gamma^5)\psi/2$, m_0 is an explicit breaking term, and the scalar VEV responsible for generating ϵ via some Yukawa interaction between scalar and N fermions is given by: $\langle\phi\rangle = e^{i\phi/f} f/\sqrt{2}$. Under the Z_N discrete symmetry: $\psi_j \rightarrow \psi_{j+1}$, $\psi_{N-1} \rightarrow \psi_0$, $\phi \rightarrow \phi + 2\pi j f/N$. The induced one-loop potential can be calculated from Eq. (98), with a cut-off: $\lambda < f$. The potential is:

$$\Delta V = - \sum_{j=0}^{N-1} \frac{M_j^4}{16\pi} \ln \left(\frac{M_j^2}{\Lambda^2} \right), \quad (100)$$

where $M_j^2 = m_0^2 \epsilon^2 + 2m_0 \epsilon \cos(\phi/f + 2\pi j/N)$. The scalar field, ϕ , is a simple example of pseudo-Nambu Goldstone Boson (pNGB), which can protect its potential naturally due to symmetries²⁸. For $N = 2$ the pNGB mass is $m_\phi \sim m_0 \epsilon/f$.

²⁷ A supersymmetric (SUSY) generalization of the Coleman-Weinberg formula will be presented in Sec. III E 1.

²⁸ Technically natural small mass scales are protected by symmetries, such that when the small mass vanishes it can not be generated in any order of perturbation theory.

C. Supersymmetry (SUSY)

The SM physics has number of pressing issues, the most compelling one is the quadratically divergent contributions to the Higgs mass, which arise in one-loop computation from the fermion contribution and quartic self interaction of the Higgs boson. The quadratic divergence is independent of the mass of the Higgs boson and cancel, exactly if $\lambda_s = \lambda_f^2$, where λ_f is the fermion Yukawa and λ_s is the quartic scalar coupling. However this procedure fails at 2-loops and one requires fine tuning of the couplings order by order in perturbations to a precision of roughly one part in 10^{17} (for the scale of gravity at $M_P \sim 10^{18}$ GeV), often known as the *hierarchy problem* or the *naturalness problem*.

In SUSY, there is a scalar of same mass associated with every fermion and the couplings are such that $\lambda_s = \lambda_f^2$. The electroweak symmetry is still broken by the Higgs mechanism, but the quadratic divergences in the scalar sector are absent. The minimal extension of the SM in SUSY is known as MSSM (minimal SUSY SM).

The matter fields of $N = 1$ SUSY are chiral superfields $\Phi = \phi + \sqrt{2}\theta\psi + \theta\theta F$, which describe a scalar ϕ , a fermion ψ and a scalar auxiliary field F . The SUSY scalar potential V is the sum of the F - and D -terms are:

$$V = \sum_i |F_i|^2 + \frac{1}{2} \sum_a g_a^2 D^a D^a \quad (101)$$

where

$$F_i \equiv \frac{\partial W}{\partial \phi_i}, \quad D^a = \phi^\dagger T^a \phi, \quad (102)$$

where W is the superpotential, and we have assumed that ϕ_i transforms under a gauge group G with the generators of the Lie algebra given by T^a . Note that all the kinetic energy terms are included in the D -term.

1. Minimal Supersymmetric Standard Model (MSSM)

In addition to the usual quark and lepton superfields, MSSM has two Higgs fields, H_u and H_d . Two Higgses are needed because H^\dagger is forbidden in the superpotential. The superpotential for the MSSM is given by, see [30, 31, 33, 34]

$$W_{MSSM} = \lambda_u Q H_u u + \lambda_d Q H_d d + \lambda_e L H_d e + \mu H_u H_d, \quad (103)$$

where H_u, H_d, Q, L, u, d, e in Eq. (103) are chiral superfields, and the dimensionless Yukawa couplings $\lambda_u, \lambda_d, \lambda_e$ are 3×3 matrices in the family space. We have suppressed the gauge and family indices. The H_u, H_d, Q, L fields are $SU(2)$ doublets, while u, d, e are $SU(2)$ singlets. The last term is the μ term, which is a SUSY version of the SM Higgs boson mass. Terms proportional to $H_u^* H_u$ or $H_d^* H_d$ are forbidden in the superpotential, since W_{MSSM} must be analytic in the chiral fields. H_u and H_d are required not only because they give masses to all the quarks and leptons, but also for the cancellation of gauge anomalies. The Yukawa matrices determine the masses and CKM mixing angles of the ordinary quarks and leptons through the neutral components of $H_u = (H_u^+, H_u^0)$ and $H_d = (H_d^0, H_d^-)$. Since the top quark, bottom quark and tau lepton are the heaviest fermions in the SM, we assume that only the third family, $(3, 3)$ element of the matrices $\lambda_u, \lambda_d, \lambda_e$ are important.

The μ term provides masses to the Higgsinos

$$\mathcal{L} \supset -\mu(\tilde{H}_u^+ \tilde{H}_d^- - \tilde{H}_u^0 \tilde{H}_d^0) + c.c., \quad (104)$$

and contributes to the Higgs $(mass)^2$ terms in the scalar potential through

$$-\mathcal{L} \supset V \supset |\mu|^2(|H_u^0|^2 + |H_u^+|^2 + |H_d^0|^2 + |H_d^-|^2). \quad (105)$$

Note that Eq. (105) is positive definite. Therefore, it cannot lead to electroweak symmetry breaking without including SUSY breaking $(mass)^2$ soft terms for the Higgs fields, which can be negative. Hence, $|\mu|^2$ should almost cancel the negative soft $(mass)^2$ term in order to allow for a Higgs VEV of order ~ 174 GeV. That the two different sources of masses should be precisely of same order is a puzzle for which many solutions has been suggested [354–357].

The most general gauge invariant and renormalizable superpotential would also include baryon number B or lepton number L violating terms, with each violating by one unit: $W_{\Delta L=1} = \frac{1}{2}\lambda^{ijk}L_i L_j e_k + \lambda^{ijk}L_i Q_j d_k + \mu^i L_i H_\mu$ and $W_{\Delta B=1} = \frac{1}{2}\lambda^{ijk}u_i d_j d_k$, where $i = 1, 2, 3$ represents the family indices. The chiral supermultiplets carry baryon number assignments $B = +1/3$ for Q_i , $B = -1/3$ for u_i, d_i , and $B = 0$ for all others. The total lepton number assignments are $L = +1$ for L_i , $L = -1$ for e_i , and $L = 0$ for all the others. Unless λ' and λ'' terms are very much suppressed, one would obtain rapid proton decay which violates both B and L by one unit.

There exists a discrete Z_2 symmetry, which can forbid baryon and lepton number violating terms, known as R -parity [358]. For each particle:

$$P_R = (-1)^{3(B-L)+2s} \quad (106)$$

with $P_R = +1$ for the SM particles and the Higgs bosons, while $P_R = -1$ for all the sleptons, squarks, gauginos, and Higgsinos. Here s is spin of the particle. Besides forbidding B and L violation from the renormalizable interactions, R -parity has interesting phenomenological and cosmological consequences. The lightest sparticle with $P_R = -1$, the LSP, must be absolutely stable. If electrically neutral, the LSP is a natural candidate for cold dark matter [359, 360]²⁹.

2. Soft SUSY breaking Lagrangian

In the MSSM there are several proposals for SUSY breaking, which we shall discuss below. However most of the time it is not important to know the exact mechanism of low energy SUSY breaking. This ignorance of the origin of SUSY breaking can always be hidden by simply writing down explicitly the soft breaking terms.

The most general soft SUSY breaking terms in the MSSM Lagrangian can be written as (see e.g. [30, 31])

$$\mathcal{L}_{soft} = -\frac{1}{2}(M_\lambda \lambda^a \lambda^a + \text{c.c.}) - (m^2)_j^i \phi_j^* \phi_i - \left(\frac{1}{2} b_{ij} \phi_i \phi_j + \frac{1}{6} a^{ijk} \phi_i \phi_j \phi_k + \text{c.c.} \right), \quad (107)$$

where M_λ is the common gaugino mass $(m^2)_j^i \sim m_0^2 \sim (\mathcal{O}(100)\text{GeV})^2$ are 3×3 matrices determining the masses for squarks and sleptons, denoted as $m_Q^2, m_u^2, m_d^2, m_L^2, m_e^2, m_{H_u}^2, m_{H_d}^2, b \sim m_0^2 \sim (\mathcal{O}(100))^2 \text{ GeV}^2$; b_{ij} is the mass term for the combination $H_u H_d$; and finally, a^{ijk} are complex 3×3 matrices in the family space which yield the A -terms $a_u, a_d, a_e \sim m_0 \sim \mathcal{O}(100) \text{ GeV}$. There are a total of 105 new entries in the MSSM Lagrangian which have no counterpart in the SM. However the arbitrariness in the parameters can be partly removed by the experimental constraints on flavor changing neutral currents (FCNC) and CP violation [361]³⁰

There are a number of possibilities for the origin of SUSY breaking [30, 31, 34]. Fayet-Iliopoulos mechanism [362] provides SUSY breaking by virtue of a non-zero D -term but

²⁹ Symmetries with the property that fields within the same supermultiplet have different transformations are called R symmetries; they do not commute with SUSY. Continuous $U(1)$ R symmetries are often employed in inflationary model-building literature. Under this symmetry a general chiral superfield transforms as $\Phi \rightarrow e^{iR\alpha} \Phi$, and in order to keep the theory R -invariant, the superpotential must have $R = 2$.

³⁰ Within global SUSY there exists a mass formula $Str \mathbf{M}^2 \equiv \sum_{j=0}^1 (-1)^j tr \mathbf{M}_j^2$, which prevents all the squarks and sleptons to have masses larger than those of quarks and leptons. This constraints on SUSY breaking scenarios in the global case, but by introducing SUGRA the above relationship will be modified.

requires a $U(1)$ symmetry. However, this mechanism does not work in the MSSM because some of the squarks and sleptons will get non-zero VEVs which may break color, electromagnetism, and/or lepton number without breaking SUSY. Therefore the contribution from the Fayet-Iliopoulos (FI) term should be negligible at low scales.

There are models of SUSY breaking by F -terms, known as O’Raifeartaigh models [363], where the idea is to pick a set of chiral supermultiplets $\Phi_i \supset (\phi_i, \psi_i F_i)$ and a superpotential W in such a way that $F_i = -\delta W / \delta \phi_i^* = 0$ have no simultaneous solution. The model requires a linear gauge singlet superfield in the superpotential. Such singlet chiral supermultiplet is not present in the MSSM. The scale of SUSY breaking has to be set by hand.

The only mechanism of SUSY breaking where the breaking scale is not introduced either at the level of superpotential or in the gauge sector is through dynamical SUSY breaking [364–367], see also [368]. In these models a small SUSY breaking scale arises by dimensional transmutation. It is customary to treat the SUSY breaking sector as a hidden sector which has no direct couplings to the visible sector represented by the chiral supermultiplets of the MSSM. The only allowed interactions are those which mediate the SUSY breaking in the hidden sector to the visible sector.

The main contenders are gravity mediated SUSY breaking, which is associated with new physics which includes gravity at the string scale or at the Planck scale [30, 369, 370], and gauge mediated SUSY breaking, which is transmitted to the visible sector by the ordinary electroweak and QCD gauge interactions [366, 371–373]. There are other variants of SUSY breaking based upon ideas on gravity and gauge mediation with some extensions, such as dynamical SUSY breaking (see [36], and references therein), and anomaly mediation (see [35, 374]).

3. *Next to MSSM (NMSSM)*

The simplest extension of the MSSM can be obtained by adding a new gauge-singlet chiral supermultiplet with even matter parity. The superpotential reads as [375–378], see also [379]:

$$W_{\text{NMSSM}} = W_{\text{MSSM}} + \lambda S H_u H_d + \frac{1}{3} \kappa S^3 + \frac{1}{2} \mu_S S^2, \quad (108)$$

where S is the new chiral supermultiplet. It is often called the next-to-minimal SUSY standard model (NMSSM). The NMSSM introduces extra coefficients, by choosing them

correctly it is possible to realize a successful electroweak symmetry breaking.

One of the virtues of the NMSSM is that it can provide a solution to the μ problem. An effective μ -term for $H_u H_d$ will arise from eq. (108), with $\mu_{\text{eff}} = \lambda s$. It is determined by the dimensionless couplings and the soft terms of order m_{soft} , instead of being a free parameter conceptually independent of SUSY breaking. In general, NMSSM also provides extra sources for large CP violation and conditions for electroweak baryogenesis [380, 381].

The singlet S contains a real $P_R = +1$, and a $P_R = -1$ Weyl fermion singlino, \tilde{S} . These fields have no gauge couplings of their own, so they can only couple to the SM particles via mixing with the neutral MSSM fields with the same spin and charge. The odd R -parity singlino \tilde{S} mixes with the four MSSM neutralinos.. The singlino could be the LSP and the dark matter candidate [382–384], in some parameter space neutralino type dark matter is also possible [385, 386]. For collider signatures, see [387].

4. Gravity mediated SUSY breaking

The gravity mediated SUSY breaking, which is associated with new physics which includes gravity at the string scale or at the Planck scale [30, 369]. It is assumed that SUSY is broken by the VEV $\langle F \rangle \neq 0$ and is communicated to the MSSM by gravity. On dimensional grounds, the soft terms in the visible sector should then be of the order $m_0 \sim \langle F \rangle / M_{\text{P}}$, see [30].

The SUGRA Lagrangian must contain the non-renormalizable terms which communicate between the hidden and the observable sectors. For the cases where the kinetic terms for the chiral and gauge fields are minimal, one obtains the following soft terms [30, 369]

$$m_{1/2} \sim \frac{\langle F \rangle}{M_{\text{P}}}, \quad m_0^2 \sim \frac{|\langle F \rangle|^2}{M_{\text{P}}^2}, \quad A_0 \sim \frac{\langle F \rangle}{M_{\text{P}}}, \quad B_0 \sim \frac{\langle F \rangle}{M_{\text{P}}}. \quad (109)$$

The gauginos get a common mass $M_1 = M_2 = M_3 = m_{1/2}$, the squark and slepton masses are $m_Q^2 = m_u^2 = m_d^2 = m_L^2 = m_e^2 = m_0^2$, and for the Higgses $m_{H_u}^2 = m_{H_d}^2 = m_0^2$. The A -terms are proportional to the Yukawa couplings while $b = B_0 \mu$.

Note that $m_{\text{soft}} \rightarrow 0$ as $M_{\text{P}} \rightarrow \infty$. In order to obtain a phenomenologically acceptable soft SUSY mass $m_{\text{soft}} \sim \mathcal{O}(100)$ GeV, one therefore requires the scale of SUSY breaking in the hidden sector to be $\sqrt{\langle F \rangle} \sim 10^{10} - 10^{11}$ GeV.

Another possibility is that the SUSY is broken via gaugino condensate $\langle 0 | \lambda^a \lambda^b | 0 \rangle =$

$\delta^{ab}\Lambda^3 \neq 0$, where Λ is the condensation scale [30, 370, 375]. If the composite field $\lambda^a\lambda^b$ belongs to the $\langle F \rangle \sim \Lambda^3/M_{\text{P}}$ -term, then again on dimensional grounds one would expect the soft SUSY mass contribution to be [30, 369]

$$m_{\text{soft}} \sim \frac{\Lambda^3}{M_{\text{P}}^2}. \quad (110)$$

In this case the nature of SUSY breaking is dynamical and the scale is given by $\Lambda \sim 10^{13}$ GeV.

Commonly gravity mediated SUSY breaking scenario is also known as minimal SUGRA (mSUGRA). In mSUGRA the number of independent parameters reduce a lot, there are $m_0, m_{1/2}, A_0$, the GUT scale value for μ_0, b_0 , and the gravitino mass. Further more $b_0 = A_0 - m_0$ and $m_{3/2} = m_0$ further reduces the parameter space. Nowadays, the popular choice of parameters is known as CMSSM (constrained MSSM), they are: $\tan\beta = \langle H_u \rangle / \langle H_d \rangle$, $m_0, A_0, m_{1/2}$ and $\text{sgn}(\mu_0)$. Within CMSSM the LSP is the lightest neutralino (the neutral higgsinos (\tilde{H}_u^0 and \tilde{H}_d^0) and the neutral gauginos (\tilde{B}, \tilde{W}^0) combine to form four mass eigenstates called neutralinos), $m_{\tilde{N}_1} < m_{\tilde{N}_2} < m_{\tilde{N}_3} < m_{\tilde{N}_4}$ is known to be the LSP, unless gravitino is the lightest, or R-parity is not conserved.

There are variants of gravity mediation, known as anomaly mediated SUSY breaking (AMSB) scenarios, where at tree level gaugino masses are not present. The masses for gauginos arise from one-loop whose origin can be traced to the super-conformal (super-Weyl) anomaly which is common to all SUGRA models [374, 388, 389]. At low energies gaugino mass parameters are given by:

$$M_i \approx \frac{b_i g_i^2}{16\pi^2} m_{3/2}. \quad (111)$$

where $m_{3/2} \sim 100 - 1000$ GeV is the gravitino mass, and b_i are the MSSM gauge beta-functions, i.e. for $SU(3), SU(2), U(1)$ gauge groups: ($b_3 = -3, b_2 = 1, b_1 = 33/5$). AMSB can naturally suppress the flavor changing processes, however at a cost of negative squared masses for the sleptons. Therefore AMSB cannot alone be the main source for SUSY breaking in the slepton sector.

5. Gauge mediated SUSY breaking

In gauge mediated SUSY breaking one employs a heavy messenger sector which couples directly to the SUSY breaking sector but indirectly to the observable sector via standard

model gauge interactions only [366, 371–373, 390]. As a result the soft terms in the MSSM arise through ordinary gauge interactions. There will still be gravitational communication, but it is a weak effect.

The simplest example is a messenger sector with a pair of $SU(2)$ doublet chiral fields l, \bar{l} and a pair of $SU(3)$ triplet fields q, \bar{q} , which couple to a singlet field z with Yukawa couplings λ_2, λ_3 , respectively. The superpotential is given by

$$W_{mess} = \lambda_2 z \bar{l} l + \lambda_3 z q \bar{q}. \quad (112)$$

The singlet acquires a non-zero VEV and a non-zero F-term $\langle F_z \rangle$. This can be accomplished either substituting z into an O’Raifeartaigh type model [371, 390], or by a dynamical mechanism [366, 372]. One may parameterize SUSY breaking in a superpotential W_{break} by $\langle \partial W_{break} / \partial z \rangle = -\langle F_z^* \rangle$. As a consequence, the messenger fermions acquire masses and a scalar potential with $\langle \partial W_{mess} / \partial z \rangle = 0$.

SUSY breaking is then mediated to the observable fields by one-loop corrections, which generate masses for the MSSM gauginos [366]. The q, \bar{q} messenger loop diagrams provide masses to the gluino and the bino, while l, \bar{l} messenger loop diagrams provide masses to the wino and the bino, i.e., $M_{a=1,2,3} = (\alpha_a / 4\pi) \Lambda$, where $\Lambda = \langle F_z \rangle / \langle z \rangle$.

For squarks and sleptons the leading term comes from two-loop diagrams, e.g. $m_\phi^2 \propto \alpha^2$. The A -terms get negligible contribution at two-loop order compared to the gaugino masses, they come with an extra suppression of $\alpha / 4\pi$ compared with the gaugino mass, therefore $a_u = a_d = a_e = 0$ is a good approximation. The Yukawa couplings at the electroweak scale are generated by evolving the RG equations.

One can estimate [366, 372] the soft SUSY breaking masses to be of order

$$m_{soft} \sim \frac{\alpha_a \langle F \rangle}{4\pi M_s}. \quad (113)$$

If $M_s \sim \langle z \rangle$ and $\sqrt{\langle F \rangle}$ are comparable mass scales, then the SUSY breaking can take place at about $\sqrt{\langle F \rangle} \sim 10^4 - 10^6$ GeV.

In gauge mediated SUSY breaking the gravitino could be the LSP, and the next-to-LSP (NLSP) could be either stau, bino-like neutralino. The NLSP decay into gravitino could be very long ranging from seconds to years, the decay process: $\tilde{\tau} \rightarrow \tau \tilde{G}$ is governed by the gravitational coupling. The decay of long lived staus are constrained by the BBN.

6. Split SUSY

An advantage of a low scale (TeV) SUSY is the gauge coupling unification, and the LSP, such as neutralino or gravitino as a dark matter candidate. Both of these virtues can be kept without any need of a TeV scale supersymmetry as shown in Refs. [391–394]. Split SUSY has light SM like Higgs bosons, the A -term and the μ -term, but super heavy squarks and sleptons. If the SUSY breaking is mediated via gravity, then the gravitino mass comes out to be:

$$m_{3/2} \geq \frac{m_0^2}{M_{\text{P}}}, \quad (114)$$

where m_0 correspond to the scalar masses, which suppresses flavor changing neutral currents and CP violations mediated via heavy squarks. While the A -term, μ -term, and gauginos are light [393, 394],

$$A, \mu, m_{1/2} \sim \frac{m_{3/2}^3}{M_{\text{P}}^2}. \quad (115)$$

Therefore the neutralino like LSP can be realizable with a very long lived gluino, which decays via off-shell squark to quark, anti-quark and LSP.

The reasoning for such a split spectrum is due to an accidental R-symmetry, which can protect the masses for gauginos, the A -term and the μ term, which gets broken via non-renormalizable interactions. The model also allows the Higgs mass to be fine tuned to be light at the weak scale. However there are other advantages, there will be no gravitino problem for BBN, as the gravitino mass can be made higher than TeV, and the proton decay upper limits can also be satisfied as the squarks are heavy.

7. Renormalization group equations in the MSSM

In many cosmological applications of flat directions, it is important to consider the running of (mass)² below M_{GUT} . For simplicity we can also assume that it is the scale where SUSY breaking is transmitted to the visible sector. The running of low-energy soft breaking masses has been studied in great detail in the context of MSSM phenomenology [30, 31, 33], see also [395], in particular in connection with radiative electroweak symmetry breaking [396]. A general form of RG equations, which to one loop can be written as:

$$\frac{\partial m_i^2}{\partial t} = \sum_g a_{ig} m_g^2 + \sum_a h_a^2 \left(\sum_j b_{ij} m_j^2 + A^2 \right), \quad (116)$$

where a_{ig} and b_{ij} are constants, m_g is the gaugino mass, h_a the Yukawa coupling, A is the A -term, and $t = \ln M_X/q$. The full RG equations have been listed in [30, 31, 33].

However, let us consider some of the salient features of the MSSM one-loop RG equations. The one-loop RG equations for the three gaugino mass parameters are determined by:

$$\frac{d}{dt}m_i = \frac{1}{8\pi^2}b_i g_i^2 m_i, \quad (b_i = 33/5, 1, -3) \quad (117)$$

where $i = 1, 2, 3$ correspond to $U(1)$, $SU(2)$, $SU(3)$. An interesting property is that the three ratios m_i/g_i^2 are RG scale independent. Therefore at the GUT scale, where the gauge couplings unify at $M_{GUT} \sim 2 \times 10^{16}$ GeV, it is assumed that gauginos masses also unify with a value $m_{1/2}$. Then at any scale:

$$\frac{m_i}{g_i^2} = \frac{m_{1/2}}{g_{GUT}^2}, \quad (118)$$

where g_{GUT} is the unified gauge coupling at the GUT scale. The RG evolution due to Yukawa interactions are small except for top. The ones relevant to flat directions, involving the Higgs doublet H_u which couples to the top quark, the right-handed stop \tilde{u}_3 , the left-handed doublet of third generation squarks \tilde{Q}_3 and the A -parameter A_t associated with the top Yukawa interaction. The RG equations read [30, 31, 33]

$$\begin{aligned} \frac{d}{dq}m_{H_u}^2 &= \frac{3h_t^2}{8\pi^2} (m_{H_u}^2 + m_{\tilde{Q}_3}^2 + m_{\tilde{u}_3}^2 + |A_t|^2) - \frac{1}{2\pi^2} \left(\frac{1}{4}g_1^2|m_1|^2 + \frac{3}{4}g_2^2|m_2|^2 \right), \\ \frac{d}{dq}m_{\tilde{u}_3}^2 &= \frac{2h_t^2}{8\pi^2} (m_{H_u}^2 + m_{\tilde{Q}_3}^2 + m_{\tilde{u}_3}^2 + |A_t|^2) - \frac{1}{2\pi^2} \left(\frac{4}{9}g_1^2|m_1|^2 + \frac{4}{3}g_3^2|m_3|^2 \right), \\ \frac{d}{dq}m_{\tilde{Q}_3}^2 &= \frac{h_t^2}{8\pi^2} (m_{H_u}^2 + m_{\tilde{Q}_3}^2 + m_{\tilde{u}_3}^2 + |A_t|^2) - \frac{1}{2\pi^2} \left(\frac{1}{36}g_1^2|m_1|^2 + \frac{3}{4}g_2^2|m_2|^2 + \frac{4}{3}g_3^2|m_3|^2 \right), \\ \frac{d}{dq}A_t &= \frac{3h_t^2}{8\pi^2}A_t - \frac{1}{2\pi^2} \left(\frac{13}{36}g_1^2m_1 + \frac{3}{4}g_2^2m_2 + \frac{4}{3}g_3^2m_3 \right). \end{aligned} \quad (119)$$

Here q denotes the logarithmic scale; this could be an external energy or momentum scale, but in the case at hand the relevant scale is set by the VEV(s) of the fields themselves. h_t is the top Yukawa coupling, while g_i and m_i are respectively the gauge couplings and soft breaking gaugino masses of $U(1)_Y \times SU(2) \times SU(3)$. If h_t is the only large Yukawa coupling (i.e. as long as $\tan \beta$ is not very large), the beta functions for (mass)² of squarks of the first and second generations and sleptons only receive significant contributions from gauge/gaugino loops. A review of these effects can be found in [33, 395].

D. F -and D -flat directions of MSSM

Field configurations satisfying simultaneously:

$$D^a \equiv X^\dagger T^a X = 0, \quad F_{X_i} \equiv \frac{\partial W}{\partial X_i} = 0. \quad (120)$$

for N chiral superfields X_i , are called respectively D -flat and F -flat. D -flat directions are parameterized by gauge invariant monomials of the chiral superfields. A powerful tool for finding the flat directions has been developed in [317–319, 365, 397–399], for a review see [90, 91] where the correspondence between gauge invariance and flat directions has been employed.

A single flat direction necessarily carries a global $U(1)$ quantum number, which corresponds to an invariance of the effective Lagrangian for the order parameter ϕ under phase rotation $\phi \rightarrow e^{i\theta}\phi$. In the MSSM the global $U(1)$ symmetry is $B-L$. For example, the LH_u -direction (see below) has $B-L = -1$. A flat direction can be represented by a composite gauge invariant operator, X_m , formed from the product of k chiral superfields Φ_i making up the flat direction: $X_m = \Phi_1\Phi_2 \cdots \Phi_m$. The scalar component of the superfield X_m is related to the order parameter ϕ through $X_m = c\phi^m$.

An example of a D -and F -flat direction is provided by

$$H_u = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \phi \end{pmatrix}, \quad L = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi \\ 0 \end{pmatrix}, \quad (121)$$

where ϕ is a complex field parameterizing the flat direction, or the order parameter, or the AD field. All the other fields are set to zero. In terms of the composite gauge invariant operators, we would write $X_m = LH_u$ ($m = 2$).

From Eq. 312 one clearly obtains $F_{H_u}^* = \lambda_u Qu + \mu H_d = F_L^* = \lambda_d H_d e \equiv 0$ for all ϕ . However there exists a non-zero F-component given by $F_{H_d}^* = \mu H_u$. Since μ can not be much larger than the electroweak scale $M_W \sim \mathcal{O}(1)$ TeV, this contribution is of the same order as the soft SUSY breaking masses, which are going to lift the degeneracy. Therefore, following [318], one may nevertheless consider LH_u to correspond to a F-flat direction.

The relevant D -terms read

$$D_{SU(2)}^a = H_u^\dagger \tau_3 H_u + L^\dagger \tau_3 L = \frac{1}{2}|\phi|^2 - \frac{1}{2}|\phi|^2 \equiv 0. \quad (122)$$

Therefore the LH_u direction is also D -flat.

The only other direction involving the Higgs fields and thus soft terms of the order of μ is $H_u H_d$. The rest are purely leptonic, such as LLe , or baryonic, such as udd , or mixtures of leptons and baryons, such as QLd . These combinations give rise to several independent flat directions that can be obtained by permuting the flavor indices. For instance, LLe contains the directions $L_1 L_2 e_3$, $L_2 L_3 e_1$, and $L_1 L_3 e_2$, let us consider a particular configuration [91]:

$$L_1 = \frac{1}{\sqrt{3}} \begin{pmatrix} \phi \\ 0 \end{pmatrix}, \quad L_2 = \frac{1}{\sqrt{3}} \begin{pmatrix} 0 \\ \phi \end{pmatrix}, \quad e_3 = \frac{1}{\sqrt{3}} \phi, \quad (123)$$

The $SU(2) \times U(1)$ D -terms are

$$V_D = \frac{g^2}{8} (|L_1|^2 - |L_2|^2)^2 + \frac{g'^2}{72} (|L_1|^2 - 3|L_2|^2 + 2|e_3|^2)^2, \quad (124)$$

where $g = e/\sin\theta_w$ is the $SU(2)$ coupling and $g' = e/\cos\theta_w$ is the $U(1)_Y$ coupling. When $L_1^1 = L_2^2 = e_3 = \phi$ the D -terms in the potential vanish as they must.

Along a flat direction gauge symmetries get broken, with the gauge supermultiplets gaining mass by super-Higgs mechanism with $m_g = g\langle\phi\rangle$. Several chiral supermultiplets typically become massive by virtue of Yukawa couplings in the superpotential; for example, in the LH_u direction one finds the mass terms $W_{\text{mass}} = \lambda_u \langle\phi\rangle Qu + \lambda_e \langle\phi\rangle H_d e$. In this respect when the flat direction VEV vanishes, i.e. $\phi = 0$, the gauge symmetry gets enhanced.

Vacuum degeneracy along a flat direction can be broken in two ways: by SUSY breaking, or by higher order non-renormalizable operators appearing in the effective low energy theory. Let us first consider the latter option.

1. Non-renormalizable superpotential corrections

Non-renormalizable superpotential terms in the MSSM can be viewed as effective terms that arise after one integrates out fields with very large mass scales appearing in a more fundamental (say, string) theory. Here we do not concern ourselves with the possible restrictions on the effective terms due to discrete symmetries present in the fundamental theory, but assume that all operators consistent with symmetries may arise. Thus in terms of the invariant operators X_m , one can have terms of the type [318, 319]

$$W = \frac{h}{dM^{d-3}} X_m^k = \frac{h}{dM^{d-3}} \phi^d, \quad (125)$$

where the dimensionality of the effective scalar operator $d = mk$, and h is a coupling constant which could be complex with $|h| \sim \mathcal{O}(1)$. Here M is some large mass, typically of the order of the Planck mass or the string scale (in the heterotic case $M \sim M_{GUT}$). The lowest value of k is 1 or 2, depending on whether the flat direction is even or odd under R -parity.

A second type of term lifting the flat direction would be of the form [318, 319]

$$W = \frac{h'}{M^{d-3}} \psi \phi^{d-1} \quad , \quad (126)$$

where ψ is not contained in X_m . The superpotential term Eq. (126) spoils F-flatness through $F_\psi \neq 0$. An example is provided by the direction $u_1 u_2 u_3 e_1 e_2$, which is lifted by the non-renormalizable term $W = (h'/M) u_1 u_2 d_2 e_1$. This superpotential term gives a non-zero contribution $F_{d_2}^* = (h'/M) u_1 u_2 e_1 \sim (h'/M) \phi^3$ along the flat direction.

Assuming minimal kinetic terms, both types discussed above in Eqs. (125,126) yield a generic non-renormalizable potential contribution that can be written as

$$V(\phi) = \frac{|\lambda|^2}{M^{2d-6}} (\phi^* \phi)^{d-1} \quad , \quad (127)$$

where we have defined the coupling $|\lambda|^2 \equiv |h|^2 + |h'|^2$. By virtue of an accidental R -symmetry under which ϕ has a charge $R = 2/d$, the potential Eq. (127) conserves the $U(1)$ symmetry carried by the flat direction, in spite of the fact that at the superpotential level it is violated, see Eqs. (125,126).

All the non-renormalizable operators can be generated from SM gauge monomials with R -parity constraint which allows only even number of odd matter parity fields (Q, L, u, d, e) to be present in each superpotential term. At each dimension d , the various $F = 0$ constraints are separately imposed in order to construct the basis for monomials.

As an example, consider flat directions involving the Higgs fields such as $H_u H_d$ and $L H_u$ directions. Even though they are already lifted by the μ term, since μ is of the order of SUSY breaking scale, for cosmological purposes they can be considered flat. At the $d = 4$ level the superpotential reads [318, 319, 399]

$$W_4 \supset \frac{\lambda}{M} (H_u H_d)^2 + \frac{\lambda_{ij}}{M} (L_i H_u) (L_j H_u) \quad . \quad (128)$$

Let us assume $\lambda, \lambda_{ij} \neq 0$. Note that $F_{H_d} = 0$ constraint implies $\lambda H_u^\alpha (H_u H_d) = 0$, which acts as a basis for the monomials. An additional constraint can be obtained by contracting $F_{H_d} = 0$ by $\epsilon_{\alpha\beta} H_d^\beta$, which forms the polynomial $H_u H_d = 0$ in the same monomial basis.

Similarly the constraint $F_{H_u} = 0$, along with the contraction yields $\lambda^{ij}(L_i H_u)(L_j H_u) = 0$. This implies that $L_i H_u = 0$ for all i . Therefore the two monomials LH_u and $H_u H_d$ can be lifted by $d = 4$ terms in the superpotential Eq. (128).

The other renormalizable flat directions are $LLe, uud, QdL, QQQL, QuQd, uude$ and $QuLe, dddLL, uuuee, QuQue, QQQQu, uudQdQd$, and $(QQQ)_4 LLLe$. The unique flat directions involving (Q, u, e) is lifted by $d = 9$, (L, d) by $d = 7$, and (L, d, e) by $d = 5$. The flat directions involving $(L, e), (u, d)$ and (L, d, e) are all lifted by $d = 6$ terms in the superpotential, while the rest of the flat directions are lifted already by $d = 4$ superpotential terms [399].

Vacuum degeneracy will also be lifted by SUSY breaking soft terms. The full flat direction potential in the simplest case reads [318, 319]

$$V(\phi) = m_0^2 |\phi|^2 + \left[\frac{\lambda A \phi^d}{d M^{d-3}} + \text{h.c.} \right] + \lambda^2 \frac{|\phi|^{2d-2}}{M^{2d-6}}, \quad (129)$$

where the SUSY breaking mass m_0 , $A \sim 100 - 1000$ GeV.

While considering the dynamics of a flat direction, in a cosmological setting, the superpotential of Eq. (126) generates a vanishing A -term. This is due to the fact that ϕ being light during inflation, i.e. $H_{inf} \gg m_0$, it obtains large VEV during inflation due to random walk. As a result ψ field gets a large mass induced by the VEV of ϕ , which drives ψ to roll down to its minimum in less than one Hubble time, i.e. $\langle \psi \rangle = 0$. The A -term being proportional to ψ vanishes in this limit, and does not play any dynamical role during and after inflation. In other words, the ψ field being super massive decouple from the dynamics.

The A -term in Eq. (129) violates the $U(1)$ carried by the flat direction and thus provides the necessary source for $B - L$ violation in AD baryogenesis. In general, the coupling λ is complex and has an associated phase θ_λ . Writing $\phi = |\phi| \exp(i\theta)$, one obtains a potential proportional to $\cos(\theta_\lambda + n\theta)$ in the angular direction. This has n discrete minima for the phase of ϕ , at each of which $U(1)$ is broken.

A very generic one-loop quantum corrections result in a logarithmic running of the soft SUSY breaking parameters. The *effective* potential for the flat direction is then given by [90, 400, 401]:

$$V_{eff}(\phi) = \frac{1}{2} m_0^2 \phi^2 \left[1 + K_1 \log \left(\frac{\phi^2}{\mu_0^2} \right) \right] - \frac{\lambda_{d,0} A_0}{d M^{d-3}} \phi^d \left[1 + K_2 \log \left(\frac{\phi^2}{\mu_0^2} \right) \right] + \frac{\lambda_{d,0}^2}{M^{2(d-3)}} \phi^{2(d-1)} \left[1 + K_3 \log \left(\frac{\phi^2}{\mu_0^2} \right) \right]. \quad (130)$$

where m_0 , A_0 , and $\lambda_{d,0}$ are the values of m_ϕ , A and λ_n given at a scale μ_0 . Here A_0 is chosen to be real and positive (this can always be done by re-parameterizing the phase of the complex scalar field ϕ), and $|K_i| < 1$ are coefficients determined by the one-loop renormalization group equations. We will provide an explicit example in section in VB 1.

2. Spontaneous symmetry breaking and the physical degrees of freedom

A flat direction VEV spontaneously breaks symmetry and gives masses to the gauge bosons/gauginos similar to the Higgs mechanism [121, 122, 318, 402, 403]. A crucial point is to identify the physical degrees of freedom and their mass spectrum in presence of a non-zero flat direction VEV. Let us consider the simplest flat direction, which includes only two fields: $H_u H_d$. This is also familiar from the electroweak symmetry breaking in MSSM. A clear and detailed discussion is given in [31].

One can always rotate the field configuration to a basis where, up to an overall phase, $H_u^1 = H_d^2 = 0$ and $H_u^2 = H_d^1 = \phi_0/\sqrt{2}$. Here superscripts denote the weak isospin components of the Higgs doublets. In this basis the complex scalar field is defined by:

$$\varphi = \frac{(H_u^2 + H_d^1)}{\sqrt{2}}, \quad (131)$$

represents a flat direction. Its VEV breaks the $SU(2)_W \times U(1)_Y$ down to $U(1)_{\text{em}}$ (in exactly the same fashion as in the electroweak vacuum). The W^\pm and Z gauge bosons then obtain masses m_W , $m_Z \sim g\phi_0$ from their couplings to the Higgs fields via covariant derivatives (g denotes a general gauge coupling). There are also:

$$\chi_1 = \frac{(H_u^2 - H_d^1)}{\sqrt{2}}, \quad \text{and} \quad \chi_2 = \frac{(H_u^1 + H_d^2)}{\sqrt{2}}. \quad (132)$$

Then χ_2 and $\chi_{1,R}$ (R and I denote the real and imaginary parts of a complex scalar field respectively) acquire masses equal to m_W and m_Z , respectively, through the D -term part of the scalar potential. Note that

$$\chi_3 = \frac{(H_u^1 - H_d^2)}{\sqrt{2}}, \quad (133)$$

and $\chi_{1,I}$ are the three Goldstone bosons, which are eaten by the gauge fields via the Higgs mechanism. Therefore, out of 8 real degrees of freedom in the two Higgs doublets, there are only two light *physical* fields: φ_R , φ_I . They are exactly massless when SUSY is not broken (and there is no μ term either).

An important point is that the masses induced by the flat direction VEV are SUSY conserving. One therefore finds the same mass spectrum in the fermionic sector. More specifically, the Higgsino fields \tilde{H}_u^1 and \tilde{H}_d^2 are paired with the Winos, while $(\tilde{H}_u^2 - \tilde{H}_d^1)/\sqrt{2}$ is paired with the Zino to acquire masses equal to m_W and m_Z , respectively, through the gaugino-gauge-Higgsino interaction terms. The fermionic partner of the flat direction $(\tilde{H}_u^2 + \tilde{H}_d^1)/\sqrt{2}$ remains massless (note that the photon and photino are also massless, but not relevant for our discussion).

SUSY being broken, φ obtains a mass $m_\varphi \sim \mathcal{O}(\text{TeV})$ from soft SUSY breaking term (the same is true for the gauginos). However, for $g\varphi_0 \gg \mathcal{O}(\text{TeV})$, which is the situation relevant to the early Universe, the mass spectrum is hierarchical: $\chi_{1, R}$, χ_2 , and gauge fields (plus their fermionic partners) are superheavy.

In a general case the total number of light scalars, N_{light} , is given by [402, 403]:

$$N_{light} = N_{total} - (2 \times N_{broken}), \quad (134)$$

where N_{total} is the total number of scalar degrees of freedom, and N_{broken} is the number of spontaneously broken symmetries. Note that the factor 2 counts for the number of eaten Goldstone bosons plus the number of degrees of freedom which have obtained large masses equal to those of the gauge bosons. In the case of $H_u H_d$ direction, Eq. (134) reads: $N_{light} = 2 = 2 \times 2 \times 2 - (2 \times 3)$. Similarly, for LH_u flat direction, $N_{light} = 2 \times 2 \times 2 - (2 \times 3) = 2$.

E. $N = 1$ Supergravity (SUGRA)

At tree level, $N = 1$ SUGRA potential in four dimensions is given by the sum of F and D -terms [30]

$$V = e^{K(\phi_i, \phi^{*i})/M_{\text{P}}^2} \left[(K^{-1})^j_i F_i F^j - 3 \frac{|W|^2}{M_{\text{P}}^2} \right] + \frac{g^2}{2} \text{Re} f_{ab}^{-1} \hat{D}^a \hat{D}^b, \quad (135)$$

where

$$F^i = W^i + K^i \frac{W}{M_{\text{P}}^2}, \quad \hat{D}^a = -K^i (T^a)_i^j \phi_j + \xi^a. \quad (136)$$

where we have added the Fayet-Iliopoulos contribution ξ^a to the D -term, and $\hat{D}^a = D^a/g^a$, where g^a is gauge coupling. Here $K(\phi_i, \phi^{*i})$ is the Kähler potential, which is a function of the fields ϕ_i , and $K^i \equiv \partial K / \partial \phi_i$. In the simplest case, at tree-level $K = \phi^{*i} \phi_i$ (and

$K_i^j = (K^{-1})_i^j = \delta_i^j$ ³¹. The kinetic terms for the scalars take the form:

$$\frac{\partial^2 K}{\partial \phi_i \partial \phi_j^*} D_\mu \phi_i D^\mu \phi_j^*. \quad (137)$$

The real part of the gauge kinetic function matrix is given by $\text{Re} f_{ab}$. In the simplest case, it is just a constant, $f_{ab} = \delta_{ab}/g_a^2$, and the kinetic terms for the gauge potentials, A_μ^a , are given by³²:

$$\frac{1}{4} (\text{Re} f_{ab}) F_{\mu\nu}^a F_a^{\mu\nu}. \quad (138)$$

SUGRA will be broken if one or more of the F_i obtain a VEV. The gravitino, spin $\pm 3/2$ component of the graviton, then absorb the goldstino component to become massive. Requiring classically $\langle V \rangle = 0$, as a constraint to obtain the zero cosmological constant, one obtains

$$m_{3/2}^2 = \frac{\langle K_j^i F_i F^{*j} \rangle}{3M_{\text{P}}^2} = e^{\langle K \rangle / M_{\text{P}}^2} \frac{|\langle W \rangle|^2}{M_{\text{P}}^4}. \quad (139)$$

In case of SUGRA at tree-level with minimal kinetic term, the super-trace formula modifies to (for D -flat directions):

$$\text{Str} \mathbf{M}^2 \equiv \sum_{\text{spin } J} (-1)^{2J} (2J+1) \text{tr} \mathbf{M}_J^2 \approx 2(n_\phi - 1) m_{3/2}^2, \quad (140)$$

where n_ϕ is the number of the chiral multiplets in the spontaneously broken SUGRA.

1. SUSY generalization of one-loop effective potential

A SUSY generalization of one-loop effective potential, Eq. (98), is given by [404, 405]:

$$\Delta V = \frac{1}{64\pi^2} \text{Str} (\mathbf{M}^0) \Lambda_c^4 \ln \left(\frac{\Lambda_c^2}{\mu^2} \right) + \frac{1}{32\pi^2} \text{Str} (\mathbf{M}^2) \Lambda_c^2 + \frac{1}{64\pi^2} \text{Str} \left(\mathbf{M}^4 \ln \left(\frac{\mathbf{M}^2}{\Lambda_c^2} \right) \right) + \dots, \quad (141)$$

with Λ_c being a momentum cut-off and μ the scale parameter. The renormalized potential will not depend on Λ_c , and the dots stand for Λ_c -independent contributions.

³¹ In general the superpotential can have non-renormalizable contributions. Similarly, the Kähler potential can be expanded as: $K = \phi_i \phi_i^* + (k_k^{ij} \phi_i \phi_j \phi_k^* + c.c.)/M_{\text{P}} + (k_{kl}^{ij} \phi_i \phi_j \phi_k^* \phi_l^* + k_l^{ijk} \phi_i \phi_j \phi_k \phi_l^* + c.c.)/M_{\text{P}}^2 + \dots$. We will discuss no-Scale SUGRA where the choice of Kähler potential plays an important role in maintaining flat potential during inflation.

³² In general, $f_{ab} = \delta_{ab}(1/g_a^2 + f_a^i \phi_i / M_{\text{P}} + \dots)$. The gauginos masses are typically given by $m_{\lambda^a} = \text{Re}[f_a^i \langle F_i \rangle] / 2M_{\text{P}}$. For a universal gaugino masses, f_a^i are the same for all the three gauge groups of MSSM.

The first term in Eq. (141), being field-independent, this term can affect the cosmological constant problem in SUGRA, but does not affect the discussion of the gauge hierarchy problem. However, this term is always absent in SUSY theories, which possess equal numbers of bosonic and fermionic degrees of freedom. In Ref. [405, 406], it was shown that for unbroken $N = 1$ global SUSY, $Str\mathbf{M}^n$ is identically vanishing for any n , due to the fermion-boson degeneracy within SUSY multiplets. It was argued that the term $Str\mathbf{M}^2$ vanishes [406], as a field identity, if global SUSY is spontaneously broken in the absence of anomalous $U(1)$ factors [405]. Anyway, the third term in Eq. (141), plays the most important role in inflationary models in lifting the flat potential for the inflaton.

It was argued in [407], that in the derivation of [405], an explicit use has been made of the fact that all first order partial derivatives of the tree-level effective potential to the fields vanish. This limits the region of applicability of the simple Coleman-Weinberg formula in the context of inflationary models. Only at extremum (or saddle) points it is justified to throw away the second term of Eq. (141) ³³.

2. *Inflaton-induced SUGRA corrections*

Since non-zero inflationary potential gives rise to SUSY breaking, the scale of which is given by the time dependent Hubble parameter. It is important to know how this will affect any other light scalar field during and after inflation, in particular the flat directions of MSSM. At early times this breaking is dominant over breaking from the hidden sector ³⁴. After the end of inflation, in most models the inflaton oscillates and its finite energy density still dominates and breaks SUSY in the visible sector. A particular class of non-renormalizable interaction terms induced by the inflaton arise if the Kähler potential has a

³³ For a particular case of SUSY model of inflation, known as the hybrid inflation (see the discussion in section IVE.1, the Coleman-Weinberg formula given by the third term in Eq. (141) is applicable only in the global minimum and along the inflationary valley, because here all derivatives in the tree-level potential vanish. If one wishes to study the field dynamics throughout the phase space, one requires special care [407].

³⁴ Note that this does not replace the soft SUSY breaking terms required to solve the problems for the low energy physics, such as addressing the hierarchy problem or the electroweak symmetry breaking within MSSM, etc.

form [318, 319, 408, 409]

$$K = \int d^4\theta \frac{1}{M_{\text{P}}^2} (I^\dagger I) (\phi^\dagger \phi), \quad (142)$$

where I is the inflaton whose energy density $\rho \approx \langle \int d^4\theta I^\dagger I \rangle$ dominates during inflation, and ϕ is the flat direction. The interaction, Eq. (142), will generate an effective mass term in the Lagrangian in the global SUSY limit, given by

$$\mathcal{L} = \frac{\rho_I}{M_{\text{P}}^2} \phi^\dagger \phi = 3H_I^2 \phi^\dagger \phi, \quad (143)$$

where H_I is the Hubble parameter during inflation.

For a minimal choice of flat direction Kähler potential $K(\phi^\dagger, \phi) = \phi^\dagger \phi$, during inflation the effective mass for the flat direction is found to be [318, 319]³⁵:

$$m_\phi^2 = \left(2 + \frac{F_I^* F_I}{V(I)} \right) H^2. \quad (144)$$

Here it has been assumed that the main contribution to the inflaton potential comes from the F -term. If there were D -term contributions $V_D(I)$ to the inflationary potential, then a correction of order $V_F(I)/(V_F(I) + V_D(I))$ must be taken into account. In purely D -term inflation there is no Hubble induced mass correction to the flat direction during inflation because $F_I = 0$. However, when D -term inflation ends, the energy density stored in the D -term is converted to an F -term and to kinetic energy of the inflaton. Thus again a mass term $m_\phi^2 = \pm \mathcal{O}(1)H^2$ appears naturally, however the overall sign is undetermined [410].

3. No-scale SUGRA

There exists a choice of Kähler potential for which there is no Hubble induced correction to the mass of the lights scalars. An example of this is provided by no-scale models, for which $K \sim \ln(z + z^* + \phi_i^\dagger \phi)$, where z belongs to SUSY breaking sector, and ϕ_i belongs to the matter sector, and both are measured in terms of reduced Planck mass [411–413] (for a review, see [414]). In no-scale models there exists an enhanced symmetry known as the Heisenberg symmetry [415], which is defined on the chiral fields as $\delta z = \epsilon^* \phi^i$, $\delta \phi^i = \epsilon^i$, and $\delta y^i = 0$, where y^i are the hidden sector fields, such that the combinations $\eta = z + z^* - \phi_i^* \phi^i$,

³⁵ If the Kähler potential has a shift symmetry, or its of type no-scale model, the Hubble induced mass correction to the flat direction does not arise at the tree-level potential.

and $y_i = 0$ are invariant. For a especial choice

$$K = f(\eta) + \ln[W(\phi)/M_{\text{P}}^3]^2 + g(y), \quad (145)$$

The $N = 1$ SUGRA potential reads [408, 416]

$$V_F = e^{f(\eta)+g(y)} \left[\left(\frac{f'^2}{f''} - 3 \right) \frac{|W|^2}{M_{\text{P}}^2} - \frac{1}{f'^2} \frac{|W_i|^2}{M_{\text{P}}^2} + g_a (g^{-1})^a_b g^b \frac{|W|^2}{M_{\text{P}}^2} \right]. \quad (146)$$

Note that there is no cross term in the potential such as $|\phi_i^* W|^2$. As a consequence any tree level flat direction remains flat even during inflation [408] (in fact it is the Heisenberg symmetry which protects the flat directions from obtaining Hubble induced masses [415]).

A particular choice of Kähler potential, i.e.

$$K = -3 \ln(\varphi + \bar{\varphi}),$$

arises quite naturally from string compactifications [417, 418]. For a constant superpotential, W_0 , the F-term of the potential yields;

$$V_F = e^{K/M_{\text{P}}^2} [(K^{-1})^j_i K^i K_j - 3] |W_0|^2. \quad (147)$$

and for the above choice of the Kähler potential, $V_F = 0$ for all φ , because the Kähler potential satisfies $(K^{-1})^j_i K^i K_j = 3$, this is a property of no-scale model. The symmetry is broken by gauge interactions or by coupling in the renormalizable part of the Kähler potential. Then the mass of the flat direction condensate arises from the running of the gauge couplings.

F. (SUSY) Grand Unified Theories

The observation that within SUSY the value of three gauge couplings nearly meet at $\sim 2 \times 10^{16}$ GeV has led to the idea ³⁶ that the three gauge groups emerge from a single (Grand Unified) group G_{GUT} with a single gauge coupling g_{GUT} ³⁷. Another motivation for

³⁶ Initial attempts were made by Pati and Salam [419] in $SU(2)_L \times SU(2)_R \times SU(4)_C$ model, where quarks and leptons are unified-a lepton becomes the fourth color of a quark. Although the model did not have gauge unification, but later models unified the couplings in a left-right symmetric model [343, 420, 421].

³⁷ The requirement of a simple gauge group can be relaxed. It is also possible to have a single high energy gauge coupling constant with a non-simple gauge group made up of products of identical groups, i.e. $G_{GUT} = H \times H \times \dots$. The single gauge coupling constant is ensured by imposing an additional symmetry that render the theory invariant under exchange of the factors of H .

such a unification beyond the SM is to explain the quantification of the electric charge, that is to explain why $3Q(\text{quark}) = Q(\text{electron})$. Various Lie group candidates are $SU(n+1)$, $SO(4n+2)$ families and E_6 , when imposing that the groups must contain anomaly free complex representations to accommodate the known fermions of the SM [422]. The smallest and most studied of these candidates are $SU(5)$, $SO(10)$, and E_6 . For reviews, see [37, 423–426] and more specifically [38] for proton decay, and [422] for group theory and useful tables on symmetry breaking.

Although the idea of gauge unification is very appealing, but this concept has additional constraints. Clearly the GUT group must contain $G_{GUT} \supset G_{SM}$ and must break spontaneously into its subgroups and finally G_{SM} at some high scales. During this breaking the extra gauge bosons of G_{GUT} , which carry quantum numbers of several groups and G_{SM} , acquire a superheavy mass, M_{GUT} , which prevent from a too rapid decay of the nucleons. The choice of G_{GUT} and the presence of SUSY affect the predictions for proton lifetime of a given model as the scale of SUSY breaking and the particles involved in Feynman graphs modify the results. Finally, the electroweak precision measurements require the particles beyond the SM to be heavy enough, for instance the famous doublet-triplet splitting of the SM higgs from the additional Higgs fields (see the $SU(5)$ example below).

1. $SU(5)$ and $SO(10)$ GUT

The most studied candidate is based on $SU(5)$ whose generators are represented by five-by-five traceless, Hermitian matrices. $SU(3) \times SU(2) \times U(1)$ is one of its maximal subgroup (of same rank) together with $SU(4) \times U(1)$. Let us highlight the main features of the fermionic, bosonic and the Higgs sectors. If the gauge bosons necessarily belong to the adjoint representation of dimension 24, the fermions and the Higgs can be accommodated in any representations that lead to the right phenomenology. The smallest representations for $SU(5)$ are the fundamental **5**, **10**, **15**, **24**, First, the 15 fermions can be put in a fundamental $\bar{\mathbf{5}}$ and **10**, a anti-symmetric traceless 5×5 matrix [427]. The decomposition of these representations under $SU(3)_c \times SU(2)_L$ lead to the quantum numbers of the fermions under the SM:

$$\bar{\mathbf{5}} = (\bar{\mathbf{3}}, \mathbf{1}) + (\mathbf{1}, \mathbf{2}) \quad \text{and} \quad \mathbf{10} = (\mathbf{3}, \mathbf{2}) + (\bar{\mathbf{3}}, \mathbf{1}) + (\mathbf{1}, \mathbf{1}) . \quad (148)$$

This accommodates a quark triplet, d_R , and a leptonic doublet, (e_L^-, ν_L) , respectively in the $\mathbf{5}$, and the quark triplets, (u_L, d_L) u_R , and e^+ , respectively in the $\mathbf{10}$. The construction of Q as a combination of two diagonal *traceless* generators of $SU(5)$ requires that the sum of all the charges of the fermions in the fundamental representation vanishes, and thus explains the quantification of the charges.

There are 24 gauge bosons of $SU(5)$ and their quantum number under the SM are given by the decomposition under $SU(5) \supset SU(3) \times SU(2)$ [422]:

$$\mathbf{24} = (\mathbf{1}, \mathbf{1}) + (\mathbf{1}, \mathbf{3}) + (\mathbf{3}, \mathbf{2}) + (\bar{\mathbf{3}}, \mathbf{2}) + (\mathbf{8}, \mathbf{1}) , \quad (149)$$

which are the photon, the triplet W^\pm , Z bosons, the so-called X and \bar{X} , and the gluon octet. The 12 gauge bosons X and \bar{X} carry both $SU(3)_c$ and $SU(2)_L$ quantum numbers, therefore they violate the baryon quantum number, leading to a proton decay.

The Higgs sector of $SU(5)$ must contain a Higgs field breaking $SU(5)$ into G_{SM} , and at least another Higgs for the electroweak breaking, as the two scales are different. The smallest representation that contains a singlet under the SM is the adjoint $\Sigma = \mathbf{24}$, see Eq. (149). Its potential is a generalization of the usual mexican hat potential:

$$V(\Sigma) = -\frac{1}{2}m_\Sigma^2 \text{Tr}\Sigma^2 + \frac{1}{4}a(\text{Tr}\Sigma^2)^2 + \frac{1}{2}b\text{Tr}\Sigma^4 , \quad (150)$$

where $m_\Sigma^2 > 0$ and a, b are constants. The most general potential for Σ would also contain a term of the form $\text{Tr}\Sigma^3$, if the invariance $\Sigma \rightarrow -\Sigma$ is not assumed. To break $SU(5)$ into the SM, the VEV has to be taken in the direction $\Sigma_{24} = \text{Diag}(-1, -1, -1, 3/2, 3/2)$ ³⁸, though depending on the sign of b , the minimum of the above potential can be obtained for VEVs $\propto \text{Diag}(1, 1, 1, 1, -4)$, instead. The breaking in this case is the other maximal subgroup, $SU(5) \rightarrow SU(4) \times U(1)$. The SM Higgs can be embedded into a fundamental, $H = \mathbf{5}$, as it contains a component (the $H_2 = (\mathbf{1}, \mathbf{2})$), a singlet under $SU(3)_c$ but doublet under $SU(2)_L$ has a potential:

$$V(H) = -\frac{m_5^2}{2}H^\dagger H + \frac{\lambda}{4}(H^\dagger H)^2 . \quad (151)$$

It is important to note that summing the two potentials $V = V(\Sigma) + V(H)$ is not sufficient to construct a realistic theory. As a general rule, the potential of all the Higgs fields contain

³⁸ We remind the reader that any field in the adjoint can be decomposed into a linear combination of the the generators of $SU(5)$, the factors are gauge singlet fields.

cross-terms that are allowed by symmetries. The correct approach is to consider a field content and minimize the multi-dimensional potential to be sure that the minima give the right phenomenology. Embedding the SM Higgs into a fundamental leads to the well-known doublet-triplet splitting problem, as a mass term for H_5 would give the same mass to the SM doublet and the additional triplet $H_3 = (\mathbf{3}, \mathbf{1})$ contained in H . This is catastrophic as H_3 possesses the right quantum number to contribute to the proton decay, thus requiring that its mass is higher than $\sim 10^{15}$ GeV. Interestingly one possible solution of the second problem is to allow cross-terms between H and Σ .

Non-SUSY $SU(5)$ unification has many problems. The three gauge couplings do not exactly meet at the unification scale at a single value. In addition, the unification scale $M_{GUT} \sim 10^{14} GeV$ is now incompatible with the most recent observations of the proton decay [428].

SUSY $SU(5)$ introduces new low mass particles in the spectrum and the existence of a SUSY breaking scale affect the running of the gauge couplings, which allow for a better unification at a larger scale $M_{GUT} \simeq 2 \times 10^{16}$ GeV. Now the proton decay is even more suppressed in the dimensional 6 channel. However the proton decay can now happen through dimensional 4 operators, that would be more dramatic. In order to suppress them one requires the R-parity (see the discussion in Sect. III C 1).

One of the major problems of SUSY $SU(5)$ is the doublet-triplet splitting. Part of the superpotential which generates the masses to the fermions is given by:

$$W \sim a\text{Tr}(\Sigma) + b\text{Tr}(\Sigma^2) + c\text{Tr}(\Sigma^3) + \lambda(H\Sigma\bar{H} + \mu H\bar{H}), \quad (152)$$

where Σ belongs to $\mathbf{45}$, H belongs to $\mathbf{5}$ and \bar{H} to $\bar{\mathbf{5}}$. Out of three degenerate vacua, the right one with the SM gauge group is given by; $\text{Diag}(\langle \Sigma \rangle) = (2, 2, 2, -3, -3)M_{GUT}$. Substituting the VEV in the above superpotential generates an effective superpotential;

$$W \sim \lambda(2M_{GUT} + \mu)\bar{H}(\bar{\mathbf{3}})H(\mathbf{3}) + \lambda(-3M_{GUT} + \mu)H(\mathbf{2})\bar{H}(\bar{\mathbf{2}}). \quad (153)$$

Since, the Higgs mass ought to be around ~ 100 GeV, the cancellation in $\mu \sim 3M_{GUT}$ has to be within one part in 10^{12} to match the observed expectation. This fine tuning is also related to the μ -problem in MSSM. This fine tuning can be replaced by a see saw mechanism in the case of SUSY $SO(10)$, where the triplet can be made naturally heavy as compared to the doublet [429]. On the cosmological front, $SU(5)$ conserves $B - L$, and baryogenesis

via leptogenesis cannot take place. This is another reason why it is desirable to go beyond $SU(5)$.

The next to the simplest gauge group candidate for the unification is $SO(10)$ [430] (see some recent reviews [431–434]), for which the entire generation of fermions can be fit into a single representation, $\mathbf{16}$, of $SO(10)$ which decompose under $SU(5) \times U(1)$ into $\bar{\mathbf{5}} + \mathbf{10} + \mathbf{1}$. It can therefore accommodate all the fermions of the MSSM along with an additional singlet fermion, with the quantum numbers of a right-handed neutrino which can fit into the fundamental spinorial representation $\mathbf{16}$ [340–343].

The SM leptons are now identified as a fourth color of quarks after the inclusion of right handed neutrino. The direct product of spinor representation gives, $\mathbf{16} \oplus \mathbf{16} = \mathbf{10} \oplus \mathbf{120} \oplus \mathbf{126}$, therefore the Higgs sector must contain either of these three, $\mathbf{10}$, $\mathbf{120}$, or $\mathbf{126}$, in order to generate mass terms for the fermions, for detailed discussion see Refs. [37, 38, 423]. In particular when $\mathbf{126}$ develops a VEV, it gives rise to the Majorana right handed neutrino masses as large as the GUT scale, in order to generate the neutrino masses one would then have to invoke the see saw mechanism [343, 435]³⁹. This also opens up naturally the possibility to realize baryogenesis via leptogenesis [320]. On the proton decay front, $SO(10)$ is also attractive as it contains a (gauged) $B - L$, the kernel of which is a Z_2 that automatically plays the role of R-parity. This requires however to break $SO(10)$ only with *safe* representations, such as $\mathbf{10}$, $\mathbf{45}$, $\mathbf{54}$, $\mathbf{120}$, $\mathbf{126}$, $\mathbf{210}$, ... but not $\mathbf{16}$, $\mathbf{144}$, $\mathbf{560}$, ... [437]. Finally in the doublet triplet front, the situation is improved compared to the $SU(5)$ case by employing the Dimopoulos-Wilczek scenario [438].

2. Symmetry breaking in SUSY GUT

The most simple superpotential to break a GUT symmetry is simply the generalization of the Higgs, one particular type of superpotential which can break G_{GUT} can be written as:

$$W = \lambda X (\text{Tr} \Sigma^2 - M_\Sigma^2) , \quad (154)$$

³⁹ The $SO(10)$ can be broken down to the MSSM via many routes, but the most popular one is through left-right symmetric group, $SO(10) \rightarrow SU(4)_c \times SU(2)_L \times SU(2)_R \rightarrow SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \rightarrow SU(3)_c \times SU(2)_L \times U(1)_Y$ (see for e.g. [436] and references therein). This can happen by a scalar belonging to $\mathbf{210}$ or a combination of the fields belonging to $\mathbf{45} + \mathbf{54}$. The next stage of breaking, i.e. left-right symmetry, can happen via the Higgs of 126 dimensional representation or $\mathbf{16}$ of $SO(10)$, see for a recent review [37].

where X is a GUT singlet. Such superpotentials play important role in inflationary cosmology, in the context of hybrid inflation at the GUT scale. In particular the most general Higgs sector (containing many scalar fields that could play constructive roles in inflation) can be constructed once the representations of all the fields present are known. For instance, $SO(10)$ can contain Higgs field in many representations, such as a fundamental $\mathbf{10}$ H_i , a four index $\mathbf{210}$ field Φ_{ijkl} , or a 3 index $\mathbf{120}$ field Ω_{ijk} . It is possible to construct a mass term, for example Φ^2 symbolizing, $\Phi_{ijkl}\Phi_{ijkl}$, but it is not possible to construct a scalar term (with all indices contracted) using three factors of H , or Ω ⁴⁰. For example, in the minimal $SO(10)$ [433], the field content which realize the breaking of $SO(10)$ are Φ_{ijkl} in $\mathbf{210}$, H_i in $\mathbf{210}$, Σ_{ijklm} and $\bar{\Sigma}_{ijklm}$ in $\mathbf{126}$ and $\overline{\mathbf{126}}$. The most general Higgs superpotential is then given by ⁴¹:

$$W_H = m_\Phi \Phi^2 + \lambda_\Phi \Phi^3 + m_H H^2 + m_\Sigma \Sigma \bar{\Sigma} + \eta \Phi \Sigma \bar{\Sigma} + \Phi H (\alpha \Sigma + \bar{\alpha} \bar{\Sigma}) \quad (155)$$

The possible symmetry breaking depends on the minima of the full superpotential, but only MSSM singlets inside each Higgs fields can take a non-vanishing VEV. For example, in the minimal $SO(10)$, there are three MSSM singlets inside Φ . The decomposition is given by: $\mathbf{210} = (\mathbf{15}, \mathbf{1}, \mathbf{1}) + (\mathbf{1}, \mathbf{1}, \mathbf{1}) + (\mathbf{15}, \mathbf{1}, \mathbf{3}) + (\mathbf{15}, \mathbf{3}, \mathbf{1}) + (\mathbf{6}, \mathbf{2}, \mathbf{2}) + (\mathbf{10}, \mathbf{2}, \mathbf{2}) + (\overline{\mathbf{10}}, \mathbf{2}, \mathbf{2})$ under the Pati-Salam subgroup $SU(4)_c \times SU(2)_L \times SU(2)_R$ [422]. The MSSM singlets are found in $(1, 1, 1)$, $(15, 1, 1)$ and $(15, 1, 3)$.

The minimal SUSY $SO(10)$ model contains 2 additional MSSM singlets in Σ and $\bar{\Sigma}$. Candidates for the inflaton should be searched for within these components. All the minima can be found by solving for the values of the singlet fields minimizing all the F -terms. The symmetries of these minima are given by the number of invariant generators of $SO(10)$.

⁴⁰ The same conclusion can be made by looking at whether the product of three $\mathbf{10}$ contains a singlet of $SO(10)$ or not.

⁴¹ The fermionic sector is also written in the form of a superpotential. The only Higgs coupling to the fermions generate Yukawa terms are in a representation \mathbf{R} such that $\mathbf{16} \times \mathbf{16} \times \mathbf{R}$ contains a singlet. In the case of a minimal $SO(10)$, this is the case only for H and $\bar{\Sigma}$ and the superpotential generating the fermion mass matrix: $W_F = \Psi_a (Y_H H + Y_\Sigma \bar{\Sigma}) \Psi_b$, where a, b are family indices and the Y are the symmetric Yukawa matrices. The most general fermionic sector contains only one additional term involving a $\mathbf{120}$ as $\mathbf{16} \times \mathbf{16} \times \mathbf{120}$. One can also enlarge the minimal $SO(10)$ model to have a more realistic fermionic mass matrices [434, 439, 440].

G. Symmetry breaking and topological defects

The formation of topological defects during symmetry breaking has been found by Kibble [441], and has rapidly gained popularity in the 80's and the 90's as it was realized that the cosmic strings - the line-like topological defects - formed at the GUT scale could generate temperature anisotropies at the level of 10^{-5} as observed by the COBE. They form during symmetry breaking, if generated via the Higgs mechanism, then their nature depend on the actual symmetry breaking and the fields involved. As presented earlier in this chapter, these symmetry breaking are assumed to have occurred in the SM of particle physics as well as many of its extensions: MSSM, (SUSY) GUTs, etc. Even string theories often give rise to a $4D$ effective theories that possesses symmetries that are larger than those of the SM, requiring then some symmetry breaking [39, 40]. In this section we will give a rapid presentation on some general properties of topological defects useful for inflation model builders, and we refer the reader to the more specialized literature for more details [441–448], for a review see [449].

1. Formation of cosmic defects during or after inflation in $4D$

As we will describe in the chapter IV, many semi-realistic models of inflation within regular $4D$ field theory consider the interaction between the inflaton ϕ and a second scalar field ψ that acquires during or at the end of inflation a non-vanishing VEV. As a consequence, depending on the precise model and inflation dynamics, it is frequent to break symmetries during or at the end of inflation, if the Higgs-type field ψ is charged under some symmetries. The hybrid inflation models is the most common class of such models (see Sec. IV).

Let us for a moment forget about the phase of inflation and illustrate the formation of topological defects. If ψ is non-trivially charged under some gauge symmetry G , a non-vanishing VEV of ψ will realize the symmetry breaking $G \rightarrow H$. The manifold of all the vacua accessible to ψ is given by the quotient group $\mathcal{M} = G/H$. For example, for the simplest abelian Higgs model, the symmetry breaking is $U(1) \rightarrow I$ and the manifold of vacua is $\mathcal{M} = U(1)$, corresponding to the circle of constant radius in the complex plane $|\phi| = \text{constant}$. What govern the formation and the type of topological defects are the topological properties of \mathcal{M} [441, 444]. The homotopy groups π_n of order n are the most

efficient way to study these properties. Each group $\pi_n(\mathcal{M})$ is composed of all classes of hypersurfaces of dimension n that can shrink to a point while staying inside \mathcal{M} [450]. If any hypersurface can shrink to a point, the homotopy group contains only one element and is said to be trivial. In particular, $\pi_0(\mathcal{M})$ is trivial if and only if \mathcal{M} is connected, $\pi_1(\mathcal{M})$ is trivial if and only if \mathcal{M} is simply connected. One can easily visualize that if \mathcal{M} is not connected (for example during the breaking of a discrete group $Z_n \rightarrow I$), uncorrelated regions of the universe will fall in different vacua and will necessarily be separated by domain walls [441, 442]. With the same reasoning, if the universe undergo a phase transition satisfying $\pi_1[G/H] \neq I$, *cosmic strings*, that is line-like defects will necessarily form, with a density given by the correlation length or the mass scale of the Higgs field responsible for the symmetry breaking. It is for example the case for $U(1) \rightarrow I$ or all symmetries of the type $G \rightarrow H \times Z_n$. In general, the formation of topological defects of space-time dimension d is governed by the non-triviality of the homotopy group:

$$\pi_{3-d} \neq I . \tag{156}$$

One can show that any symmetry breaking of the form $G \rightarrow H \times U(1)$ gives rise to the formation of monopole (point-like defects). This is the origin of the well-known monopole problem, since the Standard Model group contains a $U(1)$ factor. This formation of unwanted defects was one of the original motivation to introduce a phase of inflation [3].

Note that the above topological conditions of formation of defects only govern the formation of topologically stable defects. It was however found that defects solutions can form even when the topology is trivial [443, 445]. The most well-known example are the electroweak strings, formed during the electroweak symmetry breaking which are perturbatively stable for a range of parameters which are not realized in nature, and belong to the broader class of embedded defects.

These defects are a priori unstable though mechanisms (such as plasma effects) have been found to stabilize them. They are of interest for inflation model builders since this mechanism can allow lift the constraints from the formation of cosmic strings (see Sec. IV F on D -term inflation)

2. Formation of cosmic (super)strings after brane inflation

Recently, a new class of models of inflation was proposed, mimicking hybrid inflation within extra-dimensional theories (see VIII B). This model like D -term hybrid inflation produces cosmic string-like objects called F - and D -strings [451–453]. The nature of these objects are distinct from regular cosmic strings, as F -strings are Fundamental strings of cosmic size and D -strings are D -brane of spatial dimension 1. Fundamental strings are expected to have a Planckian size and therefore a Planckian mass-per-unit length $\mu \sim M_{\text{Pl}}^2$, leading to $G\mu \sim 1$ that is incompatible with observation. However in the context of the recently proposed large extra dimension, the fundamental Planck mass can be reduced by large warp factors and these object can be formed with a cosmic size. In fact the range of mass-per-unit length in Planckian unit for these objects is: $10^{-13} < G\mu < 10^{-6}$.

The D -strings can be formed at the end of brane inflation when a brane collide another brane of different dimension or an anti-brane, giving rise to the production of Dp -branes, with p dimensions, of which 1 is in the non-compact dimensions. This mechanism is considered as a generalization of the production of regular cosmic strings at the end of D -term inflation in $N = 1$ SUGRA [454]. The energy per unit length (or the tension) of a $D1$ -brane is given by $\mu = M_s^2/(2\pi g_s)$, where M_s is the string scale and g_s the string coupling. But for $g_s \gtrsim 1$ this can give rise to too large of a tension, considering the CMB bounds (see below). Therefore some $D(p - 2)$ -branes are assumed with $(p - 3)$ dimensions compactified to the volume V_c . Then the string tension reads the generic value [451]

$$\mu = \frac{M_s^{p-1} V_c}{(2\pi)^{p-2} g_s} = \frac{M_s^2}{4\alpha\pi} \simeq 2M_s^2, \quad (157)$$

if the gauge coupling $\alpha \simeq 1/25$. The CMB constraint, $\mathcal{P}_\zeta \sim 10^{-10}$, requires from brane inflation that $M_s \sim 10^{15}$ GeV leading to $G\mu \simeq 10^{-7}$.

3. Cosmological consequences of (topological) defects

Let's turn to the consequences of the formation of topological defects for observations, keeping in mind that a phase of inflation has to take place at some energy to explain the most recent CMB observations and solve the horizon problem. For reviews on defect evolution, see [444, 446–449]. Domain walls, that is topological defects of space-time dimension 3, are cosmologically disastrous as they evolve following $\rho_{\text{DW}} \propto t^{-1}$ and would dominate the

energy density of the universe unless they form at an energy lower than ~ 100 MeV. They cannot even have formed at an energy higher than ~ 1 MeV without producing temperature fluctuations larger than $\delta T/T \gtrsim 10^{-5}$ in the CMB [455], and without being in contradiction with CMB observations ⁴².

Point like topological defect, called (magnetic) monopoles [456, 457], with a mass of order of the GUT scale are expected to form during the early phase transition from G_{GUT} down to $SU(3) \times SU(2) \times U(1)$ if G_{GUT} is assumed simple. It was argued that unless their abundance is $n_M/s > 10^{-10}$ at the time of phase transition, their abundance can not be diluted in an adiabatic expansion of the universe [458], these ideas ultimately propelled the birth of inflation in order to dilute them.

On the other hand cosmic strings do not suffer from these problems, as a network of cosmic strings can inter commute ⁴³ and lead to the formation of closed loops from long strings. The loops oscillate due to their tension and decay into gravitational (and potentially particle) radiation, leading to a global scaling regime: their relative energy density evolve as, $\rho_{CS} \propto t^{-1}$, and do not dominate the universe (see [459] for the analytical proof and for example [460–462] for numerical confirmations, see also for more recent discussion [463, 464]. Their observational signatures are numerous (see [465] for a recent discussion); contribution to the CMB temperature fluctuations via the Kaiser-Stebbins effect [466], gravitational lensing [467, 468], generation of gravity wave background [469–472]. The amplitude of these effects are all mainly related to one property - their tension T . If the strings do not carry currents, this tension is equal to their energy per unit length in Planck units:

$$G\mu = 2\pi\epsilon(\beta)v^2/M_{\text{Pl}}^2. \quad (158)$$

where v is the VEV of the Higgs far away from the string. In the case of non-SUSY strings, or if the strings saturate the Bogomolnyi bound (they are then called “BPS strings” for Bogomolnyi-Prasad-Sommerfeld), $\epsilon(\beta) = 1$. In the context of SUSY/SUGRA, this function becomes dependent on the ratio of the Higgs to the gauge boson mass, $\beta \equiv m_\phi/m_A$. For

⁴² The defects in higher dimensions can give rise to inflation, see Sec. VIII B.

⁴³ The probability of inter-commutation, p , that is an exchange of partners when two strings intersect of one another is very close to $p = 1$ for cosmic strings from $4D$ field theory. This probability become much smaller when considering cosmic superstrings appearing from brane inflation. This is the main origin of the differences in the observational signature between cosmic strings and cosmic (super)strings.

non-BPS strings, the function $\epsilon(\beta)$ is given in [473]. This is the case for F - and D -strings originating from brane inflation.

Currently the constraints on their effects on CMB is among the most stringent observation and is directly relevant to inflationary physics when they are formed close to or at the end of inflation. In this case, their formation affects the normalization of the fluctuation power spectrum by imposing an additional contribution. This can be described by an additional contribution to the temperature quadrupole anisotropy [72, 474–476]

$$\frac{\delta T}{T}\Big|_Q^2 = \frac{\delta T}{T}\Big|_{\text{infl}}^2 + \frac{\delta T}{T}\Big|_{\text{CS}}^2, \quad (159)$$

where $(\delta T/T)_{\text{CS}} = y2\pi\epsilon(\beta)v^2/M_{\text{p}}^2$. The constant y parametrizes the density of the string network at the last scattering surface and has to be extracted from numerical simulations. The most recent simulations computing this parameter predicts $y = 8.9 \pm 2.7$ [477], though older simulations or semi-analytical calculations give $y \in [3 - 6]$ (see for e.g. [478] and references therein). The current constraints from this normalization (for example in the case of their formation at the end of F -term inflation) lead to [476, 478]:

$$v \lesssim 2 \times 10^{15} \text{ GeV}, \quad G\mu \lesssim 8 \times 10^{-7}, \quad (160)$$

In more recent studies, the effect of a cosmic string network on the CMB anisotropies have been computed at $\ell = 10$ instead of the quadrupole $\ell = 2$ [479] as the latter is polluted by a large cosmic variance error. These simulations are improved compared to previous analysis since cosmic strings are described using field theory instead of modeled. Analyzing of the presence and the impact of cosmic strings in the CMB data assuming a model of inflation can therefore be made fully consistently using Monte-Carlo methods [258]. At $\ell = 10$, a fraction $f_{10} < 0.11$ of the temperature anisotropies from cosmic strings are found compatible with the 3 years WMAP data (at 2σ), assuming a 7-parameter Λ CDM model and it is shown that the fraction of cosmic strings f_{10} is strongly degenerate with the spectral index (see Sec. II E). This constraint can be translated into $G\mu < 7 \times 10^{-7}$.

Other CMB searches, such as direct searches in spatial map for line discontinuity gives a bound on $G\mu \lesssim 3 \times 10^{-7}$ [480]. To search for cosmic string signals in future CMB data, one should also incorporate recent developments on the small-scale signal in temperature anisotropies [481, 482], on the CMB B-mode polarization signal [259–261] or from the generation of non-Gaussianities [483, 484]. Recent work have also investigated the cosmological

evolution and CMB signatures of meta-stable semi-local cosmic strings [485] as their formation is very motivated from a particle physics point of view, especially at the end of F -term hybrid inflation [436] or D -term hybrid inflation (see Sec. IV F 4).

Other very stringent constraints on the presence of cosmic strings in the universe comes from the amplitude of gravity wave background arises from the timing of the millisecond pulsar, which gives [472]

$$G\mu < 10^{-8} - 10^{-9} .$$

Note that these constraints are more model-dependent than the CMB or lensing constraints.

IV. MODELS OF INFLATION

A detailed account on inflation model building can be found in many reviews [9, 10, 153, 486, 487]. In this chapter and the next, we will review some inflationary models⁴⁴ that are motivated or that originate from particle physics, and present their successes and their challenges. By no means this section is an exhaustive description of all the models, their number being huge and still growing gradually.

A. What is the inflaton ?

There are two classes of models of inflation, which have been discussed extensively in the literature. In the first one, the inflaton field belongs to some hidden sector (not charged under the SM), such models will have at least one SM *gauge singlet* component, whose couplings to other fields and mass are chosen to match the CMB observations. This section will review some of the important ones. Models involving *gauge invariant* inflatons, charged under the SM gauge group or its extensions will be discussed in Sec. V.

All the inflationary models are tested by the required amplitude of density perturbations for the observed large scale structures [11]. Therefore the predictions for the CMB

⁴⁴ The very first attempt to build an inflation model was made in [2], where one-loop quantum correction to the energy momentum tensor due to the space-time curvature were taken into account, resulting in terms of higher order in curvature invariants. Such corrections to the Einstein equation admit a de Sitter solution [488], which was presented in [2, 489]. Inflation in Einstein gravity with an additional R^2 term was considered in [168]. A similar situation arises in theories with a variable Planck mass, i.e. in scalar tensor theories [490].

fluctuations are the most important ones to judge the merits of the models, especially the spectral index, tensor to scalar ratio, and running of the spectral tilt with the help of slow-roll parameters ϵ , η , and ξ^2 , as defined in Sec. II A ⁴⁵. We will also discuss generation of non-Gaussianities and cosmic strings (see Sec. II E for more details about the current status of the cosmological data), in order to visualize how well a given model matches the observations. Although we might not be able to pin down *the* model(s) of inflation, but certainly we would be able to rule them out from observations. From particle physics point of view, there exists an important criteria for a successful inflation; which is to end inflation in the right vacuum where the SM baryons can be excited naturally after the end of inflation in order to have a successful baryogenesis and BBN.

B. Non-SUSY one-field models

The most general form for the potential of a gauge singlet scalar field ϕ contains an infinite number of terms,

$$V = V_0 + \sum_{\alpha=2}^{\infty} \frac{\lambda_{\alpha}}{M_{\text{P}}^{\alpha-4}} \phi^{\alpha} . \quad (161)$$

In $4d$, restricting to renormalizable terms allows to prevent all terms with $\alpha \geq 4$. Furthermore, assuming extra symmetries can ensure that certain neglected terms in this series are not generated at the loop level. Note that this is the case for the constant term V_0 also. The usual example of such a symmetry is the parity Z_2 , under which $\phi \rightarrow -\phi$, which allows to prevent all terms with α odd. Most phenomenological models of inflation proposed initially assume that one or two terms in Eq. (161) dominate over the others, though some do contain an infinite number of terms.

1. Large field models

a. Power-law chaotic inflation: The simplest inflation model by the number of free parameters is perhaps the chaotic inflation [4] with the potential dominated by only one of

⁴⁵ Review of some of these models using the Hubble-flow parameters of Eq. (41) has been performed in [202, 491].

the terms in the above series

$$V = \frac{\lambda_\alpha}{M_{\text{P}}^{\alpha-4}} \phi^\alpha, \quad (162)$$

with α a positive integer. The first two slow-roll parameters are given by

$$\epsilon = \frac{\alpha^2}{2} \frac{M_{\text{P}}^2}{\phi^2}, \quad \eta = \alpha(\alpha - 1) \frac{M_{\text{P}}^2}{\phi^2}. \quad (163)$$

Inflation ends when $\epsilon = 1$, reached for $\phi_e = \alpha M_{\text{P}}/\sqrt{2}$. The largest cosmological scale becomes super-Hubble when $\phi_Q = \sqrt{2N_Q\alpha}M_{\text{P}}$, which is super Planckian; this is the first challenge for this class of models (see discussion Sec. II F). The spectral index for the scalar and tensor to scalar ratio read:

$$n_s = 1 - \frac{2 + \alpha}{2N_Q + \alpha/2}, \quad r = \frac{4\alpha}{N_Q + \alpha/4}. \quad (164)$$

The amplitude of the density perturbations, if normalized at the COBE scale, yields to extremely small coupling constants; $\lambda_\alpha \ll 1$ (for e.g. $\lambda_4 \simeq 3.7 \times 10^{-14}$). The smallness of the coupling, $\lambda_\alpha/M_{\text{P}}^{\alpha-4}$, is often considered as an unnatural fine-tuning. Even when dimension full, for example if $\alpha = 2$, the generation (and the stability) of a mass scale, $\sqrt{\lambda_2}M_{\text{P}} \simeq 10^{13}$ GeV, is a challenge in theories beyond the SM, as they require unnatural cancellations. These class of models have an interesting behavior for initial conditions with a large phase space distribution where there exists a late attractor trajectory leading to an end of inflation when the slow-roll conditions are violated close to the Planck scale [4, 275, 278, 492].

Recently, these models also suffer from an observational challenge which comes from the prediction of a high tensor to scalar ratio that is under tension by the most recent CMB+BAO+SN data [12]. The cases $\alpha \geq 4$ lie outside the 2σ region, though the case $\alpha = 2$ is still in the 1σ region for $N_Q \gtrsim 60$ (see Sec. II E).

Note that the above mentioned monomial potential can be a good approximation to describe in a certain field range for various models of inflation proposed and motivated from particle physics; natural inflation when the inflaton is a pseudo-Goldstone boson [493], or the Landau-Ginzburg potential when the inflaton is a Higgs-type field [85]. Some of these potentials will be discussed below. Chaotic inflation was also found to emerge from SUGRA theories [494–496] (see Sec. IV E 1) as well as in certain classes of brane-world models [497], where it has been claimed to give rise to a larger expansion rate due to modification in the Hubble equation rate. The necessity of super Planckian VEVs represents though a challenge to such embedding in particle physics.

Variants of these simple models have been constructed, based on the radiative corrections appearing at the loop level. First, the radiative corrections due to the inflaton self-interaction can induce a running of the mass of the inflaton with its VEV (see Sec. III B). For example for $\alpha = 2$, a “running mass” potential of the form $V(\phi) = m^2(1 + \alpha \ln(\phi/\mu))\phi^2$ has been considered in [498]. Though the dynamics is not significantly affected by this running, it was found to be able to affect the decay of the inflaton during reheating via fragmentation.

In [499], the chaotic inflation potential was extended to include a Yukawa coupling $(h/2)\phi\bar{N}_R N_R$ to the right-handed neutrino N_R . This introduces a one-loop correction to the inflation potential of the form $V_{1\text{-loop}} \simeq \kappa\phi^4 \ln(h\phi/\mu)$, where $\kappa = h^4/16\pi^2$ and μ is a renormalization scale. It was found that the radiative corrections can affect significantly the predictions of the chaotic models, namely by reducing the level of tensor-to-scalar ratio. For example, for the quartic potential, $h = 0$ gives rise to $n_s \simeq 0.95$ and $r \simeq 0.25$, whereas for $h \simeq 1.7 \times 10^{-3}$, $n_s \simeq 0.95$ and $r \simeq 0.084$ which is within the 1σ contour of the WMAP data. Note that these results are very sensitive to the value of h .

b. Exponential potential: An exponential potential also belongs to the large field models:

$$V(\phi) = V_0 \exp\left(-\sqrt{\frac{2}{p}} \frac{\phi}{M_{\text{P}}}\right). \quad (165)$$

It would give rise to a power law expansion $a(t) \propto t^p$, so that inflation occurs when $p > 1$. The case $p = 2$ corresponds to the exactly de Sitter evolution and a never ending accelerated expansion. Even for $p \neq 2$, violation of slow-roll never takes place, since $\epsilon(\phi) = 1/p$ and inflation has to be ended by a phase transition or gravitational production of particles [10, 500].

The confrontation to the CMB data yields: $n_s = 1 - 2/p$ and $r = 16/p$; the model predicts a high tensor to scalar ratio and it is within the one sigma contour-plot of WMAP (with non-negligible r) for $p \in [73 - 133]$. Multiple exponentials with differing slopes give rise to what has been dubbed as *assisted inflation* [152]. Such potentials might arise in string theories and theories with extra dimensions.

c. A combination of exponential and power law potential: Another form of potential has been found emerging from SUGRA [501–505], which is given by:

$$V(\phi) = V_0 \exp\left(\frac{\phi^2}{2M_{\text{P}}^2}\right) \left[1 - \frac{\phi^2}{2M_{\text{P}}^2} + \frac{\phi^4}{2M_{\text{P}}^4}\right], \quad (166)$$

This potential can emerge from hybrid inflation when SUGRA corrections dominate over radiative corrections, that is in the small coupling limit. In this class of models, the spectral tilt tends to exceed unity $n_s > 1$ depending on the number of e-foldings of inflation [503, 504]. It is therefore disfavored at 2σ at least, unless cosmic strings are formed at the end of inflation (see Sec. IV C 1 on hybrid inflation and Sec. III G on topological defects).

2. Small field models

Contrary to the models of the previous section, the small field models⁴⁶ take place for VEVs much smaller than the Planck scale, which is their main motivation. Their potential is of the form

$$V(\phi) = V_0 \left[1 + f\left(\frac{\phi}{\mu}\right)\right], \quad (167)$$

with the potential dominated by the constant V_0 ($\phi/\mu \ll 1$). Below we discuss some of the variants.

The most studied potential of this form is the effective hybrid model, based on

$$V(\phi) = M^4 \left[1 + \left(\frac{\phi}{\mu}\right)^p\right], \quad (168)$$

since, $p = 2$ corresponds to the effective potential for the two field hybrid model (see Sec. IV C). In that case, a second field triggers the end of inflation by fast-rolling due to a waterfall at a critical value $\phi = \phi_c$. In the large field limit $\phi \gg \mu$, one recovers the large field potential of Eq. (162). The slow-roll parameters read

$$\epsilon(\phi) = \frac{M_{\text{P}}^2 p^2 (\phi^2/\mu^2)^{p-1}}{2\mu^2 [1 + (\phi/\mu)^p]^2}, \quad \eta(\phi) = \frac{M_{\text{P}}^2 p(p-1)(\phi/\mu)^{p-2}}{\mu^2 [1 + (\phi/\mu)^p]}. \quad (169)$$

The parameter $\epsilon(\phi)$ is small both for $\phi \gg \mu$ and $\phi \rightarrow 0$, which identifies two phases of slow-roll inflation at large field and at small field. The spectral index and the ratio tensor

⁴⁶ Sometimes they are also known by the name of *modular inflation* [10, 487, 506]

to scalar in the slow-roll approximations read

$$\begin{aligned}
n_s(\phi) - 1 &= -\frac{p M_{\text{P}}^2}{\mu^2} \left(\frac{\phi}{\mu}\right)^{p-2} \frac{2 - 2p + (2+p) \left(\frac{\phi}{\mu}\right)^p}{\left[1 + \left(\frac{\phi}{\mu}\right)^p\right]^2}, \\
r &= \frac{8p^2 M_{\text{P}}^2}{\mu^2} \left(\frac{\phi}{\mu}\right)^{2p-2} \frac{1}{\left[1 + \left(\frac{\phi}{\mu}\right)^p\right]^2}.
\end{aligned}
\tag{170}$$

Let us first discuss the small field limit. For $p > 1$ and $p \neq 2$, the model is considered slightly disfavored by the recent data, since $n_s \gtrsim 1$ with negligible tensor to scalar ratio, and running of the spectral index is predicted ($r \propto (\phi/\mu)^{2p}$, $\alpha_s \propto (\phi/\mu)^{2p}$). It was pointed out however that to confront this model with CMB data, one could include the concordance model parameters and a weight for cosmic strings, since this model in its two-field version can produce cosmic strings. In that case, $n_s \sim 1$ is in agreement with observations at 1σ [258]. In the case $p = 2$, η is constant and n_s is predicted to be:

$$n_s(\phi) - 1 \simeq 2\eta \simeq \frac{4M_{\text{P}}^2}{\mu^2}.
\tag{171}$$

which is well above unity unless $\mu \gg M_{\text{P}}$. This is equivalent to having an extreme fine-tuning on a coupling constant, which is considered unnatural. The model is therefore disfavored at more than 2σ if the data are analyzed with the minimal 6-parameter model. However, if $\mu \gtrsim 7M_{\text{P}}$ then it predicts a spectral index around $1 < n_s \lesssim 1.1$, which is still in agreement with current observations if cosmic strings have a non-negligible contribution to the CMB anisotropies (see [258] and Sec. III G).

The case $p = 2$ has been studied away from the small field regime and/or without the slow-roll approximations [201, 501, 507]. It was shown that the model is in agreement with the CMB data if inflation is realized in the large field regime. This can be achieved by some mechanism (such as a waterfall triggered by an external field) independently of the parameter μ [201, 501, 507] or when $\mu \lesssim 0.32M_{\text{P}}$ [201]. In the last case, the Hubble-flow parameter ϵ_1 differs from ϵ at small VEVs and violation of slow-roll is responsible for forcing inflation to take place in the large field model without any condition on a waterfall parameter [201]. In both cases, inflation is necessarily realized for inflaton VEVs large compared to μ , which reduces the appeal of the model.

An inverted hybrid (or hiltop) inflation model has also been proposed where the curvature

of the slope is negative by construction, in order to predict a red spectrum, [508]

$$V(\phi) = M^4 \left[1 - \left(\frac{\phi}{\mu} \right)^p \right]. \quad (172)$$

This model emerges naturally when studying natural inflation [493], modular inflation [506] in the small field regime or from generic SUGRA theories, see [10, 509, 510]. Again the model differs from the chaotic limit if $\phi \ll \mu$, which is therefore the limit of interest. Another motivation to assume that the form of the potential is only valid at small VEVs is the fact the potential is not bounded from below; at large VEV, it must therefore be compensated by other terms for field theory to be well defined. In this limit, η is negative and so is $n_s - 1$, since $\epsilon \ll |\eta|$ if $p > 1$. However, n_s is very close to unity for $p \neq 2$ or well below for $p = 2$ and therefore $p = 2$ is incompatible with data [10]. Note that in the case $p = 2$, the limit of small fields could be abandoned and super-Planckian VEVs would then be necessary to obtain CMB predictions in agreement with the WMAP5 data. But in addition to the problem of super-Planckian VEV, inflation is then realized in a sector where the potential is not trustable because not bounded from below. As an example of the completion of an inverted hybrid model, in Ref. [511] authors have analyzed the potential:

$$V = V_0 - \frac{1}{2}m^2\phi^2 + \lambda\phi^4, \quad (173)$$

where the amplitude of the CMB predictions can be matched, with almost a flat spectral tilt. In this model ϵ remains negligible and $n_s - 1 \approx 2|\eta|$.

The running of one (or more) parameters of the general scalar potential can also be the origin of the function $f(\phi)$ in Eq. (167). The most common one, the “hybrid running mass” model is driven by a potential of the form [512–515]

$$V(\phi) = M^4 \left[1 + \frac{\eta_0\phi^2}{2M_{\text{P}}^2} \ln \left(\frac{\phi}{\phi_*} - \frac{1}{2} \right) \right]. \quad (174)$$

A reasonable fit to the CMB data has been found in Ref. [516], with a significant running of the spectral tilt for $\eta_0 < 0$, and ϕ_* the scale of the RG flow. The current WMAP data actually disfavors running of the spectral tilt and therefore these models are now very well constrained. The validity of the running of a quartic coupling constant has also been proposed in Ref. [10].

We will see below that another form of inflationary potential can also emerge of type [517]:

$$V(\phi) = M^4 \left[1 - \left(\frac{\phi_*}{\phi} \right)^n \right], \quad (175)$$

only valid in the large field limit $\phi \gg \phi_*$, since in the small field limit, the potential is not bounded from below and should be completed.

C. Non-SUSY models involving several fields

1. Original hybrid inflation

The most studied multi-field inflation model is the hybrid inflation first discussed in Ref. [145] (and studied extensively in [501]) as a model that differs from chaotic inflation on two main properties; it ends inflation with a waterfall triggered by a Higgs (not necessarily the SM Higgs) field coupled to the inflaton and it does not necessarily require an extremely small coupling to account for the normalization of the power-spectrum. The model is based on the potential given by [145]

$$V(\phi, \psi) = \frac{1}{2}m^2\phi^2 + \frac{\lambda}{4}(\psi^2 - M^2)^2 + \frac{\lambda'}{2}\phi^2\psi^2, \quad (176)$$

where ϕ is the inflaton and ψ is the Higgs-type field. λ and λ' are two positive coupling constants, m and M are two mass parameters. It is the most general form (omitting a quartic term $\lambda''\phi^4$) of renormalizable potential satisfying the symmetries: $\psi \leftrightarrow -\psi$ and $\phi \leftrightarrow -\phi$. Inflation is assumed to be realized in the false-vacuum along the $\psi = 0$ valley and ends with a tachyonic instability for the Higgs-type field. The critical point of instability below which the potential develops non-vanishing minimum is at

$$\phi_c = M\sqrt{\frac{\lambda}{\lambda'}}. \quad (177)$$

The system then evolves toward its true minimum at $V = 0$, $\langle\phi\rangle = 0$, and $\langle\psi\rangle = \pm M$ ⁴⁷.

The inflationary valley, for $\langle\psi\rangle = 0$, is usually assumed to be where the last 60 e-foldings take place. This is supported by numerical and analytical simulations [201, 503, 525–527], where the fine-tuning of the initial conditions were discussed. In Ref. [201] it was found that when the initial VEV of the inflaton, ϕ , is sub-Planckian, a subdominant but non-negligible part of the initial conditions for the phase space leads to a successful inflation, i.e. around less than 15% depending on the model parameters. Initial conditions with super-Planckian

⁴⁷ The hybrid inflation models were also considered in Refs. [158, 159, 309, 310, 518–523] in the context of large extra dimensions at TeV scale [524].

VEVs also lead to automatically successful inflation similarly to chaotic inflation. In the inflationary valley, $\langle\psi\rangle = 0$, the effective potential is given by:

$$V_{\text{eff}}(\phi) \simeq \frac{\lambda M^4}{4} + \frac{1}{2}m^2\phi^2, \quad (178)$$

whose phenomenology has been studied around Eq. (168), using $p = 2$, $V_0 = \lambda M^4/4$, and $V_0/M_{\text{P}}^2 = m^2/2$. Therefore, the predictions are disfavored by the data because of a blue tilt in the spectrum, i.e. $n_s > 1$, in the small field regime, $\phi_Q < M_{\text{P}}$. Note however that if its end is accompanied by the formation of cosmic strings, a slightly blue spectrum is found in agreement with the data, if these cosmic strings contribute to the CMB anisotropies around 10% [258]. It was also suggested in [499] that loop corrections to the hybrid tree level potential due to a Yukawa coupling to the right handed neutrino can render the spectral index of the model below 1, like for the chaotic model. This was confirmed recently in [528], where a red spectral tilt was found, even in the small field regime. These Yukawa couplings were also found suitable for a successful reheating phase and the generation of lepton/baryon asymmetry after inflation.

Note that a more realistic version of the model would include a quartic term for ϕ allowed by symmetries and generated by the Feynman diagrams involving loops of ψ fields. If this term dominates over the quadratic term in the inflationary valley, it was found that the spectral index would then be very close to unity [201]. Note that the coupling to a Higgs-type waterfall field has potentially important cosmological consequences [529, 530], as topological defects generically form during this symmetry breaking *after* inflation. This will be discussed in Sec. IV E 2.

2. Mutated and smooth hybrid inflation

Two variations of the hybrid inflation idea were proposed soon after the original model, both assuming that the term ϕ^2 is negligible.

The two-field scalar potentials are of the form:

$$V_{pq}(\phi, \psi) = M^4 \left[1 - \left(\frac{\psi}{m} \right)^p \right]^2 + \lambda \phi^2 \psi^q. \quad (179)$$

They share the common feature of having an inflationary trajectory during which $\langle\psi\rangle$ is varying and not vanishing. They also both reduce, in the one-field approximation to the form of Eq. (175).

The **mutated hybrid inflation** is one of them with a potential of the form of Eq. (179) with $(p, q) = (1, 2)$ [517]

$$V^{\text{mut}}(\phi, \psi) = \frac{1}{2}m^2(\psi - M)^2 + \frac{\lambda}{4}\phi^2\psi^2 . \quad (180)$$

Inflation in this case always happens for sub-Planckian VEVs. If one assumes chaotic initial values $\phi \gg \psi$, like for hybrid inflation, the potential is minimized at $\psi = 0$. While ϕ slow-rolls to smaller values, ψ settles in the local minimum satisfying, $\psi = M\alpha(\phi)/[1 + \alpha(\phi)]$, with $\alpha(\phi) = 2m^2/(\lambda\phi^2)$. At large ϕ , $\alpha \ll 1$, and its effective potential is of the form of Eq. (175), with $n = 2$:

$$V_{\text{eff}}^{\text{mut}} \simeq \frac{1}{2}m^2M^2 \left(1 - \frac{2m^2}{\lambda\phi^2} \right) + \mathcal{O}[\alpha^2(\phi)] , \quad (181)$$

while the kinetic terms, though modified, are close to minimal. In this approximation, the model predicts a red spectral index and negligible tensor to scalar ratio,

$$n_s - 1 \simeq -\frac{3}{8N_Q} \simeq 0.97 , \quad r \simeq \frac{3m}{2\lambda N_Q^{3/2}} \ll \frac{3}{8N_Q^2} \sim 10^{-4} , \quad (182)$$

if we assume $N_Q \simeq 60$. It is worth noting also that the model can emerge from a SUSY theory, e.g., from a superpotential of the form [517]

$$W = \Lambda^2 f(\Psi)\Sigma_1 + \lambda\Phi\Psi\Sigma_2 . \quad (183)$$

$f(\Psi)$ should be generated by non-perturbative process, such as gaugino condensation [30] in some hidden sector, in order to generate a Λ much smaller than the Planck scale. It also needs to satisfy $f(0) = 1$ and $f'(0) < 0$.

The **smooth hybrid inflation** model [62] also belongs to the similar class of model, with a potential of the form of Eq. (179) with $(p, q) = (4, 6)$. It therefore involves non-renormalizable terms of order M_{P}^{-2} . This model is also characterized by a ϕ -dependent minimum for ψ and, therefore a realization of inflation along a multi-field trajectory. The motivation presented in [62] is to avoid the formation of topological defects since the symmetry breaking occurs *during* inflation when all topological defects are inflated away. This is necessary when the symmetry breaking gives rise to monopoles or domain walls. Along the inflationary trajectory, the effective one-field potential is of the form of Eq. (175), with $n = 4$. The end of slow-roll inflation in this model is necessarily triggered by a violation of

the conditions; $\epsilon, \eta \ll 1$, since no waterfall transition takes place. This allows the predictions for the spectral index to be [62]

$$n_s - 1 \simeq -\frac{5}{3N_Q} \simeq 0.97, \quad (184)$$

and the ratio for tensor to scalar is found to be negligible. This model can also emerge from a SUSY framework, which helps protect the form of the potential, as discussed in Sec. IV E 7. The embedding of the model in particle physics and its predictions for reheating and leptogenesis will be discussed in that section too ⁴⁸.

3. Shifted and other variants of hybrid inflation

The **shifted hybrid inflation** model [66] shares also the features of shifting the VEV away from $\langle \psi \rangle = 0$ in the inflationary valley. Like the smooth hybrid inflation, the introduction of non-renormalizable terms in V is employed to shift the inflationary valley:

$$V^{\text{shift}}(\phi, \psi) = M^4 \left[\left(1 - \tilde{\psi}^2 + \xi \tilde{\psi}^4\right)^2 + \tilde{\phi}^2 \tilde{\psi}^2 \left(1 - 2\xi \tilde{\psi}^2\right)^2 \right]. \quad (185)$$

The parameter ξ controls the existence, locations, and number of valleys where inflation can occur. The model was proposed in the context of SUSY GUTs and will be studied in details in Sec. IV E 7. Let us for now only mention that the predicted spectral index lies within: $n_s \in [0.89, 0.99]$, and the tensor to scalar ratio $r \lesssim 10^{-5}$ can be in agreement with the observations for certain values of ξ .

The **inverted hybrid inflation** has also been proposed in a 2-field version [531]

$$V(\phi, \psi) = V_0 - \frac{1}{2}m_\phi^2\phi^2 + \frac{1}{2}m_\psi^2\psi^2 - \frac{\lambda}{2}\psi^2\phi^2 + \dots \quad (186)$$

Clearly this potential is not bounded from below at large VEVs, and the model is therefore only complete once non-renormalizable terms (contained in the dots) are assumed. Another modified version of hybrid inflation, the **complex hybrid inflation** has been proposed recently by [532]. In this model, the potential is not invariant under the change of phase

⁴⁸ Reheating in presence of a SM gauge singlet within SUSY is quite different, for the discussion see Sec. VI C 2. In many *gauge singlet* SUSY models of inflation, the role of MSSM squarks and sleptons are not taken into account appropriately. This also affects leptogenesis and in general thermal history of the universe [122].

of the waterfall field, assumed complex. This modification was proposed as a new way to generate a baryonic asymmetry.

Finally, the **thermal hybrid inflation** model has also been considered [157]. In this model, thermal corrections are assumed to generate part of the hybrid potential ⁴⁹

$$V(\phi, \psi) = V_0 + T^4 + T^2\psi^2 - \frac{1}{2}m_\psi^2\psi^2 + T^2\phi^2 . \quad (187)$$

However, as it is well known, the rapid expansion due to inflation dilute the content of the universe, exponentially rapidly driving the temperature to 0. This model therefore requires that a period of hot big-bang evolution takes place immediately before the phase of inflation. This condition can be considered rather fine tuned. In addition, inflation starts with $T \sim V_0^{1/4}$ and the temperature triggers the waterfall transition taking place at $T \sim m_\psi$. With $T \sim 1/a$ during inflation, inflation will last only 10 e-foldings and to be acceptable, this mechanism should be invoked many times.

Another way temperature effects could influence inflation has been proposed in a number of recent articles (see [533, 534]), known as **thermal inflation**. This mechanism is based on the assumption that the couplings between the inflaton field and other particles (independently required for a successful reheating) can generate a constant decay of the inflaton *during* inflation (assuming $\Gamma \sim H_{\text{inf}}$). This particle production would induce a thermal bath with a finite temperature that back-reacts on the inflaton dynamics and induces finite temperature effects on the potential. In particular this mechanism introduces a new viscosity term $C_w \dot{\phi}$ in the field dynamics Eq. (2), which slows down the rolling of the inflaton ⁵⁰. This idea was used in a number of articles to realize inflation with potentials that would be too steep for inflation without temperature effects, for example in string theory (see for example [535]). This possibility is however ad-hoc and still debated and some authors [99, 536] argue that these effects are unlikely to take place, as the viscosity term is expected to be

⁴⁹ In order to estimate the coefficients of thermal corrections one has to understand the exact particle contents. However note that ϕ belongs to a hidden sector, therefore exact particle contents are model dependent and sometimes chosen just to meet the desired results.

⁵⁰ To our knowledge the viscosity term has never been introduced consistently in the equations of motion in an expanding background. Note that the decay term usually introduced phenomenologically during the inflaton oscillations is valid *only* when the frequency of the inflaton oscillations is larger than the Hubble expansion rate. Therefore one can safely use the decay of the inflaton as in the case of a flat space-time [99].

negligible during slow-roll inflation. They argue that this mechanism could only be considered as a phenomenological idea that still lacks some theoretical support and an explicit regime where this can be realized.

4. Assisted inflation

There could be many more light fields during inflation, they could collectively assist inflation by increasing the effective friction term for all the individual fields [152, 280, 281, 537–540]. This idea can be illustrated with the help of ‘ m ’ identical scalar fields with an exponential potentials [152]:

$$V(\phi_i) = V_0 \exp\left(-\sqrt{\frac{2}{p}} \frac{\phi_i}{M_{\text{P}}}\right). \quad (188)$$

For a particular solution; where all the scalar fields are equal: $\phi_1 = \phi_2 = \dots = \phi_m$.

$$H^2 = \frac{1}{3M_{\text{P}}^2} m \left[V(\phi_1) + \frac{1}{2} \dot{\phi}_1^2 \right]; \quad (189)$$

$$\ddot{\phi}_1 = -3H\dot{\phi}_1 - \frac{dV(\phi_1)}{d\phi_1}. \quad (190)$$

These can be mapped to the equations of a model with a single scalar field $\tilde{\phi}$ by the redefinitions

$$\tilde{\phi}_1^2 = m \phi_1^2 \quad ; \quad \tilde{V} = m V \quad ; \quad \tilde{p} = mp, \quad (191)$$

so the expansion rate is $a \propto t^{\tilde{p}}$, provided that $\tilde{p} > 1/3$. The expansion becomes quicker the more scalar fields there are. In particular, potentials with $p < 1$, which for a single field are unable to support inflation, can do so as long as there are enough scalar fields to make $mp > 1$.

In order to calculate the density perturbation produced in multi-scalar field models, we recall the results from [208]:

$$\mathcal{P}_{\mathcal{R}} = \left(\frac{H}{2\pi}\right)^2 \frac{\partial N}{\partial \phi_i} \frac{\partial N}{\partial \phi_j} \delta_{ij}, \quad (192)$$

where $\mathcal{P}_{\mathcal{R}}$ is the spectrum of the curvature perturbation \mathcal{R} , N is the number of e -foldings of inflationary expansion remaining, and there is a summation over i and j . Since $N = -\int H dt$, we have

$$\sum_i \frac{\partial N}{\partial \phi_i} \dot{\phi}_i = -H, \quad (193)$$

where in our case each term in the sum is the same, yielding

$$\mathcal{P}_{\mathcal{R}} = \left(\frac{H}{2\pi}\right)^2 \frac{1}{m} \frac{H^2}{\dot{\phi}_1^2}. \quad (194)$$

Note that this last expression only contains one of the scalar fields, chosen arbitrarily to be ϕ_1 . This estimation for the spectral tilt is given by [208]:

$$n - 1 = 2 \frac{\dot{H}}{H^2} - 2 \frac{\frac{\partial N}{\partial \phi_i} \left(\frac{\dot{\phi}_i \dot{\phi}_j}{M_{\text{P}}^2 H^2} - \frac{M_{\text{P}}^2 V_{,i,j}}{V} \right) \frac{\partial N}{\partial \phi_j}}{\delta_{ij} \frac{\partial N}{\partial \phi_i} \frac{\partial N}{\partial \phi_j}}, \quad (195)$$

where there is a summation over repeated indices and the commas indicate derivatives with respect to the corresponding field component. Under our assumptions, the complicated second term on the right-hand side of the above equation cancels out, and Eq. (195) reduces to the simple form

$$1 - n = -2 \frac{\dot{H}}{H^2} = \frac{m_{\text{P1}}^2}{8\pi} \left[\frac{\frac{\partial V(\phi_1)}{\partial \phi_1}}{V(\phi_1)} \right]^2 = \frac{2}{mp}. \quad (196)$$

This result shows that the spectral index also matches that produced by a single scalar field with $\tilde{p} = mp$. The more scalar fields there are, the closer to scale-invariance is the spectrum that they produce. Note however that if the fields have such steep potentials as to be individually non-inflationary, $p < 1$, then many fields are needed before the spectrum is flat enough. The above calculation can be repeated for arbitrary slopes, p_i in Eq. (188). In which case the spectral tilt would have been given by $n = 1 - 2/\tilde{p}$, where $\tilde{p} = \sum p_i$. The above scenario has been generalized to study arbitrary exponential potentials with couplings, $V = \sum^n z_s \exp(\sum^m \alpha_{sj} \phi_j)$ in Ref. [537], see also [540]. Such potentials are expected to arise in dimensionally reduced SUGRA models [541].

One particular nice observation for m scalar potentials of chaotic type, $V \sim \sum_i f(\phi_i^n/M_{\text{P}}^{n-4})$ (for $n \geq 4$), is that inflation can now be driven at VEVs below the Planck scale [280, 281, 538, 539]⁵¹. The *effective* slow-roll parameters are given by: $\epsilon_{eff} = \epsilon/m \ll 1$ and $|\eta_{eff}| = |\eta|/m \ll 1$, where ϵ , η are the slow-roll parameters for the individual fields. Inflation can now occur for field VEVs [281]:

$$\frac{\Delta\phi}{M_{\text{P}}} \sim \left(\frac{600}{m}\right) \left(\frac{N_Q}{60}\right) \left(\frac{\epsilon_{eff}}{2}\right)^{1/2} \ll 1, \quad (197)$$

⁵¹ The double inflation model has been studied extensively with two such fields, $V = m_1^2 \phi_1^2 + m_2 \phi_2^2$, in Refs. [209, 212, 542–545]. In general one could expect: $V \sim \sum_i m_i^2 \phi_i^2$ [280, 538, 539], or $V \sim \lambda_i (\phi_i^n/M_{\text{P}}^{n-4})$, where $n \geq 4$ [281], where $\phi_i \ll M_{\text{P}}$.

where N_Q is the number of e-foldings. Obviously, all the properties of chaotic inflation can be preserved at VEVs much below the quantum gravity scale, including the prediction for the tensor to scalar ratio for the stochastic gravity waves, i.e. $r = 16\epsilon_{eff}$. For $\epsilon_{eff} \sim 0.01$ and $m \sim 100$, it is possible to realize a sub-Planckian inflation. The spectral tilt close to the flatness can be arranged in the above example $n_s - 1 = -6\epsilon_{eff} + 2\eta_{eff}$. Furthermore, realistic *assisted inflation* model can be realized with a better UV understanding in string theory with m complex structure axions arising in type IIB string theory [282, 546], Kaluza-Klein scalars [280, 538, 539], in multi-brane driven inflation [547–549], and in $SU(N)$ gauge theories [281]⁵².

However, the caveat for all these models discussed in Secs. IVC 1, IVC 2, IVC 3, IVC 4 is that the connection with the SM physics is still lacking. It is not clear whether the scalars (i.e. inflaton, waterfall fields, etc.) can carry the SM charges or not. A partial attempt has been made in the context of *assisted inflation* in Ref. [281] within $SU(N)$ gauge theories, where the inflatons are gauge invariant under SUSY $SU(N)$. Therefore, all these models bear similar uncertainties for reheating and thermalization as any other gauge singlet models of inflation, see the discussion in Sec. VIC 2.

5. Non-Gaussianities from multi-field models

With several light fields one would expect isocurvature perturbations [130, 210, 218, 551]. Isocurvature perturbations can also seed second order metric perturbations, which can yield large non-Gaussianities (see for example [22, 238, 552, 553]). The generation of non-Gaussianities in hybrid inflation has been studied in Refs [232, 553–558]. It was found that the regular hybrid inflation models do not produce large amount of non-Gaussianities, since inflation is *effectively* realized by the slow-roll of one field, while the fluctuations of the waterfall field are highly suppressed by its super-heavy mass. Some modifications of the model were proposed in Refs. [553, 554, 559], and then generalized to the “multi-brid inflation” scenarios [555, 556], where large non-Gaussianities can be generated. The model

⁵² Although it is quite plausible that conditions for late inflation can naturally be created after high scale assisted inflation, where one can have a possible signature for very long wavelength stochastic gravity waves [550].

is based on the coupling of several inflaton fields ϕ_i to a waterfall field χ [555]

$$V(\phi_i, \psi) = \frac{1}{2} \sum_i^n g_i^2 \phi_i^2 \chi^2 + \frac{\lambda}{4} \left(\chi^2 - \frac{\sigma^2}{\lambda} \right)^2. \quad (198)$$

In the two-brid model (case $n = 2$), the predictions are found very different from the original hybrid model. The value of f_{NL} is computed using the δN formalism in [555, 556, 560], and it is found to be [555]:

$$n_s = 1 - (m_1^2 + m_2^2), \quad r = \frac{8(m_1 g_1 \cos \gamma + m_2 g_2 \sin \gamma)^2}{g_1^2 \cos^2 \gamma + g_2^2 \sin^2 \gamma}, \quad f_{NL}^{\text{local}} = \frac{5g_1^2 g_2^2}{6\sigma(g_1^2 \cos^2 \gamma + g_2^2 \sin^2 \gamma)}, \quad (199)$$

where m_i are the masses of the components ϕ_i (in Planck units), and the VEVs at the end of inflation are parametrized by:

$$\phi_{1,f} = \frac{\sigma}{g_1} \cos \gamma, \quad \phi_{2,f} = \frac{\sigma}{g_2} \sin \gamma. \quad (200)$$

The presence of extra-parameters allows the model to predict large levels of non-Gaussianities. For example, for $m_1 \sim 0.005$, $m_2 \sim 0.035$, $\gamma \ll 1$ and $g_1 = g_2 \equiv g$, the model predicts $n_s \simeq 0.96$, $r \simeq 0.04$, and

$$f_{NL}^{\text{local}} \simeq \frac{5g m_2^2}{6m_1 \sigma} \sim 40 \frac{g}{\lambda^{1/4}}.$$

The stability of the model requires $\lambda^{1/4} \gg 10^{-3}$ which still allows $f_{NL}^{\text{local}} \ll 4 \times 10^4$. These results were generalized in [556] to more general potentials and for a larger parameter space.

A generalized expression for non-Gaussianity for multiple field case has been derived in [231, 244] from the δN formalism (see Sec. II C 4):

$$-\frac{6}{5} f_{NL} = \frac{r}{16} (1 + f) + \frac{\sum_{i,j} N_{,i} N_{,j} N_{,ij}}{(\sum_k N_{,k}^2)^2}, \quad (201)$$

where $N_{,i} \equiv \partial N / \partial \phi_i \approx V_i / V_i'$ by using the horizon crossing approximation [10, 244], r is the tensor to scalar ratio, and f is a geometrical factor relating to the triangular bispectrum, lying in the range, $0 \leq f \leq 5/6$ [20]. The value of r for $V = \sum_i \lambda_i \phi_i^\alpha$ is given by [247, 561]:

$$r \approx \frac{8M_{\text{P}}^2}{\sum_i (V_i / V_i')^2} \approx \frac{4\alpha}{N}, \quad (202)$$

where $N \approx M_{\text{P}}^{-2} \sum_i \int_{\phi_i}^{\phi_i^{\text{end}}} (V_i / V_i') d\phi_i \approx (\sum_i \phi_i^2 / 2\alpha M_{\text{P}}^2)$. With the help of the above expression, the value of f_{NL} can be given by [562]

$$\frac{-6}{5} f_{NL} \approx M_{\text{P}}^2 \left(\sum_j \frac{V_j^2}{V_j'^2} \right)^{-2} \sum_i \frac{V_i^2}{V_i'^2} \left(1 - \frac{V_i V_i''}{V_i'^2} \right) \approx \frac{\alpha M_{\text{P}}^2}{\sum_i \phi_i^2} \approx \frac{1}{2N} (2 + f) \approx \frac{r}{8\alpha} (2 + f). \quad (203)$$

Unfortunately, in these models the value of f_{NL} is very small and undetectable by the future experiments. The expression is also a generalization of [563], derived for the two field case.

Non-Gaussianities can also be created after inflation through tachyonic preheating at the end of inflation [240, 558, 564]. Preheating of light fields after inflation has also been considered in [238, 239, 241], and in δN formalism [242, 565]. It was found that the $\lambda\phi^4$ model can lead to levels of non-Gaussianities that are already on the verge of being excluded by current observations [238]. Also, very recently, it was pointed out that the waterfall part of the dynamics in a generic non-SUSY hybrid type inflation (or hilltop inflation) with a potential of the generic form

$$V(\phi, \psi) \sim V_0 + \frac{\eta V_0}{2M_{\text{P}}^2} \phi^2 - \lambda \phi \psi^2 + \mathcal{O}(\phi^3, \dots), \quad (204)$$

could generate a large amount of non-Gaussian fluctuations given by [566]⁵³.

$$\frac{f_{NL}}{1.3 \times 10^4} \sim \left(\frac{\gamma}{10^{-2}}\right)^{3/2} \left(\frac{V_e^{1/4}}{10^{-3} M_{\text{P}}}\right)^4 \left(\frac{10^{-2} M_{\text{P}}}{\lambda}\right) \left(\frac{10^{-2}}{\epsilon_e}\right)^{1/2}, \quad (205)$$

where the index e denotes the end of inflation and γ is a semi-analytic parameter in the range [0.03 – 0.1].

6. Challenges for non-SUSY models

For non-SUSY models of inflation there are two more challenges related at the classical and the quantum level.

- Effective couplings and symmetries:

At the classical level inflaton can couple to other light scalar fields during inflation. In any of the effective potentials considered so far, there is no symmetry argument which will not allow couplings of type; $\phi^2 \sum \chi_i^2$, or some non-renormalizable couplings to other fields (belonging to other hidden sectors) such as $\sim \phi^n \chi^m / M_{\text{P}}^{n+m-4}$ for $n, m > 2$.

⁵³ A word of caution when we use δN formalism to estimate non-Gaussianity during parametric resonance or during tachyonic preheating. Note that preheating is a violent and non-adiabatic process, which can happen at time scales much shorter than one Hubble time. Especially, during tachyonic preheating, the field displacement can be very negligible as compared to one Hubble time, during which the $\delta N \approx 0$, therefore in a separate universe approach, where the Hubble patches evolve homogeneously and smoothly, one can not trust the δN formalism.

There may be some discrete symmetries which will forbid some terms or some combinations, but it cannot render all the couplings to be vanishing. Any such light field other than the inflaton would introduce isocurvature perturbations during inflation, which at the classical level leaves such models vulnerable to quantum predictions of the CMB fluctuations. A simple calculation assuming adiabatic nature of density perturbations will not suffice in such cases.

Furthermore, the same couplings can dump almost *all the inflaton energy density* into some other non-SM degrees of freedom (or hidden sectors) upon reheating or preheating. One must assume that SM degrees of freedom are excited, but such assumptions are always hard to justify if the particle contents are unknown.

- Quantum stability of the potential:

The inflaton cannot be considered free from matter couplings, any coupling of the inflaton to fermions and gauge bosons would introduce loop corrections at the perturbative level [352]. This will spoil the classical flatness of the inflaton potential even when the scale of inflation is far below the scale of gravity [285, 347, 349]⁵⁴. Beyond destabilizing or modifying the shape of the potential, radiative corrections can substantially alter the CMB predictions of the models.

SUSY helps stabilizing the classical potential, as the leading quantum corrections are logarithmic in nature. The another classic example is the pseudo Nambu Goldstone Boson (pNGB) as an inflaton, where the potential explicitly breaks the global symmetry with small couplings [353, 493, 567, 568]. In Ref. [568], it was observed that if the global symmetry is explicitly broken by a combination of couplings, then *loop contributions to pNGB masses must involve all of the couplings, and therefore one-loop contribution cannot be quadratically divergent*. This is due to collective or non-local symmetry breaking as discussed in Ref. [569].

Let us follow the discussion of [568], where they begin by considering the simplest model which involves a pNGB θ which comes from the breaking of a global $SO(2)$

⁵⁴ However, it is interesting to note that the potential of Eq. (162) with $\alpha = 2$ or $\alpha = 4$ have some accidental stability with respect to loop corrections due to the absence of a self-coupling or a mass term respectively. This is not the case anymore if both the terms are present, if $\alpha = 3$, or in the case of a coupling with other fields, for example in hybrid inflation and its variants.

symmetry. After integrating out the “radial” degree of freedom and pushing the cutoff of this non-linear sigma model to the point where the interactions become strongly coupled, namely $\Lambda \sim 4\pi f$. The inflaton, Φ , can be parameterized as

$$\Phi = \begin{pmatrix} \cos(\theta/f) & \sin(\theta/f) \\ -\sin(\theta/f) & \cos(\theta/f) \end{pmatrix} \begin{pmatrix} -1 \\ 1 \end{pmatrix} \times \frac{f}{\sqrt{2}} \quad (206)$$

Let us consider the tree-level potential to be:

$$V = \lambda (\sigma^T \sigma - v^2)^2 + \frac{g_1}{4} (\sigma^T \Phi)^2 + \frac{g_2}{4} (\sigma^T \tau_1 \Phi)^2 \quad (207)$$

where $\sigma^T = (\sigma_1 \sigma_2)$ and τ_1 is the first Pauli matrix. Let us consider a simple situation when $g_1, g_2 = g \neq 0$. From expanding out the Φ s in the potential, one finds:

$$V = \frac{gf^2}{4} (\sigma_1^2 + \sigma_2^2 - 2\sigma_1\sigma_2 \cos(2\theta/f)) \quad (208)$$

Now computing the one-loop corrections to the mass of θ , the authors of Ref. [568] obtained that there is no one-loop quadratic divergent contribution to a θ mass. This is because θ only couples to the combination $\sigma_1\sigma_2$ making it impossible to close a loop with only one vertex. There is a logarithmic divergence at one loop proportional to $g_1g_2 = g^2$

$$\begin{aligned} V_{1-loop} &= \frac{g^2}{128\pi^2} \log\left(\frac{\Lambda^2}{m_\theta^2}\right) (\Phi^T \tau_1 \Phi)^2 + \dots \\ &= \frac{g^2 f^4}{128\pi^2} \log\left(\frac{\Lambda^2}{m_\theta^2}\right) \cos^2(2\theta/f) + \dots \end{aligned} \quad (209)$$

The value of Λ could be as large as M_P or below, but the corrections to the potential is only logarithmic dependent. The pNGB inflaton could also originate in SUSY inflation models and in extra dimensional models [568, 570, 571].

Some of the above mentioned challenges can be addressed if inflation is explicitly embedded within an observable sector. One such example of inflaton is the SM Higgs in a non-SUSY context.

D. SM Higgs as the inflaton

It is natural to study if the SM Higgs can play the role of the inflaton field. This question has been discussed long ago [572], and has regained interest in the last few years [85, 285, 573–576].

1. *Dynamics of the SM Higgs inflation*

It has been proposed long ago to improve this situation by abandoning the universal coupling to gravity [85, 572, 577, 578], in order to flatten the Higgs potential at high energies⁵⁵. The lagrangian is extended to contain a non-minimal coupling to gravity (only) for a Higgs field H

$$\mathcal{L} = \mathcal{L}_{\text{SM}} - \frac{M^2}{2}R - \xi H^\dagger H R . \quad (210)$$

This non-minimal coupling can be motivated by the renormalizability of the $\lambda\phi^4$ potential [591]. The very form of this Lagrangian might represent the first challenge of the model, since the equivalence principal is lost; all particles are not coupled in the same way to gravity.

The above Lagrangian has been studied in the past [572, 592], where H is a GUT Higgs field and was applied in [85] to the SM Higgs field. If h is the Higgs field in the unitary gauge, the resulting action for h , in the Jordan frame, reads

$$\mathcal{S}_J = \int d^4x \sqrt{-g} \left\{ \frac{M^2 + \xi h^2}{2} R + \frac{1}{2} \partial_\mu h \partial^\mu h - \frac{\lambda}{4} (h^2 - v^2)^2 \right\} . \quad (211)$$

Since the coupling to gravity is not minimal in S_J , studying the phenomenology of the model is simplified once the conformal transformation is applied [85, 572]

$$\begin{aligned} \Omega^2 &= 1 + \xi h^2 , & g_{\mu\nu} &\rightarrow \hat{g}_{\mu\nu} = \Omega^2 g_{\mu\nu} , \\ h &\rightarrow \chi \text{ with } \frac{d\chi}{dh} = \sqrt{\frac{\Omega^2 + 6\xi^2 h^2/M_{\text{P}}^2}{\Omega^4}} , \end{aligned} \quad (212)$$

such that the action in the Einstein frame reads

$$\mathcal{S}_E = \int d^4x \sqrt{-\hat{g}} \left\{ \frac{M_{\text{P}}^2}{2} \hat{R} + \frac{1}{2} \partial_\mu \chi \partial^\mu \chi - U(\chi) \right\} . \quad (213)$$

In order to keep canonical kinetic terms once the metric is redefined, the potential U for the new field χ now reads

$$U(\chi) = \frac{1}{\Omega(\chi)^4} \frac{\lambda}{4} [h^2(\chi) - v^2]^2 . \quad (214)$$

⁵⁵ Such attempts were previously made in connection with scalar tensor theories of inflation to flatten the potential [572, 579–590]. Typically the gravitational part of the action is given by: $S = \int d^4x \sqrt{-g} [\frac{1}{2} M^2 f(\phi) R - \frac{1}{2} \partial_\mu \phi \partial^\mu \phi]$. The action is dynamically equivalent to a theory in which the gravitational action is the usual one, via the conformal transformation: $\hat{g}_{\mu\nu} = f(\phi) g_{\mu\nu}$, where we use the bar to indicate a quantity in the new frame. The new action looks like: $S_E = \frac{1}{2} \int d^4x \sqrt{-\hat{g}} [M^2 \hat{R} - K(\phi) (\hat{\partial}\phi)^2]$, where, $K(\phi) \equiv \frac{2f(\phi) + 3M^2 f'(\phi)}{2f^2(\phi)}$.

At low energies, that is for small VEVs, $\Omega^2 \simeq 1$, $h \simeq \chi$ and the two frames are indistinguishable. At high energies, $h \propto \exp \chi$, and the potential tends to

$$U(\chi) \simeq \frac{\lambda M_{\text{P}}^4}{4\xi^2} \left[1 + \exp \left(\frac{-2\chi}{\sqrt{6}M_{\text{P}}} \right) \right]. \quad (215)$$

Once the potential is known at high energies, it is straightforward to compute the CMB predictions for the model. From Eq. (215), the slow-roll parameters read [85]

$$\epsilon(\chi) \simeq \frac{4M_{\text{P}}^4}{3\xi^2 h^4}, \quad \eta(\chi) \simeq -\frac{4M_{\text{P}}^2}{3\xi h^2}, \quad \zeta \simeq \frac{16M_{\text{P}}^4}{9\xi^2 h^4}, \quad (216)$$

which requires inflation to take place in the range $h \in [1.07 - 9.4]M_{\text{P}}/\sqrt{\xi}$. A correct normalization to the COBE data imposes $\xi \simeq 5 \times 10^4 \sqrt{\lambda}$, and the model predicts, at the classical level [85, 575],

$$\begin{aligned} n_s &\simeq 1 - 8 \frac{(4N_Q + 9)}{(4N_Q + 3)^2} \simeq 0.97, & r &\simeq \frac{192}{(4N_Q + 3)^2} \simeq 0.0033, \\ \alpha_s &\simeq -5.2 \times 10^{-4}, \end{aligned} \quad (217)$$

which is in agreement with the most recent WMAP data.

The main challenge to this model is to evaluate the quantum corrections to the inflationary potential at high energy to evaluate if the flatness of the inflationary potential can be destabilized by them. Both the quantum gravity corrections and the quantum corrections due to SM particles can be evaluated, though the full quantum gravity corrections cannot be rigorously computed. It has been argued in Ref. [85] that the model is safe since $U(\chi)/M_{\text{P}}^4 \lambda/\xi^2 \ll 1$ and $d^2U/d\chi^2 \ll M_{\text{P}}^2$.

The leading log method [85] as well as renormalization group (RG) methods have been implemented [574, 575] in both frames to compute corrections to V , and ξ . The corrections from SM particles to V using RG improved calculation with 2-loop beta-function are also found in Ref. [575] to be very small and do not spoil the model (see Fig. 3). However, they introduce a dependence of the CMB predictions on the SM particle masses, in particular on the Higgs mass m_h and the top quark mass m_t as detailed below.

The stability of the present classical action could also be affected by the presence of non-renormalizable operators. This represents another serious challenge to the model [285, 576], though this argument can be applied to other high scale models of inflation. Indeed, it is shown that the effective cutoff of the Lagrangian of Eq. (210) is M_{P}/ξ , whereas the energy scale of inflation is $M_{\text{P}}/\sqrt{\xi}$. One therefore should expect non-renormalizable contributions

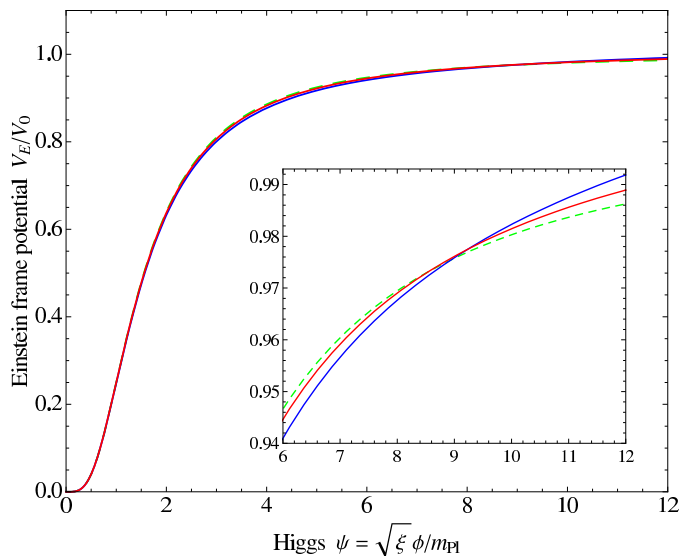


FIG. 3: Normalized scalar potential of SM inflation in the Einstein frame, in the classical approximation (green), or taking into account quantum corrections for $m_h = 126.5$ GeV and $m_h = 128$ GeV, in blue and red respectively. Figure is taken from [575].

to the action to be relevant at energies well below inflation. This represents a challenge to the model since the implicit assumption is that the SM of particle physics is the effective theory at least up to the inflationary scale. In the absence of symmetries to prevent the appearance of non-renormalizable contributions the model should be considered as fine-tuned, so that these contributions do not spoil the flatness of the model.

2. SM Higgs inflation and implications for collider experiments

As mentioned above, the important feature of this model of inflation is that masses of SM particles enter the quantum corrections to the scalar potential, and thus impact the CMB predictions n_s , r and α_s . In particular the most important masses that affect the predictions are the Higgs mass m_h as it enters the tree level potential, and then the top mass, the highest mass that enters in loops of SM particle. Independently of inflation, current accelerator experiments allow these masses to range in [312] $m_h \in [114.4 - 182]$ GeV and $m_t \in [169 - 173]$ GeV. The evolution of the CMB predictions with respect to these masses is given in details in [575]. We will here only mention that compared to the classical values (equivalent to large m_h), n_s , r and $|\alpha_s|$ increases when m_h decreases or when m_t increases. In particular, for the spectral index to be within the 1σ contour of WMAP

($n_s < 0.99$), the mass of the Higgs is found [575] to be larger than

$$m_h \gtrsim 125.7 + 3.8 \left(\frac{m_t - 171 \text{ GeV}}{2 \text{ GeV}} \right) - 1.4 \left(\frac{\alpha_{\text{SU}(3)}(M_Z) - 0.1176}{0.0020} \right) \pm 2 \text{ GeV} , \quad (218)$$

where $\alpha_{\text{SU}(3)}$ is the strong coupling and the $\pm 2 \text{ GeV}$ is due to theoretical uncertainties from higher order corrections.

Testing this model in the future will therefore require the discovery of the Higgs particle at the LHC, which is expected if its mass is within the above range, together with an improvement of the error bar on n_s as given by the PLANCK and some improvement on the top quark mass measurement at the LHC and the ILC. From phenomenological point of view SM Higgs inflation is a welcoming news, the Higgs can directly produce the SM degrees of freedom as shown in Refs. [123, 593].

E. SUSY models of inflation

Historically, SUSY inflation was first introduced to cure the flatness problem and associated fine tuning of new inflation [594, 595], but since then utilizing SUSY as a tool for inflation has gained in popularity. The fundamental reason is similar to the resolution of the hierarchy problem for which SUSY was introduced, i.e. to protect the flatness of the potential.

The scalar sector is now described by a superpotential W and a Kähler potential K , instead of just a scalar potential (see Sec. III C). Another difference with non-SUSY effective field theory concerns the range of VEVs allowed, which is now below the Planck scale. Close to the Planckian VEVs SUGRA corrections become important. One of course recovers the global SUSY in the limit when $M_{\text{P}} \rightarrow \infty$.

In four dimensions, the $N = 1$ SUSY potential receives two contributions; one from the F -terms, describing interaction between chiral superfields, and the second from the D -terms, which contains the gauge interactions. The scalar potential derived from W and K has to be non-vanishing to support inflation, therefore breaking (transiently) SUSY. Therefore two classes of models driven by F -terms or by D -terms have been proposed. We will discuss first old and new properties of F -and D -term hybrid inflation, which is by far the most popular amongst the model builders. Some of these models have also been reviewed in Refs. [10, 596].

1. Chaotic inflation in SUSY

Embedding chaotic inflation within SUSY is a challenging problem, discussed long ago and is still under investigation by many [486, 494–496, 597–599]. As discussed in Sec. IV B 1, chaotic inflation in its non-SUSY description requires super-Planckian VEV to last long enough. By construction SUSY and SUGRA are only valid when $\phi \ll M_{\text{P}}$. Another challenge comes from the fact that chaotic inflation requires a UV-complete theory that SUGRA does not provide [486].

Even discarding the question of validity, if inflation was driven by the F -terms, the exponential of the Kähler potential potential can generically spoil the flatness of the potential. To tackle this problem, non-minimal Kähler potentials with a logarithmic form have been proposed [494], which can be combined to a mass-term superpotential, $W = M\Phi^2$, to generate the right scalar potential, driven by F -terms. It has also been shown that a chaotic potential can also be embedded within D -terms [495, 496]. They assume 4 chiral superfields and that the symmetries of the theory are $U(1)_{\text{gauge}} \times U(1)_R$ ⁵⁶. A renormalizable superpotential and a minimal Kähler potential can generate a scalar potential possessing many F -flat valleys lifted by D -terms required for generating chaotic inflation. It was pointed out in Ref. [599] that the mechanism requires a fine-tuned moduli sector which is hard to embed within string theory. As a conclusion, the question of managing super-Planckian VEVs and densities within SUGRA does not make chaotic models appealing from particle physics point of view⁵⁷.

⁵⁶ A challenging problem for these kind of attempts is that the inflaton sector, though it resides in a hidden sector, must couple to the observable sector, i.e. MSSM. This modifies the D -flatness conditions for any extra $U(1)_{\text{gauge}}$ sector added to the SM gauge group, which opens up new D -flat directions and this will eventually modify the inflationary potential by lifting such combinations with the help of F -term. Such contributions are often ignored in the literature [495, 496, 599].

⁵⁷ There are attempts to embed natural inflation realizable at low scales within SUGRA, see [161, 350, 600–602]. The generic potential has a form: $V = \Lambda^4(1 + \beta|\phi|^2 + \gamma|\phi|^3 + \delta|\phi|^4 + \dots)$, where β , γ , δ are model dependent coefficients. Challenge for these models is to justify why inflation starts at $\phi \approx 0$ VEV and $\dot{\phi} \approx 0$. A prior phase of inflation may justify such initial conditions.

2. Hybrid inflation from F -terms

The most well-known model of SUSY inflation driven by F -terms is of the hybrid type and based on the superpotential [61]

$$W^F = \kappa S(\Phi\bar{\Phi} - M^2) . \quad (219)$$

where, S is an absolute gauge singlet while Φ and $\bar{\Phi}$ are two distinct superfields belonging to complex conjugate representation, and κ is an arbitrary constant fixed by the CMB observations⁵⁸. This form of potential is protected from additional destabilizing contributions with higher power of S , if S , Φ and $\bar{\Phi}$ carrying respectively the charges $+2$, α and $-\alpha$ under R -parity. Then W carries a charge $+2$ so that the action $\mathcal{S} = \int d^2\theta W + \dots$ is invariant.

The tree level scalar potential derived from Eq. (219) reads

$$V_{\text{tree}}(S, \phi, \bar{\phi}) = \kappa^2 |M^2 - \bar{\phi}\phi|^2 + \kappa^2 |S|^2 (|\phi|^2 + |\bar{\phi}|^2)^2 + D - \text{terms} , \quad (220)$$

where we have denoted by $S, \phi, \bar{\phi}$ the scalar components of $S, \Phi, \bar{\Phi}$. It has a form similar to the original hybrid inflation model, though $m = 0$, and both λ and λ' are replaced by only one coupling constant κ^2 . In what follows, it is assumed that $\phi^* = \bar{\phi}$ along this direction, the D -terms vanish and that the kinetic terms for the superfields are minimal, which is equivalent to a minimal Kähler potential, $K = |S|^2 + |\Phi|^2 + |\bar{\Phi}|^2$.

We can defined two effective real scalar fields canonically normalized, $\sigma \equiv \sqrt{2}\text{Re}(S)$, and $\psi \equiv 2\text{Re}(\Phi) = 2\text{Re}(\bar{\Phi})$. The two-field scalar potential can then be put to the form [596]

$$V_{\text{tree}}(\sigma, \psi) = \kappa^2 \left(M^2 - \frac{\psi^2}{4} \right)^2 + \frac{\kappa^2}{4} \sigma^2 \psi^2 . \quad (221)$$

Like in the non-SUSY version, the global minimum of the potential is located at $S = 0$, $\phi\bar{\phi} = M^2$, though at large VEVs, $S > S_c \equiv M$, the potential also possesses a local valley of minima (at $\langle\psi\rangle = 0$) in which the field σ , identified as the inflaton from now on, lies in a flat direction, $V_{\text{tree}} = V_0 \equiv \kappa^2 M^4$. This non-vanishing value of the potential both sustain the inflationary dynamics and induces a SUSY breaking. Chaotic initial conditions are usually assumed [61], which provide for a large inflaton VEV at the ‘‘beginning’’ of inflation (We

⁵⁸ It is desirable to obtain an effective singlet S superfield arising from a higher gauge theory such as GUT, however to our knowledge it has not been possible to implement this idea, see Sect. IV G 3. Typically S would have other (self)couplings which would effectively ruin the flatness.

will return later to the issue of initial conditions in this model). The end of inflation is then triggered by slow-roll violation and the system rapidly settles at the bottom of one of the global minima, breaking the symmetry G , potentially forming topological defects, and restoring SUSY.

Since, $V(\psi = 0) \neq 0$, SUSY is broken. This induces a splitting in the mass of the fermionic and bosonic components of the superfields Φ and $\bar{\Phi}$, with $m_B^2(S) = \kappa^2|S|^2 \pm \kappa^2 M^2$ and $m_F^2 = \kappa^2|S|^2$. Note that this description is valid only as long as S is sufficiently slow-rolling such that $\kappa^2|S|^2|\Phi|^2$ can be considered as a mass term. Therefore radiative corrections do not exactly cancel out [61, 67], and provide a one-loop potential:

$$V_{1\text{-loop}}^F(S) = \frac{\kappa^4 \mathcal{N} M^4}{32\pi^2} \left[2 \ln \frac{s^2 \kappa^2}{\Lambda^2} + (z+1)^2 \ln(1+z^{-1}) + (z-1)^2 \ln(1-z^{-1}) \right], \quad (222)$$

using the Coleman-Weinberg formula [352]. In this expression

$$z = \frac{|S|^2}{M^2} \equiv x^2, \quad (223)$$

Λ represents a non-physical energy scale of renormalization and \mathcal{N} denotes the dimensionality⁵⁹ of the Higgs fields Φ and $\bar{\Phi}$. When discussing the predictions of the model and the dynamics at the end of inflation, it is important to keep in mind that the perturbative approach of Coleman and Weinberg breaks down when close to the inflection point at $z \simeq 1$.

3. CMB predictions and constraints

The predictions of the model differ strongly from the original model, because the potential is concave down due to the radiative correction is the origin for the slope in the potential. For small coupling κ , the slow-roll conditions (for η) are violated infinitely close to the critical point, $z = 1$, which ends inflation. Thus, the quadrupole value for the inflaton, $z_Q \equiv S_Q^2/M^2$, is obtained by solving

$$N_Q = \frac{32\pi^3 M^2}{\kappa^2 \mathcal{N} M_P^2} \int_1^{z_Q} \frac{dz}{z f(z)}, \quad (224)$$

with $f(z) = (z+1) \ln(1+z^{-1}) + (z-1) \ln(1-z^{-1})$.

⁵⁹ To be very precise, the value of \mathcal{N} is less or equal to the dimensionality of Φ or $\bar{\Phi}$. This factor should count the number of degrees of freedom in Φ that are light enough to be affected by the value of S . This can depend on the precise mass spectrum of the gauge group, for instance a specific GUT model. See discussion in [72].

The normalization to COBE allows to fix the scale M as a function of κ . If the breaking of G does not produce cosmic strings, the contribution to the quadrupole anisotropy simply comes from the inflationary contribution (see Eq. (34)) and the observed value can be obtained even with a coupling κ close to unity [61]⁶⁰. However it has been shown that the formation of cosmic strings at the end of F -term inflation is highly probable when the model is embedded in SUSY GUTs [436]. In that case, the normalization to COBE receives two contributions [63, 475], one from inflation $(\delta T/T)_{\text{infl}} \propto V^{3/2}/V'$, and the other from cosmic strings $(\delta T/T)_{\text{CS}} \propto G\mu$, where μ is the mass per unit length of the strings (see Sec. III G). The relation between M and κ from the normalization of the power-spectrum is now obtained by solving [475]

$$\left(\frac{\delta T}{T}\right)_{\text{COBE}}^2 = \left(\frac{\delta T}{T}\right)_{\text{Infl}}^2 + \left(\frac{\delta T}{T}\right)_{\text{CS}}^2, \quad (225)$$

which affects the relation $M(\kappa)$ at large κ , and imposes new stringent bounds on $M \lesssim 2 \times 10^{15}$ GeV, and [476, 478]

$$\kappa \lesssim 7 \times 10^{-7} \frac{126}{\mathcal{N}}, \quad (226)$$

coming from imposing that the weight of cosmic strings in the WMAP3 data is less than $\lesssim 10\%$ [258]. This constraint can be made less stringent if some other sector of the Higgs potential imposes that some components of $\Phi, \bar{\Phi}$ acquire an ultra-large mass before inflation, during previous symmetry breaking [478].

Once M is fixed, the spectral index n_s can also be computed as a function of the coupling constant. The range found is $n_s \in [0.98, 1]$ whether cosmic strings form or not [478, 607]. This represents the most important difference with Linde's original model, where a blue tilt is generated. Note however that, when cosmic strings form, the coupling constant has to be suppressed (see Eq. (226)) the predicted value for n_s is very close to unity, $n_s \simeq 1^-$. This value is precisely in agreement with the observations when cosmic strings are formed, if they contribute significantly to the generation of the anisotropies [258].

⁶⁰ Small values of κ can render the scale of inflation very low, as low as the TeV scale [603–606].

4. SUGRA corrections to F -term inflation

When the VEVs are not negligible compared to the Planck scale, SUGRA effects become important and may ruin the flatness of the inflaton potential. Indeed, the potential is now given by Eq. (135), the F -terms containing an additional exponential factor. The soft breaking mass of the scalar fields are typically [318, 319, 494, 501, 597, 599, 608–613]

$$m_{soft}^2 \sim \frac{V}{3M_{\text{P}}^2} \sim \mathcal{O}(1)H^2. \quad (227)$$

Once the inflaton gains a mass $\sim H$, the contribution to the second slow-roll parameter η is of order unity and the field simply rolls down to the minimum of the potential and inflation stops,

$$|\eta| \equiv M_{\text{P}}^2 \frac{V''}{V} \sim \frac{m_{SUGRA}^2}{H^2} \sim \mathcal{O}(1), \quad (228)$$

where $m_{SUGRA}^2 \approx m_{susy}^2 + (V_{susy}/3M_{\text{P}}^2) \sim m_{susy}^2 + \mathcal{O}(1)H^2$, where $m_{susy} \sim \mathcal{O}(100)$ GeV contains soft-SUSY breaking mass term for low scale SUSY breaking scenarios. For VEVs smaller than the Planck scale it is always possible to obtain $\epsilon \ll 1$, but in SUGRA η can never be made less than one for a single chiral field with minimal kinetic terms⁶¹. This is known as the η problem in SUGRA models of inflation [318, 319, 501].

When there are more than one chiral superfields, as in the F -term hybrid model, it can be possible to cancel the dominant $\mathcal{O}(1)H$ correction to the inflaton mass by choosing an appropriate Kähler term [501, 611]. In hybrid inflation models derived from an F -term the dominant $\mathcal{O}(1)H$ correction in the mass term can be cancelled if $|S| = 0$ exactly, which however seems to lead to an initial condition problem, as discussed above. The fact that the superpotential is linear in S (as in Eq. (219)) guarantees the cancellation of the dominant contribution in the mass term for a minimal Kähler, $K_{\min}(\Psi_i, \Psi_i^*) = \sum \Psi_i \Psi_i^* = |S|^2 + |\Psi|^2 + |\bar{\Psi}|^2$ [501],

$$\begin{aligned} V_{\text{tree}}^{\text{F-SUGRA}} &= \kappa^2 \exp\left(\frac{s^2 + \psi^2}{2M_{\text{P}}^2}\right) \\ &\times \left\{ \left(\frac{\psi^2}{4} - M^2\right)^2 \left(1 - \frac{s^2}{2M_{\text{P}}^2} + \frac{s^4}{4M_{\text{P}}^4}\right) \right. \\ &\left. + \frac{s^2\psi^2}{4} \left[1 + \frac{1}{M_{\text{P}}^2} \left(\frac{1}{4}\psi^2 - M^2\right)\right]^2 \right\}. \end{aligned} \quad (229)$$

⁶¹ Except when $m_{inf} \gg H_{inf}$. This can be realizable in an inflection point inflation as in the case of MSSM inflation, see Sec. V B 2.

Note that these corrections affect the dynamics at large field, though at small VEVs, the radiative corrections are the dominant origin of the dynamics. This is generically the case during the last 60 e-foldings of inflation [505].

5. Non-minimal kinetic terms and the SUGRA η problem

For Kähler potentials such as

$$K = |S|^2 + |\Phi|^2 + |\bar{\Phi}|^2 + \kappa_S |S|^4 / M_{\text{P}}^2 + \dots, \quad (230)$$

the kinetic terms $K^{ij} \partial_\mu \Phi_i \partial^\mu \Phi_j^*$ are non-minimal because $K^{ij} \neq \delta^{ij}$. One obtains in particular $(\partial_{S S^*} K)^{-1} \sim 1 - 4\kappa_S |S|^2 / M_{\text{P}}^2 + \dots$ that leads to a problematic $\kappa_S \times \mathcal{O}(1)H$ contribution to the inflaton mass, and therefore on the second slow-roll parameter η , unless some suppression is invoked. Several mechanisms have been proposed to tackle this η -problem.

- The first one is to constrain the parameter of the leading corrections, imposing $\kappa_S \sim 10^{-3}$ which is sufficient to keep the model safe, but without much physical motivation. For a generic inflationary model it is not possible to compute κ_S at all (see the discussion on the footnote below). In this model $|S| \ll M_{\text{P}}$ ensures that higher order terms are negligible [596].
- Safe non-minimal Kähler potentials have also been proposed [614–617] making use of the shift symmetry⁶² [598, 618] to protect the Kähler potential of the form $K(\Phi, \bar{\Phi}) \rightarrow K(\Phi + \bar{\Phi})$. This symmetry generates an exactly flat direction for an inflaton field and a non-invariance of the superpotential induces some slope to its potential to allow slow-roll at the loop level.

Another symmetry - the Heisenberg symmetry - has also been invoked to protect the form of the Kähler potential, see for a recent discussion in [619], generating a model where the effective Kähler is a no-scale potential, that is of the form $K = \ln(\Phi_i)$. This obviously solves the η -problem by canceling the exponential term $\exp(K)$ ⁶³.

⁶² Under this symmetry, a superfield $S \rightarrow S + iC$, where C is a constant.

⁶³ There is a word of caution here, it is assumed that one can take the inflaton VEV above the Planck scale, as in the case of Heisenberg symmetry of the Kähler potential, in order to realize chaotic type models [598, 615, 618, 619]. However, this assumes that the Kähler potential does not obtain any quantum corrections.

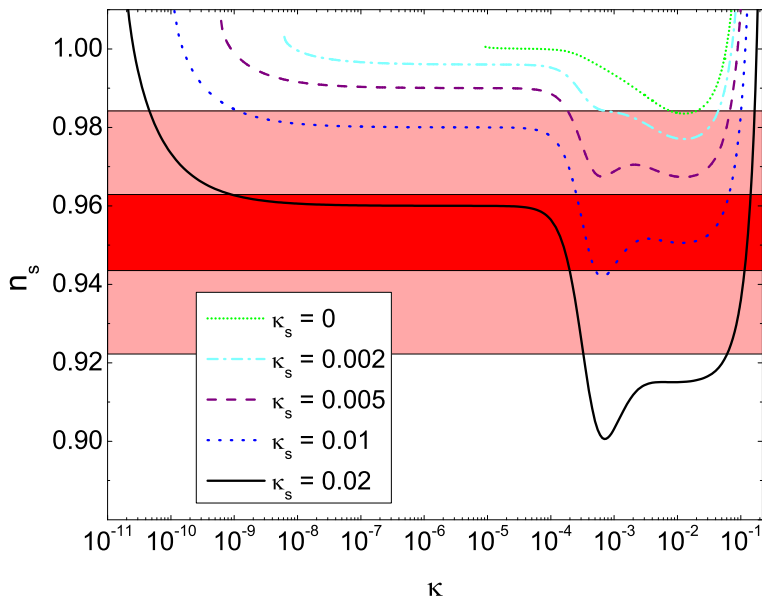


FIG. 4: Spectral index for F -term hybrid inflation with minimal kinetic terms (top green curve) and non-renormalizable corrections to the Kähler. Figure is taken from [81].

The presence of non-renormalizable correction to the kinetic terms has important consequences on the CMB predictions of the F -term hybrid model. In particular it can be used to realize a better agreement with the WMAP 5 measurement on the spectral index [71, 81].

It has been shown that the presence of the κ_s term in Eq. (230) allow the model to;

- 1) lower the value of M responsible for the normalization of the spectrum for a given κ and thus lower the influence of cosmic strings on the CMB, and
- 2) predict a spectral index lower than 0.98, easily in the range $n_s \in [0.9, 1]$ as represented in Fig. (4).

Further note that non-renormalization theorem can only protect the superpotential terms [620], but the Kähler potential generically obtains quantum corrections, which have been computed explicitly in some cases [621–623]. In string motivated models it is hard to realize chaotic type models of inflation with VEVs larger than the $4d$ Planck scale.

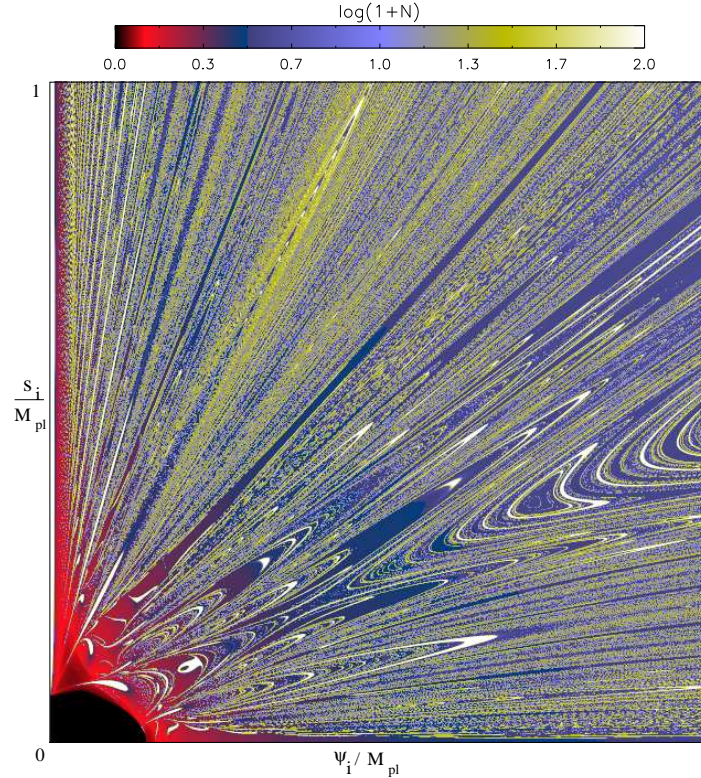


FIG. 5: Initial condition space for F -term hybrid inflation in minimal SUGRA, when restricting to the sub-Planckian ($M_{\text{pl}} = M_{\text{P}}$) initial VEVs and vanishing velocities. The color code represents the number of e-foldings generated for each initial VEVs. This confirms that hybrid inflation is successful when the onset of inflation occurs close to the inflationary valley (the white narrow band along the y -axis), but shows a subdominant space for other trajectories where inflation is also successful for the sub-Planckian VEVs [505].

6. Initial conditions for F -term hybrid inflation

We have seen in Sec. IV C that initial conditions for hybrid inflation was considered one of the challenges for the hybrid models, and it was considered as a problem for the F -term inflation in [526]. It was argued that for inflation to be successful, the initial field value for the waterfall field ψ_i had to be fine-tuned to an almost vanishing value, so as to start close enough to the inflationary valley. Note that in this reference, the radiative corrections to the inflationary potential has been approximated by a mass term, similarly to the non-SUSY case (see Sec. IV C). This model has been studied more recently with a high precision, taking into account of the SUGRA corrections induced by a minimal Kähler potential [505].

In [505], the space of successful initial conditions is found composed of a non-fractal set of successful points but with fractal boundaries. Such a set is represented in Fig. (5), where for given initial VEVs, the number of e-foldings of inflation realized is represented. It was pointed out that similar to non-SUSY hybrid inflation, inflation is realized generically by a first phase of fast roll down the potential and when the velocity vector is correctly oriented at the bottom of the potential, the inflaton climbs up and slow-rolls back down the inflationary valley around $\langle\psi\rangle = 0$.

7. Other hybrid models and effects of non-renormalizable terms

The superpotential of F -term hybrid inflation given in Eq. (219) contains only renormalizable terms. However VEVs close to the UV cutoff of the theory, (necessarily smaller than the reduced Planck mass), non-renormalizable terms play a non-negligible role. Two models have been proposed to study these effects: the “smooth” [62] and “shifted” [66] hybrid inflation models.

The initial motivation for both of these models was to implement hybrid inflation in a GUT model based on the Pati-Salam gauge group $G_{PS} \equiv SU(4)_C \times SU(2)_L \times SU(2)_R$ without the formation of monopoles at the end of inflation. The idea is therefore to break the symmetry *before* the phase of inflation, which can be achieved by introducing one non-renormalizable term in the superpotential.

- Smooth hybrid inflation:

In this model [62], the superpotential has to satisfy a new Z_2 symmetry in addition to the R-symmetry and G_{GUT} under which the pair of Higgs superfields would transform following $\Phi\bar{\Phi} \rightarrow -\Phi\bar{\Phi}$. This forbids the second term in the superpotential of standard F -term inflation Eq. (219), but allows one of the first non-renormalizable term

$$W^{\text{smooth}} = \kappa S \left[-M^2 + \frac{(\bar{\Phi}\Phi)^2}{M_{\text{P}}^2} \right]. \quad (231)$$

the scalar potential reads

$$V^{\text{smooth}}(S, \Phi) = \kappa^2 \left| -M^2 + \frac{(\bar{\Phi}\Phi)^2}{M_{\text{P}}^2} \right|^2 + \kappa^2 |S|^2 \frac{|\Phi|^2 |\bar{\Phi}|^2}{M_{\text{P}}^4} (|\Phi|^2 + |\bar{\Phi}|^2), \quad (232)$$

where we denote by the same letter the superfield and its scalar component ($\theta = 0$).

We remind the reader that $\bar{\Phi}$, Φ are 2 fields charged under G_{GUT} . If we follow the original motivation, $G_{\text{GUT}} = G_{\text{PS}}$ and we want Φ to be non-trivially charged under the factors $SU(4)_{\text{C}} \times SU(2)_{\text{R}}$ in order to generate the breaking scheme $G_{\text{PS}} \rightarrow G_{\text{SM}}$. The simplest possibility to realize this is to assign Φ to the representation $(\mathbf{4}, \mathbf{1}, \mathbf{2})$. It is then necessary to assign $\bar{\Phi}$ to its complex conjugate representation $(\bar{\mathbf{4}}, \mathbf{1}, \mathbf{2})$ so that the superpotential is invariant under G , S being necessarily an absolute gauge singlet belongs to a hidden sector.

We can define two real scalar fields, s and ϕ , as being the relevant component of the representation of the S , Φ , $\bar{\Phi}$ fields such that the potential can be rewritten [67]

$$V^{\text{smooth}}(s, \phi) = \kappa^2 \left(M^2 - \frac{\phi^4}{M_{\text{P}}^2} \right)^2 + 2\kappa^2 s^2 \frac{\phi^6}{M_{\text{P}}^4} . \quad (233)$$

This modifies the picture drastically, since now the valley $\phi = 0$ still represents a flat direction for s , but is also a local maximum in the ϕ direction. As a consequence, inflation will be realized for non-vanishing values of ϕ , which induces the symmetry breaking *during* inflation. The minimum of the potential at fixed s is indeed reached for

$$\phi^2 = \frac{4}{3} \frac{M^2 \mu^2}{s^2} , \quad \text{for } s \gg \mu M , \quad (234)$$

which correspond to the two symmetric minima of the potential.

Inside the inflationary trajectory described above, the effective one-field potential is $V(s) = \mu^4(1 - (2/27)\mu^2 M^2/s^4)$ in the limit $s \gg \mu M$, a form similar to mutated hybrid inflation. The predictions of the model have been studied in [62], assuming an embedding within SUSY GUTs, that is with a unification scale of 2×10^{16} GeV and a gauge coupling constant of ~ 0.7 . The normalization to the COBE data imposes the mass scales of inflation is found lower than in the F -term case $\mu \simeq 9 \times 10^{14}$ GeV and the cutoff M scale is found close to the reduced Planck mass $M \sim M_{\text{P}}$. The spectral index is then given by [62]

$$n_s \simeq 1 = \frac{5}{3N_Q} \simeq 0.97 \quad (\text{for } N_Q = 60) , \quad (235)$$

and a negligible running spectral index.

- Shifted hybrid inflation:

The shifted inflation model is similar to the smooth model except that the additional

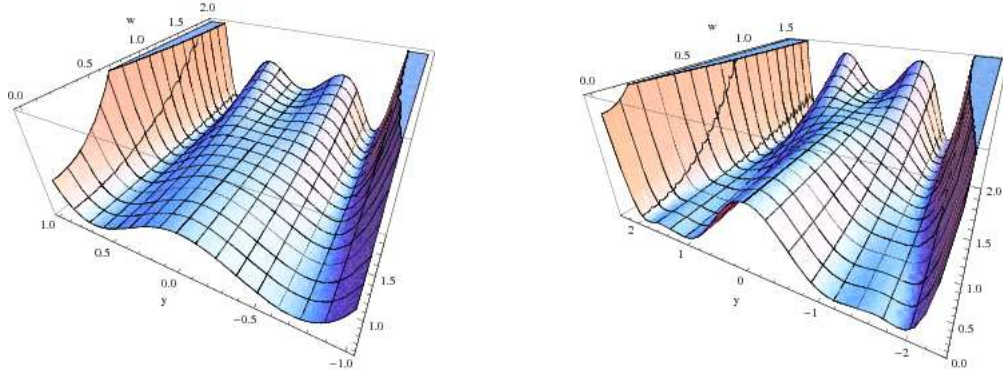


FIG. 6: Shifted hybrid inflation potential of Eq. (238) for $\xi > 1/4$ (left) and $1/6 < \xi < 1/4$ (right) in the reduced variable space.

Z_2 symmetry is not imposed. As a consequence, the superpotential reads [66]

$$W^{\text{shifted}} = \kappa S [\bar{\Phi}\Phi - M^2] - \beta \frac{S(\bar{\Phi}\Phi)^2}{M_{\text{P}}^2}, \quad (236)$$

where $\Phi, \bar{\Phi}$ are two distinct superfields that belong to non-trivial representations of the Pati-Salam group $G_{\text{PS}} = SU(4)_c \times SU(2)_L \times SU(2)_R$: $\Phi \in (\bar{\mathbf{4}}, \mathbf{1}, \mathbf{2})$, $\bar{\Phi} \in (\mathbf{4}, \mathbf{1}, \mathbf{2})$. This gives rise to the following F -terms (in global SUSY)

$$V_{F\text{-terms}}^{\text{shifted}} = \left| -\kappa M^2 + \kappa \bar{\Phi}\Phi - \beta \frac{(\bar{\Phi}\Phi)^2}{M_{\text{P}}^2} \right|^2 + \kappa^2 |S|^2 (|\bar{\Phi}|^2 + |\Phi|^2) \left| 1 - \frac{2\beta}{\kappa M_{\text{P}}^2} \bar{\Phi}\Phi \right|^2, \quad (237)$$

where we have denoted by the same letter the superfield and its $\theta = 0$ component in the superspace.

Following [67], we define the relevant fields $\bar{\phi}$ and ϕ as the component of $\bar{\Phi}$ and Φ that generates the symmetry breaking $G_{\text{PS}} \rightarrow G_{\text{SM}}$, that is to give a VEV to a component of Φ that is charged under G_{PS} but not under G_{SM} . For the inflaton field, we will define $s \equiv |S|$. We can also choose to set $\beta > 0$ and $\bar{\phi}^* = \phi$, so that the potential is D -flat and becomes

$$V^{\text{shifted}}(s, \phi) = \kappa^2 \left[-M^2 + |\phi|^2 - \frac{\beta}{\kappa M_{\text{P}}^2} |\phi|^4 \right]^2 + 2\kappa^2 s^2 |\phi|^2 \left[1 - \frac{2\beta}{\kappa M_{\text{P}}^2} |\phi|^2 \right]^2, \quad (238)$$

$$\frac{V^{\text{shifted}}(w, y)}{\kappa^2 M^4} = (y^2 - 1 - \xi y^4)^2 + 2w^2 y^2 (1 - 2\xi y^2)^2.$$

The second form is for normalized fields, $w = s/M$, $y = \phi/M$ and $\xi \equiv \beta M^2/\kappa M_{\text{S}}$. The potential is represented by Fig. (6).

We can observe that the potential contains three (respectively two) local minima at high (respectively small) values of the inflaton field w , if $\xi \geq 1/4$ is of the order of unity (Fig. (6) left). They are located at $y = 0$ for the central one while the other valleys are “shifted” away from $y = 0$, with a trajectory function of the inflaton field $y = f(w)$. The number of valleys goes up to four at small inflaton VEVs and intermediate values of $1/7.2 < \xi < 1/4$ (Fig. (6) right): two new shifted valley at $y = \pm 1/\sqrt{2\xi}$ appears. At smaller values of ξ , both shifted valleys become indistinguishable. If inflation is realized in one shifted valley, like for smooth hybrid inflation, the symmetry G is broken during inflation and no topological defect can affect the post-inflationary cosmology. This is the motivation of the model and also an implicit assumption. Note that the dynamics could impose inflation to take place in the central valley, at vanishing ψ . It would then be possible that the number of e-foldings produced after the symmetry breaking is small enough that topological defects have some influence on the CMB. This is still an open question.

If inflation is realized in the $y = \pm 1/\sqrt{2\xi}$ valley, it is possible to compute the full mass spectrum of the model [66]. The classical contribution to the potential is $V_0 = \kappa^2 \tilde{M}^4$, where $\tilde{M}^4 = M^4(1/4\xi - 1)^2$. The one-loop quantum corrections appear like in the F -term case from the splitting in mass, $2\kappa^2 \tilde{M}$, between fermions and bosons of the superfield $\Phi, \bar{\Phi}$. Consequently, the effective scalar potential in this valley is identical to Eq. (222) for the original F -term model replacing M by \tilde{M} . The CMB predictions are then derived; the scale of inflation and the spectral index are found as a function of κ , for a fixed value of β and M_S identified as the string scale. For $\kappa \in [10^{-2}, 10^{-3}]$, they are predicted in the ranges $n_s \in [0.89, 0.99]$ and $V_{\text{inf}} \in [2, 7] \times 10^{14}$ GeV, which is in agreement with the most recent CMB measurements [66]. The level of predicted tensor perturbations with these parameters, $r \lesssim 10^{-5}$, is however beyond the reach of planned experiments.

- Non-renormalizable terms and SUGRA effects:

The analysis of the previous models driven by F -terms are based on the presence of non-renormalizable terms (smooth and shifted hybrid inflation), which can be expected to become unstable and suffer from the η -problem when formulated in SUGRA. It actually turns out not to be a major problem, the presence of extra non-renormalizable

term changes the shape of the tree-level potential and avoid the η -problem as discussed in the previous section. Moreover, we are allowed to assume non-minimal (non-renormalizable) Kähler potentials, which introduce more parameters and more freedom to flatten the potential [71, 81].

For example in the case of a smooth hybrid inflation (with the superpotential given by Eq. (231)) the effective potential in minimal SUGRA is of the form ⁶⁴ [70]

$$V(s) \simeq \mu^4 \left[1 - \frac{2\mu^2 M^2}{27s^4} + \frac{s^4}{8M_{\text{P}}^4} \right]. \quad (239)$$

The last term in this expression comes from the SUGRA corrections. Assuming this expression is valid for relevant scales, it is found that the spectral index is raised from $n_s \simeq 0.97$ in global SUSY to larger values $n_s \in [0.98, 1.03]$ depending on the scale $\tilde{M} = \sqrt{\mu M}$ [70].

Introducing the non-renormalizable corrections to the Kähler potential, $K = K_{\text{min}} + \lambda S^4/(4M) + \dots$, lead to an effective potential [81, 502]

$$V(s) \simeq \mu^4 \left[1 - \frac{2\mu^2 M^2}{27s^4} - \lambda \frac{s^2}{2M} + \frac{\gamma_S}{8} \frac{s^4}{M_{\text{P}}^4} \right]. \quad (240)$$

This allows the model to reduce the tension by increasing the cutoff scale M and reduce of the predicted spectral index to values inside $n_s \in [0.95, 0.99]$.

We conclude this section by pointing out that non-renormalizable corrections can also affect significantly other predictions such as the running spectral index, even if inflation takes place well below the scale of new physics [624, 625]. The reason is that if the inflationary valley is flat at tree level, all the dynamics is determined by the lifting of a non-renormalizable term. It was first found in [624] that smooth hybrid inflation could predict a non-negligible amount of running by balancing the non-renormalizable contributions to W and K with the SUGRA effects. This prediction however requires the modification of the superpotential to include a phase of new inflation, since the parameter range to generate a large running imposes a short (less than 60 e-foldings) phase of smooth inflation. A model independent study of SUSY hybrid type models

⁶⁴ Note that this expression has a rather small range of validity, since it is valid only if $s^2 \gg \mu M$ and $s \ll M_{\text{P}}$.

with potentials flat at tree level, and lifted by radiative corrections was considered in [625],

$$V(\phi) = V_0 + \beta \ln \frac{m(\phi)}{\mu} + \phi^4 \frac{\phi^{2N}}{M^{2N}} ,$$

where the last term contains an arbitrary non-renormalizable operator. It was found that typically running of the spectral tilt is negligible, $\alpha_s \ll 1$, in renormalizable models if a large number of e-foldings is realized. However they also pointed out that if the non-renormalizable mass scale, M is larger than the inflationary scale can generate a large running $\mathcal{O}(-0.05)$.

- Extensions of hybrid F -term inflation:

Many models have been proposed, that generalize or extend the idea of the hybrid inflaton driven by F -terms. In Ref. [605, 606], the realization of hybrid inflation has been illustrated in conjunction with solving the μ -problem of the MSSM within the NMSSM. They proposed that the waterfall field coupled to the inflaton also induce the mass term for the electroweak Higgs pairs of the MSSM:

$$W^{\phi\text{NMSSM}} = \lambda N H_1 H_2 - \kappa S N^2 . \quad (241)$$

At the cost of adding two scalar singlets to the MSSM, this model is found to solve the μ -problem, prevents from domain walls to form during the electro-weak symmetry breaking (see [626]), and give rise to a phase of inflation of the (non-SUSY) hybrid type with an effective potential of the form $V_0 + m^2 \phi^2$.

Other examples of extensions of the hybrid model via F -terms are the smooth and shifted scalar potentials obtained with only renormalizable operators [69, 82]. They were named “new smooth” and “new shifted” hybrid inflation, and require the introduction of additional fields interacting with the inflaton and the waterfall field.

For example, new shifted hybrid inflation is still based on a singlet S of the Pati-Salam gauge group G_{PS} coupled to $\Phi, \bar{\Phi}$ in $(\bar{\mathbf{4}}, \mathbf{1}, \mathbf{2})$ and $(\mathbf{4}, \mathbf{1}, \mathbf{2})$. But the model also assumes the introduction of new superfields $\Psi, \bar{\Psi}$ both in $(\mathbf{15}, \mathbf{1}, \mathbf{3})$ to which the inflaton S would be coupled. The introduction of these extra-fields has motivations [68] from the fermion spectrum, in particular the predicted mass of the bottom quark, that becomes in better agreement with experimental measurements, compared to the minimal Pati Salam model.

The inflaton sector now relies on the following superpotential [69]

$$W^{\text{new shifted}} = \kappa S(\Phi\bar{\Phi} - M^2) - \beta S\Psi^2 + m\Psi\bar{\Psi} + \lambda\bar{\Psi}\Phi\bar{\Phi} , \quad (242)$$

which leads to the following F -term contribution to the scalar potential

$$V = |\kappa(\Phi\bar{\Phi} - M^2) - \beta\Psi^2|^2 + |2\beta\Psi - m\bar{\Psi}|^2 + |m\Psi + \lambda\Phi\bar{\Phi}|^2 + |\kappa S + \lambda\bar{\Psi}|^2 \left(|\Phi|^2 + |\bar{\Phi}|^2 \right) , \quad (243)$$

The flat directions of the potential are at $\langle\Phi\rangle = \langle\bar{\Phi}\rangle = \langle\Psi\rangle = \langle\bar{\Psi}\rangle = 0$ for the trivial one, and at

$$\langle\Phi\rangle = \langle\bar{\Phi}\rangle = v , \quad \langle S\rangle > 0 , \quad \langle\Psi\rangle = -\frac{M}{2} \sqrt{\frac{\kappa}{\beta\xi}} , \quad \langle\bar{\Psi}\rangle = -S\frac{\kappa}{\lambda} , \quad (244)$$

where

$$v^2 \equiv \frac{2\kappa^2(1/4\xi + 1) + \lambda^2/\xi}{2(\kappa^2 + \lambda^2)} \xi .$$

Inflation is assumed to take place in this shifted valley, and thus in this model breaking of the Pati-Salam group to the SM group (and the formation of monopoles associated with it) is realized *during* inflation. Indeed, the VEV of Φ breaks completely the Pati-Salam gauge group only leaving the SM group unbroken (the doublet of $SU(2)_R$ breaks it completely and the $\mathbf{4}$ of $SU(4)_c$ necessarily breaks its $U(1)_{B-L}$ subgroup [422]).

The presence of many free parameters in this model only allows to confirm that with reasonable parameter values (coupling of order 10^{-2} , and masses and unification scales around $10^{15} - 10^{16}$ GeV), the normalization to COBE, around 60 e-foldings of inflation, and a spectral index around $n_s \simeq 0.98$ can be obtained [69]. The motivations and the mechanism behind the “new smooth hybrid inflation” [82] are identical, with a similar potential given by Eq. (242). The predictions of the model are: $n_s \simeq 0.969$, $r \simeq 9.4 \times 10^{-7}$, $\alpha_s \simeq -5.8 \times 10^{-4}$ [82].

A “semi-shifted hybrid inflation” was also proposed [83], with a similar framework of the extended Pati-Salam group, as for new shifted or new smooth hybrid inflation and the same (super)potential. In this model, however, the chosen inflationary valley is different, since it takes advantage of a third flat direction appearing only if $\tilde{M}^2 \equiv M^2 - m^2/2\kappa^2 > 0$

$$\langle\Phi\rangle = \langle\bar{\Phi}\rangle = 0 , \quad \langle\Psi\rangle = \pm\tilde{M} , \quad \langle\bar{\Psi}\rangle = -\frac{2\kappa\langle\Psi\rangle}{m} S , \quad \langle S\rangle > 0 . \quad (245)$$

This inflation is then called semi-shifted, since only the VEVs of $\Psi, \bar{\Psi}$ are shifted away from 0. As a consequence the breaking during inflation is $G_{PS} \rightarrow SU(3)_c \times SU(2)_L \times U(1)_R \times U(1)_{B-L}$ leaving the second symmetry breaking $SU(3)_c \times SU(2)_L \times U(1)_R \times U(1)_{B-L} \rightarrow G_{SM}$ for the end of inflation. As a consequence, the model predicts the formation of B-L cosmic strings, which can have important phenomenological consequences [627], including on the CMB predictions of the model. When taking into account of the SUGRA corrections, the spectral index [83] is found in the range $n_s \in [0.98, 1.05]$ for $m \in [0.5, 2.5] \times 10^{15}$ GeV, in agreement with WMAP data in presence of cosmic strings. The running and the ratio of tensor to scalar is again found well below expected detection limits.

F. Inflation from D -terms in SUSY and SUGRA

It was first noticed in Ref. [611] that inflation with a perfectly flat inflaton potentials in SUSY/SUGRA can be constructed using a constant contribution to the D -term, and a rather complicated superpotential used to drive the field dependent contributions to the D -terms to 0. In [628, 629], a very simple superpotential was proposed to achieve the similar result, where it was noticed that the radiative correction would lift this flat direction and drive inflation.

In addition, it was noticed that the η problem arising in F -term models does not appear for D -terms driven inflation even for the non-minimal Kähler potential because the D -sector of the potential does not receive exponential contributions from non-minimal SUGRA. The model requires the presence of a Fayet-Iliopoulos (FI) term ξ , and therefore a $U(1)_\xi$ symmetry that allows or generates it.

This model rapidly became one of the most studied models of inflation because of its stability in SUGRA, and its stability when embedded in other high energy frameworks such as SUSY GUTs (see for example [63, 65]), and SUGRA from superconformal field theory [454]. The model was also found to be a good low-energy description of brane inflation [630, 631]) proposed in the context of extra-dimensional cosmology.

Finally, the presence of anomalous $U(1)$ symmetries in weakly coupled string theories [632–635] generated a lot of hope, though there are problems in fully embedding the model and the generation of the FI term [636, 637]. Also, the model tends to require large

inflaton values (Planckian, even super-Planckian in part of the parameter space) unlike F -term inflation, which represents another challenge for the model, and thus necessitates D -term inflationary models to be studied in the context of SUGRA with a VEV below M_{P} .

1. Minimal hybrid inflation from D -terms

Let us consider both F - and D -terms contribute to the scalar potential. Given a Kähler potential $K(\Phi_m, \Phi_n)$, the D -terms

$$D^a = -g_a [D_a = \phi_i (T_a)^i_j K^j + \xi_a]$$

(where $K^m \equiv \partial K / \partial \Phi_m$) give rise to a scalar potential within $N = 1$ SUGRA:

$$V^{\text{SUGRA}}(\phi, \phi^*) = \frac{1}{2} [\text{Re} f(\phi)]^{-1} \sum D^a D_a + \text{F-terms} \quad (246)$$

where g_a and T^a are respectively the gauge coupling constants and the generators of each factors of the symmetry of the action, ' a ' running over all factors of the symmetry, and $f(\phi)$ is the gauge kinetic function. If this symmetry contains a factor $U(1)_\xi$, not originating from a larger non-abelian group, the most general action allows for the presence of an additional constant contribution ξ . Below we will assume that such an abelian symmetry is the only symmetry of the inflaton sector.

The simplest realization of D -term inflation reproduces the hybrid potential with three chiral superfields, S , ϕ_+ , and ϕ_- with non-anomalous $U(1)_\xi$ (an abelian theory is said to be anomalous if the trace of the generator is non-vanishing $\sum q_n \neq 0$) charges $q_n = 0, +1, -1$ [628, 629]. The superpotential can be written as

$$W^D = \lambda S \phi_+ \phi_- . \quad (247)$$

In what follows, we assume the minimal structure for $f(\Phi_i)$ (i.e., $f(\Phi_i)=1$) and take the minimal Kähler potential⁶⁵. Then the scalar potential reads

$$\begin{aligned} V_{\text{tree}}^{\text{D-SUGRA}} = \lambda^2 \exp \left(\frac{|\phi_-|^2 + |\phi_+|^2 + |S|^2}{M_{\text{P}}^2} \right) & \left[|\phi_+ \phi_-|^2 \left(1 + \frac{|S|^4}{M_{\text{P}}^4} \right) + |\phi_+ S|^2 \left(1 + \frac{|\phi_-|^4}{M_{\text{P}}^4} \right) \right. \\ & \left. + |\phi_- S|^2 \left(1 + \frac{|\phi_+|^4}{M_{\text{P}}^4} \right) + 3 \frac{|\phi_- \phi_+ S|^2}{M_{\text{P}}^2} \right] + \frac{g_\xi^2}{2} (|\phi_+|^2 - |\phi_-|^2 + \xi)^2 , \end{aligned} \quad (248)$$

⁶⁵ This is the simplest SUGRA model and in general the Kähler potential can be a more complicated function of the superfields.

where g_ξ is the gauge coupling of $U(1)_\xi$. Here, we have assumed a minimal Kähler potential $K = |\phi_-|^2 + |\phi_+|^2 + |S|^2$. The global minimum of the potential is obtained for $\langle S \rangle = 0$ and $\langle \Phi_- \rangle = \sqrt{\xi}$, which is SUSY preserving but induces the breaking of $U(1)_\xi$. For $S > S_{inst} \equiv g_\xi \sqrt{\xi} / \lambda$ the potential is minimized for $|\phi_+| = |\phi_-| = 0$ and therefore, at the tree level, the potential exhibits a flat inflationary valley, with vacuum energy $V_0 = g_\xi^2 \xi^2 / 2$. The radiative corrections depend on the splitting between the effective masses of the components of the superfields Φ_+ and Φ_- , because of the transient D -term SUSY breaking. Extracting the quadratic terms from the potential Eq. (248), the scalar components ϕ_+ and ϕ_- get squared masses:

$$m_\pm^2 = \lambda^2 |S|^2 \exp\left(\frac{|S|^2}{M_{\text{Pl}}^2}\right) \pm g_\xi^2 \xi, \quad (249)$$

while the squared mass of the Dirac fermions reads

$$m_f^2 = \lambda^2 |S|^2 \exp\left(\frac{|S|^2}{M_{\text{Pl}}^2}\right). \quad (250)$$

The radiative corrections are given by the Coleman-Weinberg expression [352] and the full potential inside the inflationary valley reads

$$V_{\text{eff}}^{\text{D-SUGRA}} = \frac{g_\xi^2 \xi^2}{2} \left\{ 1 + \frac{g_\xi^2}{16\pi^2} \left[2 \ln \frac{\lambda^2 |S|^2}{\Lambda^2} \exp\left(\frac{|S|^2}{M_{\text{Pl}}^2}\right) + (z+1)^2 \ln(1+z^{-1}) + (z-1)^2 \ln(1-z^{-1}) \right] \right\}, \quad (251)$$

with

$$z = \frac{\lambda^2 |S|^2}{g_\xi^2 \xi} \exp\left(\frac{|S|^2}{M_{\text{Pl}}^2}\right). \quad (252)$$

Inflation ends when the slow-roll conditions break down, that is for $z_{\text{end}} \simeq 1$, and the predictions for the inflationary parameters are very similar to the previous discussion on F -term inflation. D -term inflation based on an anomalous $U(1)_a$ symmetry (which could appear in string theory [633–635]) is no different. More than one anomalous $U(1)_a$'s can also give rise to a multiple phase of hybrid inflation, see [155].

2. Constraints from CMB and cosmic strings

The CMB phenomenology is very similar to the F -term inflation model described in the earlier sections. The fitting to the CMB observables can be done by simply setting the values for energy scale $\sqrt{\xi}$, superpotential coupling λ , and gauge coupling g_ξ of $U(1)_\xi$.

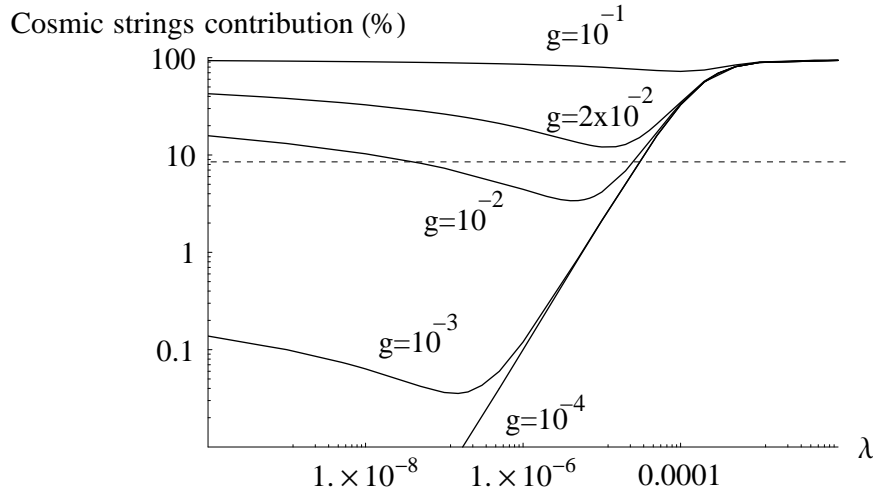


FIG. 7: Cosmic strings contribution to the CMB quadrupole anisotropies as a function of the superpotential coupling λ and gauge coupling $g \equiv g_\xi$. Figure is taken from [73].

By construction, the model leads to the formation of cosmic strings, since the inflationary phase ends by the breaking of the abelian symmetry $U(1)_\xi$ [63, 628]. Their formation affects the normalization of the fluctuation power spectrum by imposing an additional contribution. This can be described by an additional contribution to the temperature quadrupole anisotropy

$$\left. \frac{\delta T}{T} \right|_Q^2 = \left. \frac{\delta T}{T} \right|_{\text{infl}}^2 + \left. \frac{\delta T}{T} \right|_{\text{CS}}^2, \quad (253)$$

where $(\delta T/T)_{\text{CS}} = y2\pi\xi/M_{\text{P}}^2$, because the D -term strings are BPS. The contributions from inflation and cosmic strings are therefore proportional to the same energy scale $\sqrt{\xi}$. When using $y = 9$, it was found [72, 474–476] that the contribution of strings to the anisotropies varies with the coupling constants; λ and g_ξ , as represented in Fig. 7.

Thus the formation of cosmic strings only imposes a suppressed superpotential coupling for a successful D -term inflation;

$$\lambda \lesssim 10^{-5}. \quad (254)$$

This conclusion was found valid also with next-to-minimal Kähler potentials [72, 73, 80]. Three mechanisms have been proposed to lift this (slight) fine-tuning; adding some symmetries to the Higgs Φ_\pm or adding more fields to make the strings unstable [454, 638], modifying the superpotential to produce them during inflation or finally introducing couplings for the inflaton to flatten its potential [639]. They will be discussed in Sec. IV F 4.

The predicted spectral index of the model can be computed in a similar way to the F -term model. In the inflationary valley, the potential of Eq. (251) allows to compute n_s at the quadrupole scale for given values of ξ , z_Q , λ and g_ξ . The general logarithmic slope of the potential being concave down, the second slow-roll parameter is negative and thus the spectrum turns out to be red.

More precisely, the normalization of the spectrum and imposing 60 e-foldings of inflation between the field values responsible for the quadrupole z_Q and z_{end} , leaves two parameters unconstrained (say λ and g_ξ) out of the four unknowns. The model possess two regimes; at large coupling λ , $z_Q, z_{\text{end}} \gg 1$, and the spectral index can be approximated by [72]

$$n_s = 1 - \frac{2\lambda}{2g_\xi N_Q + \lambda}, \quad (255)$$

which can be much smaller than unity. In the small coupling limit, $z_Q, z_{\text{end}} \simeq 1$, and $n_s - 1 \simeq 0$, which is slightly disfavored in the WMAP 5-years data. Note also that in this regime the computation of loop corrections to the potential using the Coleman-Weinberg [352] formula breaks down, since the relevant quantities are computed very close to the inflection point $z = 1$, at which V'' and thus η , n_s diverge. A more accurate description of this regime might require the use of renormalization group improvements (see Sec. III B).

Finally, an additional problem for D -term inflation is the super Planckian VEVs for the relevant parameter space $S_Q^2/M_{\text{P}}^2 \gtrsim \text{PLog}[g_\xi^2 \xi / \lambda^2]$ ⁶⁶. For a gauge coupling, $g_\xi \gtrsim 10^{-3}$, and a small superpotential coupling in agreement with a low weight of cosmic strings, the inflaton VEV already shoots up above the Planck scale for 60 e-foldings.

The minimal version of D -term inflation does not predict a detectable amount of non-Gaussianity, as only one field effectively rolls and fluctuates during inflation, at least. The reason is that in the inflationary valley, the waterfall fields coupled to the inflaton have mass of the order of the GUT scale, $m_B^2 \simeq \lambda^2 S^2 \pm g_\xi^2 \xi$, much heavier than the Hubble scale during inflation. But in case of the presence of a light (super)field S_2 with mass μ_2 in the theory, both S and S_2 contribute to the primordial fluctuations, and can create a large amount of non-Gaussianities, well above the level of one field inflation [640]. Furthermore, this will also induce large isocurvature perturbations which would modify the spectral tilt and the primordial spectrum ⁶⁷.

⁶⁶ $\text{Plog}[x]$ represents the inverse function of $x \rightarrow xe^x$.

⁶⁷ Note that during the tachyonic stage of the waterfall field the non-Gaussianity was found to be small [564],

3. *D-term inflation from superconformal field theory*

The standard picture described in the previous section has been modified in [454] to take into account of the possibility that SUGRA is constructed from a superconformal field theory (see [641] for a review). In this framework the theory is described by a conformal Kähler \mathcal{N} and a conformal superpotential \mathcal{W} . In this formulation, which allows to embed any SUGRA of $N \leq 4$, the field content are some chiral superfield Φ_i and a field Y , called the “conformon”, whose modulus fixes the Planck scale:

$$|Y|^2 = M_{\text{P}}^2 \exp[K(\Phi_i, \Phi_i^*)/3M_{\text{P}}^2] , \quad (256)$$

where K is the SUGRA Kähler potential, related to the conformal Kähler by

$$-\frac{1}{3}\mathcal{N}(Y, \Phi_i) = |Y|^2 \exp[-K(\Phi_i, \Phi_i^*)/3M_{\text{P}}^2] .$$

The theory is fully described once a conformal superpotential \mathcal{W} is chosen, related to the SUGRA superpotential, W , by

$$\mathcal{W}(Y, \Phi_i) \equiv Y^3 M_{\text{P}}^3 W(\Phi_i) , \quad (257)$$

The phase of the conformon, Λ_Y , is free and should be fixed to break the invariance of the theory under the Kähler transformations that leave the Lagrangian invariant. Λ_Y can be fixed by imposing that the superconformal superpotential is real, $\mathcal{W} = \mathcal{W}^*$. This leads to the regular description of $N = 1$ SUGRA, where the Lagrangian is fully described by the function $G \equiv K/M_{\text{P}}^2 + \ln(|W|^2/M_{\text{P}}^6)$. But during D -term inflation, this choice is meaningless as the superpotential is vanishing [454]. Alternatively, another choice to fix the gauge is to assume that Y is real [641], $Y = Y^*$ (see [641] for the full description of the Lagrangian with this choice).

The transformation of Y under $U(1)_\xi$ can be written as an imaginary constant,

$$\delta_\alpha Y = i \frac{g_\xi \xi}{3M_{\text{P}}^2} , \quad (258)$$

where ξ is the FI term. From Eq. (257), it is clear that imposing the invariance of the conformal superpotential, $\delta_\alpha \mathcal{W} = 0 = 3\delta_\alpha Y + \delta_\alpha W$, implies that the existence of the FI term requires the superpotential W not to be invariant, $\delta_\alpha W = ig_\xi \xi / M_{\text{P}}^2$.

but see contradictory results found in [566].

For $W = S\Phi_+\Phi_-$, $\delta_\alpha W = ig_\xi \sum q_i$, and the presence of a FI term imposes the anomaly (non-vanishing sum of charges) of the $U(1)_\xi$ symmetry, which is given by ξ/M_{P}^2 . This can be accommodated by modifying the charges from 0, +1, -1, for example to 0, $(1 - \xi/2M_{\text{P}}^2)$ and $(-1 - \xi/2M_{\text{P}}^2)$ for S , Φ_+ and Φ_- , respectively. This leads to a tree level potential of the form:

$$V_{\text{tree}}^D(\phi_+, \phi_-) = \frac{g^2}{2} \left[\left(1 - \frac{\xi}{2M_{\text{P}}^2}\right) |\phi_+|^2 - \left(1 + \frac{\xi}{2M_{\text{P}}^2}\right) |\phi_-|^2 + \xi \right]^2, \quad (259)$$

This affects only the D -terms of the potential, which modify the effective masses of the components of Φ_\pm involving the expression for the one-loop contribution to $V(S)$, but note that the amplitude, ξ/M_{P}^2 , is very small [73, 454], since $\sqrt{\xi}$ is found at least three orders of magnitude below the Planck mass, from the normalization of the CMB anisotropies and the non-observation of cosmic strings. It is worth noting that this construction makes the D -term inflation more robust against non-renormalizable corrections to W , since the new charge assignment prevents, by symmetry, all terms of the form $S(\Phi_+\Phi_-)^n/M_{\text{P}}^{2n-2}$ [73, 454].

4. D -term inflation without cosmic strings

Several modifications of the original D -term model have also been proposed to avoid the formation of cosmic strings and therefore lift the constraint on the coupling constant, λ given in Eq. (254). These works were partly motivated by the fact that the model was thought incompatible with the observations because the cosmic strings had been found responsible for most of the temperature anisotropies (75%), in contradiction with observations ($\lesssim 10\%$) [63]. Other motivations include the embedding of D -term inflation in strings theory, where FI term are generically present, but at a much lower scale than that imposed by the COBE normalization. They either assume that extra symmetry or extra fields are present to render the defects unstable, or add couplings in the superpotential to produce the strings before or during inflation or add couplings to allow for a normalization to COBE with lighter cosmic strings (then reducing their impact on the CMB).

It is first possible to assume that the Higgs fields are charged not only under $U(1)_\xi$, but also under some additional non-abelian local symmetry [454]. For example, if Φ_- that takes a non-vanishing VEV, $\sqrt{\xi}$, is also a doublet under some other symmetry $SU(2)_a$, the

symmetry breaking at the end of inflation follows:

$$SU(2)_a \times U(1)_\xi \rightarrow U(1)' , \quad (260)$$

and no topologically stable cosmic strings form (though embedded strings would form, like during the electro-weak symmetry breaking). Rendering the cosmic strings unstable by enlarging the symmetries of the theory can also be achieved by assuming two additional fields [454, 638]. For example, if a second pair of Higgs fields, $\tilde{\phi}_\pm$, carry some charges identical to ϕ_\pm under $U(1)_\xi$. Thus the theory possesses an extra accidental global symmetry $SU(2)$, since the potential (written here in global SUSY) is given by ⁶⁸

$$V(\phi_\pm, \tilde{\phi}_\pm) = \frac{g^2}{2} \left(|\phi_+|^2 + |\tilde{\phi}_+|^2 - |\phi_-|^2 - |\tilde{\phi}_-|^2 - \xi \right)^2 + \lambda^2 S^2 \left(|\phi_+|^2 + |\tilde{\phi}_+|^2 + |\phi_-|^2 + |\tilde{\phi}_-|^2 \right) + \left| \phi_+ \phi_- - \tilde{\phi}_+ \tilde{\phi}_- \right|^2 , \quad (261)$$

which is now invariant under the exchange $\phi_+ \leftrightarrow \tilde{\phi}_+$ and $\tilde{\phi}_- \leftrightarrow \phi_-$. When the Higgs fields settle in the global minimum, a global $U(1)$ symmetry is left unbroken and the symmetry breaking at the end of inflation is of the form:

$$SU(2)_{\text{glob}} \times U(1)_\xi \rightarrow U(1)_{\text{glob}} , \quad (262)$$

producing again no topologically stable strings, but only embedded (or semi-local) strings. Note that their stability in this case depends on various parameters; the ratio of the scalar mass over the $U(1)_\xi$ gauge field mass, the existence of zero modes and the details of the symmetry breaking [638]. As a conclusion, it is not excluded that these objects have an interesting impact on cosmology. Both of these ingredients can be found in string theory [454, 638].

Other mechanisms have been proposed to eliminate the impact of cosmic strings by breaking the $U(1)$ symmetry before inflation. For example, authors in [642] modified the D -term superpotential with the introduction of a pair of vector-like fields [under $U(1)_\xi$] $\Psi, \bar{\Psi}$ and two singlets X, σ [642]

$$W = \lambda \Phi_+ \Phi_- + X(\Psi \bar{\Psi} - M) + \sigma \Phi_+ \Psi \quad (263)$$

⁶⁸ In [454], the same idea is proposed, though they employ only 1 extra Higgs fields ϕ' with a charge identical to ϕ_- .

Inflation then takes place with $\langle X \rangle = \langle \sigma \rangle = 0$, $\langle \Psi \rangle = \langle \bar{\Psi} \rangle = M$, therefore breaking $U(1)_\xi$ before or during inflation, and diluting the strings.

The Refs. [639, 643], explored various ways to reduce the impact of cosmic strings on the CMB, proposing the “sneutrino modified D -term inflation”. The superpotential is modified by introducing a coupling of the inflaton superfield S to the right-handed sneutrino N_R field [639, 643]

$$W = \lambda S \Phi_+ \Phi_- + \lambda_\nu \Phi H_u L + \frac{M_R}{2} N_R^2, \quad (264)$$

where the second and third terms are responsible for generating neutrino masses via see-saw mechanism. The model also requires a coupling through a non-renormalizable Kähler potential:

$$K_{\min} \rightarrow K_{\min} + \frac{c S^\dagger S N_R^\dagger N_R}{\Lambda^2}, \quad (265)$$

which affects the main conclusions of the model concerning the impact of cosmic strings via the predictions for the spectral index. The potential in Eq. (251) is modified by an additional term, $-\kappa s^2/2$, where $\kappa = (c - 1)M_R^2|N_R|^2/\Lambda^2$, if the coupling is assumed to be $\lambda \gtrsim 0.1$. For such large couplings, inflation is realized with $z \gg 1$ and the effective potential reduces to $V \simeq V_0 + \frac{g^4 \xi}{16\pi^2} \ln(s^2/\Lambda^2) - \kappa s^2/2$. For $c > 1$, this reduces the spectral index and allows for a better fit to the observations. In addition, the normalization of the power spectrum of primordial fluctuations is enhanced, leading to a lowering the energy scale of inflation, therefore a reducing the energy per unit length for cosmic strings.

Similarly, the “sneutrino D -term inflation” proposed in [644] assumed the sneutrino to be the inflaton. The superpotential is given by:

$$W = \frac{\lambda}{M_{\text{P}}} N_R^2 \Phi_+ \Phi_- + \lambda_\nu N_R H_u L + \frac{M_R}{2} N_R^2, \quad (266)$$

where N_R is assumed to be the lightest right handed (s)neutrino, with $U(1)_R$ charge $+1$ and no $U(1)_\xi$ charge. Therefore, the tree level D -terms in the potential of Eq. (248) are not affected, though the additional couplings will affect the radiative correction, and therefore the dynamical properties of the model. Even with a minimal Kähler potential, inflation is found to be successful in the regime, $M_R^2 N_R \ll g_\xi^2 \xi^2$. The model predicts an almost scale-invariant power spectrum, $n_s \simeq 1$, and the constraint from the cosmic string tension is relaxed as compared to the standard case.

5. F_D -term hybrid inflation

There are also models where both F and D -terms are contributing to the inflationary potential. A mixture of the F - and D -term inflation was proposed by [645, 646], built as an extension to the NMSSM. The model is constructed in such a way that the inflaton field S involved in a F -term like superpotential also generates the μ term of the MSSM, and it is also coupled to the right-handed neutrinos, generating the Majorana mass scale.

The symmetries of the model are also extended to $G_{SM} \times U(1)_\xi$, the additional abelian factor allowing for the presence of a FI term. This subdominant contribution to the D -terms is employed to control the decay rate of superheavy fields such as the waterfall fields X_i and the inflaton field into gravitinos. In this model, the potential is dominated by the F -terms. The renormalizable superpotential for the F_D -terms hybrid model is given by:

$$W = \kappa S (X_1 X_2 - M^2) + \lambda S H_u H_d + \frac{\rho_{ij}}{2} S N_i N_j + h_{ij}^\nu L_i H_u N_j + W_{\text{MSSM}}^{(\mu=0)}, \quad (267)$$

where $W_{\text{MSSM}}^{(\mu=0)}$ denotes the MSSM superpotential without the μ -term, S is the SM-singlet inflaton superfield, N_i are the right-handed Majorana neutrinos and $X_{1,2}$ is a chiral multiplet pair with opposite charges under some $U(1)_\xi$ gauge group. Consequently, the D -term contribution to the scalar potential is given by: $V_D = (g^2/8)(|X_1|^2 - |X_2|^2 - \xi)^2$. The soft SUSY-breaking sector can be obtained from Eq. (267) and reads:

$$-\mathcal{L}_{\text{soft}} = M_S^2 S^* S + \left(\kappa A_\kappa S X_1 X_2 + \lambda A_\lambda S H_u H_d + \frac{\rho}{2} A_\rho S \tilde{N}_i \tilde{N}_i - \kappa a_S M^2 S + \text{H.c.} \right), \quad (268)$$

where M_S , $A_{\kappa,\lambda,\rho}$ and a_S are soft SUSY-breaking mass parameters of order $M_{\text{SUSY}} \sim 1$ TeV.

The second term in Eq. (267) induces the μ -term when the scalar component of S acquires a VEV, $\mu = \lambda \langle S \rangle \approx \frac{\lambda}{2\kappa} |A_\kappa - a_S|$, where the VEVs of $H_{u,d}$ are neglected compared to the VEV of $X_{1,2}$. The third term in Eq. (267), $\frac{1}{2} \rho_{ij} S N_i N_j$, gives rise to an effective lepton-number-violating Majorana mass matrix, i.e. $M_S = \rho_{ij} v_S$. Assuming that ρ_{ij} is approximately SO(3) symmetric, viz. $\rho_{ij} \approx \rho \mathbf{1}_3$, one obtains 3 nearly degenerate right-handed neutrinos $\nu_{1,2,3R}$, with mass $m_N = \rho v_S$. If λ and ρ are comparable in magnitude, then the μ -parameter and the SO(3)-symmetric Majorana mass m_N are tied together, i.e. $m_N \sim \mu$, thus leading to a scenario where the singlet neutrinos $\nu_{1,2,3R}$ can naturally have TeV or electroweak-scale masses.

The renormalizable superpotential Eq. (267) of the model may be uniquely determined by imposing the continuous R symmetry: $S \rightarrow e^{i\alpha} S, L \rightarrow e^{i\alpha} L, Q \rightarrow e^{i\alpha} Q$ with $W \rightarrow$

$e^{i\alpha}W$, whereas all other fields remain invariant under an R transformation. Notice that the R symmetry forbids the presence of higher-dimensional operators of the form $X_1X_2N_iN_j/M_{\text{P}}$.

The crucial observation is that the superpotential Eq. (267) is symmetric under the permutation of the waterfall fields, $X_1 \leftrightarrow X_2$. This permutation symmetry persists, even after the spontaneous SUSY breaking of $U(1)_\xi$, since in the ground state, $\langle X_1 \rangle = \langle X_2 \rangle = M$. Hence, there is an exact discrete symmetry acting on the gauged waterfall sector, similar to the D -parity. In order to break this unwanted D -parity, a subdominant FI D -term is required. As a D -parity conservation, heavy particles with mass gM are stable and can be considered cosmologically bad, if they are overproduced after the end of inflation.

Furthermore, in order to avoid the SUGRA- η problem, the Kähler potential has to be chosen of the form $K = S^2 + \kappa_s |S|^4/4M_{\text{P}}^2$, where the Hubble induced mass correction to S field turns out to read $\pm 3\kappa_s H^2 S^2$ [645, 646], and the tree level potential is similar to F -term inflation models, reading in the limit $S \gg M$,

$$V_{\text{infl}} \simeq \kappa^2 M^4 \left[1 + \frac{1}{64\pi^2} (4\kappa^2 + 8\lambda^2 + 6\rho^2) \ln(|S|^2/M^2) \right] + M_S^2 S^2 - (\kappa a^2 M^2 S + \text{h.c.}) + \kappa^2 M^4 \frac{|S|^4}{2M_{\text{P}}^2}. \quad (269)$$

where the last term corresponds to the SUGRA correction assuming a minimal Kähler potential. The cosmological predictions of the model are typical of any F -term inflation with so many free parameters at disposal, it is always possible to get the desired spectral tilt and the power spectrum.

6. Embedding D -term models in string theory

Attempts were made to embed D -term models within an explicit SUSY $SU(6)$ model [65]. This will be discussed with other embedding of inflation in SUSY GUT below at section IV G. More recent modifications of the D -term models were motivated by the possibility that the model is generically realized at large field VEV. This opens up the importance of non-renormalizable corrections to the Kähler potential [73, 501, 647], or corrections which are motivated from string theory [648–650]. It was found that the dynamics of the model remains mostly unchanged when considering any non-renormalizable corrections of order M_{P}^{-2} in Kähler potential [73, 647]. It was observed in Ref. [73] that despite the new contributions to the potential, the non-observation of cosmic strings still requires a suppressed superpotential coupling constant $\lambda \lesssim 10^{-4}$.

The D -term inflation models arising from string theories have generated a lot of activity in the past (within weakly coupled string theories) [10, 636, 637, 651], and more recently within a brane description or with a more phenomenological approach studying the moduli corrections to the model. The original interest for D -term inflation was in part due to the observation that anomalous $U(1)$, generically appear in weakly coupled string theories with $\sum q_i < 0$. There are many problems to circumvent destabilizing the model by vacuum shifting due to $\sum q_i < 0$, or to generate the right amplitude for the FI term and the string coupling, see for discussion in [10, 636].

The D -term inflation has also been found to be the low energy description of brane inflation [454, 630, 631]. The existence of branes in string theory allows to construct a new class of inflationary models (see Sec. VIII), where the inflaton becomes a modulus describing the distance between two branes. The most studied examples are the $D3/D7$ and the $D3 - \overline{D3}$ [630, 652–654] brane systems. It has been shown that such a system give rise to a D -term model of inflation, the inter-brane distance possessing a flat direction at tree level, and the open string degrees of freedom between the two branes playing the role of the waterfall fields Φ_{\pm} , as one of them becomes tachyonic below a certain inter-brane separation, i.e. near the string length scale. The formation of cosmic (super)strings occurs at the end, in the form of $D1$ -branes or fundamental (F -)strings [452, 655, 656].

A more phenomenological approach is to study the modifications of D -term models when embedded in string inspired SUGRA [648–650]. These corrections arise from the coupling (at least gravitationally) to moduli fields, originating from the string theory compactification. A volume modulus can be stabilized using the following form of the potentials [649, 657, 658]:

$$W_{\text{mod}}(T, \chi) = W_0 + \frac{Ae^{-aT}}{\chi^b}, \quad K_{\text{mod}} = -3 \ln(T + \bar{T} - |\chi|^2 + \delta_{\text{GS}} V_2), \quad (270)$$

where T is a volume modulus, the modulus, χ , is a matter field introduced to allow for an additional contribution to the superpotential which is charged under a new $U(1)$ abelian symmetry, with V_2 its gauge superfield, and δ'_{GS} is the Green-Schwarz parameter. The moduli potential derived contains the D -terms of the form:

$$V_{\text{mod}}^{\text{D}}(T, \chi) = \frac{\{3\delta'_{\text{GS}}[1 + (a/b)|\chi|^2]\}^2}{8\text{Re}(f(T)) \exp^2(-K/3)}, \quad (271)$$

where $f_{\text{mod}}(T)$ is the gauge kinetic function for that sector. Interestingly, the above stabilization potential has a non-vanishing vacuum which can generate an effective FI term for

inflation, if T and χ are also charged under the $U(1)_\xi$, giving

$$\xi = \frac{3\delta_{\text{GS}}^\xi(1 + a/b|\chi|^2)}{4\text{Re}(T) - 2|\chi|^2} ,$$

with δ_{GS}^ξ the Green-Schwarz parameter of $U(1)_\xi$.

The form of the Kähler potential can be invariant under the shift symmetry of the inflaton field

$$K_{\text{infl}} = |S - \bar{S}|^2/2 + |\Phi_+|^2 + |\Phi_-|^2, \quad (272)$$

as the minimal potential would induce a SUGRA- η problem, due to non-vanishing F -terms of the moduli sector during inflation. However, if we assume that the total Kähler is simply the sum, $K = K_{\text{mod}} + K_{\text{infl}}$, the contribution from that sector to the (effective mass)² of the waterfall field would spoil the graceful waterfall exit from inflation. A total Kähler potential of the form:

$$K = -3\ln(T + \bar{T} - |\chi|^2 - K_{\text{infl}}/3 + \delta'_{GS}V_2 + \delta_{\text{GS}}^\xi V_1) , \quad (273)$$

would preserve the general behavior of the D -term inflationary model, as long as the amplitude for the moduli are small. The current observations constrain the parameter space of this model, leaving 3 classes of models, all predicting a spectral index below or close to unity. One last modification due to the moduli sector is the nature of the cosmic strings formed at the end of inflation, which are not being BPS anymore and potentially containing massive fermionic currents.

7. Hybrid inflation in $N = 2$ SUSY: P -term inflation

There are attempts to embed hybrid inflation in $N = 2$ SUSY [474, 659, 660]. In Ref. [474], the authors have unified F and D -terms within P -term inflation in the context of a global $SU(2, 2|2)$ superconformal gauge theory, which also corresponds to a dual gauge theory of $D3/D7$ branes [661]. The idea is to break the $SU(2, 2|2)$ symmetry down to $N = 1$ SUSY by adding the $N = 2$ FI terms. The bosonic part of the superconformal action is given by [474]

$$\mathcal{L} = D_\mu\Phi_3 D^\mu\Phi_3^* - \frac{1}{4}F_{\mu\nu}^2 + \frac{1}{2}\mathbf{P}^2 + D_\mu\Phi^A D^\mu\Phi_A + F^A F_A + g\Phi^A\sigma_A{}^B\mathbf{P}\Phi_B - 2g^2\Phi^A\Phi_A\Phi_3\Phi_3 . \quad (274)$$

where two complex scalar fields forming a doublet under $SU(2)$, Φ^A and $\Phi_A = (\Phi^A)^*$. $N = 2$ gauge multiplet consists of a complex scalar Φ_3 , a vector A_μ , (all singlets in $SU(2)$), a spin-1/2 doublet $\lambda^A = \varepsilon^{AB}\gamma_5 C \bar{\lambda}_B^T$ (gaugino) and an auxiliary field P^r , triplet in $SU(2)$. There is also a doublet of dimension 2 auxiliary fields, F^A with $F_A = (F^A)^*$. The covariant derivatives on the hyperderivatives are given by [474]:

$$D_\mu \Phi_A = \partial_\mu \Phi_A + ig A_\mu \Phi_A, \quad D_\mu \Phi^A = \partial_\mu \Phi^A - ig A_\mu \Phi^A. \quad (275)$$

After adding the $N = 2$ FI terms, the potential is given by:

$$V_{N=2}^P = 2g^2 \left[\Phi^\dagger \Phi |\Phi_3|^2 + \frac{1}{4} (\Phi^\dagger \sigma \Phi - \xi)^2 \right]. \quad (276)$$

where the terms in the potential arises after solving the equations of motion for the auxiliary fields, $P = -g\Phi^A(\sigma)_{AB}\Phi_B$ and $F^A = 0$. Note that the FI term proportional to ξ has already been added to the potential now.

Let us isolate the gauge singlet, $S = \Phi_3$, and rest of the charged fields, $\Phi_1 = \Phi_+$ ($\Phi_2^* = \Phi_-$) for the positively (negatively) charged scalars, the total potential can be written as [474]

$$V_{N=2}^P = 2g^2 \left(|S\Phi_+|^2 + |S\Phi_-|^2 + \left| \Phi_+ \Phi_- - \frac{\xi_+}{2} \right|^2 \right) + \frac{g^2}{2} \left(|\Phi_+|^2 - |\Phi_-|^2 - \xi_3 \right)^2. \quad (277)$$

It was observed that the P -term potential now corresponds to an $N = 1$ model, $V = |\partial W|^2 + \frac{g^2}{2} D^2$, with an appropriate superpotential and a D -term given by: $W = \sqrt{2}gS(\Phi_+ \Phi_- - \xi_+/2)$, $D = |\Phi_+|^2 - |\Phi_-|^2 - \xi_3$, where $\xi \equiv \sqrt{|\xi|^2} = \sqrt{\xi_+ \xi_- + (\xi_3)^2}$, $\xi_\pm \equiv \xi_1 \pm i\xi_2$ ⁶⁹.

It was noticed in [474] that the potential for $N = 2$ SUSY gauge theory at $\xi_+ = \xi_- = 0$, $\xi_3 = |\xi|$ are similar to the case of D -term inflation studied before in [628] with $W = \lambda S \Phi_+ \Phi_-$ and $D = |\Phi_+|^2 - |\Phi_-|^2 - \xi$, for which the potential is given by:

$$V_{N=2}^D = 2g^2 \left(|S\Phi_+|^2 + |S\Phi_-|^2 + |\Phi_+ \Phi_-|^2 \right) + \frac{g^2}{2} \left(|\Phi_+|^2 - |\Phi_-|^2 - \xi \right)^2. \quad (278)$$

with an assumption, $\lambda = \sqrt{2}g$. If instead, $\xi_+ = \xi_- = 2M^2 = \xi$, one recovers a potential of an F -term inflation model with, $W = \lambda S(\Phi'_+ \Phi'_- - M^2)$, and, $D = |\Phi'_+|^2 - |\Phi'_-|^2$, with a potential:

$$V_{N=2}^F = 2g^2 \left(|S\Phi'_+|^2 + |S\Phi'_-|^2 + |\Phi'_+ \Phi'_- - M^2|^2 \right) + \frac{g^2}{2} \left(|\Phi'_+|^2 - |\Phi'_-|^2 \right)^2. \quad (279)$$

⁶⁹ In $D3/D7$ brane construction the three FI terms ξ are provided by a magnetic flux triplet $\sigma(1 + \Gamma_5) F_{ab} \Gamma^{ab}$, where F_{ab} is the field strength of the vector field living on D7 brane in the Euclidean part of the internal space with $a = 6, 7, 8, 9$. The spectrum of $D3 - D7$ strings depends only on $|\xi|$ [474, 661–663].

The first term of the potential now coincides with the F -term inflation potential in $N = 1$ theory, proposed by [61], under the assumption that the gauge coupling $g = \lambda/\sqrt{2}$.

The P -term model could be coupled to $N = 1$ SUGRA and the inflationary potential is given by [454, 474]

$$V = 2g^2 e^{\frac{|S|^2}{M_p^2}} \left[|\Phi_+ \Phi_- - \xi_+/2|^2 \left(1 - \frac{S\bar{S}}{M_p^2} + \left(\frac{S\bar{S}}{M_p^2} \right)^2 \right) + |S\Phi_+|^2 + |S\Phi_-|^2 \right] + \frac{g^2}{2} \left(|\Phi_+|^2 - |\Phi_-|^2 - \xi_3 \right)^2. \quad (280)$$

The inflating trajectory takes place at $\Phi_+ = \Phi_- = 0$. After adding the 1-loop correction from gauge fields, one obtains [474]

$$V = \frac{g^2 \xi^2}{2} \left(1 + \frac{g^2}{8\pi^2} \ln \frac{|S^2|}{|S_c^2|} + f \frac{|S|^4}{2M_p^4} + \dots \right), \quad (281)$$

where $f = \frac{\xi_1^2 + \xi_2^2}{\xi^2}$, $0 \leq f \leq 1$ and \dots stands for terms $|S|^6/2M_p^6$ and higher order gravitational corrections. Special case $f = 0$ corresponds to D-term inflation, and $f = 1$ corresponds to F-term inflation. A general P-term inflation model has an arbitrary $0 \leq f \leq 1$. The running of the spectral index from blue to red and the amplitude of the perturbations are similar to what has been studied in [502].

G. Embedding inflation in SUSY GUTs

Embedding inflation within GUT has a long history. We give here an overview of some of the old attempts and the current status of some of these models, and then turn to more recent developments.

1. Inflation in non-SUSY GUTs

The original idea of inflation was built on $SU(5)$ GUT, and the motivation was to dilute the unwanted relics, i.e. GUT monopoles, besides predicting the universe as large and as homogeneous on the largest scales as possible. Guth [1] first suggested (see also [486, 664]) that the GUT phase transition is first order, driven by the potential of the GUT Higgs field Φ in the adjoint representation **24**. Once the finite temperature effects are taken into account, [486]

$$V^{\text{old}}(\phi, T) = -\frac{N\pi T^4}{90} - \frac{\mu^2 - \beta T^2}{2} \phi^2 - \alpha_i T \phi^3 + \gamma_i \phi, \quad (282)$$

where $\phi^2 \equiv \text{Tr}\Phi^2$ and α_i and γ_i are constants of the theory. Assuming an initial temperature much larger than μ and the GUT scale, the cooling of the universe induces the GUT phase transition $SU(5) \rightarrow G_{SM}$, which could be used to generate a false vacuum inflation if the universe was trapped in a local minimum of the potential, typically at $\phi = 0$. The false vacuum leads to the so called “old inflation”, which is terminated by the formation of bubbles of true vacuum, as the phase transition takes place. This idea was abandoned because it was plagued with many problems. In particular, inside a bubble of new vacuum, the energy density of the false vacuum is transferred into kinetic energy, inducing the bubble expansion and the collisions due to bubble walls. This leads to a highly inhomogeneous and anisotropic universe in strong contradiction with the observations.

In Refs. [3, 5], the “new inflation” scenario was proposed to avoid such problems and also implemented within $SU(5)$, as it was suggested that the field ϕ trapped in the false vacuum $\phi = 0$ slowly rolls down its potential described by the Coleman-Weinberg potential [352] at finite high temperature $T \gg M_X$, [486]

$$V^{\text{new}}(\phi, T) = \frac{9M_X^4}{32\pi^2} + \frac{5}{8}g^2T^2\phi^2 + \frac{25g^4\phi^4}{128\pi^2} \left(\ln \frac{\phi}{\phi_0} - \frac{1}{4} \right) + cT^4, \quad (283)$$

$$\simeq V(0) - \frac{\lambda}{4}\phi^4,$$

where the second expression describes the effective inflationary potential close to the origin when the temperature has dropped to $T \sim H$. In this expression, $V(0) = 9M_X^4/32\pi^2$ and $\lambda \simeq 25g^4/32\pi^2 (\ln H/\phi_0 - 1/4)$. This model evaded the old inflation problem but predicted the wrong amplitude of anisotropies $\delta\rho/\rho \sim 110\sqrt{\lambda} \gg 10^{-4}$.

An improved version of this idea followed, the Shafi-Vilenkin model and the Pi model [665–668], based on an $SU(5)$ theory containing an additional $SU(5)$ singlet χ , which is driving inflation in order to obtain an effective potential similar to Eq. (283) with an appropriate level of CMB temperature anisotropies. The theory is based on the following potential (Φ still represents the GUT Higgs in the adjoint, and H_5 represents the Higgs in the fundamental representation which is realizing the electro-weak breaking) [665] (see also [486, 669, 670] for recent reviews)

$$V^{\text{new}}(\chi, \Phi, H_5) = \frac{1}{4}a(\text{tr}\Phi^2)^2 + \frac{1}{2}b\text{tr}\Phi^4 - \alpha(H_5^\dagger H_5)\text{tr}\Phi^2 + \frac{\gamma}{4}(H_5^\dagger H_5)^2 - \beta H_5^\dagger \Phi^2 H_5$$

$$+ \frac{\lambda_1}{4}\chi^4 - \frac{\lambda_2}{2}\chi^2\text{tr}\Phi^2 + \frac{\lambda_3}{2}\chi^2 H_5^\dagger H_5. \quad (284)$$

The inflaton develops a Coleman-Weinberg potential due to its coupling to Φ and H_5 . Its precise expression is obtained by minimizing the above potential for Φ which settles the

system in the inflationary valley. Indeed, the breaking $SU(5) \rightarrow G_{SM}$ is realized in the usual $T_{24} \propto \text{Diag}(1, 1, 1, -3/2, -3/2)$ direction, the VEV of Φ being a function of that of χ because of the coupling λ_2 ,

$$\langle \Phi \rangle = \sqrt{\frac{2}{15}} \phi \text{Diag}(1, 1, 1, -3/2, -3/2), \quad \text{with} \quad \phi^2 = (2\lambda_2/\lambda_c)\chi^2. \quad (285)$$

($\lambda_c \equiv a + 7b/15$ represents the mixture of the Φ^4 terms in V .) Discarding the pure H_5 sector (relevant at the EW scale) and computing the masses of the triplet and doublet in H_5 that enter the Coleman-Weinberg formula, one can reduce the potential to $V(\phi, \chi)$ and then to the effective inflationary potential using Eq. (285) [665]

$$V_{\text{eff}}^{\text{new}}(\chi) = A\chi^4 \left[\ln \left(\frac{\chi}{\chi_0} \right) - \frac{1}{4} \right] + \frac{A\chi_0^4}{4}, \quad (286)$$

where χ_0 is the position of the minimum of $V_{\text{eff}}^{\text{new}}(\chi)$ and A is a function of the couplings λ_2 , λ_c and the gauge coupling g_5 . The system after inflation is trapped in the global minimum at $\chi = \chi_0$ and $\phi = \phi_0 = \sqrt{2\lambda_2/\lambda_c}$. The mass of the superheavy gauge bosons inducing the proton decay is proportional to ϕ_0 ,

$$M_X = \sqrt{\frac{5}{3}} \frac{g\phi_0}{2}, \quad (287)$$

Thus the phase of inflation take place at an energy close to the mass scale involved in the proton decay, $M_X \sim 2V_0^{1/4}$, and its stability constrains the inflationary scale.

The predictions for $SU(5)$ singlet inflation [3, 5, 665, 666, 668] is similar to that of the potential; $V = V_0[1 - \lambda_\chi(\chi/\mu)^4]$, with $\lambda_\chi = A \ln \chi/\chi_0$. The predictions depend on A or alternatively on V_0 , and for $V_0^{1/4} \in [2 \times 10^{15}, 4 \times 10^{16}]$ GeV, they are found in the range: $n_s \in [0.93, 0.96]$, $r \in [10^{-5} - 10^{-1}]$, $\alpha_s \in [0.6, 1.3] \times 10^{-3}$. These predictions are found within the 2σ limit of the WMAP data provided that V_0 is large, [670, 671], though this requires that inflation taking place for super Planckian VEVs [670]. A large scale $V_0 \sim 10^{16}$ GeV also implies a proton lifetime roughly estimated in the range $\tau(p \rightarrow \pi^0 e^+) \in [10^{34}, 10^{38}]$ years [670]. It was also proposed in Ref. [670] to realize a chaotic-like inflation with the same potential but at large field $\chi > \chi_0$, but this also imposes super Planckian VEVs. Taking the VEV above the Planck scale does not make any physical meaning as it would be hard to rely on the Coleman-Weinberg one-loop corrected potential away from the renormalization scale.

Other challenges for this model makes it unappealing or unrealistic. The analysis of the parameter space (a, b, g) in [672] revealed that the potential possess a local minimum with symmetries $SU(4) \times U(1)$ in which the system gets trapped, even when starting close to the SM minimum (with symmetries $SU(3) \times SU(2) \times U(1)$). Obtaining the standard model at low energy is therefore only possible after another first order transition that leads to the problems of old inflation. Other problems were found related to the reheating and the generation of baryon asymmetry (see for e.g. [486]) ⁷⁰.

2. Hybrid inflation within SUSY GUTs and topological defects

The embedding of inflation in SUSY GUTs has mostly been studied for hybrid inflation as those models consider a coupling between the inflaton sector and the GUT Higgs sector. It was therefore natural to first inquire if some GUT Higgs field already present in theories which can play the role of the waterfall fields Φ_{\pm} . For the D -term model, the presence of a constant FI term would require that the group $U(1)_{\xi}$ is not the subset of a non-abelian group [63]. As a consequence, if the SM is embedded in a model based on the group G_{GUT} , the whole theory is based on $G_{\text{GUT}} \times U(1)_{\xi}$ and inflation takes place in the chain [63],

$$G_{\text{GUT}} \times U(1)_{\xi} \rightarrow \dots H \times U(1)_{\xi} \xrightarrow{\text{Infl}} H \rightarrow \dots \rightarrow G_{\text{SM}} , \quad (288)$$

where ‘‘Infl’’ identify the symmetry breaking that triggers the waterfall at the end of inflation. This represents the lowest possible level of embedding of inflation within SUSY GUTs, since both the inflaton field and the Higgs fields are introduced in addition to the field content motivated by the particle physics.

⁷⁰ One can also think of realizing a brief period of thermal inflation close to the electroweak scale with the help of the GUT singlet [156, 157]. The idea is that thermal effects will keep the singlet scalar field close to the symmetric phase, but once inflation begins, the universe would cool down and the field would roll down the potential and settle in its minimum, which is close to the GUT scale. The simplest potential will be like: $V \sim V_0 - m^2 \phi^2$. The idea is appealing, but challenging to realize. The model requires a very light GUT singlet, i.e. $m \sim 100$ GeV to be kept in thermal equilibrium with other light degrees of freedom to get $+T^2 \phi^2$ correction. However the singlet should break the GUT group down to the SM when it develops a VEV and excite the SM degrees of freedom alone at the electroweak scale. If $V_0^{1/4} \sim 10^{16}$ GeV, then the initial temperature of the universe prior to such a phase transition ought to be very high fairly close to the GUT temperatures and final temperature should be close to the electroweak scale to get sufficient inflation, i.e. roughly 7 – 10 e-foldings of inflation, which is sufficient to dilute the unwanted relics such as excess gravitons or damping the moduli oscillations, etc. To our knowledge there is no explicit light GUT singlet which has been constructed to execute this idea.

The F -term inflation model does not contain the restriction due to the presence of a FI term, therefore its embedding within SUSY GUT has a richer phenomenology. The superfields Φ and $\bar{\Phi}$ are assumed to belong to a non-trivial representation and the complex conjugate representation of some group G_{infl} , whereas S is a singlet of G_{infl} , in such a way that $W = \kappa S(\Phi\bar{\Phi})$ is invariant under G_{infl} .

The general picture is when inflation takes place, within the cascade of spontaneous symmetry breaking induced by the Higgs sector is therefore [63],

$$G_{\text{GUT}} \rightarrow \dots H \times G_{\text{infl}} \xrightarrow{\text{Infl}} H \rightarrow \dots \rightarrow G_{\text{SM}} . \quad (289)$$

Cosmological observations can constrain what G_{infl} can and cannot be, i.e. one of the motivations to introduce inflation in GUT was to solve the monopole problem. Therefore, the breaking of G_{infl} , and all subsequent SUSY breaking cannot give rise to the formation of monopoles. Following the argument of Ref. [64] for $SO(10)$, a systematic study has been done in Ref. [436] for all possible models that can be constructed using SUSY GUTs based on all possible GUT groups of rank lower than 8 (including $SO(10)$, $SU(n)$, E_6). It was assumed that a discrete Z_2 symmetry is left unbroken at low energies to protect the proton from a too rapid decay. It was shown that generically the waterfall at the end of inflation gives rise to the formation of cosmic strings, since almost all possible ways to break G_{GUT} down to G_{SM} involve the generation of a $U(1)$ symmetry (leading to monopoles) or its breaking (leading to cosmic strings).

In addition, for an $SO(10)$ or E_6 based models, the waterfall accompanies generically the breaking of the $U(1)_{B-L}$ symmetry [436]. Consequently, the Higgs coupled to the inflaton can also be involved in the see-saw mechanism. Assuming an $SO(10)$ model that preserves the R-parity, and using a minimum number of fields to realize this breaking, this is realized generically employing a pair $\Phi = \mathbf{126}, \bar{\Phi} = \overline{\mathbf{126}}$ [437], though $\Phi = \mathbf{16}, \bar{\Phi} = \overline{\mathbf{16}}$ is also possible, and does not preserve the R-parity. Since the $U(1)_{B-L}$ symmetry breaking scale is also commonly used to generate the Majorana mass for right-handed neutrinos, this opens up the possibility to combine constraints from neutrino measurement and CMB constraints on the parameter space in specific models with these ingredients.

3. Embedding inflation within GUT

Several embeddings of F -term models of inflation in a specific and realistic SUSY GUT model have been proposed, see Refs [72, 75, 673, 674]. Most of these references [72, 75, 673] have considered SUSY $SO(10)$ models that are capable of accounting for enough proton stability, a (semi-)realistic mass matrix for fermions, and a doublet-triplet (D-T) splitting usually through the Dimopoulos-Wilczek mechanism. We will discuss such models in this section and also discuss a more minimal model of $SO(10)$ [674] in the next subsection.

Imposing that the R-parity is unbroken at low energy is usually assumed in order to protect the proton from a too rapid decay through dimension 4 operators involving sparticles (see Sec. III F). $SO(10)$ contains a Z_2 symmetry subgroup of $U(1)_{B-L} \subset SU(4)_C \subset SO(10)$ that can play this role provided only “safe” Higgs representations are employed to realize $SO(10) \rightarrow G_{SM}$, namely via **10**, **45**, **54**, **126**, $\overline{\mathbf{126}}$, **210**, etc. Therefore, in Ref. [673], the model has the following field content; two pairs of adjoint **45** and **54** denoted $A_{45}, A'_{45}, S_{54}, S'_{54}$ are assumed to break $SO(10)$ down to $3_c 2_L 1_R 1_{B-L}$ while a pair $\mathbf{126} + \overline{\mathbf{126}}$ denoted $\Phi + \bar{\Phi}$ is used to break the $B - L$ symmetry and obtain the MSSM $G_{SM} \times Z_2^R$. An additional pair (H, H') of fundamental **10** are assumed for the electroweak symmetry breaking and one last **45** denoted A'' is also required to avoid dangerous light degrees of freedom.

Finally F -term hybrid inflation is realized adding an $SO(10)$ singlet \mathcal{S} to this field content. The superpotential then contains 5 sectors, the first two implementing the breaking of $SO(10)$ and the electroweak symmetry breaking respectively, and the D-T splitting at the same time, [673]

$$W_1 = m_A A^2 + m_S S^2 + \lambda_S S^3 + \lambda_A A^2 S, \quad W_2 = H A H + m'_H H'^2. \quad (290)$$

The third sector is a replica of the first sector for the field S' and A' . If each sector 1 and 3 can break individually $SO(10)$ down to $3_c 2_L 2_R 1_{B-L}$, their combination can break $SU(2)_R$ further down to $U(1)_R$. The fourth sector breaks the $B - L$ symmetry dynamically, realizing the F -term hybrid inflation at the same time using

$$W_4 = \kappa \mathcal{S} (\Phi \bar{\Phi} - M^2). \quad (291)$$

It is argued that though the inflaton was not required to break the B-L symmetry, its presence can be motivated by the fact that then the symmetry breaking is realized dynamically and

at a scale close to the GUT scale. One last sector has to be introduced to avoid massless goldstone bosons $W_5 = AA'A''$. The VEV in the appropriate direction for all those fields can break $SO(10)$ to the $MSSM \times Z_2$ and it is clear that at least one of the global minima of the potential emerging from, $W_1 + W_2 + W_3 + W_4 + W_5$, possess the right symmetries. However it seems unclear from what part of the initial conditions in field space the dynamical evolution of the system would indeed end up on the inflationary valley and give rise to F -term inflation. Indeed the risk is the presence of tachyonic instabilities in this multi-dimensional potential that would destabilize the dynamics and avoid the inflationary valley.

Another problem that may arise is the stability of the superpotential, since many other terms are allowed by the symmetries of the model, for example coupling the inflaton to itself or with other fields would have potential to ruin the inflationary success, as it would introduce mass terms or quartic terms for the inflaton potential destroying the flat direction of the potential ensured by the linearity of W in \mathcal{S} . Note however that the non-renormalizability theorem protects the form of a given superpotential against the generation of other terms from radiative corrections.

A similar attempt to embed F -term hybrid inflation in the Barr-Raby model has also been described in the appendix of [72] with a different field content; 1 adjoint, $A = \mathbf{45}$, instead of 2, the same pair of fundamental (H, H') , the breaking of $B - L$ symmetry employing one of the two pairs of $\mathbf{16}, \overline{\mathbf{16}}$ (therefore breaking the R-parity), and 4 singlets instead of 1 are assumed. The D-T splitting is then realized giving a VEV along B-L to the adjoint and the presence of the singlets is required to avoid the presence of massless pseudo-goldstone bosons. For the symmetry breaking, like in the previous model, the VEVs of the adjoint and one pair of spinors is enough to break $SO(10)$ down to the MSSM, the inflaton S being again coupled to the pair of spinors $\mathbf{16}, \overline{\mathbf{16}}$ that breaks $B - L$. The superpotential of the theory contains non-renormalizable terms and reads:

$$\begin{aligned}
W = & S(\Phi\bar{\Phi} - M^2) + \frac{\alpha}{4M_S}A^4 + \frac{1}{2}M_A A^2 + T_1 A T_2 + M_T T_2^2 + \bar{\Phi}' \left[\zeta \frac{PA}{M_S} + \zeta_Z Z_1 \right] \Phi \\
& + \bar{\Phi} \left[\xi \frac{PA}{M_S} + \xi_Z Z_1 \right] \Phi' + M_\Phi \bar{\Phi}' \Phi' .
\end{aligned} \tag{292}$$

where the spinors are denoted by: $\Phi, \bar{\Phi}, \Phi', \bar{\Phi}'$ and S, P, Z_1, Z_2 being the 4 gauge singlets. Plugging in the VEV required to achieve the symmetry breaking of $SO(10)$ allows to identify which of the components of $\mathbf{16} + \overline{\mathbf{16}}$ stay light during inflation, and which are coupled to the GUT Higgs fields and thus acquire a GUT scale mass. This is important for the predictions of

the F -term hybrid inflation model, since only the light states will contribute to the radiative corrections and to the effective \mathcal{N} in front of the \ln term (see Sec. IV E 2 and Eq. (222)). In this particular model, Φ is a 16-dimensional spinor, only two of its components will remain light, although having such light states is rather model dependent issue.

Finally, an embedding of shifted hybrid inflation in a very elaborated $SO(10)$ model has been proposed in [75], and an embedding of D -term hybrid model has been proposed in [65].

4. Origin of a gauge singlet inflaton within SUSY GUTs

It is desirable to seek an answer to the question, whether F -term hybrid inflation *coupled* or *embedded* within SUSY GUTs? Independently of the assumed GUT group, because of the form of the superpotential Eq. (219), the inflaton superfield, S , is necessarily an *absolute gauge singlet*, since SM^2 cannot be a gauge invariant if S is not a singlet superfield [674].

To fully embed the inflation model within SUSY GUTs, one could generate M by the VEV of some other field, though this has not yet been done so far, and attempts in this direction within the minimal $SO(10)$ model of [433] are unsuccessful [674].

These models are based on a minimal number of superfields, and a protected R-parity at low energies. It contains only a $\Sigma \equiv \mathbf{210}$, a pair $\Phi \equiv \mathbf{126}, \bar{\Phi} \equiv \overline{\mathbf{126}}$ and a fundamental $H = \mathbf{10}$, and the most general superpotential with this field content given by [433]:

$$W_{\min}^{SO(10)} = m_{\Sigma}\Sigma^2 + m_{\Phi}\Phi\bar{\Phi} + \lambda_{\Sigma}\Sigma^3 + \eta\Sigma\Phi\bar{\Phi} + m_H H^2 + \Sigma H(\alpha\Phi + \bar{\alpha}\bar{\Phi}) . \quad (293)$$

One can show that any VEV that would generate the mass term M^2 in the inflaton superpotential would also generate a mass for the inflaton field [674]. As a conclusion it is not possible to generate *exactly* the F -term model, at least within this minimal version of $SO(10)$.

Another concerning issue is the stability of the superpotential [674]. The form of the superpotential of Eq. (219) is supposedly protected by a $U(1)_R$ symmetry, under which S is doubly charged, while Φ and $\bar{\Phi}$ have opposite charges. This property is important for example to prevent contributions to W such as quadratic or cubic terms in the inflaton superfield, that would spoil the flatness of the inflationary potential. Furthermore, assigning R -charges to obtain flatness of the inflaton potential in presence of the Higgs sector is also challenging. For example, the minimal $SO(10)$ model of [433], whose most general

superpotential cannot be invariant under such an R -symmetry. Trying to accommodate a phase of F -term inflation in such model would receive destabilization from various sources.

- Quadratic and cubic terms in the inflaton potential, which would a-priori ruin the flatness of the potential. It is possible to arrange the parameters to realize a saddle point or an inflection point inflation. More work is required in this direction.
- Additional couplings between the inflaton, S , and other singlets (including $S\Phi_i^2$ for all the superfields Φ_i in the theory), which would demand additional assumptions such that these Φ_i superfields remain heavy during inflation via their couplings.
- Like the original hybrid model, those additional couplings would generate dangerous quadratic and cubic terms at one-loop level, even if they were assumed vanishing at the tree level. One would have to ensure that extra terms do not spoil the flatness of the potential.

Note that these stability problems are also present in earlier attempts to embed F -term hybrid inflation in SUSY GUTs. If these stability issues are left aside, it is interesting to note that among all possible global minima of the minimal $SO(10)$ theory, one and only one of them can accommodate a phase of F -term inflation, when the stability of the potential against VEV shifting and the formation of topological defect formation are taken into account [674].

5. Other inflationary models within SUSY GUTs

Inspite of the challenge to seek an origin of a gauge singlet inflaton within the GUT group, there have been many examples to embed the waterfall field within SUSY GUTs. Here we briefly sketch some examples of the embedding.

- Shifted and smooth hybrid inflation:

As we discussed in Sec. IV E 7, the shifted hybrid inflation model was built embedded in the Pati-Salam subgroup $G_{PS} \equiv SU(4)_C \times SU(2)_L \times SU(2)_R$ of $SO(10)$ [66, 675]. The idea is to break the gauge group during inflation in a single step which also avoids the monopole problem arising in the breaking of the Pati-Salam group down to the MSSM group, breaking that can be done using a pair of Higgs superfields,

$H^c = (\bar{\mathbf{4}}, \mathbf{1}, \mathbf{2})$ and $\bar{H}^c = (\mathbf{4}, \mathbf{1}, \mathbf{2})$. These Higgs fields also give through their VEVs a Majorana mass to the right-handed neutrino directions, inducing the see-saw mechanism to account for the tiny mass of the left-handed neutrinos. This pair of Higgs fields is also assumed to be the one coupled to the inflaton field, realizing the symmetry breaking during inflation, and supporting inflation through their F -terms. The vanishing of the D -term part of the potential constraints, $|\bar{H}^c| = |H^c|$. The inflationary potential is embedded in a potential where there are many more fields, which are relevant for the computation of one-loop corrected inflationary potential. The actual potential also is invariant under the global $U(1)_R$ of conventional SUSY and under a Peccei-Quinn symmetry, $U(1)_{PQ}$. Like for the original F -term inflation, the R -symmetry prevents from non-linear terms for S , such as S^2, S^3 terms in the superpotential, which would ruin the flat direction for the inflaton at tree level.

In Ref. [75], the authors embedded this shifted model within an explicit model based on $SO(10)$ and also implemented the Dimopoulos-Wilczek mechanism [429], and addressed the MSSM μ -problem. The inflationary superpotential is given by [66, 75]

$$\begin{aligned}
W_{\text{infl}} &\approx -\kappa S \left[M_{B-L}^2 - \mathbf{16}_H \bar{\mathbf{16}}_H + \frac{\rho}{\kappa M_*^2} (\mathbf{16}_H \bar{\mathbf{16}}_H)^2 \right. \\
&\quad \left. - \frac{\kappa_1}{\kappa} P \bar{P} + \frac{\rho_1}{\kappa M_*^2} (P \bar{P})^2 - \frac{\kappa_2}{\kappa} Q \bar{Q} + \frac{\rho_2}{\kappa M_*^2} (Q \bar{Q})^2 \right] \\
&\equiv -\kappa S M_{\text{eff}}^2,
\end{aligned} \tag{294}$$

where a gauge singlet S is the inflaton, $\mathbf{16}_H, \bar{\mathbf{16}}_H$, additional singlets are P, \bar{P}, Q , and \bar{Q} are displaced from the present values, for which two eigenvalues can potentially act like a moduli, while two of them are heavy as the GUT scale with a large VEV. The dimensionless constants are κ, κ_i, ρ [75].

It is assumed that initially, $|\langle S \rangle|^2 \approx M_{B-L}^2 [1/(4\zeta) - 1]/2$, with $1/4 < \zeta < 1/7.2$, and $\langle \mathbf{16}_H \rangle, \langle \bar{\mathbf{16}}_H \rangle, \langle P \rangle, \langle \bar{P} \rangle, \langle Q \rangle, \langle \bar{Q} \rangle \neq 0$, where $M_{\text{eff}}^2 \sim M_{B-L}^2$. With $D_S W \approx -\kappa M_{\text{eff}}^2 (1 + |S|^2/M_P^2)$, the F -term part of the potential is given by:

$$V_F \approx \left(1 + \sum_k \frac{|\phi_k|^2}{M_P^2} + \dots \right) \left[\kappa^2 M_{\text{eff}}^4 \left(1 + \frac{|S|^4}{2M_P^4} \right) + \left(1 + \frac{|S|^2}{M_P^2} + \frac{|S|^4}{2M_P^4} \right) \sum_k |D_{\phi_k} W|^2 \right], \tag{295}$$

where all scalar fields except S contribute to ϕ_k . The factor $(1 + \sum_k |\phi_k|^2/M_P^2 + \dots)$ in front originates from e^{K/M_P^2} . Provided, $|D_{\phi_k} W|/M_P$ are much smaller than the

Hubble scale ($\sim \kappa M_{\text{eff}}^2/M_{\text{P}}$), the flatness of S will be guaranteed even after including SUGRA corrections. This can be realized by the choice $\kappa \ll 1$, and $\kappa/\rho \ll 1$. The $U(1)$ R -symmetry ensures the absence of terms proportional to S^2 , S^3 , etc. in the superpotential, which otherwise could spoil the slow-roll conditions. The spectral tilt arising from this model is very close to one, i.e. $n_s = 0.99 \pm 0.01$ and $|dn_s/d \ln k| \ll 0.001$.

The smooth hybrid inflation model was also originally built within $SO(10)$ [62, 675] though to our knowledge this model has not been embedded into a specific SUSY GUTs model. Its proximity to the shifted model allows us to guess that there should not be any problem as long as an additional singlet is assumed to play the role of the inflaton. As mentioned earlier in this chapter, these two models can also be realized without the non-renormalizable superpotential terms leading to the new shifted [69], and the new smooth hybrid inflation [82] models, at the cost of adding more fields in the picture, see Sec.IV E 7.

- Flipped $SU(5)$:

Inflation in flipped $SU(5)$ ($= SU(5) \times U(1)_X$), which is a maximal subgroup of $SO(10)$ with a chiral superfield in the spinorial representation $\mathbf{16}$ per family, was also considered in [77]. The breaking of $SU(5) \times U(1)_X$ to the MSSM gauge group happens when $\mathbf{10}_H$ and $\overline{\mathbf{10}}_H$ develops a VEV. The relevant superpotential is given by [77]

$$W = \kappa S [\mathbf{10}_H \overline{\mathbf{10}}_H - M^2] + \lambda_1 \mathbf{10}_H \mathbf{10}_H \mathbf{5}_h + \lambda_2 \overline{\mathbf{10}}_H \overline{\mathbf{10}}_H \overline{\mathbf{5}}_h \quad (296)$$

$$+ y_{ij}^{(d)} \mathbf{10}_i \mathbf{10}_j \mathbf{5}_h + y_{ij}^{(u,\nu)} \mathbf{10}_i \overline{\mathbf{5}}_j \overline{\mathbf{5}}_h + y_{ij}^{(e)} \mathbf{1}_i \overline{\mathbf{5}}_j \mathbf{5}_h,$$

where $\kappa, \lambda_1, \lambda_2$ are constants and the appropriate Yukawas are given by Y_{ij} , i.e. $Y_{ij}^{(\mu,\nu)}$, provide masses to up-type quarks and neutrino Dirac masses. The $U(1)_R$ symmetry eliminates terms such as S^2 and S^3 from the superpotential. Higher dimensional baryon number violating operators such as $\mathbf{10}_i \mathbf{10}_j \mathbf{10}_k \overline{\mathbf{5}}_l \langle S \rangle / M_{\text{P}}^2$, $\mathbf{10}_i \overline{\mathbf{5}}_j \overline{\mathbf{5}}_k \mathbf{1}_l \langle S \rangle / M_{\text{P}}^2$, etc. are suppressed as a consequence of $U(1)_R$. The proton decay proceeds via dimension six operators mediated by the superheavy gauge bosons. The dominant decay mode is $p \rightarrow e^+/\mu^+, \pi^0$ and the estimated lifetime is of order 10^{36} yrs. [676, 677]. This Z_2 ensures that the LSP is absolutely stable. Inflation happens for $\kappa \leq 10^{-2}$

and matches the standard predictions, i.e. $n_s \approx 1$ with negligible gravity waves and running of the spectral tilt [74, 77] for $M \sim 10^{16}$ GeV.

- 5D $SO(10)$:

Inflationary models from 5D $SO(10)$ were also constructed in Refs. [76, 678]. There are certain advantages of orbifold constructions of five dimensional (5D) SUSY GUTs, in which $SO(10)$ can be readily broken to its maximal subgroup H [679–681], with the doublet-triplet splitting problem addressed due to construction where $SO(10)$ is compactified on $S^1/(Z_2 \times Z'_2)$ (where Z_2 reflects, $y \rightarrow -y$, and Z'_2 reflects $y' \rightarrow -y'$ with $y' = y + y_c/2$). The two orbifold points are $y = 0$ and $y = y_c/2$). In order to realize inflation in 4D, the $N = 2$ SUSY in the 5D bulk is broken down to $N = 1$ SUSY on the orbifold fixed points, below the compactification scale π/y_c , where the branes are located.

The F -term inflation potential can be constructed on the branes, assuming that the inter brane separation is fixed. The two branes preserve different symmetries, on one the full $SO(10)$ is preserved while in the other $SU(4)_c \times SU(2)_L \times SU(2)_R$ [682]. There also exists a bulk scalar field, S , which couples to the singlets of $SO(10)$ on the branes. For instance, on the brane where $SO(10)$ is preserved, the superpotential would be given by: $W = \kappa S(Z\bar{Z} - M_1^2)$, while in the second brane $W = \kappa S(\phi\bar{\phi} - M_2^2)$, where $\phi, \bar{\phi}$ belong to $(\bar{\mathbf{4}}, \mathbf{1}, \mathbf{2})$ and $(\mathbf{4}, \mathbf{1}, \mathbf{2})$ [678]. Similar constructions were made in Ref. [76] where on one brane $SU(5) \times U(1)_X$ and on the other $SU(5)' \times U(1)'_X$ were preserved, inflationary potential arises to the breaking of $U(1)$ at a scale close to the GUT scale.

In all the above examples, see Refs. [66, 69, 75, 76, 82, 675, 678, 683], it is possible to excite non-thermal leptogenesis either from the direct decay of the inflaton or the Higgs coupled to it or from non-perturbative excitation from the coherent oscillations of the inflaton.

6. Inflation, neutrino sector and family replication

The right handed Majorana sneutrino as an inflaton has been proposed as a particle physics candidate for inflation [494, 684]. These initial models were based on a chaotic type

potential for the inflaton with a superpotential ⁷¹

$$W = \frac{1}{2}MN_iN_i + \mu H_u H_d + h^{ij}N_iL_jH_u + k^{ij}e_iL_jH_d, \quad (297)$$

where the right handed Majorana neutrino superfield, N , has been treated as a gauge singlet. In order to avoid higher order contributions such as N^3 term in the superpotential, the right handed neutrino can be assigned odd under R-parity. The lightest right handed electron sneutrino acts as an inflaton with an initial VEV larger than M_{P} . After the end of inflation the coherent oscillations of the sneutrino field generates lepton asymmetry with the interference between tree-level and one-loop diagrams. The largest lepton asymmetry is proportional to the reheat temperature, $n_L/s \sim \epsilon(3T_{\text{R}}/4M_{\text{P}})$. The CP asymmetry, $\epsilon \sim (\ln 2/8\pi)\text{Im}h_{33}^{*2}$. Models of inflation and non-thermal leptogenesis were also considered in Refs. [332, 685, 686].

Sneutrino hybrid inflation was constructed in Refs. [78, 79]. In [78] the following superpotential has been used to generate inflation and the masses for the right handed neutrinos:

$$W = \kappa S \left(\frac{\Phi^4}{M'^2} - M^2 \right) + \frac{(\lambda_N)_{ij}}{M_*} N_i N_j \Phi \Phi + \dots, \quad (298)$$

where $\kappa, (\lambda_N)_{ij}$ are dimensionless Yukawa couplings, M, M', M_* are three independent mass scales. Φ, S, N are all gauge singlets. The waterfield is Φ generates masses for the right handed sneutrinos. During inflation N can take large VEVs as Φ is stuck at the zero VEV. The Kähler potential was given by [78]:

$$K = |S|^2 + |\phi|^2 + |N|^2 + \kappa_S \frac{|S|^4}{4M_{\text{P}}^2} + \kappa_N \frac{|N|^4}{4M_{\text{P}}^2} + \kappa_\phi \frac{|\phi|^4}{4M_{\text{P}}^2} \\ + \kappa_{S\phi} \frac{|S|^2|\phi|^2}{M_{\text{P}}^2} + \kappa_{SN} \frac{|S|^2|N|^2}{M_{\text{P}}^2} + \kappa_{N\phi} \frac{|N|^2|\phi|^2}{M_{\text{P}}^2} + \dots, \quad (299)$$

and the F -term potential is given by:

$$V = \kappa^2 \left(\frac{|\phi|^4}{M'^2} - M^2 \right)^2 \left(1 + (1 - \kappa_{S\phi}) \frac{|\phi|^2}{M_{\text{P}}^2} + (1 - \kappa_{SN}) \frac{|N|^2}{M_{\text{P}}^2} - \kappa_S \frac{|S|^2}{M_{\text{P}}^2} \right) \\ + \frac{4\lambda_N^2}{M_*^2} (|N|^4|\phi|^2 + |N|^2|\phi|^4) + \dots, \quad (300)$$

Inflation is driven by the singlet field, S , where ϕ has a zero VEV. By virtue of the coupling between ϕ and N , the N field remains massless during inflation and therefore subject to

⁷¹ We are using the superfield and the field notation to be the same for the right handed neutrinos, i.e. N .

random fluctuations during inflation. The authors assumed that the last 60 e-foldings of inflation arises when the sneutrino field is rolling slowly down the potential given by:

$$V \approx \kappa^2 M^4 \left(1 + (1 - \kappa_{SN}) \frac{|N|^2}{2M_{\text{P}}^2} + \delta \frac{|N|^4}{4M_{\text{P}}^4} \right) + \dots, \quad (301)$$

where $\delta = 0.5 + \kappa_{SN}^2 - \kappa_{SN}\kappa_N + 1.25\kappa_N + \dots$. The predictions of the model is typical of a hybrid inflation with a nearly flat spectrum. The WMAP data constraints parameters such as $|(1 - \kappa_{SN})| \leq 0.02$, also the running in the spectral tilt and tensor to scalar ratio are negligible. The model also generates isocurvature fluctuations, since both S and N in principle can be light during inflation, but they have assumed that S obtains a heavy mass and therefore settled down to its minimum in one Hubble time or so during inflation [78].

Hybrid inflation model with Dirac neutrinos was constructed in Refs. [79, 687, 688], in a specific Type-I string theory with the help of anisotropic compactification [688]. The model explains the smallness of the μ -term, strong-CP and the Dirac nature of neutrinos. The relevant part of the superpotential for inflation is given by:

$$W = \lambda\phi H_u H_d + \kappa\phi N^2, \quad (302)$$

where $\lambda \sim \kappa \sim 10^{-10}$. Including the soft SUSY breaking terms, the F -term potential is given by [79, 687]

$$V = V_0 + \lambda A_\lambda \phi H_u H_d + \kappa A_\kappa \phi N^2 + \text{h.c.} + m_0^2 (|H_u|^2 + |H_d|^2 + |N|^2) + m_\phi \phi^2, \quad (303)$$

where the origin of V_0 arises from the Peccei-Quinn breaking scale, $\Lambda = 2\pi f_a$, where $10^{10} \leq f \leq 10^{13}$ GeV.

The phase transition associated with the spontaneous breaking of family symmetry, by flavons, which is responsible for the generation of the effective quark and lepton Yukawa couplings could also be responsible for inflation [84].

In order to understand the origin of fermion masses and mixing, one can extend the SM by some horizontal family symmetry G_F , which may be continuous or discrete, and gauged or global. It must be broken at high scales with the help of flavons, ϕ , whose VEV will break the family symmetry. The Yukawa couplings are generically forbidden by the family symmetry G_F , but once it is broken, effective Yukawa couplings can be generated by non-renormalizable operators, i.e. $(\phi/M_c)^n \psi \psi^c H$. This gives rise to an effective Yukawa coupling $\varepsilon^n \psi \psi^c H$, where $\varepsilon = \langle \phi \rangle / M_c \sim \mathcal{O}(0.1)$ and ψ, ψ^c are SM fermion fields, H is a SM Higgs field, and M_c is the messenger scale.

The relevant superpotential for inflation can be given by [84, 689]:

$$W = \kappa S \left[\frac{(\phi_1 \phi_2 \phi_3)^n}{M_*^{3n-2}} - \mu^2 \right] \quad (304)$$

for $n \geq 1$ and $\kappa \sim 1$. The fields, $\phi = (\phi_1, \phi_2, \phi_3)$ is in the fundamental triplet $\underline{3}$ representation of A_4 or Δ_{27} . At the global minimum of the potential, the ϕ_i components get VEVs of order $M = M_* \left(\frac{\mu}{M_*} \right)^{2/3n}$ ⁷².

The Kähler potential is of the non-minimal form [84, 689]

$$K = |S|^2 + |\phi|^2 + \kappa_2 \frac{|S|^2 |\phi|^2}{M_P^2} + \kappa_1 \frac{|\phi|^4}{4M_{Pl}^2} + \kappa_3 \frac{|S|^4}{4M_{Pl}^2} + \dots \quad (305)$$

Along the D -flat direction, the VEVs are $\phi_1 = \phi_2 = \phi_3 \ll M \ll M_P$. One can further make an assumption such that $\kappa_3 < -1/3$, so that the S field becomes heavy compared to the Hubble expansion rate, and therefore the field relaxes to its minimum VEV, $S = 0$, within one e-folding. By assuming $|\phi_i| = \varphi/\sqrt{2}$, $\beta = (\kappa_2 - 1)$, $\lambda = (\beta(\beta + 1) + 1/2 + \kappa_1/12)$ and $\gamma = 2/(6)^{3n/2} \lesssim 0.14$, the potential along φ is given by [84]

$$V \simeq \mu^4 \left[1 - \frac{\beta}{2} \frac{\varphi^2}{M_P^2} + \frac{\lambda}{4} \frac{\varphi^4}{M_P^4} - \gamma \frac{\varphi^{3n}}{M^{3n}} + \dots \right]. \quad (306)$$

One needs to suppress the quartic term in the potential, i.e. φ^4 term, otherwise for $|\gamma \varphi^{3n}/M^{3n}| \ll |(\lambda/4)(\varphi^4/M_P^4)|$, it turns out that $M \geq M_P$. For a specific parameter space, $M \approx 10^{15} - 10^{16}$ GeV and $\mu \approx 10^{13} - 10^{14}$ GeV and $n = 2, 3, 4$ and $\beta \leq 0.03$, it is possible to match the amplitude of the perturbations and the spectral tilt within $n_s = 0.96 \pm 0.014$ for $N = 60$ e-foldings of inflation. The model requires pre-inflationary phase to set the initial conditions for $\varphi \approx 0$ to be realizable.

⁷² The initial motivations were proposed in [330, 689] in order to obtain a phenomenologically viable new inflationary potential from the superpotential: $W = S(-\mu^2 + (\bar{\Phi}\Phi)^m/M_*^{2m-2})$. Here $\bar{\Phi}(\Phi)$ denotes a conjugate pair of superfields charged under some gauge group, and S is a gauge singlet. There is an $U(1)_R$ symmetry under which $\Phi \rightarrow \Phi$, $\bar{\Phi} \rightarrow \bar{\Phi}$, $S \rightarrow e^{i2\alpha}S$ and $W \rightarrow e^{i2\alpha}W$. Under these symmetries $m = n$ for odd n and $m = n/2$ otherwise. Along the D -flat direction, the kähler potential is given by: $K = |S|^2 + (2|\Phi|^2 + \kappa_1|\Phi|^4/4M_P^2 + \kappa_2|S|^2|\Phi|^2/M_P^2) + \kappa_3|S|^4/4M_P^4 \dots$. The resultant potential is given by: $V \approx \mu^4(1 - \kappa_3|S|^2/M_P^2 + 2(1 - \kappa_2)|\phi|^2/M_P^2 - 2|\phi|^{2m}/M_*^{2m} + \dots)$. For $\kappa_3 < -1/3$, the S field obtains heavy mass compared to the Hubble expansion rate and the field rolls to its minimum in one e-foldings leaving behind the dynamics of ϕ field to slowly roll over the potential; $V \approx \mu^4(1 - 0.5(\kappa_2 - 1)\phi^2/M_P^2 - 2\phi^{2m})$ [690]. The spectral can match the CMB data for values of $m = 2 - 5$ with a negligible running of the spectral index $dn_s/d \ln k \leq 10^{-3}$. One particular issue is that the initial condition for ϕ field which needs close to zero VEV, this is a nontrivial initial condition, one proposal is to have a pre-inflationary phase.

Another example of flavon is to consider a vacuum alignment potential as studied in the SU(3) family symmetry model of [84, 691]. It was assumed that $\langle \phi_{23} \rangle \propto (0, 1, 1)^T$ and $\langle \Sigma \rangle = \text{diag}(a, a, -2a)$ are already at their minima, and that the relevant part of the superpotential which governs the final step of family symmetry breaking is given by [691]

$$W = \kappa S(\bar{\phi}_{123}\phi_{123} - M^2) + \kappa' Y_{123}\bar{\phi}_{23}\phi_{123} + \kappa'' Z_{123}\bar{\phi}_{123}\Sigma\phi_{123} + \dots \quad (307)$$

where a singlet, S , is the driving superfield for the flavon ϕ_{123} and the non-minimal Kähler potential is given by:

$$K = |S|^2 + |\phi_{123}|^2 + |\bar{\phi}_{123}|^2 + |Y_{123}|^2 + |\bar{\phi}_{23}|^2 + |\phi_{23}|^2 + |Z_{123}|^2 + |\Sigma|^2 + \kappa_S \frac{|S|^4}{4M_{\text{P}}^2} + \kappa_{SZ} \frac{|S|^2|Z_{123}|^2}{4M_{\text{P}}^2} + \dots \quad (308)$$

During inflation the fields with larger VEVs Y_{123} , ϕ_{123} , $\bar{\phi}_{123}$ do not evolve, the inflationary potential is dominated by

$$V = \kappa^2 M^4 \left[1 - \gamma \frac{\xi^2}{2M_{\text{P}}^2} - 2\kappa_S \frac{\sigma^2}{2M_{\text{P}}^2} + \dots \right], \quad (309)$$

where $|S| = \sigma/\sqrt{2}$, $|Z_{123}| = \xi/\sqrt{2}$ and $\gamma = \kappa_{SZ} - 1$. Inflation can happen if the coefficients in front of the mass terms for σ and/or ξ are sufficiently small.

The inflaton could be σ if $\gamma < -1/3$, the mass of ξ becomes heavy compared to the Hubble scale during inflation. For $\kappa_S \approx (0.005 - 0.01)$ and $\kappa \approx (0.001 - 0.05)$, the spectral index is consistent with the current data, $n_s = 0.96 \pm 0.014$ [71]. Finally, the scale, $M \sim 10^{15}$ GeV, of family symmetry breaking along the $\langle \phi_{123} \rangle$ -direction will be determined by the temperature anisotropy in the CMB.

So far, as shown in details regarding F and D -term hybrid inflation models, all of them have to assume an existence of a hidden sector physics, an extra gauge singlet playing the role of the inflaton field, whose mass and couplings are constrained only from the CMB data. There are of course sneutrino driven models of inflation which employ a SM gauge singlet with an additional motivation of connecting inflationary sector to the neutrino physics. However, it is desirable to seek models of inflation which are truly embedded within an observable sector particle physics. Such models are based on beyond the SM physics within a robust framework, where the shape of the potential, interactions and various parameters are well motivated from the low energy particle physics point of view. Furthermore, such an observable sector model of inflation can be directly put to the test by both LHC and CMB data from PLANCK.

V. MSSM GAUGED INFLATONS

A. Inflation due to MSSM flat directions

Since MSSM introduces so many squarks and sleptons, one obvious question is why can't they be an inflaton? Indeed the squarks and sleptons, being light compared to the high scale model of inflation, can also be displaced away from their minimum. However, since they do not minimize F - and D -terms of the total potential, they cost more energetically as compared to a *gauge invariant* combination of squarks and sleptons. In the SUSY limit both F and D -contributions are vanishing for *gauge invariant* flat directions, which maintain their D -flatness, but they can be lifted by the F -term contribution away from the point of *enhanced symmetry*.

A simple observation has been made in [86–88], where the inflaton properties are directly related to the soft SUSY breaking mass term and the A -term. In the limit of unbroken SUSY the flat directions have exactly vanishing potential. This situation changes when soft SUSY breaking and non-renormalizable superpotential terms of the type [90, 91]

$$W_{non} = \sum_{n>3} \frac{\lambda_n}{n} \frac{\Phi^n}{M^{n-3}}, \quad (310)$$

are included. Here Φ is a *gauge invariant* superfield which contains the flat direction. Within MSSM all the flat directions are lifted by non-renormalizable operators with $4 \leq n \leq 9$ [399], where n depends on the flat direction. Let us focus on the lowest order superpotential term in Eq. (310) which lifts the flat direction. Softly broken SUSY induces a mass term for ϕ and an A -term so that the scalar potential along the flat direction reads [86, 87]

$$V = \frac{1}{2} m_\phi^2 \phi^2 + A \cos(n\theta + \theta_A) \frac{\lambda_n \phi^n}{n M_P^{n-3}} + \lambda_n^2 \frac{\phi^{2(n-1)}}{M_P^{2(n-3)}}, \quad (311)$$

Here ϕ and θ denote respectively the radial and the angular coordinates of the complex scalar field $\Phi = \phi \exp[i\theta]$, while θ_A is the phase of the A -term (thus A is a positive quantity with dimension of mass). Note that the first and third terms in Eq. (311) are positive definite, while the A -term leads to a negative contribution along the directions whenever $\cos(n\theta + \theta_A) < 0$.

1. *Inflaton candidates*

As discussed in [86–89], among nearly 300 flat directions [399], there are two that can lead to a successful inflation along the lines discussed above.

One is udd , up to an overall phase factor, which is parameterized by:

$$u_i^\alpha = \frac{1}{\sqrt{3}}\phi, \quad d_j^\beta = \frac{1}{\sqrt{3}}\phi, \quad d_k^\gamma = \frac{1}{\sqrt{3}}\phi. \quad (312)$$

Here $1 \leq \alpha, \beta, \gamma \leq 3$ are color indices, and $1 \leq i, j, k \leq 3$ denote the quark families. The flatness constraints require that $\alpha \neq \beta \neq \gamma$ and $j \neq k$.

The other direction is LLe , parameterized by (again up to an overall phase factor)

$$L_i^a = \frac{1}{\sqrt{3}} \begin{pmatrix} 0 \\ \phi \end{pmatrix}, \quad L_j^b = \frac{1}{\sqrt{3}} \begin{pmatrix} \phi \\ 0 \end{pmatrix}, \quad e_k = \frac{1}{\sqrt{3}}\phi, \quad (313)$$

where $1 \leq a, b \leq 2$ are the weak isospin indices and $1 \leq i, j, k \leq 3$ denote the lepton families. The flatness constraints require that $a \neq b$ and $i \neq j \neq k$. Both these flat directions are lifted by $n = 6$ non-renormalizable operators [399],

$$W_6 \supset \frac{1}{M_{\text{P}}^3}(LLe)(LLe), \quad W_6 \supset \frac{1}{M_{\text{P}}^3}(udd)(udd). \quad (314)$$

The reason for choosing either of these two flat directions is twofold:

1. *within MSSM, a non-trivial A-term arises, at the lowest order, only at $n = 6$, and*
2. *we wish to obtain the correct COBE normalization of the CMB spectrum.*

Since LLe and udd are independently D - and F -flat, inflation could take place along any of them but also, at least in principle, simultaneously. The dynamics of multiple flat directions are however quite involved [281, 692].

Those MSSM flat directions which are lifted by operators with dimension $n = 7, 9$ are such that the superpotential term contains at least two monomials, i.e. is of the type [318, 319, 399]

$$W \sim \frac{1}{M_{\text{P}}^{n-3}}\Psi\Phi^{n-1}. \quad (315)$$

If ϕ represents the flat direction, then its VEV induces a large effective mass term for ψ , through Yukawa couplings, so that $\langle \psi \rangle = 0$. Hence Eq. (315) does not contribute to the A -term.

The scalar component of Φ superfield, denoted by ϕ , is given by

$$\phi = \frac{u + d + d}{\sqrt{3}} \quad , \quad \phi = \frac{L + L + e}{\sqrt{3}} \quad , \quad (316)$$

for the udd and LLe flat directions respectively.

After minimizing the potential along the angular direction θ , we can situate the real part of ϕ by rotating it to the corresponding angles θ_{\min} . The scalar potential is then found to be [86, 87, 89]

$$V(\phi) = \frac{1}{2}m_\phi^2 \phi^2 - A \frac{\lambda \phi^6}{6 M_{\text{P}}^6} + \lambda^2 \frac{\phi^{10}}{M_{\text{P}}^6} \quad , \quad (317)$$

where m_ϕ and A are the soft breaking mass and the A -term respectively (A is a positive quantity since its phase is absorbed by a redefinition of θ during the process).

2. Inflection point inflation

Provided that

$$\frac{A^2}{40m_\phi^2} \equiv 1 + 4\alpha^2 \quad , \quad (318)$$

where $\alpha^2 \ll 1$, there exists a point of inflection in $V(\phi)$

$$\phi_0 = \left(\frac{m_\phi M_{\text{P}}^3}{\lambda \sqrt{10}} \right)^{1/4} + \mathcal{O}(\alpha^2) \quad , \quad (319)$$

$$V''(\phi_0) = 0 \quad , \quad (320)$$

at which

$$V(\phi_0) = \frac{4}{15}m_\phi^2 \phi_0^2 + \mathcal{O}(\alpha^2) \quad , \quad (321)$$

$$V'(\phi_0) = 4\alpha^2 m_\phi^2 \phi_0 + \mathcal{O}(\alpha^4) \quad , \quad (322)$$

$$V'''(\phi_0) = 32 \frac{m_\phi^2}{\phi_0} + \mathcal{O}(\alpha^2) \quad . \quad (323)$$

From now on we only keep the leading order terms in all expressions. Note that in gravity-mediated SUSY breaking, the A -term and the soft SUSY breaking mass are of the same order of magnitude as the gravitino mass, i.e. $m_\phi \sim A \sim m_{3/2} \sim (100 \text{ GeV} - 1 \text{ TeV})$. Therefore the condition in Eq. (318) can indeed be satisfied. We then have $\phi_0 \sim \mathcal{O}(10^{14} \text{ GeV})$. Inflation occurs within an interval

$$|\phi - \phi_0| \sim \frac{\phi_0^3}{60M_{\text{P}}^2} \quad , \quad (324)$$

in the vicinity of the point of inflection, within which the slow-roll parameters $\epsilon \equiv (M_{\text{P}}^2/2)(V'/V)^2$ and $\eta \equiv M_{\text{P}}^2(V''/V)$ are smaller than 1. The Hubble expansion rate during inflation is given by

$$H_{\text{MSSM}} \simeq \frac{1}{\sqrt{45}} \frac{m_\phi \phi_0}{M_{\text{P}}} \sim (100 \text{ MeV} - 1 \text{ GeV}). \quad (325)$$

The amplitude of density perturbations δ_H and the scalar spectral index n_s are given by [86, 87, 693, 694]:

$$\delta_H = \frac{8}{\sqrt{5}\pi} \frac{m_\phi M_{\text{P}}}{\phi_0^2} \frac{1}{\Delta^2} \sin^2[N_Q \sqrt{\Delta^2}], \quad (326)$$

and

$$n_s = 1 - 4\sqrt{\Delta^2} \cot[N_Q \sqrt{\Delta^2}], \quad (327)$$

respectively where

$$\Delta^2 \equiv 900\alpha^2 \mathcal{N}_Q^{-2} \left(\frac{M_{\text{P}}}{\phi_0} \right)^4. \quad (328)$$

N_Q is the number of e-foldings between the time when the observationally relevant perturbations are generated till the end of inflation and follows: $N_Q \simeq 66.9 + (1/4)\ln(V(\phi_0)/M_{\text{P}}^4) \sim 50$, provided that the universe is immediately thermalized after the end of inflation [154, 155]. We note that reheating after MSSM inflation is very fast, due to gauge couplings of the inflaton to gauge/gaugino fields, and results in a radiation-dominated universe within few Hubble times after the end of inflation [86, 87].

3. Parameter space for MSSM inflation

A remarkable property of MSSM inflation, which is due to inflation occurring near a point of inflection, is that it can give rise to a wide range of scalar spectral index. This is in clear distinction with other models (for example, chaotic inflation, hybrid inflation, natural inflation, etc.) and makes the model very robust. Indeed it can yield a spectral index within the whole 2σ allowed range by 5-year WMAP data $0.934 \leq n_s \leq 0.988$. Note that for $\alpha^2 = 0$, Eqs. (326,327) are reduced to the case of a saddle point inflation, for which the spectral index is strictly 0.92, for details see [88, 89, 693]⁷³.

⁷³ Varying range of spectral tilt is in concordance with the statistical nature of the vacua at low energies below the cut-off. MSSM harbors a mini-landscape [89] with a moduli space of 37 complex dimensions [399], which has more than 700 gauge invariant monomials [695]. Although, its much smaller compared to the

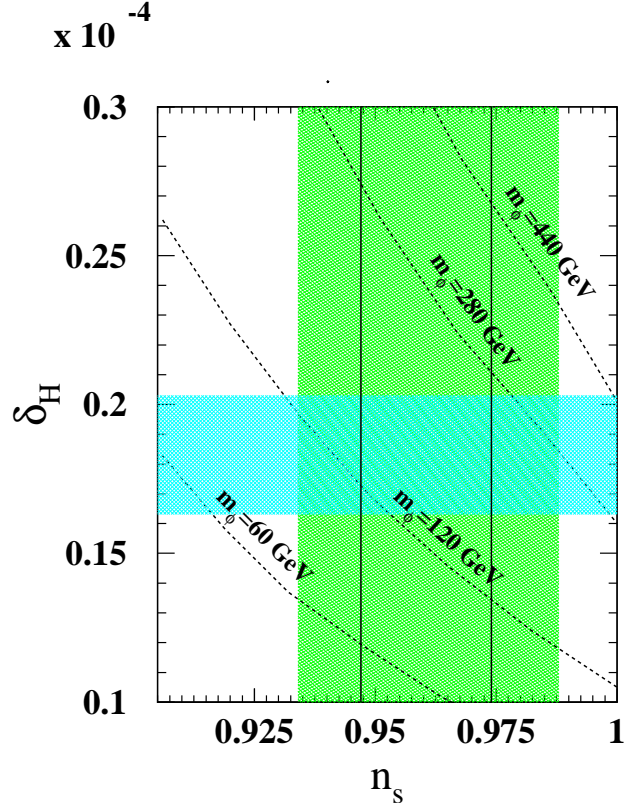


FIG. 8: n_s is plotted as a function of δ_H for different values of m_ϕ . The 2σ region for δ_H is shown by the blue horizontal band and the 2σ allowed region of n_s is shown by the vertical green band. The 1σ allowed region of n_s is within the solid vertical lines. We choose $\lambda = 1$ [89].

For $\alpha^2 < 0$, the spectral index will be smaller than the 0.92, which is more than 3σ away from observations. The more interesting case, as pointed out in [89, 693], happens for $\alpha^2 > 0$. This happens for

$$2 \times 10^{-6} \leq \Delta^2 \leq 5.2 \times 10^{-6}. \quad (329)$$

In Fig. (8), we have shown δ_H as a function of n_s for different values of m_ϕ . The horizontal blue band shows the 2σ experimental band for δ_H . The vertical green shaded region is the 2σ experimental band for n_s . The region enclosed by solid lines shows the 1σ experimental allowed region. Smaller values of m_ϕ are preferred for smaller values of n_s . Note that the allowed range of m_ϕ is 90 – 330 GeV for the experimental ranges of n_s and δ_H . This figure is drawn for $\lambda \simeq 1$, which is natural in the context of effective field theory (unless it is suppressed due to some symmetry). Smaller values of λ will lead to an increase in m_ϕ , see

string landscape, but one would naturally expect a distribution of discrete values of non-renormalizable A -terms along with the soft breaking terms. This would inevitably give rise to many realizations of our universe with varied range of spectral tilts.

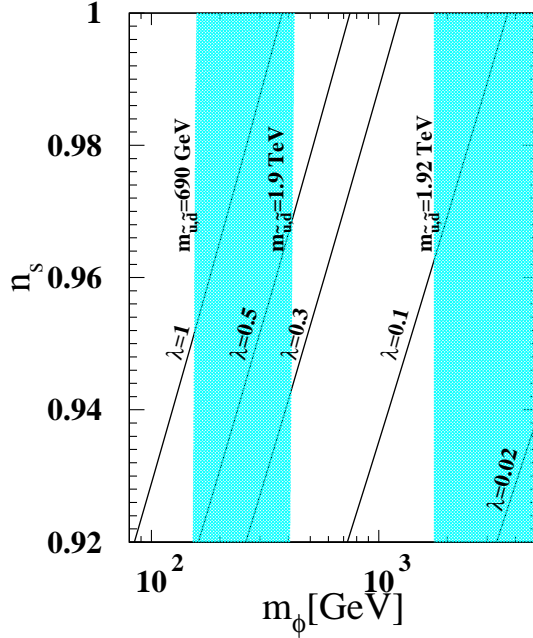


FIG. 9: Contours of λ for $\delta_H = 1.91 \times 10^{-5}$ in the n_s - m_ϕ plane. The blue band on the left is due to the stau-neutralino coannihilation region for $\tan\beta = 10$ and the blue band on the right (which continues beyond the plotting range) denotes the focus point region [696].

Fig. (9), where we have plotted (n_s, m_ϕ) for different values of λ for the case of udd direction with different masses of m_u , m_d , and fixed value of $\tan(\beta)$. We will explain the blue bands when we discuss the parameter space of MSSM inflation at low energies close to the dark matter production scale [89, 696].

4. Embedding MSSM inflation in $SU(5)$ or $SO(10)$ GUT

By embedding MSSM inflation in GUT makes a mild assumption that there exists new physics which encompasses MSSM beyond the unification scale M_G . We remind the readers that inflation occurs around a flat direction VEV $\phi_0 \sim 10^{14}$ GeV. Since $\phi_0 \ll M_G$, heavy GUT degrees of freedom play no role in the dynamics of MSSM inflation, and hence they can be ignored. Here we wish to understand how such embedding would affect inflationary scenario.

It is generically believed that gravity breaks global symmetries [697]. Then all *gauge invariant* terms which are M_P suppressed should appear with $\lambda \sim \mathcal{O}(1)$. Obviously the

above terms in Eq. (314) are invariant under the SM. Once the SM is embedded within a GUT at the scale M_G , where gauge couplings are unified, the gauge group will be enlarged. Then the question arises whether such terms in Eq. (314) are invariant under the GUT gauge group or not. Note that a GUT singlet is also a singlet under the SM, however, the vice versa is not correct.

- $SU(5)$:

We briefly recollect representations of matter fields in this case: L and d belong to $\bar{\mathbf{5}}$, while e and u belong to $\mathbf{10}$ of $SU(5)$ group. Thus under $SU(5)$ the superpotential terms in Eq. (314) read [696]

$$\frac{\bar{\mathbf{5}} \times \bar{\mathbf{5}} \times \mathbf{10} \times \bar{\mathbf{5}} \times \bar{\mathbf{5}} \times \mathbf{10}}{M_{\text{P}}^3}. \quad (330)$$

This product clearly includes a $SU(5)$ singlet. Therefore in the case of $SU(5)$, we expect that M_{P} suppressed terms as in Eq. (310) appear with $\lambda \sim \mathcal{O}(1)$. If we were to obtain the $(LLe)^2$ term by integrating out the heavy fields of the $SU(5)$ GUT, then $\lambda = 0$. This is due to the fact that $SU(5)$ preserves $B - L$.

- $SO(10)$:

In this case all matter fields of one generation are included in the spinorial representation $\mathbf{16}$ of $SO(10)$. Hence the superpotential terms in Eq. (310) are $[\mathbf{16}]^6$ under $SO(10)$, which does not provide a singlet. A *gauge invariant* operator will be obtained by multiplying with a 126-plet Higgs. This implies that in $SO(10)$ the lowest order *gauge invariant* superpotential term with 6 matter fields arises at $n = 7$ level:

$$\frac{\mathbf{16} \times \mathbf{16} \times \mathbf{16} \times \mathbf{16} \times \mathbf{16} \times \mathbf{16} \times \mathbf{126}_H}{M_{\text{P}}^4}. \quad (331)$$

Once $\mathbf{126}_H$ acquires a VEV, $SO(10)$ can break down to a lower ranked subgroup, for instance $SU(5)$. This will induce an effective $n = 6$ non-renormalizable term as in Eq. (310) with

$$\lambda \sim \frac{\langle \mathbf{126}_H \rangle}{M_{\text{P}}} \sim \frac{\mathcal{O}(M_{\text{GUT}})}{M_{\text{P}}}. \quad (332)$$

Hence, in the case of $SO(10)$, we can expect $\lambda \sim \mathcal{O}(10^{-2} - 10^{-1})$ depending on the scale where $SO(10)$ gets broken.

By embedding MSSM in $SO(10)$ naturally implies that $\lambda \ll 1$. Smaller values of n_s (within the range $0.92 \leq n_s \leq 1$) point to smaller λ , as can be seen from figure 6. This, according to Eq. (332), implies a scale of $SO(10)$ breaking, i.e. $\langle \mathbf{126}_H \rangle$, which is closer to the GUT scale. Further note that embedding the MSSM within $SO(10)$ also provides an advantage for obtaining a right handed neutrino. It was concluded in Ref. [696] that if we include the RH neutrinos, then udd direction is preferred over LLe .

5. Gauged inflaton in $SM \times U(1)_{B-L}$

If we augment MSSM with three right-handed (RH) neutrino multiplets, then it is possible to realize neutrinos of Dirac type with an appropriate Yukawa coupling. Whether the nature of neutrino is Dirac or Majorana can be determined in the future neutrinoless double beta decay experiment.

For various reasons, which will become clear, the inclusion of a gauge symmetry under which the RH (s)neutrinos are not singlet is crucial. As far as inflation is concerned, a singlet RH sneutrino would not form a gauge-invariant inflaton along with the Higgs and slepton fields. The simplest extension of the SM symmetry, $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}$, which is also well motivated: anomaly cancelation requires that three RH neutrinos exist, so that they pair with LH neutrinos to form three Dirac fermions.

The relevant superpotential term is

$$W \supset hNH_uL. \tag{333}$$

Here N , L and H_u are superfields containing the RH neutrinos, left-handed (LH) leptons and the Higgs which gives mass to the up-type quarks, respectively.

In this model there is an extra Z boson (Z') and one extra gaugino (\tilde{Z}'). The $U(1)_{B-L}$ gets broken around TeV by new Higgs fields with $B - L = \pm 1$. This also prohibits a Majorana mass for the RH neutrinos at the renormalizable level (note that NN has $B - L = 2$). The Majorana mass can be induced by a non-renormalizable operator, but it will be very small.

The value of h needs to be small, i.e. $h \leq 10^{-12}$, in order to explain the light neutrino mass, $\sim \mathcal{O}(0.1 \text{ eV})$ corresponding to the atmospheric neutrino oscillations detected by Super-Kamiokande experiment. Note that the NH_uL monomial represents a D -flat direction under

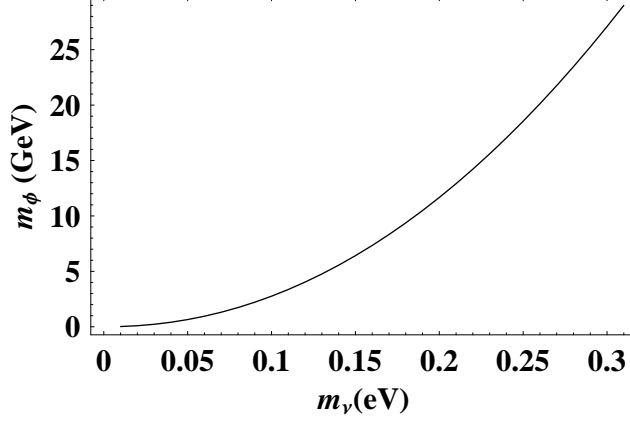


FIG. 10: The inflaton mass m_ϕ is plotted as a function of the neutrino mass m_ν [698].

the $U(1)_{B-L}$, as well as the SM gauge group [87, 698].

$$\phi = \frac{\tilde{N} + H_u + \tilde{L}}{\sqrt{3}}, \quad (334)$$

where \tilde{N} , \tilde{L} , H_u are the scalar components of corresponding superfields. Since the RH sneutrino \tilde{N} is a singlet under the SM gauge group, its mass receives the smallest contribution from quantum corrections due to SM gauge interactions, and hence it can be set to be the lightest SUSY particle (LSP). Therefore the dark matter candidate arises from the RH sneutrino component of the inflaton, see Eq. (334). The potential along the flat direction, after the minimization along the angular direction, is found to be [87, 698],

$$V(|\phi|) = \frac{m_\phi^2}{2}|\phi|^2 + \frac{h^2}{12}|\phi|^4 - \frac{Ah}{6\sqrt{3}}|\phi|^3. \quad (335)$$

The flat direction mass m_ϕ is given in terms of \tilde{N} , H_u , \tilde{L} masses: $m_\phi^2 = \frac{m_{\tilde{N}}^2 + m_{H_u}^2 + m_{\tilde{L}}^2}{3}$. For $A \approx 4m_\phi$, there exists an *inflection point* for which $V'(\phi_0) \neq 0$, $V''(\phi_0) = 0$, where inflation takes place

$$\phi_0 = \sqrt{3} \frac{m_\phi}{h} = 6 \times 10^{12} m_\phi \left(\frac{0.05 \text{ eV}}{m_\nu} \right), \quad V(\phi_0) = \frac{m_\phi^4}{4h^2} = 3 \times 10^{24} m_\phi^4 \left(\frac{0.05 \text{ eV}}{m_\nu} \right)^2. \quad (336)$$

Here m_ν denotes the neutrino mass which is given by $m_\nu = h\langle H_u \rangle$, with $\langle H_u \rangle \simeq 174 \text{ GeV}$. For neutrino masses with a hierarchical pattern, the largest neutrino mass is $m_\nu \simeq 0.05 \text{ eV}$ in order to explain the atmospheric neutrino oscillations [699], while the current upper bound on the sum of the neutrino masses from cosmology, using WMAP and SDSS data alone, is 0.94 eV [700].

The amplitude of density perturbations follows [87, 698].

$$\delta_H \simeq \frac{1}{5\pi} \frac{H_{inf}^2}{\dot{\phi}} \simeq 3.5 \times 10^{-27} \left(\frac{m_\nu}{0.05 \text{ eV}} \right)^2 \left(\frac{M_P}{m_\phi} \right) N_Q^2. \quad (337)$$

Here m_ϕ denotes the loop-corrected value of the inflaton mass at the scale ϕ_0 in Eqs. (336,337). In Fig. (10), we have shown the neutrino mass as a function of the inflaton mass for $\delta_H = 1.91 \times 10^{-5}$. We see that the neutrino mass in the range 0 to 0.30 eV corresponds to the inflaton mass of 0 to 30 GeV. The spectral tilt as usual has a range of values $0.90 \leq n_s \leq 1.0$ [87, 698].

6. Inflection point inflation in gauge mediation

In GMSB the two-loop correction to the flat direction potential results in a logarithmic term above the messenger scale, i.e. $\phi > M_S$ [35, 90, 91, 701]. Together with the A -term this leads to the scalar potential [702]

$$V = M_F^4 \ln \left(\frac{\phi^2}{M_S^2} \right) + A \cos(n\theta + \theta_A) \frac{\lambda_n \phi^n}{n M_{\text{GUT}}^{n-3}} + \lambda_n^2 \frac{\phi^{2(n-1)}}{M_{\text{GUT}}^{2(n-3)}}, \quad (338)$$

where $M_F \sim (m_{\text{SUSY}} \times M_S)^{1/2}$ and $m_{\text{SUSY}} \sim 1$ TeV is the soft SUSY breaking mass at the weak scale. For $\phi > M_F^2/m_{3/2}$, usually the gravity mediated contribution, $m_{3/2}^2 \phi^2$, dominates the potential where $m_{3/2}$ is the gravitino mass. Here we will concentrate on the VEVs such that $M_s \ll \phi \leq M_F^2/m_{3/2}$. A successful inflation can be obtained near the point of inflection;

$$\phi_0 \approx \left(\frac{M_{\text{GUT}}^{n-3} M_F^2}{\lambda_n} \sqrt{\frac{n}{(n-1)(n-2)}} \right)^{1/(n-1)}, \quad (339)$$

$$A \approx \frac{4(n-1)^2 \lambda_n}{n M_{\text{GUT}}^{n-3}} \phi_0^{n-2}. \quad (340)$$

In the vicinity of the inflection point, the total energy density is given by

$$V(\phi_0) \approx M_F^4 \left[\ln \left(\frac{\phi_0^2}{M_S^2} \right) - \frac{3n-2}{n(n-1)} \right], \quad (341)$$

There are couple of interesting points, first of all note that the scale of inflation is extremely low, the Hubble scale during inflation is given by: $H_{\text{inf}} \sim M_F^2/M_P \sim 10^{-3} - 10^{-1}$ eV for $M_F \sim 1 - 10$ TeV. The relevant number of e-foldings is $N_Q \sim 40$ [702]. For $M_F \sim 10$ TeV, and $\phi_0 \sim 10^{11}$ GeV it is possible to match the CMB temperature anisotropy and the required tilt in the spectrum [702].

The validity of Eq. (338) for such a large VEV requires that $M_F^2 > (10^{11} \text{ GeV}) \times m_{3/2}$. For $M_F \sim 10 \text{ TeV}$ this yields a bound on the gravitino mass, $m_{3/2} < 1 - 10 \text{ MeV}$, which is compatible with the warm dark matter constraints.

B. Quantum stability

1. Radiative correction

Since the MSSM inflaton candidates are represented by *gauge invariant* combinations of squarks and sleptons, the inflaton parameters receive corrections from gauge interactions which can be computed in a straightforward way. Quantum corrections result in a logarithmic running of the soft SUSY breaking parameters m_ϕ and A . The effective potential at phase minimum $n\theta_{\min} = \pi$ is then given by [88]:

$$V_{eff}(\phi, \theta_{min}) = \frac{1}{2}m_0^2\phi^2 \left[1 + K_1 \log \left(\frac{\phi^2}{\mu_0^2} \right) \right] - \frac{\lambda_{n,0}A_0}{nM^{n-3}}\phi^n \left[1 + K_2 \log \left(\frac{\phi^2}{\mu_0^2} \right) \right] + \frac{\lambda_{n,0}^2}{M^{2(n-3)}}\phi^{2(n-1)} \left[1 + K_3 \log \left(\frac{\phi^2}{\mu_0^2} \right) \right]. \quad (342)$$

where m_0 , A_0 , and $\lambda_{n,0}$ are the values of m_ϕ , A and λ_n given at a scale μ_0 . Here A_0 is chosen to be real and positive (this can always be done by re-parameterizing the phase of the complex scalar field ϕ), and $|K_i| < 1$ are coefficients determined by the one-loop renormalization group equations.

In the limit when $|K_i| \ll 1$, one finds a simple relationship [88]

$$A^2 = 8(n-1)m_\phi^2(\phi_0) \left(1 + K_1 - \frac{4}{n}K_2 + \frac{1}{n-1}K_3 \right), \quad (343)$$

$$\phi_0^{n-2} = \frac{M^{n-3}m_\phi(\phi_0)}{\lambda_n\sqrt{2(n-1)}} \left(1 + \frac{1}{2}K_1 - \frac{1}{2(n-1)}K_3 \right). \quad (344)$$

For $n = 6$, the coefficient is $A^2 \sim 40m_\phi^2$, where ϕ_0 denotes the point of inflection. The coefficients K_i need to be solved from the renormalization group equations at the inflationary scale $\mu = \phi_0$. Since K_i are already one loop corrections, taking the tree-level value as the renormalization scale is sufficient ⁷⁴.

⁷⁴ In general there is no prospect of measuring the non-renormalizable A_6 term, because interactions are suppressed by M_P . However, a knowledge of SUSY breaking sector and its communication with the observable sector may help to link the non-renormalizable A -term under consideration to the renormalizable ones. In

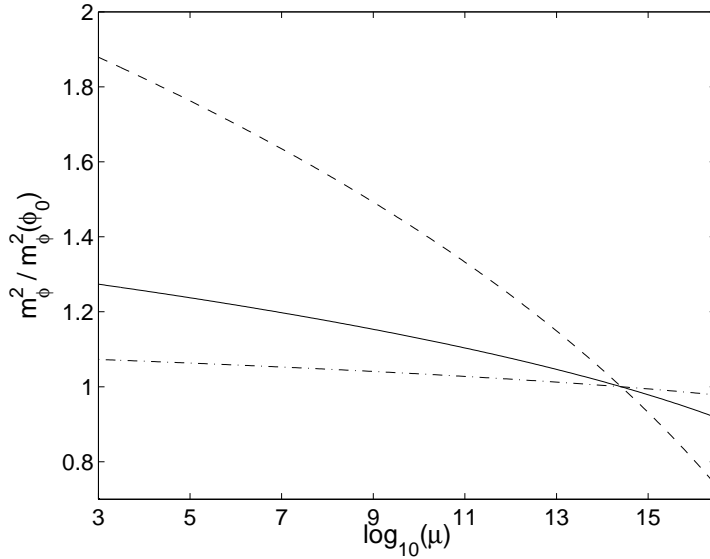


FIG. 11: The running of m_ϕ^2 for the LLe inflaton when the saddle point is at $\phi_0 = 2.6 \times 10^{14} \text{GeV}$ (corresponding to $n = 6$, $m_\phi = 1 \text{ TeV}$ and $\lambda = 1$). The three curves correspond to different values of the ratio of gaugino mass to flat direction mass at the GUT scale: $\xi = 2$ (dashed), $\xi = 1$ (solid) and $\xi = 0.5$ (dash-dot) [88].

The radiative corrections do not remove the inflection point nor shift it to unreasonable values. The existence of an inflection point is thus insensitive to radiative corrections.

One can analytically obtain the values of K_i for the LLe flat direction. For LLe the one-loop RG equations governing the running of m_ϕ^2 , A , and λ with the scale μ are given by [30, 31, 88]

$$\begin{aligned}
\mu \frac{dm_\phi^2}{d\mu} &= -\frac{1}{6\pi^2} \left(\frac{3}{2} \tilde{m}_2^2 g_2^2 + \frac{3}{2} \tilde{m}_1^2 g_1^2 \right), \\
\mu \frac{dA}{d\mu} &= -\frac{1}{2\pi^2} \left(\frac{3}{2} \tilde{m}_2^2 g_2^2 + \frac{3}{2} \tilde{m}_1^2 g_1^2 \right), \\
\mu \frac{d\lambda}{d\mu} &= -\frac{1}{4\pi^2} \lambda \left(\frac{3}{2} g_2^2 + \frac{3}{2} g_1^2 \right).
\end{aligned} \tag{345}$$

Here \tilde{m}_1 , \tilde{m}_2 denote the mass of the $U(1)_Y$ and $SU(2)_W$ gauginos respectively and g_1 , g_2 are the associated gauge couplings. The running of gauge couplings and gaugino masses obey

the case of a Polonyi model where a general A -term at a tree level is given by: $m_{3/2}[(a-3)W + \phi(dW/d\phi)]$, with $a = 3 - \sqrt{3}$ [30, 31]. One then finds a relationship between A -terms corresponding to $n = 6$ and $n = 3$ superpotential terms, denoted by A_6 and A_3 respectively, at high scales [88]: $A_6 = \frac{3-\sqrt{3}}{6-\sqrt{3}} A_3$.

the usual equations [30, 31, 88]:

$$\begin{aligned}\mu \frac{dg_1}{d\mu} &= \frac{11}{16\pi^2} g_1^3, \\ \mu \frac{dg_2}{d\mu} &= \frac{1}{16\pi^2} g_2^3, \\ \frac{d}{d\mu} \left(\frac{\tilde{m}_1}{g_1^2} \right) &= \frac{d}{d\mu} \left(\frac{\tilde{m}_2}{g_2^2} \right) = 0.\end{aligned}\tag{346}$$

By solving the above equations, one finds:

$$\begin{aligned}K_1 &\approx -\frac{1}{4\pi^2} \left[\left(\frac{\tilde{m}_2}{m_{\phi_0}} \right)^2 g_2^2 + \left(\frac{\tilde{m}_1}{m_{\phi_0}} \right)^2 g_1^2 \right], \\ K_2 &\approx -\frac{3}{4\pi^2} \left[\left(\frac{\tilde{m}_2}{A_0} \right) g_2^2 + \left(\frac{\tilde{m}_1}{A_0} \right) g_1^2 \right], \\ K_3 &\approx -\frac{3}{8\pi^2} \lambda_0 [g_2^2 + g_1^2],\end{aligned}\tag{347}$$

where the subscript 0 denotes the values of parameters at the high scale μ_0 .

For universal boundary conditions, as in minimal grand unified SUGRA, the high scale is the GUT scale $\mu_X \approx 3 \times 10^{16}$ GeV, $\tilde{m}_1(\mu_X) = \tilde{m}_2(\mu_X) = \tilde{m}$ and $g_1 = \sqrt{\pi/10} \approx 0.56$, $g_2 = \sqrt{\pi/6} \approx 0.72$. With the help of RG equations to run the coupling constants and masses to the scale of the saddle point $\mu_0 = \phi_0 \approx 2.6 \times 10^{14}$ GeV for $M_P = 2.4 \times 10^{18}$ GeV, $m_{\phi_0} = 1$ TeV, $\lambda_0 = 1$. With these values one obtains [88]

$$K_1 \approx -0.017\xi^2, \quad K_2 \approx -0.0085\xi, \quad K_3 \approx -0.029.\tag{348}$$

where $\xi = \tilde{m}/m_\phi$ is calculated at the GUT scale. Similar calculation can be performed for the $NH_u L$ flat direction also [87].

2. SUGRA η problem, trans-Planckian, and moduli problems

SUGRA corrections often destroy the slow-roll predictions of inflationary potentials. In general, the effective potential depends on the Kähler potential K as $V \sim \left(e^{K(\varphi^*, \varphi)/M_P^2} V(\phi) \right)$ so that there is a generic SUGRA contribution to the flat direction potential of the type for minimal choice of K ,

$$V(\phi) = H^2 M_P^2 f \left(\frac{\phi}{M_P} \right),\tag{349}$$

where f is some function (typically a polynomial). Such a contribution usually gives rise to a Hubble induced correction to the mass of the flat direction with an unknown coefficient,

which depends on the nature of the Kähler potential. If the Kähler potential has a shift symmetry, i.e. no scale type, then at tree level there is no Hubble induced correction.

Let us compare the non-gravitational contribution, Eq. (311), to that of Hubble induced contribution, Eq. (349). Writing $f \sim (\phi/M_{\text{P}})^p$ where $p \geq 1$ is some power, we see that non-gravitational part dominates whenever [88]

$$H_{\text{inf}}^2 M_{\text{P}}^2 \left(\frac{\phi}{M_{\text{P}}} \right)^p \ll m_\phi^2 \phi_0^2, \quad (350)$$

so that the SUGRA corrections are negligible as long as $\phi_0 \ll M_{\text{P}}$, as is the case here (note that $H_{\text{inf}} M_{\text{P}} \sim m_\phi \phi_0$). The absence of SUGRA corrections is a generic property of this model. Note also that although non-trivial Kähler potentials give rise to non-canonical kinetic terms of squarks and sleptons, it is a trivial exercise to show that at sufficiently low scales, $H_{\text{inf}} \ll m_\phi$, and small VEVs, they can be rotated to a canonical form without affecting the potential.

The same reason, i.e. $H_{\text{inf}} \ll m_\phi$ also precludes any large **trans-Planckian** correction. Any such correction would generically go as $(H_{\text{inf}}/M_*)^2 \sim (m_\phi/M_*) \ll 1$, where M_* is the scale at which one would expect trans-Planckian effects to kick in [286, 288, 293]. Note that in our case the initial vacuum is the Bunch-Davis and the evolution of the modes is adiabatic. The latter condition is important to make sure that unknown physics at the high scale is less and less sensitive to the low energy world [293, 346].

Finally, we also make a comment on the cosmological **moduli problem** [703–706]. The moduli are generically displaced from their true minimum if their mass is less than the expansion rate during inflation. In our case $H_{\text{inf}} \ll m_{\text{moduli}} \sim \mathcal{O}(\text{TeV})$. This implies that quantum fluctuations cannot displace the moduli from their true minima during the inflationary epoch driven by MSSM flat directions. Moreover, any oscillations of the moduli will be exponentially damped during the inflationary epoch. Therefore, MSSM inflation can naturally address the infamous moduli problem [88].

C. Exciting SM baryons and cold dark matter

Interesting aspect of MSSM inflaton is that inflation takes place away from the point of *enhanced gauge symmetry*. Keep in mind that the VEV of the MSSM flat direction inflaton breaks the gauge symmetry spontaneously, for instance udd breaks $SU(3)_C \times U(1)_Y$ while

LLe breaks $SU(2)_W \times U(1)_Y$, therefore, induces a SUSY conserving mass $\sim g\langle\phi(t)\rangle$ to the gauge/gaugino fields in a similar way as the Higgs mechanism, where g is a gauge coupling. When the flat direction goes to its minimum, $\langle\phi(t)\rangle = 0$, the gauge symmetry is restored. In this respect the origin is a point of enhanced symmetry.

After the end of inflation, the flat direction starts rolling towards its global minimum. At this stage the dominant term in the scalar potential will be: $m_\phi\phi^2/2$. Since the frequency of oscillations is $\omega \sim m_\phi \sim 10^3 H_{\text{inf}}$, the flat direction oscillates a large number of times within the first Hubble time after the end of inflation. Hence the effect of expansion is negligible. Further note that the motion is strictly along the radial direction, i.e. one dimensional.

In all the examples inflaton has gauge couplings to the gauge/gaugino fields and Yukawa couplings to the Higgs/Higgsino fields. As we will see particles with a larger couplings are produced more copiously during inflaton oscillations ⁷⁵.

D. Particle creation and thermalization

There are distinct phases of particle creation in this model, here we briefly summarize them below ⁷⁶.

⁷⁵ The flat direction is coupled to the scalar fields through gauge and Yukawa interactions. For instance, in the LLe case since the lepton Yukawas are $\leq 10^{-2}$ we can safely neglect them. The gauge couplings arise from the D -term part of the scalar potential. The D -terms corresponding to $SU(2)_W$ and $U(1)_Y$ symmetries follow: $V_D \supset \frac{1}{2}g^2[\sum_{i=1}^3(\tilde{L}_1^\dagger T^i \tilde{L}_1 + \tilde{L}_2^\dagger T^i \tilde{L}_2)^2] + \frac{1}{2}g'^2[(|\tilde{e}_3^*|^2 - \frac{1}{2}|\tilde{L}_1|^2 - \frac{1}{2}|\tilde{L}_2|^2)]$. Here T^1, T^2, T^3 are the $SU(2)$ generators (i.e. 1/2 times the Pauli matrices) and g, g' are the gauge couplings of $SU(2)_W, U(1)_Y$, respectively.

Similarly, couplings of the flat direction to the gauge fields are obtained from the flat direction kinetic terms: $\mathcal{L} \supset (D^\mu \tilde{L}_1)^\dagger (D_\mu \tilde{L}_1) + (D^\mu \tilde{L}_2)^\dagger (D_\mu \tilde{L}_2) + (D^\mu \tilde{e}_3)^\dagger (D_\mu \tilde{e}_3)$, where $D_\mu \tilde{L}_1 = (\partial_\mu + \frac{i}{2}g'B_\mu - igW_{1,\mu}T^1 - igW_{2,\mu}T^2 - igW_{3,\mu}T^3)\tilde{L}_1$, $D_\mu \tilde{L}_2 = (\partial_\mu + \frac{i}{2}g'B_\mu - igW_{1,\mu}T^1 - igW_{2,\mu}T^2 - igW_{3,\mu}T^3)\tilde{L}_2$, and $D_\mu \tilde{e}_3 = (\partial_\mu - ig'B_\mu)\tilde{e}_3$, where $W_{1,\mu}, W_{2,\mu}, W_{3,\mu}$ and B_μ are the gauge fields of $SU(2)_W$ and $U(1)_Y$, respectively.

The flat direction couplings to the fermions are found from the following part of the Lagrangian: $\mathcal{L} \supset \sqrt{2}g\sum_{i=1}^3[\tilde{L}_1^\dagger \tilde{W}_i^t T^i (i\sigma_2 L_1) + \tilde{L}_2^\dagger \tilde{W}_i^t T^i (i\sigma_2 L_2)] + \sqrt{2}g'[\tilde{e}_3^\dagger \tilde{B}^t (i\sigma_2 e_3) - \frac{1}{2}\tilde{L}_1^\dagger \tilde{B}^t (i\sigma_2 L_1) - \frac{1}{2}\tilde{L}_2^\dagger \tilde{B}^t (i\sigma_2 L_2)] + \text{h.c.}$, where $\tilde{W}_1, \tilde{W}_2, \tilde{W}_3$ and \tilde{B} are the gauginos of $SU(2)_W$ and $U(1)_Y$ respectively. Superscript t denotes transposition, and σ_2 is the second Pauli matrix. The field content of L_1, L_2, e_3 are given by: $L_1 = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}, L_2 = \begin{pmatrix} \psi_3 \\ \psi_4 \end{pmatrix}, e_3 = \psi_5$, where ψ_i are left-handed Wyle spinors. Similarly, one can also work out all the relevant couplings of udd .

⁷⁶ Readers may wish to revisit this section after reading the following Secs. VIB3, VIB2, VIC, VID4, and VID5.

- Tachyonic preheating:

Right after the end of inflation, the second derivative is negative in the case of inflection point inflation. The inflaton fluctuations with a physical momentum $k \lesssim m_\phi$ will have a tachyonic instability, see Sec. VIB3. Moreover $V'' < 0$ only at field values which are $\sim \phi_0 \sim 10^{14}$ GeV. Tachyonic effects are therefore expected to be negligible since, unlike the case in [707], the homogeneous mode has a VEV which is hierarchically larger than m_ϕ , and oscillates at a frequency $\omega \sim m_\phi$. Further note fields which are coupled to the inflaton acquire a very large mass $\sim h\phi_0$ from the homogeneous piece which suppresses non-perturbative production of their quanta at large inflaton VEVs. Therefore tachyonic effects, although genuinely present, do not lead to significant particle production in this case ⁷⁷.

- Instant preheating:

An efficient bout of particle creation occurs when the inflaton crosses the origin, which happens twice in every oscillation. The reason is that fields which are coupled to the inflaton are massless near the *point of enhanced symmetry* (see Sec. VIB2). Mainly electroweak gauge fields and gauginos are then created as they have the largest coupling to the flat direction. The production takes place in a short interval, $\Delta t \sim (gm_\phi\phi_0)^{-1/2}$, where $\phi_0 \sim 10^{14}$ GeV is the initial amplitude of the inflaton oscillation, during which quanta with a physical momentum $k \leq (gm_\phi\phi_0)^{1/2}$ are produced. The number density of gauge/gaugino degrees of freedom is given by [88, 117]

$$n_g \approx \frac{(gm_\phi\phi_0)^{3/2}}{8\pi^3}. \quad (351)$$

As the inflaton VEV is rolling back to its maximum value ϕ_0 , the mass of the produced quanta $g\langle\phi(t)\rangle$ increases. The gauge and gaugino fields can (perturbatively) decay to the fields which are not coupled to the inflaton, for instance to (s)quarks. Note that (s)quarks are not coupled to the flat direction, hence they remain massless throughout

⁷⁷ One interesting observation for the *LLe* direction is that its VEV naturally gives masses to the Hypercharged fields, thus breaking the conformal invariance required for the photons [708, 709]. The excited hypercharge can be converted into normal electromagnetism after the electroweak phase transition. This would seed vector perturbations for the observed large scale magnetic field in the intergalactic medium [710].

the oscillations. The total decay rate of the gauge/gauginos fields is then given by $\Gamma = C (g^2/48\pi) g\phi$, where $C \sim \mathcal{O}(10)$ is a numerical factor counting for the multiplicity of final states.

The decay of the gauge/gauginos become efficient when $\langle\phi\rangle \simeq (48\pi m_\phi \phi_0 / C g^3)^{1/2}$, where we have used $\langle\phi(t)\rangle \approx \phi_0 m_\phi t$, which is valid when $m_\phi t \ll 1$, and $\Gamma \simeq t^{-1}$, where t represents the time that has elapsed from the moment that the inflaton crossed the origin. Note that the decay is very quick compared with the frequency of inflaton oscillations, i.e. $\Gamma \gg m_\phi$. It produces relativistic (s)quarks with an energy [88]:

$$E = \frac{1}{2} g \phi(t) \simeq \left(\frac{48\pi m_\phi \phi_0}{C g} \right)^{1/2}. \quad (352)$$

The ratio of energy density in relativistic particles thus produced ρ_{rel} with respect to the total energy density ρ_0 follows from Eqs. (351), (420):

$$\frac{\rho_{rel}}{\rho_0} \sim 10^{-1} g, \quad (353)$$

where we have used $C \sim \mathcal{O}(10)$. This implies that a fraction $\sim \mathcal{O}(10^{-1})$ of the inflaton energy density is transferred into relativistic (s)quarks every time that the inflaton passes through the origin. Within 10 – 50 oscillations the inflaton would loose its energy into relativistic MSSM degrees of freedom.

- Reheating and thermalization:

A full thermal equilibrium is reached when (a) *kinetic* and (b) *chemical equilibrium* are established. The maximum (hypothetical) temperature attained by the plasma would be given by:

$$T_{max} \sim V^{1/4} \sim (m_\phi \phi_0)^{1/2} \approx 10^9 \text{ GeV}. \quad (354)$$

However, not all the MSSM degrees of freedom can be in thermal equilibrium at such a high temperature. Depending on the nature of a flat direction inflaton, the final reheat temperature can be quite low.

For instance, if LLe is the inflaton then udd can acquire a large VEV independently. The VEV of udd will spontaneously generate masses to the gluons and gluinos, i.e.

$$m_G \sim g \langle\varphi(t)\rangle < g \phi_0. \quad (355)$$

To develop and maintain such a large VEV for udd , it is not necessary that udd potential has a saddle point, or point of inflection as well. It can be trapped in a false minimum during inflation, which will then be lifted by thermal corrections when the inflaton decays [88, 134]. The above inequality arises due to the fact that the VEV of φ cannot exceed that of the inflaton ϕ , since its energy density should be subdominant to the inflaton energy density.

If $g\varphi_0 \gg T_{max}$ the gluon/gluino fields will be too heavy and not kinematically accessible to the reheated plasma. Here φ_0 is the VEV of udd at the beginning of inflaton oscillations. In a radiation-dominated Universe the Hubble expansion redshifts the flat direction VEV as $\langle\varphi\rangle \propto H^{3/4}$, which is a faster rate than the change in the temperature $T \propto H^{1/2}$. Once $g\langle\varphi\rangle \simeq T$, gluon/gluino fields come into equilibrium with the thermal bath. When this happens the final reheat temperature is generically small, i.e. $T_R \leq 10^7$ GeV [88, 121]. See Secs. VID 4, VID 5, and VIE.

1. Benchmark points for MSSM inflation and dark matter abundance

In this section we explore the available parameter space for inflation in conjunction with a thermal cold dark matter abundance within the minimal SUGRA model. Remarkably for the inflaton, which is a combination of squarks and sleptons, there is a stau-neutralino coannihilation region below the inflaton mass 500 GeV for the observed density perturbations and the tilt of the spectrum. For such a low mass of the inflaton the LHC is capable of discovering the inflaton candidates within a short period of its operation. Inflation is also compatible with the focus point region which opens up for the inflaton masses above TeV.

Since m_ϕ is related to the scalar masses, sleptons (LLe direction) and squarks (udd direction), the bound on m_ϕ will be translated into the bounds on these scalar masses which are expressed in terms of the model parameters [88].

Note that CMB constraints m_ϕ at $\phi_0 \sim 10^{14}$ GeV which is two orders of magnitude below the GUT scale. From this m_ϕ , m_0 and $m_{1/2}$ are determined at the GUT scale by solving the RGEs for fixed values of A_0 and $\tan\beta$. The RGEs for m_ϕ are

$$\begin{aligned} \mu \frac{dm_\phi^2}{d\mu} &= \frac{-1}{6\pi^2} \left(\frac{3}{2} M_2^2 g_2^2 + \frac{9}{10} M_1^2 g_1^2 \right), \quad (\text{for } LLe) \\ \mu \frac{dm_\phi^2}{d\mu} &= \frac{-1}{6\pi^2} \left(4M_3^2 g_3^2 + \frac{2}{5} M_1^2 g_1^2 \right), \quad (\text{for } udd). \end{aligned} \quad (356)$$

M_1 , M_2 and M_3 are $U(1)$, $SU(2)$ and $SU(3)$ gaugino masses respectively. After determine m_0 and $m_{1/2}$ from m_ϕ , one can determine the allowed values of m_ϕ from the experimental bounds on the mSUGRA parameters space [711–714].

The models of mSUGRA depend only on four parameters and one sign. These are m_0 (the universal scalar soft breaking mass at the GUT scale M_G); $m_{1/2}$ (the universal gaugino soft breaking mass at M_G); A_0 (the universal trilinear soft breaking mass at M_G); $\tan\beta = \langle H_u \rangle \langle H_d \rangle$ at the electroweak scale (where H_u gives rise to u quark masses and H_d to d quark and lepton masses); and the sign of μ , the Higgs mixing parameter in the superpotential ($W_\mu = \mu H_u H_d$).

Unification of gauge couplings within SUSY suggests that $M_G \simeq 2 \times 10^{16}$ GeV. The model parameters are already significantly constrained by different experimental results. Most important constraints are: (1) The light Higgs mass bound of $M_{h^0} > 114.0$ GeV from LEP [715]. (2) The $b \rightarrow s\gamma$ branching ratio [716]: $2.2 \times 10^{-4} < \mathcal{B}(B \rightarrow X_s \gamma) < 4.5 \times 10^{-4}$. (3) In mSUGRA the $\tilde{\chi}_1^0$ is the candidate for CDM. (4) The 2σ bound from the WMAP [12] gives a relic density bound for CDM to be $0.095 < \Omega_{\text{CDM}} h^2 < 0.129$. (5) The bound on the lightest chargino mass of $M_{\tilde{\chi}_1^\pm} > 104$ GeV from LEP [717]. (6) The possible 3.3σ deviation (using e^+e^- data to calculate the leading order hadronic contribution) from the SM expectation of the anomalous muon magnetic moment from the muon $g - 2$ collaboration [718].

The allowed mSUGRA parameter space has mostly three distinct regions: (i) the stau-neutralino ($\tilde{\tau}_1 - \tilde{\chi}_0^1$), coannihilation region where $\tilde{\chi}_0^1$ is the LSP, (ii) the $\tilde{\chi}_0^1$ having a dominant Higgsino component (focus point) and (iii) the scalar Higgs (A^0, H^0) annihilation funnel ($2M_{\tilde{\chi}_0^1} \simeq M_{A^0, H^0}$).

The mSUGRA parameter space in Figs. (12), for $\tan\beta = 10$ and 40 with the udd flat direction using $\lambda = 1$. In the figures, we show contours correspond to $n_s = 1$ for the maximum value of $\delta_H = 2.03 \times 10^{-5}$ (at 2σ level) and $n_s = 1.0, 0.98, 0.96$ for $\delta_H = 1.91 \times 10^{-5}$. It is also interesting to note that the allowed region of m_ϕ , as required by the inflation data for $\lambda = 1$ lies in the stau-neutralino coannihilation region which requires smaller values of the SUSY particle masses. See Fig. (9), where both co-annihilation and focus point regions have been illustrated for $\lambda \sim 1 - 0.02$.

The SUSY particles in this parameter space are, therefore, within the reach of the LHC very quickly. The detection of the region at the LHC has been considered in [719]. From the figures, one can also find that as $\tan\beta$ increases, the inflation data along with the dark

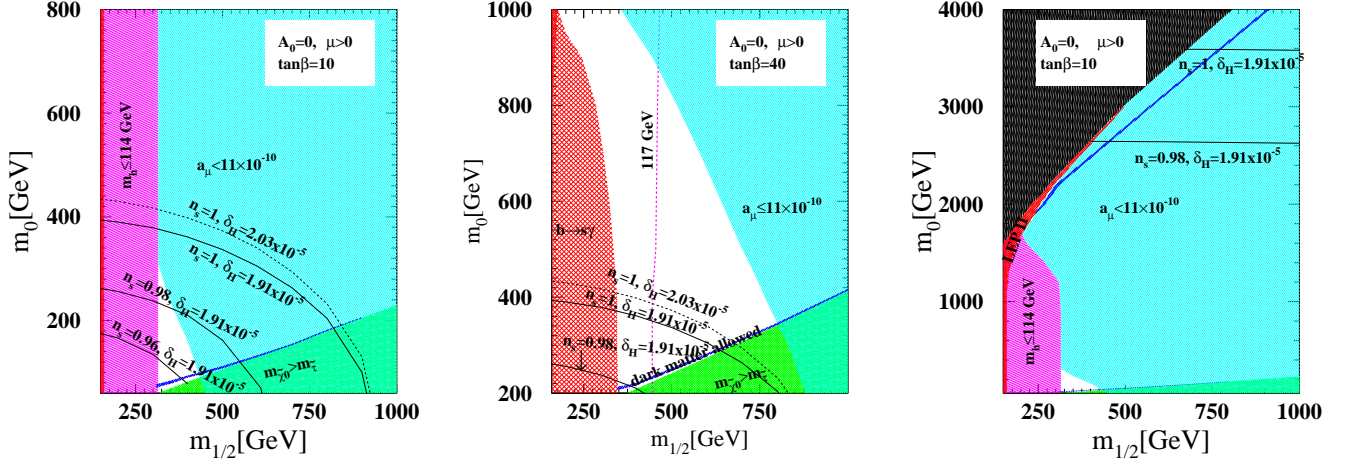


FIG. 12: The contours for different values of n_s and δ_H are shown in the $m_0 - m_{1/2}$ plane for $\tan\beta = 10, 40$, and $\lambda = 1$ for the contours. We show the dark matter allowed region narrow blue corridor, $(g-2)_\mu$ region (light blue) for $a_\mu \leq 11 \times 10^{-8}$, Higgs mass ≤ 114 GeV (pink region) and LEP II bounds on SUSY masses (red). In the third panel $\lambda = 0.1$ were chosen, The black region is not allowed by radiative electroweak symmetry breaking, and $m_t = 172.7$ GeV for this graph. Note that black curved lines denote the cosmological parameters, (δ_H, n_s) , within 95% c.l. Note that smaller values of $n_s < 1$ is preferred by the dark matter abundance in this scheme of parameters. The plots are taken from [696].

matter, rare decay and Higgs mass constraint allow smaller ranges of $m_{1/2}$. For example, the allowed ranges of gluino masses are 765 GeV-2.1 TeV and 900 GeV-1.7 TeV for $\tan\beta = 10$ and 40 respectively. Now if λ is small, i.e. $\lambda \lesssim 10^{-1}$, the allowed values of m_ϕ would be large. In this case the dark matter allowed region requires the lightest neutralino to have larger Higgsino component in the mSUGRA model.

2. Can dark matter be the inflaton?

If the reheat temperature of the universe is higher than the mass of the inflaton, then the plasma upon reheating will, in addition, have a thermal distribution of the inflaton quanta. If the inflaton is absolutely stable, due to some symmetry, then it can also serve as the cold dark matter. One such example, $NH_u L$ as an observable sector inflaton is an interesting scenario, as it can explain successful inflation, exciting SM quarks and leptons, the observed

neutrino masses via Dirac coupling and the right handed sneutrino, \tilde{N} , as a dark matter candidate [698].

It is well known that for particles with gauge interactions, the unitarity bound puts an absolute upper bound on the dark matter mass to be less than ~ 100 TeV [334]. Once the temperature drops below the inflaton mass, its quanta undergo thermal freeze-out and yield the correct dark matter abundance. Furthermore, scatterings via the new $U(1)_{B-L}$ gauge interactions also bring the right handed sneutrino into thermal equilibrium. Note that part of the inflaton, i.e. its \tilde{N} component see Eq. (334), has never decayed; only the coherence in the original condensate that drives inflation is lost. However, the neutrino Yukawa, h , is way too small to allow acceptable thermal dark matter. Note that \tilde{N} would dominate the universe right after the end of inflation if it had no gauge interactions.

In order to calculate the relic abundance of the RH sneutrino, it is necessary to know the masses of the additional gauge boson Z' and its SUSY partner \tilde{Z}' , the new Higgsino masses, Higgs VEVs which break the new $U(1)$ gauge symmetry, the RH sneutrino mass, the new gauge coupling, and the charge assignments for the additional $U(1)$. Assuming that the new gauge symmetry is broken around 2 TeV, and the existence of two new Higgs superfields to maintain the theory anomaly free.

The primary diagrams responsible to provide the right amount of relic density are mediated by \tilde{Z}' in the t -channel. In Fig. (13), we show the relic density values for smaller masses of sneutrino. In the case of a larger sneutrino mass in this model, the correct dark matter abundance can be obtained by annihilation via Z' pole [698, 720]. A sneutrino mass in the 1 – 2 TeV range provides a good fit to the PAMELA data and a reasonable fit to the ATIC data [721].

Since the dark matter candidate, the RH sneutrino, interacts with quarks via the Z' boson, it is possible to see it via the direct detection experiments. The detection cross sections are not small as the interaction diagram involves Z' in the t -channel. The typical cross section is about 2×10^{-8} pb for a Z' mass around 2 TeV, makes it possible to probe the dark matter candidate in direct detection [722].

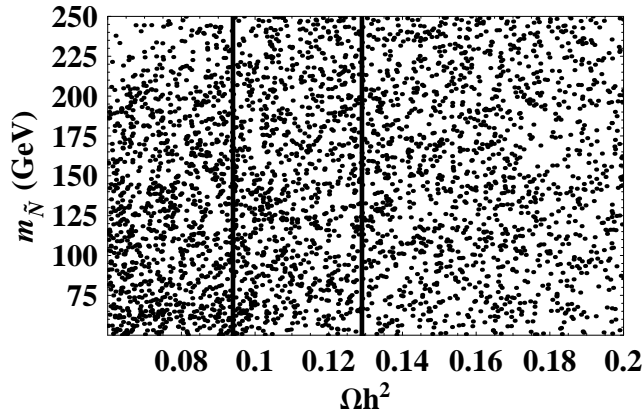


FIG. 13: Ωh^2 vs $m_{\tilde{N}}$. The solid lines from left to right are for $\Omega h^2 = 0.094$ and 0.129 respectively. The Z' -ino mass is equal to the Bino mass since the new $U(1)$ gauge coupling is the same as the hypercharge gauge coupling.

E. Stochastic initial conditions for low scale inflation

It is conceivable that the universe gets trapped in a metastable vacuum at earlier stages, irrespective of how it began. Metastable vacua are ubiquitous in string theory, see the discussion in section VIII D. Inflation can be driven from one vacuum to another, either via quantum tunneling or via transient phase of non-inflationary dynamics [155]. In any case, it is important to note that these high scale inflation provides a natural initial condition for MSSM inflation.

Let us imagine that there are almost degenerate metastable vacua. Once the energy density of the false vacuum dominates, inflation begins and the universe undergoes accelerated expansion with a constant Hubble rate H_{false} . False vacuum is not stable and decays via bubble nucleation. The rate per volume for the decay of a metastable vacuum to the true vacuum is given by:

$$\Gamma/V = C \exp(-\Delta S_E) , \quad (357)$$

where C is a one-loop determinant and ΔS_E is the difference in Euclidean actions between the instanton and the background with larger cosmological constant. The determinant C can at most be $C \lesssim M_{\text{P}}^4$, simply because M_{P} is the largest scale available, and estimates (ignoring metric fluctuations) give a value as small as $C \sim r_0^{-4}$, with r_0 the instanton bubble

radius [723, 724]. Therefore a typical decay rate in a (comoving) Hubble volume is given by

$$\Gamma \lesssim \frac{M_{\text{P}}^4}{H_{\text{false}}^3} \exp(-\Delta S_E). \quad (358)$$

Especially with a large Hubble scale H_{false} , the associated decay time is much longer than H_{false}^{-1} , given that typically $\Delta S_E \gg 1$. This implies that most of the space is locked in the false vacuum and inflate forever, while bubble nucleation creates pockets of true vacuum whose size grow only linearly in time. Here we will concentrate on one such true vacuum, where MSSM flat directions can be excited.

1. Quantum fluctuations of MSSM flat directions

During false vacuum inflation quantum fluctuations displace any scalar field whose mass is smaller than H_{false} . The question is whether these fluctuations can push the MSSM inflaton sufficiently to the plateau of its potential around the point of inflection ϕ_0 , see Eqs. (319,324). If MSSM inflaton begins with a small VEV $\phi < \phi_0$, the mass term in Eq. (311) dominates. Hence, for any $H_{\text{false}} > \mathcal{O}(\text{TeV})$, it obtains a quantum jump, induced by the false vacuum inflation, of length $H_{\text{false}}/2\pi$, within each Hubble time [486]. Typically the quantum fluctuations have a Gaussian distribution, and the r.m.s (root mean square) value of the modes which exit the inflationary Hubble patch within one Hubble time is given by $H_{\text{false}}/2\pi$. These jumps superimpose in a random walk fashion, which is eventually counterbalanced by the classical slow-roll due to the mass term, resulting in [173, 181–183, 191, 486]

$$\langle \phi^2 \rangle = \frac{3H_{\text{false}}^4}{8\pi^2 m_\phi^2} \left[1 - \exp\left(-\frac{2m_\phi^2}{3H_{\text{false}}}t\right) \right], \quad (359)$$

which for $t \rightarrow \infty$ yields

$$\phi_{r.m.s} = \sqrt{\frac{3}{8\pi^2} \frac{H_{\text{false}}^2}{m_\phi}}. \quad (360)$$

If $\phi_{r.m.s} \geq \phi_0$, then ϕ will lie near ϕ_0 , see Eq. (319), in many regions of space. This requires that

$$H_{\text{false}} \geq \left(\frac{8\pi^2}{3}\right)^{1/4} (m_\phi \phi_0)^{1/2} \gtrsim 10^8 \text{ GeV}, \quad (361)$$

where $m_\phi \sim 100 \text{ GeV}$ and $\phi_0 \sim 3 \times 10^{14} \text{ GeV}$. $\phi_{r.m.s}$ settles at its final value when $t > 3H_{\text{false}}/2m_\phi^2$, which amounts to

$$N_{\text{false}} > \frac{3}{2} \left(\frac{H_{\text{false}}}{m_\phi}\right)^2 \gtrsim 10^{16}, \quad (362)$$

e-foldings of false vacuum inflation. The large number of e-foldings required is not problematic as inflation in the false vacuum is eternal in nature.

2. Inflection point as a dynamical attractor

In general the MSSM inflaton can have a VEV above the point of inflection $\phi > \phi_0$ and/or a large velocity $\dot{\phi}$ at the beginning of false vacuum inflation [86–89, 279]. In this case the classical motion of the field becomes important. There are typically three regimes in the evolution of ϕ field [89]; (1) Oscillatory regime: If the initial VEV of ϕ , denoted by ϕ_i , is such that $V''(\phi_i) > H_{\text{false}}^2$, then it starts in the oscillatory regime, (2) Kinetic energy dominance regime: If $\phi_i < \phi_{\text{tr}}$, then $V''(\phi) < H_{\text{false}}^2$, and the potential is flat during false vacuum inflation, then the dynamics of ϕ in this case depends on its initial velocity denoted by $\dot{\phi}_i$. If $\dot{\phi}_i^2 > 2V(\phi_i)$, the kinetic dominance prevails. In principle the inflaton will overshoot the point of inflection only if it begins very close to ϕ_0 and has a large negative velocity initially. It is interesting to note that ϕ can end up above the point of inflection even if $\phi_i < \phi_0$, provided that $\dot{\phi}_i > 3H_{\text{false}}(\phi_0 - \phi_i)$. The most important regime is (3) the slow-roll regime: which we will discuss below.

Once an initial phase of oscillations or kinetic energy dominance ends, ϕ starts a slow-roll motion towards the inflection point, ϕ_0 . The equation of motion in this regime is $3H_{\text{false}}\dot{\phi} + V'(\phi) \approx 0$. Initially the field is under the influence of the non-renormalizable potential, i.e. $V(\phi) \propto \phi^{10}$, see Eq. (317), however, as ϕ moves toward ϕ_0 , the ϕ^{10} term becomes increasingly smaller. Eventually, for $\phi \approx \phi_0$, we have $V'(\phi) = V'(\phi_0) + V'''(\phi_0)(\phi - \phi_0)^2/2$. It happens that this is the longest part of ϕ journey. It is desirable to find how long does it take for ϕ to reach the edge of the plateau in Eq. (324). Outside the plateau, $V'(\phi_0)$ is subdominant, see Eqs. (322, 323), and hence:

$$\dot{\phi} = -\frac{32m_\phi^2(\phi - \phi_0)^2}{3H_{\text{false}}\phi_0}. \quad (363)$$

This results in

$$(\phi - \phi_0) \approx \frac{3H_{\text{false}}\phi_0}{32m_\phi^2 t}, \quad (364)$$

for large t . Therefore the inflection point acts as an attractor for the classical equation of motion. After using Eq. (324), $t_{\text{slow}} \gtrsim 10^{10}$ e-foldings of false vacuum inflation (note that $\phi_0 \sim 10^{14}$ GeV, and $H_{\text{false}} > m_\phi \sim \mathcal{O}(100)$ GeV). After this time ϕ is settled within the

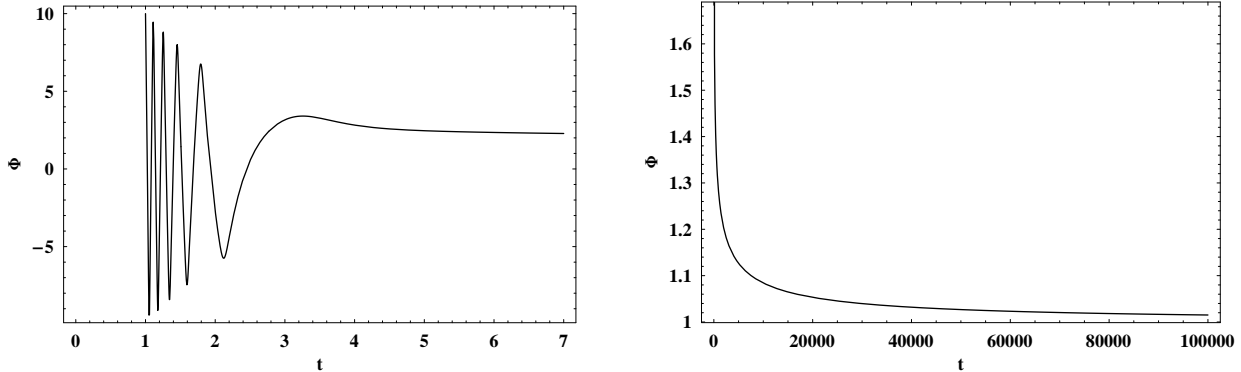


FIG. 14: The plot shows ϕ (scaled by ϕ_0) as a function of time for $\phi_i = 10$ and $\dot{\phi}_i = 100$, with $H_{\text{false}} = 10^3 m_\phi$. There is an initial oscillatory phase since $V''(\phi_i) \geq H_{\text{false}}^2$. It ends quickly as the amplitude of oscillations decreases fast due to Hubble expansion. Then the slow-roll motion begins which lasts much longer. In the second plot the slow-roll phase for the same initial conditions is depicted. It lasts very long but the field asymptotes to the *point of inflection* ϕ_0 with $\dot{\phi} \rightarrow 0$. The plots are taken from [89].

plateau in the bulk of the inflating space. This implies that the bubbles which nucleate henceforth have the right initial conditions for a subsequent stage of MSSM inflation.

3. Inflating the MSSM bubble

Let us consider the bubbles that have the right initial conditions for MSSM inflation, i.e. ϕ has settled in the plateau around a point of inflection according to Eq. (324). The initial size of the bubble is $r_0 < H_{\text{false}}^{-1}$. Inside of an expanding bubble has the same geometry as an open FRW universe. The Hubble rate inside the bubble is therefore given by [279]

$$H^2 = \frac{V_\phi + V_\varphi}{3M_{\text{P}}^2} + \frac{1}{a^2}, \quad (365)$$

where a is the scale factor of the universe and V_φ is the energy density in the φ field. Note that φ is the field responsible for forming the false vacuum, which could either arise within MSSM or from some other sector. There are examples of φ field as an MSSM flat direction.

Since ϕ is inside the plateau of its potential, its dynamics is frozen, hence $V(\phi) \sim V(\phi_0)$ as long as $H > H_{\text{MSSM}}$. Right after tunneling, $H \equiv \dot{a}/a = r_0^{-1} > H_{\text{false}}$. This implies that the last term on the right-hand side of Eq. (365) dominates over the first two terms, and hence

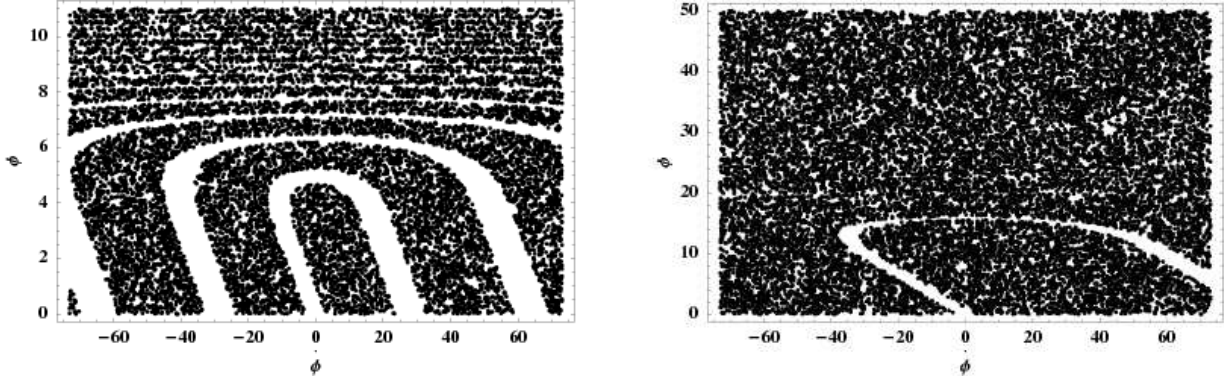


FIG. 15: The attractor behavior holds for a wide range of initial values of ϕ and $\dot{\phi}$. In the left panel the initial values of ϕ versus $\dot{\phi}$ for $H_{\text{false}} = 10^2 m_\phi$ is shown. The dots show the initial values for which ϕ settles to $\pm\phi_0$ and the white bands \cap show the critically damped regions where ϕ settles to zero at late times. The situation improves a lot for a larger $H_{\text{false}} = 10^4 m_\phi$ as shown in the right hand panel. The plots are symmetric under $\phi \rightarrow -\phi$. Here we have shown the upper half of $\phi - \dot{\phi}$ plane where $\phi \geq 0$. The plots are taken from [89].

the universe is curvature dominated. The φ field oscillates around the true vacuum of its potential at the origin, and quickly decays to radiation whose energy density is redshifted $\propto a^{-4}$. On the other hand the curvature term is redshifted $\propto a^{-2}$, while $V(\phi)$ remains essentially constant (due to extreme flatness of the inflaton potential) for $H > H_{\text{MSSM}}$. As a result, the universe inside the bubble will remain curvature dominated until $H \simeq H_{\text{MSSM}}$.

At this point the inflaton field ϕ dominates the energy density and a phase of MSSM inflation begins. This blows the open universe inside the bubble and inflates away the curvature term. As long as the total number of e-foldings is N_Q plus few, the observable part of the universe looks like flat today (within the limits of 5 year WMAP data) [12]. Perturbations of the correct size with acceptable spectral index will be generated during the slow-roll phase, and the SM degrees of freedom will be created from the decay of ϕ field in the post-inflationary phase.

F. Other examples of gauge invariant inflatons

Within MSSM:

So far we have studied the flat direction inflaton represented by a monomial superfield, Φ ,

instead one can also imagine a polynomial I spanned by the Higgses and the sleptons as an example [692],

$$I = \nu_1 H_u L_1 + \nu_2 H_u L_2 + \nu_3 H_u L_3 \quad (366)$$

where ν_i are complex coefficients, H_u is the up-type Higgs and L_i are the sleptons. For a following field configuration, the polynomial I has a vanishing matter current and vanishing gauge fields [692],

$$L_i = e^{-i\chi/2} \phi_i \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad H_u = e^{i\chi/2} \sqrt{\sum_i |\phi_i|^2} \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad (367)$$

where ϕ_i are complex scalar fields and the phase χ is a real field constrained by

$$\partial_\mu \chi = \frac{\sum_j J_j^\phi}{2i \sum_k |\phi_k|^2}, \quad J_i^\phi = \phi_i^* \partial_\mu \phi_i - \phi_i \partial_\mu \phi_i^*. \quad (368)$$

The field configuration in Eq. (367) leads to an effective Lagrangian for the flat direction fields ϕ_i ,

$$\mathcal{L} = |D_\mu H_u|^2 + \sum_{i=1}^3 |D_\mu L_i|^2 - V = \frac{1}{2} \partial_\mu \Phi^\dagger \left(1 + P_1 - \frac{1}{2} P_2 \right) \partial^\mu \Phi - V, \quad (369)$$

where D_μ is a gauge covariant derivative that reduces to the partial derivative when the gauge fields vanish, P_1 is the projection operator along Φ and P_2 along Ψ , where

$$\bar{\phi} = (\phi_1 \quad \phi_2 \quad \phi_3)^T, \quad \Phi = \begin{pmatrix} \bar{\phi} \\ \bar{\phi}^* \end{pmatrix}, \quad \Psi = \begin{pmatrix} \bar{\phi} \\ -\bar{\phi}^* \end{pmatrix}, \quad (370)$$

and the corresponding equation of motion

$$\begin{aligned} \partial_\mu \partial^\mu \Phi + 3H\dot{\Phi} + \left(1 - \frac{1}{2} P_1 + P_2 \right) \frac{\partial V}{\partial \Phi^\dagger} - R^{-2} \left[\partial_\mu \Psi (\Psi^\dagger \partial^\mu \Phi) \right. \\ \left. + \Psi (\partial_\mu \Psi^\dagger P_2 \partial^\mu \Phi) + \frac{1}{2} \Phi \partial_\mu \Phi^\dagger \left(1 - P_1 - \frac{3}{2} P_2 \right) \partial^\mu \Phi \right] = 0, \end{aligned} \quad (371)$$

where $R = \sqrt{\Phi^\dagger \Phi}$. We are interested in the background dynamics where all the fields are homogeneous in time, and for simplicity we study only the radial motion, such that $\Phi = R \hat{e}_\Phi$, where $\dot{\hat{e}}_\Phi = 0$ (the dot denotes derivative w.r.t time). Then the equation of motion simplifies to

$$\ddot{R} + 3H\dot{R} + \frac{1}{2} \frac{\partial V}{\partial R} = 0, \quad (372)$$

A notable feature is that the fields have non-minimal kinetic terms, since the field manifold defined by the flat direction is curved, actually a hyperbolic manifold. This results into the usual equation of motion for one scalar field with a potential for the radial mode except for the factor $1/2$, which makes the potential effectively flatter in this direction. This can be traced to the square root nature of H_u in Eq. (367).

As far as our example of LH_u is concerned there are only three families which we can account for. The flattest MSSM direction, $QuQue$, is lifted by $n = 9$ superpotential operator, $QuQuQuH_d ee$. The flat direction $QuQue$ is an 18 complex dimensional manifold, Ref. [399]. The largest D-flat direction is only 37 complex dimensional [399]. One can imagine a larger representation which will have larger number of D -flat directions which can mimic *assisted inflation* [281]⁷⁸.

Beyond MSSM:

A gauge invariant inflationary model has been proposed sometime ago in in Ref. [728]. The idea is that $(SU(3))^3$ gauge group is spontaneously broken down to $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$. The quartic contribution to the superpotential is given by a D -flat direction of $(SU(3))^3$, Φ , which is invariant under 27,

$$W \sim \frac{\lambda}{M_P} (27\bar{27})^2, \quad (373)$$

where λ is a small number determined by matching the amplitude of the CMB observations, and Φ is the monomial representing, N, \bar{N} or $\nu^c, \bar{\nu}^c$. The potential along such a D -flat direction is given by [728]:

$$V(\phi) \approx -M_S^2 |\phi|^2 + \frac{\lambda^2}{3} \frac{|\phi|^6}{M_P^2}, \quad (374)$$

⁷⁸ Let us consider M fields H_i and $N - 1$ fields G_j in the fundamental representation N of the gauge group $SU(N)$. Note that the matter content is also enhanced, which has a total $N - 1 + M$ degrees of freedom. Then there exists a D -flat direction described by a gauge invariant polynomial $I = \sum_{j=1}^M \alpha_j \epsilon_{d_1 \dots d_{N-1}} H_1^{d_1} \dots H_{N-1}^{d_{N-1}} G_j^e$, which after solving the constraint equations $\partial I / \partial H_j^a = CH_j^{a*}$, $\partial I / \partial G_i^a = CG_i^{a*}$ produces a vacuum configuration: $H_j^a = \delta_N^a \phi_j$, for $j = 1, \dots, M$, and $G_i^a = \delta_i^a \sqrt{\sum_{j=1}^M |\phi_j|^2}$, for $i = 1, \dots, N - 1$. When one substitutes these into D -terms, one finds that all D -terms vanish. The Lagrangian for the flat direction is given by $\mathcal{L} = c \sum_{j=1}^M |D_\mu H_j|^2 + c \sum_{i=1}^{N-1} |D_\mu G_i|^2 - V(\{H_i, G_j\})$, where $c = 1/2$ for the real fields, and $c = 1$ for the complex fields [281, 725]. The lagrangian reduces to: $\mathcal{L} = \frac{1}{2} \partial_\mu \Phi^\dagger [1 + (N - 1)P] \partial^\mu \Phi - V(\Phi)$ where $P = \Phi \Phi^\dagger / (\Phi^\dagger \Phi)$ is the projection operator, and the field configurations of the real fields are: $\Phi = (\phi_1, \dots, \phi_M)^T$, for $\phi_i \in \mathcal{R}$. Similar generalizations can be made for $N \times N$ non-commutative hermitian matrices, see [726, 727].

where $M_S \sim 10^3$ GeV, denotes the soft SUSY breaking scale. It was argued that the negative mass squared term would appear due to running in presence of strong dynamics [728]. Inflation happens near $\phi \sim 0$, and ends with a VEV, $\phi \sim M \sim \lambda^{-1/2} \sqrt{M_P M_S}$ GeV. In order to match the CMB temperature anisotropy, $\Delta T/T \approx 0.023 \lambda N_Q^2$, where $N_Q = (2\pi/3)(\phi/M_P)^2$ is the number of e-foldings before the end of inflation. We require $M \sim 10^{15}$ GeV and $\lambda \sim 10^{-7}$ [728]. The spectral index for the scalar perturbations tend to be small $n \simeq 0.92-0.88$, while the ratio of the tensor to the scalar ratio is given by; $r \approx 0.4-0.7$.

VI. INFLATON DECAY, REHEATING AND THERMALIZATION

A. Perturbative decay and thermalization

For a plasma which is in full thermal equilibrium, the energy density, ρ , and the number density, n , of relativistic particles are given by [25, 95]

$$\begin{aligned} \rho &= (\pi^2/30) T^4, & n &= (\zeta(3)/\pi^2) T^3, & & \text{(Boson)}, \\ \rho &= (7/8) (\pi^2/30) T^4, & n &= (3/4) (\zeta(3)/\pi^2) T^3, & & \text{(Fermion)}, \end{aligned} \quad (375)$$

where T is the temperature of a thermal bath. Note that in a full equilibrium the relationships, $\langle E \rangle \sim \rho^{1/4}$, and $n \sim \rho^{3/4}$ hold, with $\langle E \rangle = (\rho/n) \simeq 3T$ being the average particle energy. On the other hand, right after the inflaton decay has completed, the energy density of the universe is given by: $\rho \approx 3(\Gamma_d M_P)^2$. For a perturbative decay, which generates entropy, we have $\langle E \rangle \approx m_\phi \gg \rho^{1/4}$. Then, from the conservation of energy, the total number density is found to be, $n \approx (\rho/m_\phi) \ll \rho^{3/4}$. Hence the *complete inflaton decay* results in a dilute plasma which contains a small number of very energetic particles. This implies that the universe is far from full thermal equilibrium initially [92, 93, 96–99, 119, 121, 729].

Reaching full equilibrium requires re-distribution of the energy among different particles, *kinetic equilibrium*, as well as increasing the total number of particles, *chemical equilibrium*.

Therefore both the number-conserving and the number-violating reactions must be involved.

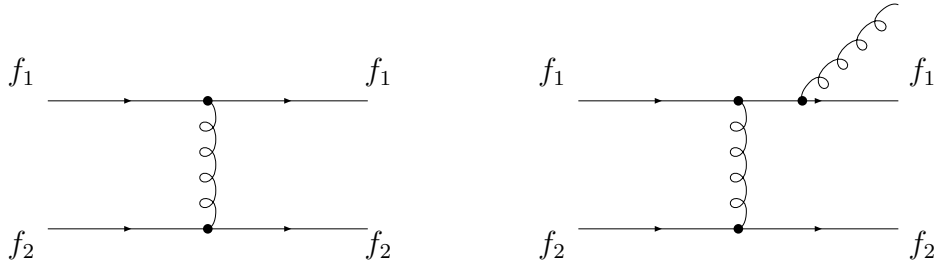


Fig. A Typical scattering diagram which builds kinetic equilibrium in the reheat plasma. Note that the t -channel singularity which results in a cross-section $\propto |t|^{-1}$. The second panel shows a typical scattering diagram which increase the number of particles.

- Kinetic equilibrium among SM fermions:

The most important processes are $2 \rightarrow 2$ scatterings with gauge boson exchange in the t -channel, shown in Fig. (A). The cross-section for these scatterings is $\sim \alpha|t|^{-1}$. Here " t " is related to the exchanged energy, ΔE , and the momentum, $\vec{\Delta p}$, through $t = \Delta E^2 - |\vec{\Delta p}|^2$. The fine structure constant is denoted by α (note that $\alpha \geq 10^{-2}$ in the SM/MSSM). This cross section can be understood as follows: the gauge boson propagator introduces a factor of $|t|^{-2}$, while phase space integration results in an extra factor of $|t|$. Scalar exchange in t -channel diagrams are usually suppressed, similarly a fermion-fermion-scalar vertex, which arises from a Yukawa coupling, flips the chirality of the scattered fermion, are also suppressed. Due to an infrared singularity, these scatterings are very efficient even in a dilute plasma [119, 121].

- Chemical equilibrium:

In addition one also needs to achieve chemical equilibrium by changing the number of particles in the reheat plasma. The *relative* chemical equilibrium among different degrees of freedom is built through $2 \rightarrow 2$ annihilation processes, occurring through s -channel diagrams. Hence they have a much smaller cross-section $\sim \alpha s^{-1}$. More importantly the total number of particles in the plasma must also change. It turns out from Eq. (375) that in order to reach full equilibrium, the total number of particles must *increase* by a factor of: n_{eq}/n , where $n \approx \rho/m_\phi$ and the equilibrium value is: $n_{\text{eq}} \sim \rho^{3/4}$. This can be a very large number, i.e. $n_{\text{eq}}/n \sim \mathcal{O}(10^3)$. It was

recognized in [119, 120], see also [730–732], that the most relevant processes are $2 \rightarrow 3$ scatterings with gauge-boson exchange in the t -channel. Again the key issue is the infrared singularity of such diagrams shown in Fig. (A). The cross-section for emitting a gauge boson, whose energy is $|t|^{1/2} \ll E$, from the scattering of two fermions is $\sim \alpha^3 |t|^{-1}$. When these inelastic scatterings become efficient, i.e., their rate exceeds the Hubble expansion rate, the number of particles increases very rapidly [733], because the produced gauge bosons subsequently participate in similar $2 \rightarrow 3$ scatterings. Decays (which have been considered in [732]) are helpful, but in general they cannot increase the number of particles to the required level.

The full thermal equilibrium will be established shortly after the $2 \rightarrow 3$ scatterings become efficient. For this reason, to a very good approximation, one can use the rate for inelastic scatterings as a thermalization rate of the universe Γ_{thr} . Since the inflaton decay products have SM gauge interactions, the universe reaches full thermal equilibrium immediately after the inflaton decay. The reason is that the $2 \rightarrow 3$ scatterings with gauge boson exchange in the t -channel are very efficient, see [119–121].

Even before all inflatons decay, the decay products form a plasma can very quick thermalize, and the plasma has the instantaneous temperature given by Eq. (76). The plasma can reach its maximum T_{max} soon after the inflaton field starts to oscillate around the minimum of its potential, which happens for a Hubble parameter $H_I \leq m_\phi$. During this era the energy density of the universe is still dominated by the (non-relativistic) inflatons that haven't decayed yet. The scale factor of the universe a then varies as $a \propto T^{-8/3}$ [95]. The universe remains in this phase as long as $H > \Gamma_d$. During this phase one can produce massive long-lived or stable weakly interacting massive particles (WIMPS) [120, 734–741], see also gravitational production of particles [742, 743]. There are three possible scatterings which have been discussed in the literature.

Particle creation via Soft-soft scatterings were investigated in detail in Refs. [734–738, 741], where the relevant Boltzmann equations governing the production and annihilation of stable particles, χ 's, are solved both numerically and analytically. In Refs. [734–736, 741], out of equilibrium production of χ from scatterings in the thermal bath were studied and the final result is found to be (the superscript “ss” stands for χ production from “soft-soft”

scatterings)⁷⁹:

$$\Omega_\chi^{\text{ss}} h^2 \sim \left(\frac{200}{g_*}\right)^{3/2} \alpha_\chi^2 \left(\frac{2000 T_{\text{R}}}{m_\chi}\right)^7. \quad (376)$$

Here Ω_χ is the χ mass density in units of the critical density, and h is the Hubble constant in units of 100 km/(s·Mpc). The cross section for χ pair production or annihilation is given by: $\sigma \simeq \alpha_\chi^2/m_\chi^2$. Most χ particles are produced at $T \simeq m_\chi/4$. The density of earlier produced particles is strongly red-shifted, while χ production at later times is suppressed by the Boltzmann factor.

B. Non-perturbative inflaton scatterings

Many studies have been devoted to understand non-perturbative effects during reheating. Various non-thermal and non-perturbative effects may lead to a rapid transfer of the inflaton energy to other degrees of freedom by the process known as *preheating*. The requirement is that the inflaton quanta couple to other (essentially massless) field χ through, i.e. terms like $\phi^2\chi^2$. The quantum modes of χ may then be excited during the inflaton oscillations via a *parametric resonance*. Preheating has been treated both analytically [96–99, 111–118, 123, 530, 744–753] (for an elaborate discussion on reheating and preheating, see [99]), and numerically on lattice simulations [100–110, 754]. Like bosons, fermions can also be excited during preheating [94, 331, 755–758]. In fact, it has been argued that fermionic preheating is more efficient than bosonic preheating, however these fermions can not be related to the chiral fermions of the SM. The SM fermions can only couple to a gauge singlet inflaton via non-renormalizable dimensional 5 operators, therefore the effective couplings are very small. During preheating it is possible to excite gravity waves [709, 759–768], magnetic field [769, 770], gravitino abundance with spin $\pm 1/2$, $\pm 3/2$ [641, 771–776], moduli and non-thermal stringy relics [777], phase transitions [707, 778]. A successful cold electroweak baryogenesis were also studied in the context of preheating [779–781].

During preheating, it is also possible to excite the perturbed FRW metric potential, see [210, 763, 782–785], however as shown in Ref. [786], it is hard to excite large metric

⁷⁹ Particle creation via hard-soft scatterings, and hard-hard scatterings were also considered in [120, 740]. It was found that the χ production through hard-hard scattering is most efficient *before* thermalization is completed.

perturbations in $g^2\phi^2\chi^2$ theory. For large g and large inflaton VEV, ϕ , the initial χ perturbations in the vacuum are very much suppressed, see also [787]. The second order metric perturbations can also leave non-Gaussian signature during preheating [238–241, 565], which may put severe constraint on a simple $\lambda\phi^4$ inflation model [238].

1. Parametric Resonance

Let us briefly review the initial stages of a gauge singlet inflaton decay, which happens typically non-perturbatively, i.e. preheating, in most of the non-SUSY cases. In SUSY, there are complications with the potential itself, as well as the presence of MSSM flat directions.

Our focus is on bosonic preheating which acts most efficiently in transferring the energy density from the inflaton oscillations. We consider models of large field inflation, such as chaotic inflation or hybrid models, for which bosonic preheating is most pronounced. The relevant renormalizable couplings between the inflaton ϕ and a scalar field χ will read from the following potential:

$$V = \frac{1}{2}m_\phi^2\phi^2 + \frac{1}{2}m_\chi^2\chi^2 + \sigma\phi\chi^2 + h^2\phi^2\chi^2 + \lambda\chi^4, \quad (377)$$

where we have considered ϕ and χ to be real. Here σ is a coupling which has a [mass] dimension. The only scalar field in the SM is the Higgs doublet. Therefore in a realistic case χ denotes the real and imaginary parts of the Higgs components. The cubic interaction term is required for a complete inflaton decay. The quartic self-coupling of χ is required to bound the potential from below along the χ direction. The dimensionless couplings σ/m_ϕ and h (as well as λ) are not related to each other, hence either of the cubic or the quartic terms can dominate at the beginning of inflaton oscillations (i.e. when the Hubble expansion rate is $H(t) \simeq m_\phi$ and the amplitude of oscillations is $\hat{\phi} \sim \mathcal{O}(M_{\text{P}})$).

In a non-SUSY case efficient preheating happens over a narrow window $3 \times 10^{-4} \leq h \leq 10^{-3}$. The reason is that the $h^2\phi^2\chi^2$ term yields a quartic self-coupling for the inflaton at a one-loop level which is constrained by the CMB normalization of the density perturbations, i.e. $\lambda \leq 10^{-12}$. However, in SUSY this correction is canceled out by that from fermionic partner of χ , so in principle one could expect a rather broader range of parameter space.

Neglecting the self interaction for χ field, the equation of motion for χ_k quanta is given by:

$$\ddot{\chi}_k + 3\frac{\dot{a}}{a}\dot{\chi}_k + \left(\frac{k^2}{a^2} + m_\chi^2 + 2(\sigma\phi + h^2\phi^2)\right)\chi_k = 0. \quad (378)$$

It is assumed that the inflaton oscillations are homogeneous, $\phi(t) = \hat{\phi}(t)\sin(m_\phi t)$, where $\hat{\phi}(t) \approx (M_{\text{P}}/\sqrt{3\pi}m_\phi t)$, for chaotic inflation with mass m_ϕ . The occupation number for the excited χ_k is given by:

$$n_k = \frac{\omega_k}{2} \left(\frac{|\dot{\chi}_k|^2}{\omega_k^2} + |\chi_k|^2 \right) - \frac{1}{2}, \quad (379)$$

It was observed in Refs. [96–99], that in general one can have a *narrow resonance*, when expansion of the universe and the trilinear interaction are neglected, then the evolution for χ_k yields a Mathieu equation, which has well known instability bands, during which the mode grows exponentially, $\chi_k \propto \exp(\mu_k^n z)$, where μ_k^n is set by the instability band Δ_k^n labeled by an integer n , and $z = m_\phi t$. The resonance occurs for $k = 0.5m_\phi(1 \pm q/2)$, where μ_k vanishes at the edges and takes the maximum value $\mu_k = q/2$, where $q = g^2(\hat{\phi}^2/4m_\phi^2)$. Thus the occupation number grows exponentially. The situation changes quite dramatically when one switches the expansion rate of the universe, the evolution of the scalar field during the first 10 – 50 oscillations modifies to:

$$\phi(t) \simeq \frac{M_{\text{P}}}{\sqrt{3\pi}} \frac{\cos(m_\phi t)}{m_\phi t}, \quad (380)$$

where t is the physical time. The presence of the t at the denominator shows the damping of the oscillations due to the expansion of the universe. During this period the *stochastic resonance* come into the picture [99], where there are resonance bands as well as decrease in the particle number due to quantum effects.

In either case (expanding or non-expanding background), based on initial VEV of σ there would be two distinct cases.

- $\sigma \ll h^2 M_{\text{P}}$:

In this regime the $h^2\phi^2\chi^2$ term is dominant at the beginning of the inflaton oscillations. This case has been studied in detail in first two references of [98, 99]. For a nominal value of the inflaton mass, i.e. $m_\phi = 10^{13}$ GeV in chaotic inflation case, non-perturbative χ production during every oscillations of ϕ field, with a physical momentum, $k \lesssim (hm_\phi\hat{\phi})^{1/2}$ (where $\hat{\phi} \sim M_{\text{P}}$), takes place if $h > 10^{-6}$. Particle production is particularly efficient if $h > 3 \times 10^{-4}$, and results in an explosive transfer of

energy to χ quanta. The number density of χ_k quanta increases exponentially. The parametric resonance ends when re-scatterings destroy the inflaton condensate. The whole process happens over a time scale $\sim 150m_\phi^{-1}$, which depends logarithmically on h [99, 101].

- $\sigma \gg h^2 M_{\text{P}}$:

In this regime the cubic term $\sigma\phi\chi^2$ dominates. This case was recently considered in Refs. [102, 788], where the χ field becomes tachyonic during half of each oscillation. For $\sigma > m_\phi^2/M_{\text{P}}$ (which amounts to $\sigma > 10^7$ GeV for $m_\phi = 10^{13}$ GeV) this tachyonic instability transfers energy from the oscillating condensate very efficiently to the χ quanta with a physical momentum $k \lesssim (\sigma\hat{\phi})^{1/2}$. Particle production ceases when the back-reaction from χ self-coupling induces a mass-squared $\gtrsim \sigma\hat{\phi}$. Depending on the size of λ , most of the energy density may or may not be in χ quanta by the time back reaction becomes important [789].

Couple of points to note here. In the borderline regime $\sigma \sim h^2 M_{\text{P}}$, the cubic and quartic interaction terms are comparable. The inflaton decay happens due to a combination of resonant and tachyonic instabilities. If $h \ll m_\phi/M_{\text{P}}$ and $\sigma \ll m_\phi^2/M_{\text{P}}$, the inflaton decays perturbatively via the cubic interaction term. However this requires very small couplings; h , $(\sigma/m_\phi) < 10^{-6}$. Therefore, unless the inflaton is only gravitationally coupled to other fields, the initial stage of its decay will be generically non-perturbative.

Resonant particle production and re-scatterings lead to the formation of a plasma consisting of ϕ and χ quanta with typical energies $\sim 10^{-1} (hm_\phi M_{\text{P}})^{1/2}$, see [99, 114, 707, 778]. This plasma is in kinetic equilibrium but full thermal equilibrium is established over a much longer time scale than preheating [101, 103].

The occupation number of particles in the preheat plasma is $\gg 1$ (which is opposite to the situation after the perturbative decay). This implies that the number density of particles is larger than its value in full equilibrium, while the average energy of particles is smaller than the equilibrium value. It gives rise to large effective masses for particles which, right after preheating, is similar to their typical momenta [99, 114, 778]. Large occupation numbers also lead to important quantum effects due to identical particles and significant off-shell effects in the preheat plasma. Because of all these, a field theoretical study of thermalization is considerably more complicated in case of preheating. Due to the large

occupation numbers, one can consider the problem as thermalization of classical fields at early stages [101–103]. In the course of evolution towards full equilibrium, however, the occupation numbers decrease. Therefore a proper (non-equilibrium) quantum field theory treatment will be inevitably required at late stages when occupation numbers are close to one.

Preheating ends due to back reaction as well as the expansion of the universe. Preheating does not destroy the zero mode of the inflaton condensate completely, although the amplitude of the inflaton oscillations diminish, but the inflaton decay is completed when the zero mode perturbatively decays into the SM or some other degrees of freedom, see [96–99].

One of the most interesting effects of preheating is the copious production of particles which have a mass greater than the inflaton mass m_ϕ . Such processes are impossible in perturbation theory and in the theory of narrow parametric resonance. However, superheavy χ -particles with mass $M \gg m_\phi$ can be produced in the regime of a broad parametric resonance. For very small $\phi(t)$ the change in the frequency of oscillations $\omega(t)$ ceases to be adiabatic when the adiabaticity condition is violated [99]

$$\frac{d\omega(t)}{dt} \geq \omega^2(t). \quad (381)$$

The momentum dependent frequency, $\omega_k(t)$ violates the above condition when

$$k^2 + m_\chi^2 \lesssim (h^2 \phi m_\phi \hat{\phi})^{2/3} - h^2 \hat{\phi}^2. \quad (382)$$

The maximal range of momenta for which particle production occurs corresponds to $\phi(t) = \phi_*$, where $\phi_* \approx \frac{1}{2} \sqrt{\frac{m_\phi \hat{\phi}}{h}}$. The maximal value of momentum for particles produced at that epoch can be estimated by $k_{\max}^2 + m_\chi^2 = \frac{hm_\phi \hat{\phi}}{2}$. The resonance becomes efficient for $hm_\phi \hat{\phi} \gtrsim 4m_\chi^2$. Thus, the inflaton oscillations may lead to a copious production of superheavy particles with $m_\chi \gg m$ if the amplitude of the field ϕ is large enough, $h\hat{\phi} \gtrsim 4m_\chi^2/m_\phi$.

During the second stage of preheating both m_ϕ and $\hat{\phi}$ change very rapidly, but their product remains almost constant because the energy density of the field ϕ , which is proportional to $m_\phi^2 \hat{\phi}^2/2$, practically does not change until the very end of preheating. Therefore it is sufficient to check that $hm\hat{\phi} \gtrsim 4m_\chi^2$ at the end of the first stage of preheating. One can represent this criterion in a simple form [99]:

$$m_\chi \lesssim \frac{m_\phi}{\sqrt{2}} q^{1/4} \approx m_\phi \left(\frac{hM_p}{3m_\phi} \ln^{-1} \frac{10^{12} m_\phi}{h^5 M_p} \right)^{1/2}. \quad (383)$$

For example, one may take $m_\chi = 2m_\phi$ and $h \approx 0.007$, which corresponds to $q_0 = h^2 \hat{\phi}^2 / m_\phi^2 = 10^6$. The production of χ -particles with $m_\chi = 10m_\phi$ is possible for $h \gtrsim 10^{-2}$. Anyway, as we shall see in a realistic SUSY case, the existence of heavy mass of χ induced by the flat direction VEV of MSSM can kinematically block resonant preheating altogether [122], see Sec. VID 1.

2. Instant preheating

Let us focus solely on the interaction $h^2 \phi^2 \chi^2$. In instant preheating the particle production occurs during one oscillation of the inflaton [117]. The particle production occurs when the inflaton passes through the minimum of the potential $\phi = 0$. In this case the process can be approximated by writing $\phi = \dot{\phi}_0(t - t_0)$, where $\dot{\phi}_0$ is the velocity of the field when it passes through the minimum of the potential at time t_0 . The time interval within which the production of σ quanta occurs is [117] $\Delta t_* = (g|\dot{\phi}_0|)^{-1/2}$, which is much smaller than the Hubble expansion rate; thus expansion can be neglected. The occupation number of produced particles jumps from its initial value zero to a non-zero value during $-\phi_* \leq \phi \leq \phi_*$. In the momentum space the occupation number is given by [117] $n_k = \exp\left(-\frac{\pi k^2}{g|\dot{\phi}_0|}\right)$, and the largest number density of produced particles in x -space reads [117]

$$n_\chi \approx \frac{(h|\dot{\phi}_0|)^{3/2}}{8\pi^3}, \quad (384)$$

with the particles having a typical energy of $(g|\dot{\phi}_0|/\pi)^{1/2}$, so that their total energy density is given by

$$\rho_\chi \sim \frac{1}{2}(\delta^{(1)}\dot{\chi})^2 \sim \frac{(g|\dot{\phi}_0|)^2}{8\pi^{7/2}}. \quad (385)$$

These expressions are valid if $m_\chi^2 < g|\dot{\phi}_0|$. Instant preheating has applications especially when gauge fields and fermions are involved [88, 706]. One particular interesting point is when the modulus is carrying the SM gauge charges and passing through the point of *enhanced gauge symmetry*, i.e. $\langle \phi \rangle \approx 0$ (see Sec. VD). Where the gluons are nearly massless, then they can be excited with a similar abundance given by Eq. (384). When the modulus is displaced away from the point of enhanced gauge symmetry, the gluons become heavy due to modulus induced VEV dependent mass $\sim g\langle \phi \rangle$, where g is the gauge coupling. As the gluons become heavy they rapidly decay into fermions to reheat the plasma [121]. Transferring the

inflaton energy through this mechanism is quite fast and efficient, see the discussion on reheating due to MSSM inflaton, Sec. VD.

3. Tachyonic preheating

The second scenario is known as *tachyonic preheating*. Let us consider a simple example of tachyonic potential

$$V = V_0 - \frac{1}{2}m^2\chi^2 + \frac{\lambda}{4}\chi^4 \quad (386)$$

The rolling of a tachyon in itself results in an exponential instability in the perturbations of χ with physical momenta smaller than the mass. The tachyonic growth takes place within a short time interval, $t_* \sim (1/2m)\ln(\pi^2/\lambda)$ (see [707]). During this short period the occupation number of χ quanta grows exponentially for modes $k < m$ up to $n_k \sim \exp(2mt_*) \sim \exp(\ln(\pi^2/\lambda)) \sim \pi^2/\lambda$. For very small self-coupling, which is required for a successful inflation, the occupation number, which depends inversely on the coupling constant, can become much larger than one. First, the number density of the produced particles in x -space is given by $n_\chi \sim m^3/(8\pi\lambda)$. Hence the total energy density stored in produced χ quanta is given by [707]

$$\rho_\chi \sim \frac{1}{2}(\delta^{(1)}\dot{\chi})^2 \sim mn_\chi \sim \frac{1}{8\pi} \frac{m^4}{\lambda}. \quad (387)$$

The plasma from the non-perturbative inflaton decay eventually reaches full thermal equilibrium, though, at time scales much longer than that of preheating itself [101, 103]. The occupation number of particles is $f_k \gg 1$ in the meantime. This implies that dangerous relics (such as gravitino and moduli) can be produced much more copiously in the aftermath of preheating than in full thermal equilibrium. This is a negative aspect of an initial stage of preheating. One usually seeks a late stage of entropy release, in order to dilute the excess of relics. As we shall show, SUSY naturally provides us a tool to undo preheating [122], see Sec. VID 1

4. Fermionic preheating

The resonant and instant preheating calculations can be reanalyzed for a fermionic coupling, $h\phi\bar{\psi}\psi$. In both the cases one would expect a large exponential growth in particle

creation. It is also possible to excite superheavy fermions from resonant preheating [331, 756–758]. However note that both ϕ and ψ are SM gauge singlets.

For an inflaton field coherently oscillating about the minimum of the potential $V = \frac{1}{2}m_\phi^2\phi^2$, If one neglects the back reaction of the created particles, then after few oscillations, the inflaton evolves according to the formula Eq. (380). Thus, there exists a final time after which $|\phi| < m_\psi/h$, and the total mass no longer vanishes, then the resonant production of fermion ends.

The Dirac equation (in conformal time η) for a fermionic field is given by:

$$\left(\frac{i}{a}\gamma^\mu\partial_\mu + i\frac{3}{2}H\gamma^0 - m(\eta)\right)\psi = 0, \quad (388)$$

where a is the scale factor of the universe, $H = a'/a^2$ the Hubble rate and \prime denotes derivative w.r.t. η , and $m(\eta) = m_\psi + h\phi(\eta)$, where m_ψ is the bare mass of the fermion. The particle density per physical volume $V = a^3$ at time η is given by:

$$n(\eta) \equiv \langle 0|\frac{N}{V}|0\rangle = \frac{1}{\pi^2 a^3} \int dk k^2 |\beta_k|^2, \quad (389)$$

where α_k, β_k are the Bogolyubov's coefficients satisfying: $|\alpha_k|^2 + |\beta_k|^2 = 1$. The occupation number of created fermions is thus given by $n_k = |\beta_k|^2$, and the above condition ensures that the Pauli limit $n_k < 1$ is respected. One important physical quantity is the scaling of the total energy

$$\rho_\psi \propto m_\psi N_\psi \propto q m_\psi^{1/2} \quad (390)$$

which is linear in $q = h^2\hat{\phi}^2/m_\phi^2$, as generally expected [331, 756–758], but also note that $m_\psi(t) \propto q^{1/2}$.

In a realistic case, since the SM fermions are chiral, if the inflaton is a SM gauge singlet, then it can only couple via dimension-5 operators, i.e.

$$\frac{\lambda}{M_{\text{P}}}\phi(H\bar{q}_l)q_R, \quad (391)$$

where $\lambda \sim \mathcal{O}(1)$, H is the SM Higgs doublet and q_l, q_R are the $SU(2)_l$ doublet and the right handed SM fermions, respectively. As a result preheating of SM fermions from a gauge singlet inflaton becomes less important due to weak coupling.

In Ref. [331], it was argued that an inflaton coupling to right handed neutrino, $h\phi\bar{N}N$, where N is right handed neutrino, will induce non-thermal leptogenesis, where the right

handed neutrinos were treated gauge singlets. Anyway, if we embed the right handed neutrinos in a gauge sector, where they get their masses via some Higgs mechanism, then one requires non-renormalizable couplings like Eq. (391).

Similar argument holds for coupling to the SM gauge bosons, where the inflaton can only couple via non-renormalizable operator, i.e.

$$\frac{\lambda}{M_{\text{P}}}\phi F_{\mu\nu}F^{\mu\nu}, \quad (392)$$

where $\lambda \sim \mathcal{O}(1)$. Therefore, exciting the SM gauge bosons and the SM fermions through parametric resonance of a gauge singlet inflaton is a daunting task. Inflaton would rather prefer perturbative decay ⁸⁰.

5. Fragmentation of the inflaton

One very curious aspect of fermionic coupling to the inflaton is fragmentation of the inflaton to form an inflating non-topological solitons, known as Q-balls ⁸¹. Let us illustrate this idea by studying a chaotic inflation model where the inflaton field is not real but complex. Provided the fermions live in a larger representation than the bosons, the inflaton mass obtains a Logarithmic correction ⁸²:

$$V = m^2|\Phi|^2 \left[1 - K \log \left(\frac{|\Phi|^2}{M^2} \right) \right], \quad (393)$$

where the value of K is determined by the Yukawa coupling h with $K = -C(h^2/16\pi^2)$, where C is some number. If $K < 0$, the inflaton condensate feels a negative pressure for field values $\phi \ll M$, we find:

$$V(\phi) \simeq \frac{1}{2}m_{3/2}^2\phi^2 \left(\frac{\phi^2}{2M^2} \right)^K \propto \phi^{2+2K}. \quad (394)$$

⁸⁰ The only way one can excite SM fermions and gauge fields copiously, if they are directly excited by the oscillations of the SM Higgs boson. This can happen in low scale electroweak baryogenesis [779–781], or in the context of SM Higgs inflation [123]. During the Higgs oscillations the SM degrees of freedom can be excited via parametric resonance, instant preheating and also via tachyonic preheating. All three phases of preheating are present. The other notable example is the MSSM inflation discussed in Sec. VD, where gluons and MSSM fermions were excited via instant preheating.

⁸¹ The Q-balls known to evaporate from their surface, see for a review [90], therefore suppressing the reheating and thermalization time scale.

⁸² Similar corrections to the potential arises for the MSSM flat directions in a gravity mediated scenarios, where $m \sim \mathcal{O}(\text{TeV})$ and $K \sim \frac{\alpha}{8\pi} \frac{m_{1/2}^2}{m_i^2}$, where $m_{1/2}$ is the gaugino mass and m_i is the slepton mass. Fragmentation of such flat direction can excite Q-balls and also gravity waves, see [765, 766].

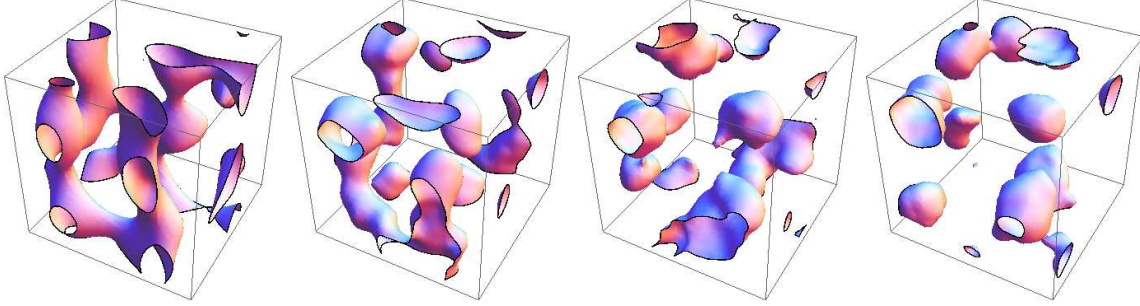


FIG. 16: Lumps of Q-matter are formed during the fragmentation of a condensate with a potential given by Eq. (393).

where we assume $|K| \ll 1$. The equation of state for a field rotating in such a potential is

$$p \simeq \frac{K}{2+K}\rho \simeq -\frac{|K|}{2}\rho, \quad (395)$$

where p and ρ is a pressure and energy density of the scalar field, respectively. Evidently, the negative value of K corresponds to the negative pressure, which signals the instability of the condensate. A linear perturbation analysis [498] shows that the fluctuations grow exponentially if the following condition is satisfied:

$$\frac{k^2}{a^2} \left(\frac{k^2}{a^2} + 2m_{3/2}^2 K \right) < 0. \quad (396)$$

Clearly, the instability band exists for negative K , as expected from the negative pressure arguments [90]. The instability band, k , is in the range [498] $0 < \frac{k^2}{a^2} < \frac{k_{max}^2}{a^2} \equiv 2m_{3/2}^2|K|$, where a is the expansion factor of the universe. The most amplified mode lies in the middle of the band, and the maximum growth rate of the perturbations is determined by $\dot{\alpha} \sim |K|m_{3/2}/2$ [90]. When $\delta\phi/\phi_0 \sim \mathcal{O}(1)$, the fluctuations become nonlinear. This is the time when the homogeneous condensate breaks down into Q-balls and anti-Q-balls⁸³.

6. Non-perturbative creations of gravity waves

The gravity waves are generated during preheating, as the excitations involve inhomogeneous, non-spherical, anisotropic motions of the excited scalar modes. As a result, the

⁸³ In general K and h are not independent quantities but are related to each other by $|K| \sim C(h^2/16\pi^2)$. In this regime the evaporation rate is saturated by: $\Gamma_Q = \frac{1}{Q} \frac{dQ}{dt} \simeq \frac{1}{|K|^{3/2}} \left(\frac{m}{M_P} \right)^2 m$ [498]. Even though coupling is large, i.e. $h \sim \mathcal{O}(0.1)$, the decay rate mimics that of a Planck suppressed interaction of the inflating Q -ball with matter fields.

stress energy tensor receives anisotropic stress-energy contribution. The generation of gravity waves were studied in Refs. [709, 759–768]. Typically, the peak frequency of the gravity waves is such that they correspond to the sub-Hubble wavelengths at the time of production. Gravity waves excitations can be studied numerically by following the transverse-traceless (TT) components of the stress-energy momentum tensor. By perturbing the Einstein's equation, we obtain the evolution of the tensor perturbations, see [761]:

$$\ddot{h}_{ij} + 3H\dot{h}_{ij} - \frac{\nabla^2}{a^2}h_{ij} = 16\pi G\Pi_{ij}, \quad (397)$$

where $\partial_i\Pi_{ij} = \Pi_{ii} = 0$ and $\partial_i h_{ij} = h_{ii} = 0$. The TT part of the spatial components of a symmetric anisotropic stress-tensor $T_{\mu\nu}$ can be found by using the spatial projection operators, $P_{ij} = \delta_{ij} - \hat{k}_i\hat{k}_j$, with $\hat{k}_i = k_i/k$:

$$\Pi_{ij}(k) = \Lambda_{ij,mn}(\hat{k})T_{mn}(k), \quad (398)$$

where $\Lambda_{ij,mn}(\hat{k}) \equiv \left(P_{im}(\hat{k})P_{jn}(\hat{k}) - (1/2)P_{ij}(\hat{k})P_{mn}(\hat{k})\right)$. The TT perturbation is written as $h_{ij}(t, \hat{k}) = \Lambda_{ij,lm}(\hat{k})u_{ij}(t, k)$, where

$$\ddot{u}_{ij} + 3H\dot{u}_{ij} - \frac{1}{a^2}\nabla^2 u_{ij} = 16\pi GT_{ij}. \quad (399)$$

The source terms for the energy momentum tensor in our case are just the gradient terms of the scalar field χ involved during preheating.

$$T_{ij} = \frac{1}{a^2}(\nabla_i\chi_1\nabla_j\chi_1 + \nabla_i\chi_2\nabla_j\chi_2), \quad (400)$$

where χ_1 and χ_2 represent the real and imaginary parts of ϕ , respectively. The gravitational wave (GW) energy density is given by:

$$\rho_{GW} = \frac{1}{32\pi G} \frac{1}{V} \int d^3\mathbf{k} \dot{h}_{ij}\dot{h}_{ij}^* \approx \frac{1}{32\pi GV} \int d^3\mathbf{x} \dot{u}_{ij}\dot{u}_{ij}^*. \quad (401)$$

where V is the volume of the lattice. As an application, let us consider exciting gravity waves in hybrid inflation model of inflaton, with inflaton, ϕ , and the Higgs field, χ , see [761].

The coupled evolution equations that have to be solved numerically on a lattice for the hybrid model of inflation are given by [761]⁸⁴:

$$\ddot{\chi} - \nabla^2\chi + (g^2|\phi|^2 + \mu^2)\chi = 0, \quad (402)$$

$$\ddot{\phi}_a - \nabla^2\phi_a + (g^2\chi^2 + \lambda|\phi|^2 - m^2)\phi_a = 0, \quad (403)$$

⁸⁴ Note that the weakness of gravity renders negligible on scalar fields.

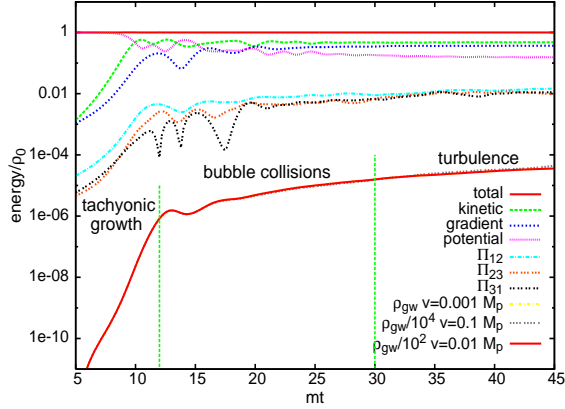


FIG. 17: The time evolution of the different types of energy (kinetic, gradient, potential, anisotropic components and gravitational waves for different lattices), normalized to the initial vacuum energy, after hybrid inflation, for a model with $v = 10^{-3} M_P$. One can clearly distinguish here three stages: tachyonic growth, bubble collisions and turbulence. The plot is taken from [761].

where χ is the inflaton and the ϕ_a are the complex water-fields.

The initial energy density at the end of hybrid inflation is given by: $\rho_0 = m^2 v^2 / 4$, with $m^2 = \lambda v^2$, where v is the VEV of the Higgs field (water-field), so the fractional energy density in gravitational waves is roughly given by [761]:

$$\frac{\rho_{\text{GW}}}{\rho_0} = \frac{4t_{00}}{v^2 m^2} = \frac{1}{8\pi G v^2 m^2} \langle \dot{h}_{ij} \dot{h}^{ij} \rangle_V, \quad (404)$$

where $\langle \dot{h}_{ij} \dot{h}^{ij} \rangle_V$, defined as a volume average like $\frac{1}{V} \int d^3x \dot{h}_{ij} \dot{h}^{ij}$, is extracted from the numerical simulations.

There are three stages of preheating which contribute to gravity waves. First, an exponential growth driven by the tachyonic instability of the long-wave modes of the Higgs field. Second, the Higgs field oscillates around the true vacuum, as the Higgs' bubbles collide and scatter off each other. Third, a period of turbulence is reached, during which the inflaton oscillates around its minimum and the Higgs is already settled in its vacuum [707, 761, 778].

Gravity wave production has also been studied in a potential given by Eq. (393). The fractional energy density can be estimated by following Eq. (401), given by [766]:

$$\Omega_{\text{GW}} = \frac{\rho_{\text{GW}}}{m^2 \phi(t)^2} \sim |K|^2 \left(\frac{\phi(t)}{M_{\text{Pl}}} \right)^2. \quad (405)$$

The scalar field is dominating the energy density of the universe at the time of fragmentation. For physically motivating parameters, we have chosen; $m \sim 100$ GeV, $\phi(t) \sim 10^{16}$ GeV, and $H(t) \sim 1$ GeV, therefore, for a reasonable value of $K \sim 0.1$, we obtain, $\Omega_{\text{GW}} \sim 10^{-6}$. Note

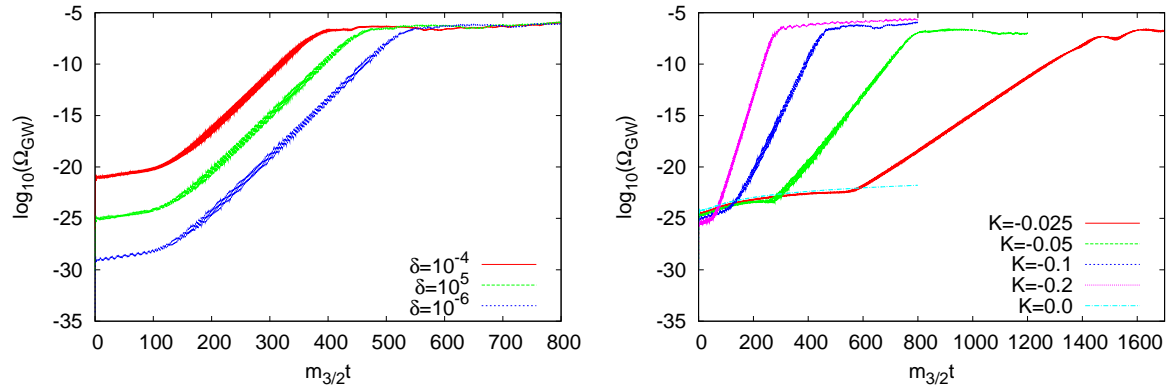


FIG. 18: The final amplitude of the gravity waves does not depend on the initial perturbations, δ , and The final amplitude of gravity waves saturates for different values of K . Note for $K = 0$ there is no fragmentation at all, therefore there is no gravity waves. The field value is: $\phi_0 = 10^{16}$ GeV and $m \sim m_{3/2} = 100$ GeV.

that Ω_{GW} depends on the value of K , for $K = 0$ there are no excitations of gravity waves, see Fig. (18).

7. Non-perturbative production of gauge fields

Let us now consider non-perturbative effects on gauge fields. For simplicity, let us consider an example where the tachyonic field, χ , is charged under $U(1) \times U(1)$, which arises quite naturally in a brane-anti-brane inflation [709]. Here F^+ and F^- are the gauge fields that live in the world volume of the brane and anti-brane respectively. The tachyon, χ , is a bi-fundamental field that couples only to a linear combination of the two gauge fields: $D_\mu \chi = \partial_\mu \chi - (A_\mu^+ - A_\mu^-) \chi$, and ϕ is the inflaton field⁸⁵. The Lagrangian is given by:

$$\mathcal{L} = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi + \frac{1}{2} D_\mu \chi D^\mu \chi^* + \frac{1}{2} m_\phi^2 \phi^2 + \frac{1}{2} m_T^2 |\chi|^2 (\phi^2 - \phi_0^2) + \frac{\lambda}{4} |\chi|^4 - \frac{1}{4} F^2 \quad (406)$$

The tachyon mass is fixed in terms of the string scale, $m_T = M_s$, while the mass of the inflaton is determined by the amplitude of the temperature anisotropy, which turns out to

⁸⁵ The physical situation is quite similar to that of a hybrid inflation. The tachyon field in brane-anti-brane inflation plays a dynamical role of a water-fall field or the Higgs field.

be: $m_\phi = 0.01M_s$. The stress-energy tensor now gets contributions from both the charged tachyon, χ and uncharged inflaton, ϕ , as well as the gauge fields, which is given by [709]:

$$\Pi_{ij} = F_{iC}F_j^C - \frac{1}{3}\delta_{ij}F_{kC}F^{kC} - D_i\chi D_j\chi^* + \frac{1}{3}\delta_{ij}D_k\chi D^k\chi^* - \partial_i\phi\partial_j\phi^* + \frac{1}{3}\delta_{ij}\partial_k\phi\partial^k\phi^* \quad (407)$$

The energy density of the gravitational waves is simply given by the t'_{00} component of the energy-momentum tensor of the gravitational waves calculated in a synchronous gauge, i.e. $h_{0i} = 0$, $h_{00} = 0$. In Fig. (19), the energy pumped into the gauge fields and the gravity waves are shown for $\lambda = 1$, $m_\phi = 0.01M_s$ and the tachyon mass is given by $M_s = 10^{-4}M_p$. Of course, larger the value of a tachyon mass, greater is the instability, and the growth in the respective perturbations, but compare the energy densities in gauge fields and gravity waves.

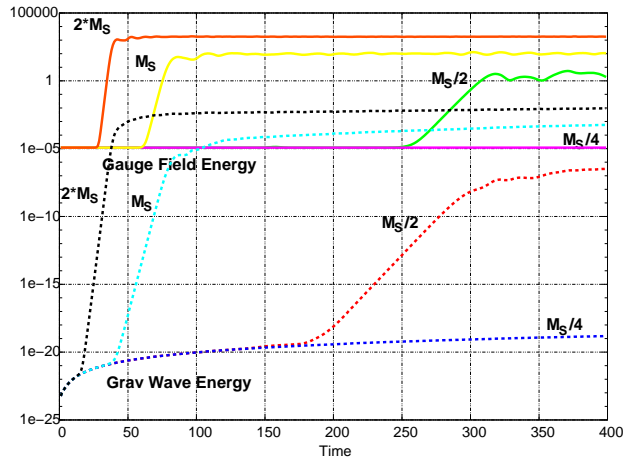


FIG. 19: Different colors show how the gravitational wave energy and gauge field energy grow for different values of the tachyonic mass.

In Figs. (20), 4 snap-shots of the iso-surface of the constant energy density of gauge field, gravity waves, tachyon and inflaton are depicted. All the fields except the inflaton show a remarkable departure in the homogeneity. Except the inflaton, all fields undergo long wavelength excitations (they all look relatively smooth on small scales), while the inflaton obtains the largest inhomogeneity on the smaller scales. This is due to the fact that there is no long wavelength amplification for the inflaton in this case [709].

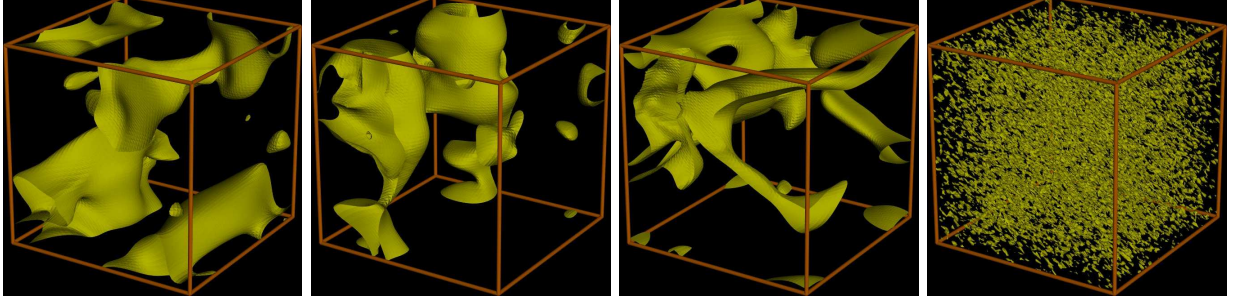


FIG. 20: Snap-shots of iso-surface of the energy density for gauge field, gravity waves, tachyon and the inflaton (from left-to-right) at a particular instant of time, $t = 300$. The plots are taken from [709].

8. SM Higgs preheating

On particular realistic example of preheating [123, 593] can be illustrated by the SM Higgs inflation [85]. After inflation the Higgs field evolves in time, $h = h(\chi(t))$, see Eq. (215), so the effective masses of the fermions and of the gauge bosons also obtain time dependence.

$$m_W = m_Z \cos \theta_W = \frac{1}{2} g_2 h(\chi(t)), \quad m_f = \frac{1}{2} y_f h(\chi(t)), \quad (408)$$

where θ_W is the Weinberg angle $\theta_W = \tan^{-1}(g_1/g_2)$, and y_f , g_1 and g_2 are the Yukawa and the $U(1)_Y$ and $SU(2)_L$ couplings, respectively. The time dependent Higgs VEV spontaneously breaks the gauge symmetry, and give masses to W and Z gauge bosons masses. The relevant interactions are given by the charged and neutral Currents, coupling the SM fermions to gauge bosons through the J_μ^\pm , J_μ^Z currents, and the Yukawa sector, and coupling the SM fermions with the Higgs:

$$S_{SSB} = \int d^4x \sqrt{-g} \left\{ m_W^2 W_\mu^+ W^{\mu-} + \frac{1}{2} m_Z^2 Z_\mu Z^\mu \right\}, \quad S_Y = \int d^4x \sqrt{-g} \left\{ m_d \bar{\psi}_d \psi_d + m_u \bar{\psi}_u \psi_u \right\},$$

$$S_{CC} + S_{NC} = \int d^4x \sqrt{-g} \left\{ \frac{g_2}{\sqrt{2}} W_\mu^+ J_\mu^- + \frac{g_2}{\sqrt{2}} W_\mu^- J_\mu^+ + \frac{g_2}{\cos \theta_W} Z_\mu J_\mu^Z \right\}. \quad (409)$$

One can redefine the fields and masses with a specific conformal weight as to keep the canonical kinetic terms for the gauge fields and fermions, provided: $\tilde{W}_\mu^\pm \equiv W_\mu^\pm / \Omega$, $\tilde{Z}_\mu \equiv Z_\mu / \Omega$, $\tilde{\psi}_d \equiv \psi_d / \Omega^{3/2}$, $\tilde{\psi}_u \equiv \psi_u / \Omega^{3/2}$, $\tilde{m}_W^2 = \tilde{m}_Z^2 \cos^2 \theta_W = m_W^2 / \Omega^2 = (g_2^2 M_P^2 / 4\xi) (1 - e^{-\alpha\kappa|\chi|})$, $\tilde{m}_f \equiv m_f / \Omega = (y_f M_P / \sqrt{2\xi}) (1 - e^{-\alpha\kappa|\chi|})^{1/2}$, where $\Omega^2 \approx \exp(\alpha\kappa\chi)$ with $\alpha = \sqrt{2/3}$ and $\kappa = 1/M_P$. The oscillations of the Higgs field can be approximated by a simple quadratic potential $U(\chi) \approx (1/2)M^2\chi^2$, where $M = \sqrt{\lambda/3}M_P/\xi \sim 10^{-5}M_P$.

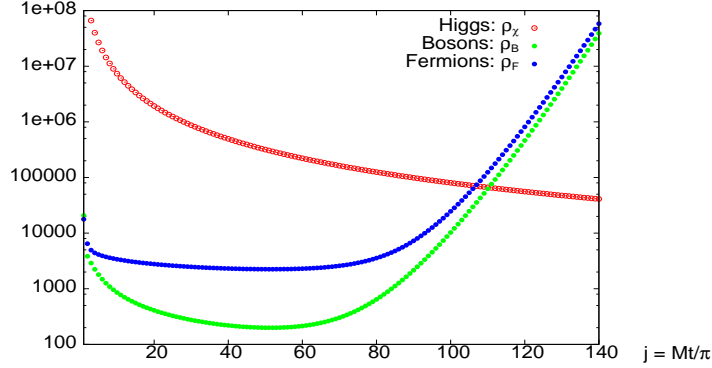


FIG. 21: Evolution of the energy density transferred into the gauge bosons and into the fermions as a function of j , for $\lambda = 0.2$ and $\xi = 44700\sqrt{\lambda}$. All densities are in units of M^4 [123].

The oscillations evolve with $\chi(t) \approx X(t) \sin(Mt)$, where $X(t) \propto (Mt)^{-1}$, like in the matter-dominated case. There is a small departure from a quadratic potential, which is given by Eq. (215), $U(\chi) = (1/2)M^2\chi^2 + \Delta U(\chi)$, but during reheating this correction is negligible. As χ passes through the minimum, by virtue of its couplings, the gauge fields and fermions become massless, therefore they can be excited through parametric resonance or via instant preheating during every oscillations⁸⁶. However during one cycle of oscillation, the gauge bosons and fermions become heavy, as χ goes away from the minimum and obtains a time dependent VEV. During this epoch the gauge fields can decay into lighter fermions, but there is a kinematical blocking. The process of preheating is not as efficient as one would have thought. Nevertheless, the Higgs energy can be transferred at a faster rate compared to that of the perturbative decay of the Higgs, as shown in the Fig. (21) [123]. Furthermore, in order to obtain a full thermalization, Higgs must decay completely, which happens in a time scale similar to that of a perturbative decay rate.

C. SUSY generalization of reheating and preheating

Reheating, preheating and thermalization issues are quite different once SUSY is introduced. SUSY introduces new degrees of freedom and new parameters. Cosmology also acts as a test bed where some of the SUSY particles can be tested from the success of BBN, a well known example is the *gravitino problem* in the context of a SUSY cosmology.

⁸⁶ This feature was first discussed in the context of MSSM inflation, where instant preheating excites the gluons abundantly near the point of enhanced gauge symmetry [88].

1. Gravitino problem

The gravitino is a spin-3/2 partner of a graviton, which is coupled to the SM particles with the gravitational strength. Gravitinos with both the helicities can be produced from a thermal bath. There are many scattering channels which include fermion, sfermion, gauge and gaugino quanta all of which have a cross-section $\propto 1/M_{\text{P}}^2$ [302–304], and [305], which results in a gravitino abundance (up to a logarithmic correction):

$$\begin{aligned} \text{Helicity } \pm \frac{3}{2} : \frac{n_{3/2}}{s} &\simeq \left(\frac{T_{\text{R}}}{10^{10} \text{ GeV}} \right) 10^{-12}, \\ &\text{(full equilibrium)} \\ \text{Helicity } \pm \frac{1}{2} : \frac{n_{3/2}}{s} &\simeq \left(1 + \frac{M_{\tilde{g}}^2}{12m_{3/2}^2} \right) \left(\frac{T_{\text{R}}}{10^{10} \text{ GeV}} \right) 10^{-12}; \end{aligned} \quad (410)$$

where $M_{\tilde{g}}$ is the gluino mass. Note that for $M_{\tilde{g}} \leq m_{3/2}$ both the helicity states have essentially the same abundance, while for $M_{\tilde{g}} \gg m_{3/2}$ production of helicity $\pm 1/2$ states is enhanced due to their Goldstino nature ⁸⁷.

An unstable gravitino decays to particle-particle pairs, and its decay rate is given by $\Gamma_{3/2} \simeq m_{3/2}^3/4M_{\text{P}}^2$, see [304]. If $m_{3/2} < 50$ TeV, the gravitinos decay during or after BBN [25, 790], which can ruin its successful predictions for the primordial abundance of light elements. If the gravitinos decay radiatively, the most stringent bound, $(n_{3/2}/s) \leq 10^{-14} - 10^{-12}$, arises for $m_{3/2} \simeq 100$ GeV – 1 TeV [313]. On the other hand, much stronger bounds are derived if the gravitinos mainly decay through the hadronic modes. In particular, for a hadronic branching ratio $\simeq 1$, and in the same mass range, $(n_{3/2}/s) \leq 10^{-16} - 10^{-15}$ will be required [306, 307].

For a radiatively decaying gravitino the tightest bound $(n_{3/2}/s) \leq 10^{-14}$ arises when $m_{3/2} \simeq 100$ GeV [313]. Following Eq. (410) the bound on reheat temperature becomes: $T_{\text{R}} \leq 10^{10}$ GeV. For a TeV gravitino which mainly decays into gluon-gluino pairs (allowed when $m_{3/2} > M_{\tilde{g}}$) a much tighter bound $(n_{3/2}/s) \leq 10^{-16}$ is obtained [306, 307], which requires quite a low reheat temperature: $T_{\text{R}} \leq 10^6$ GeV.

The gravitino will be stable if it is the LSP, where R -parity is conserved. The gravitino

⁸⁷ Since the cross-section for the gravitino production is $\propto M_{\text{P}}^{-2}$, the production rate at a temperature, T , and the abundance of the gravitinos produced within one Hubble time will be $\propto T^3$ and $\propto T$ respectively. This implies that the gravitino production is efficient at the highest temperature of the radiation-dominated phase of the universe, i.e. T_{R} .

abundance will in this case be constrained by the dark matter limit, $\Omega_{3/2}h^2 \leq 0.12$, leading to

$$\frac{n_{3/2}}{s} \leq 5 \times 10^{-10} \left(\frac{1 \text{ GeV}}{m_{3/2}} \right). \quad (411)$$

For $m_{3/2} < M_{\tilde{g}}$, the helicity $\pm 1/2$ states dominate the total gravitino abundance. As an example, consider the case with a light gravitino, $m_{3/2} = 100 \text{ KeV}$, which can arise very naturally in gauge-mediated models [701]. If $M_{\tilde{g}} \simeq 500 \text{ GeV}$, see Eq. (410), a very severe constraint, $T_R \leq 10^4 \text{ GeV}$, will be obtained on the reheat temperature ^{88 89}.

Note that the above discussions assume that all the MSSM degrees of freedom are in thermal equilibrium instantly right after inflation, however this basic assumption is in contradiction in presence of MSSM flat directions [121], which alters the thermal history of the universe and also affects the gravitino abundance.

2. Gauge singlet inflaton couplings to MSSM

Within MSSM there exists two gauge-invariant combinations of only two superfields:

$$H_u H_d, \quad H_u L. \quad (412)$$

The combinations which include three superfields are:

$$H_u Q_u, \quad H_d Q_d, \quad H_d L_e, \quad Q L_d, \quad u d d, \quad \text{and} \quad L L e. \quad (413)$$

⁸⁸ Gravitinos could also be produced by non-perturbative processes, as was first described in [771] for helicity $\pm 3/2$ component of gravitino. Later the production of the helicity $\pm 1/2$ state were also studied [641, 772–776, 791]. The helicity $\pm 1/2$ component obtains a major contribution from the inflatino (superpartner of inflaton), however the inflatino decays with the same rate as that of the inflaton, therefore the $\pm 1/2$ abundance does not any major role [774–776, 791]. A very late decay of inflatino could however be possible, as argued in [774–776]. In [792], it was argued that if the inflatino and gravitino were not LSP, then late off-shell inflatino and gravitino mediated decays of heavy relics could be significant.

⁸⁹ Recently, non-thermal abundance of gravitino from a modulus decay has been revisited [793–796]. The gravitino abundance is given by: $Y_{3/2} \sim B_{3/2} \frac{3T_R}{4m_\phi}$, where $B_{3/2}$ is the branching ratio into gravitino and would be $B_{3/2} = 10^{-2} - 1$ with the mixing between modulus and the SUSY-breaking field, where we have used an approximation $n_\phi/s \sim (3T_R/4m_\phi)$, essentially the moduli decay is creating all the entropy of the universe. The branching ratio of the gravitino production from a modulus decay is little more contentious than one would naively expect. For $B_{3/2} \sim 1$, there is a possibility of overproducing gravitinos. However, as it was pointed out in Ref. [795], the decay rate generically obtains a helicity suppression. The precise value depends on the details of the SUSY breaking hidden sector. There are well known examples of hidden sectors, where one naturally obtains the expected value of $B_{3/2} \sim 10^{-2}$ [795].

SUSY together with gauge symmetry requires that the inflaton (a SM gauge singlet) superfield be coupled to these combinations. The superpotential terms $\Phi H_u H_d$ and $\Phi H_u L$ have dimension four, and hence are renormalizable. On the other hand, the interaction terms that couple the inflaton to the combinations with three superfields have dimension five and are non-renormalizable. In following we focus on renormalizable interactions of the inflaton with matter which play the dominant role in its decay. Further note that terms representing gauge-invariant coupling of the inflaton to the gauge fields and gauginos are also of dimension five, and hence preheating into them will be suppressed⁹⁰.

Preserving R -parity at the renormalizable level further constrains inflaton couplings to matter. Note that $H_u H_d$ is assigned $+1$ under R -parity, while $H_u L$ has the opposite assignment -1 . Therefore only one of the couplings preserves R -parity: $\Phi H_u H_d$ if $R_\Phi = +1$, and $\Phi H_u L$. If $R_\Phi = -1$ (such as models where the RH sneutrino plays the role of the inflaton [765]). Therefore the renormalizable inflaton coupling to matter can be represented as [122]:

$$2g\Phi H_u \Psi \tag{414}$$

where $\Psi = H_u$ if $R_\Phi = +1$ and $\Psi = L$ if $R_\Phi = -1$. Taking into account of the inflaton superpotential mass term: $(m_\phi/2)\Phi\Phi$, and defining $X_{1,2} = (H_u \pm \Psi)/\sqrt{2}$, the *renormalizable part of the potential*, which is relevant for the inflaton decay into MSSM scalars is given by:

$$V \supset \frac{1}{2}m_\phi^2\phi^2 + g^2\phi^2\chi^2 \pm \frac{1}{\sqrt{2}}gm_\phi\phi\chi^2, \tag{415}$$

where χ denotes the scalar component of $X_{1,2}$ superfields, and we have only considered the real parts of the inflaton, ϕ , and χ field. Further note that the cubic interaction term appears with different signs for χ_1 and χ_2 , but this is irrelevant during inflaton oscillations.

In addition to the terms in Eq. (415), there are also the self- and-cross-couplings,

$$V_D \supset \left(\frac{g^2}{4}\right)(\chi_1^2 - \chi_2^2)^2 + \alpha\chi_1^2\chi_2^2, \tag{416}$$

arising from the superpotential and D -terms respectively (α is a gauge fine structure constant). Therefore even in the simplest SUSY set up the scalar potential is more involved than

⁹⁰ It is possible that the inflaton mainly decays to another singlet (for example, the RH neutrinos) superfield, however, the underlying interactions of a gauge singlet to the MSSM superfields do not change. One must transfer the inflaton energy into the MSSM sector at any cost.

the non-SUSY case given in Eq. (377), which can alter the picture of preheating presented in the literature, see for the detailed discussion in Refs. [121, 122]⁹¹.

D. MSSM flat directions, reheating and thermalization

The MSSM flat directions have important role to play in SUSY reheating and thermalization [121, 122]. Consider a MSSM flat direction, φ , with the corresponding superfield denoted by φ (only for flat directions we are denoting the superfield and the field with the same notation in this section). Note that since φ and X superfields are linear combinations of the MSSM superfields (defined in the earlier subsection after Eq. (414)), and hence are coupled through the MSSM superpotential in Eq. (103).

$$W \supset \lambda_1 H_u \varphi \Sigma_1 + \lambda_2 \Psi \varphi \Sigma_2 + \dots, \quad (417)$$

where $\Sigma_{1,2}$ are some MSSM superfields such that $\Sigma_1 \neq \Psi$ and $\Sigma_2 \neq H_u$, since φ is a non-gauge-singlet.

For example consider the case where φ is a flat direction classified by the udd monomial, and $\Psi = H_d$. In this case $\Sigma_{1,2}$ are Q superfields and $\lambda_{1,2}$ correspond to λ_u and λ_d respectively. Then with the help of $X_{1,2} = (H_u \pm \Psi)/\sqrt{2}$, part of MSSM superpotential can be written as: $W \supset \frac{\lambda_1}{\sqrt{2}} X \varphi \Sigma_1 + \frac{\lambda_2}{\sqrt{2}} X \varphi \Sigma_2$. This results in [122]:

$$V \supset \lambda^2 |\varphi|^2 \chi^2, \quad \lambda \equiv \left(\frac{\lambda_1^2 + \lambda_2^2}{8} \right)^{1/2}, \quad (418)$$

where we have again considered the real part of χ . Note that the first generation of (s)leptons and (s)quarks have a Yukawa coupling $\sim \mathcal{O}(10^{-6} - 10^{-5})$, while the rest of the SM Yukawa couplings are: $\lambda \geq 3 \times 10^{-4}$.

The most important point is to note that 60–70 e-foldings of inflation is sufficient for the MSSM flat directions to take large VEVs during and after inflation by virtue of stochastic jumps during inflation, see the discussion in Sec. VE 1 and the Refs. [90, 91]. This however requires that the MSSM flat directions do not obtain positive Hubble induced corrections during inflation.

⁹¹ A remarkable feature in Eq. (415) is that SUSY naturally relates the strength of cubic $\phi\chi^2$ and quartic $\phi^2\chi^2$ interactions, which is required for complete decay of the inflaton field. One can also include couplings of the inflaton to fermionic partners of χ . Regarding the prospects for fermionic preheating the same conclusions hold as that of a bosonic case.

1. kinematical blocking of preheating

In order to understand the preheating dynamics it is important to take into account of χ coupling to the inflaton ϕ , as well as to the MSSM flat direction, φ , which is displaced away from its minimum (towards large VEVs) during inflation. The governing potential can be obtained from Eqs. (415,418):

$$V = \frac{1}{2}m_\phi^2\phi^2 + g^2\phi^2\chi^2 + \frac{g}{\sqrt{2}}m_\phi\phi\chi^2 + \lambda^2\varphi^2\chi^2. \quad (419)$$

As mentioned in the previous section, we generically have $\lambda \geq 3 \times 10^{-4}$, and g can be as large as $\sim \mathcal{O}(1)$. After mode decomposition of the field χ , the energy of the mode with momentum k , denoted by χ_k , is given by:

$$\omega_k = \left(k^2 + 2g^2\langle\phi\rangle^2 + \sqrt{2}gm_\phi\langle\phi\rangle + 2\lambda^2\langle\varphi\rangle^2 \right)^{1/2}. \quad (420)$$

Let us freeze the expansion of the universe first. Including the expansion will not change our conclusions anyway. Let us even consider the most opportunistic case for preheating with a large inflaton VEV, i.e. $\langle\phi\rangle > M_{\text{P}}$. Therefore if $g > 10^{-6}$, the inflaton induces a large mass $g\langle\phi\rangle > H_{\text{inf}}$ for χ during inflation. As a result, χ , quickly settles down to the minimum, i.e. $\langle\chi\rangle = 0$, even if it is initially displaced, and remains there. Therefore, φ , does not receive any mass corrections from its coupling to χ during inflation. Note that the VEV of the flat direction, φ , induces a large mass, $\lambda\varphi_0$, to the χ field during inflation.

In the interval $m_0 \leq H(t) \leq m_\phi$, where $m_0 \sim \mathcal{O}(100)$ GeV is the mass of the MSSM flat direction, the flat direction VEV slides very slowly because of the under damped motion due to large Hubble friction term, the flat direction effectively slow rolls. Non-perturbative production of χ quanta will occur if there is a non-adiabatic time-variation in the energy, i.e. that $d\omega_k/dt \gtrsim \omega_k^2$. The inflaton oscillations result in a time-varying contribution to ω_k , while the flat direction coupling to χ yields a virtually *constant* piece. The piece induced by the flat direction VEV weakens the non-adiabaticity condition. Indeed time-variation of ω_k will be adiabatic at all times: $d\omega_k/dt < \omega_k^2$, provided $\lambda^2\langle\varphi\rangle^2 > g\hat{\phi}m_\phi$, where $\hat{\phi}$ is the amplitude of the inflaton oscillations. There will be no resonant production of χ quanta, provided that

$$\varphi_0 > \lambda^{-1}(gM_{\text{P}}m_\phi)^{1/2}, \quad \text{Typically } \lambda \geq 3 \times 10^{-4}, \quad (421)$$

except the first generation of (s)leptons and (s)quarks which have a Yukawa coupling $\sim \mathcal{O}(10^{-6} - 10^{-5})$, and $g \geq 10^{-6}$, in order to have large inflaton couplings to matter, see

Eq. (414). This surmounts to a kinematical blocking of preheating by inducing a piece (which is virtually constant at time scales of interest) to the mass of inflaton decay products due to their couplings to a flat direction which has a large VEV. Similar argument holds for kinematical blocking for fermionic preheating, as the symmetry between bosons and fermions implies similar equations for the momentum excitations, see Eq. (420).

2. Late inflaton decay in SUSY

The inflaton decay at the leading order will be kinematically forbidden if $y|\varphi| \geq m_\phi/2$ (y is a SM gauge or Yukawa coupling) [121, 122]. One should then wait until the Hubble expansion has redshifted, $|\varphi|$, down to $(m_\phi/2y)$. The decay happens when (note that $|\varphi| \propto H$, after the flat direction starts oscillating and before the inflaton decays):

$$H_1 = \min \left[\left(\frac{m_\phi}{y\varphi_0} \right) m_0, \Gamma_d \right], \quad (\Gamma_d \equiv \text{total inflaton decay width}), \quad (422)$$

where $m_0 \sim \mathcal{O}(100)$ GeV is the MSSM flat direction mass. The inflaton also decays at higher orders of perturbation theory to particles which are not directly coupled to it [121, 740]. This mode is kinematically allowed at all times, but the rate is suppressed by a factor of $\sim (m_\phi/y|\varphi|)^2 \Gamma_d$. It becomes efficient at:

$$H_2 \sim \left(\frac{m_\phi m_0}{\varphi_0} \right)^{2/3} \Gamma_d^{1/3}. \quad (423)$$

Therefore, if the decay products are coupled to a flat direction with a non-zero VEV, the inflaton will actually decay at a time when the expansion rate of the universe is given by [121]

$$H_d = \max [H_1, H_2]. \quad (424)$$

In general it is possible to have, $H_d \ll \Gamma_d$, particularly for large values of φ_0 . Flat directions can therefore significantly delay inflaton decay on purely kinematical grounds.

3. Decay of a flat direction

There is a crucial difference between a rotating and an oscillating flat direction when it comes to decay. It is well known that an oscillating condensate can decay non-perturbatively via preheating, as we discussed in the earlier subsections.

In the case of a rotating condensate, there is the conservation of global charges associated with the net particle number density in fields [317–319]⁹². This ensures that the total number density of quanta will not decrease and, consequently, the average energy of quanta will not increase. The actual decay of a rotating flat direction into other fields happens perturbatively, through the F -term couplings, as originally envisaged by Affleck and Dine [317–319], see also [797, 798].

It was argued that there could be possible non-perturbative effects [799–801], stemming from the D -terms of the potential for a rotating flat direction condensate. However it was shown in [402, 403] that such non-perturbative effects have no bearing for the decay of energy density in rotating flat direction(s). In the case of a rotating condensate all that can happen is a mere redistribution of the energy among the fields on the D -flat subspace.

There is a no-go theorem for a rotating condensate [402, 403], which states; for MSSM flat direction(s), which are represented by gauge-invariant combinations of fields $\Phi_i = e^{i\alpha_i}\phi_i$, possible non-perturbative particle production from time-variation in the mass eigenstates caused by the D -terms:

(1) cannot change the net particle number density in Φ_i , denoted by $n_i = i\dot{\Phi}_i^*\Phi_i + h.c.$, and hence the total baryon/lepton number density stored in the condensate.

(2) cannot decrease the total *comoving* particle number density in ϕ_i , denoted by \tilde{n}_i , thus the total number density of quanta $\tilde{n} = \sum_i \tilde{n}_i$ in the condensate. As a direct consequence of the conservation of energy density, non-perturbative effects will not increase the average energy of quanta E_{ave} .

The theorem also applies to elliptical trajectories, where the condensate will mainly contain particles (or anti-particles), but it will also contain a small mixture of anti-particles (or particles). The theorem is applicable for the subsequent evolution of the plasma formed after the phase of particle production. This implies that possible non-perturbative effects do not lead to the decay of a rotating condensate. They merely redistribute the energy which is initially stored in the condensate among the fields on the D -flat subspace [402].

The marked difference between rotating and an oscillating flat direction case can be un-

⁹² Charges identified by the net particle number in fields, which are included in a flat direction are most notably baryon and lepton number, which are preserved by the D -terms

derstood from the trajectory of motion (i.e. circular for rotation versus linear for oscillation). An oscillating condensate ϕ , whose trajectory of motion is a line, can be written as

$$\phi = \frac{\varphi}{2} \exp(i\theta) + \frac{\varphi}{2} \exp(-i\theta), \quad (425)$$

and the conserved charge associated with the global $U(1)$ (corresponding to phase θ) is given by

$$n = i\dot{\phi}^*\phi + \text{h.c.} = 0. \quad (426)$$

This is not surprising since an oscillation is the superposition of two rotations in opposite directions, which carry exactly the same number of particles and anti-particles respectively. Therefore the net particle number density stored in an oscillating condensate is zero.

Now consider non-perturbative particle production from an oscillating condensate. One can think of this process as a series of annihilations among N particles and N anti-particles in the condensate, $N > 1$, into an energetic particle-anti-particle pair. This is totally compatible with conservation of charge, see Eq. (426); $n = 0$ after preheating as well as in the condensate.

On the other hand, a (maximally) rotating condensate consists of particles or anti-particles *only* [402, 403]. Conservation of the net particle number density then implies that $N \rightarrow 2$ annihilations ($N > 2$) are forbidden: annihilation of particle (or anti-particle) quanta cannot happen without violating the net particle number density. Therefore the total number density of quanta will not decrease, and the average energy will not increase⁹³.

Any possible non-perturbative particle production in the rotating condensate case will result in a plasma which is at least as dense as the initial condensate. All that can happen is a redistribution of the energy density among the fields on the D -flat subspace. These fields have masses comparable to the flat direction mass m_0 , as they all arise from SUSY breaking. Since the average energy is $E_{\text{ave}} \leq m_0$, the resulting plasma essentially consists of non-relativistic quanta. Its energy density $\rho = \tilde{n}E_{\text{ave}}$ is therefore redshifted $\propto a^{-3}$. The decay of a rotating condensate happens quite late [402]

$$H_{\text{dec}} \sim m_0 \left(\frac{m_0}{y\varphi_0} \right) \quad (\text{m. d.}), \quad H_{\text{dec}} \sim m_0 \left(\frac{m_0}{y\varphi_0} \right)^{4/3} \quad (\text{r. d.}), \quad (427)$$

⁹³ Note that an increase in the total particle number density, through creation of an equal number of particles and anti-particles will be in agreement with the conservation of the net particle number density. In this case the resulting plasma will be even denser than the condensate.

where m.d. corresponds to matter domination and r.d. corresponds to radiation domination, and y is the Yukawa coupling of the flat direction to MSSM matter. The decay happens essentially perturbatively for $y\varphi_0 > m_0$, as envisaged by [317–319]

4. SUSY thermalization

Flat directions VEV can dramatically affect thermal history of the universe. The reason is that the MSSM flat direction VEV spontaneously breaks the SM gauge group. The gauge fields of the broken symmetries then acquire a SUSY conserving mass, $m_g \sim g|\varphi|$, from their coupling to the flat direction, where g is a gauge coupling constant. In which case a flat direction can crucially alter thermal history of the universe by suppressing thermalization rate of the reheat plasma. Note that, m_g , provides a physical infrared cut-off for scattering diagrams with gauge boson exchange in the t -channel shown in Fig. (A). The thermalization rate will then be given by (up to a Logarithmic “bremsstrahlung” factor):

$$\Gamma_{\text{thr}} \sim \alpha^2 \frac{n}{|\varphi|^2}, \quad (428)$$

where we have used $m_g^2 \simeq g^2|\varphi|^2$ where $\alpha \sim g^2/4\pi$. After the flat direction starts its oscillations at $H \simeq m_0 \sim \mathcal{O}(100)$ GeV, the Hubble expansion redshifts, $|\varphi|^2 \propto a^{-3}$, where a is the scale factor of the FRW universe. The interesting point is that, $n \propto a^{-3}(t)$, as well, and hence Γ_{thr} remains constant while H decreases for $H(t) < m_0 \sim \mathcal{O}(100)$ GeV. This implies that Γ_{thr} eventually catches up with the expansion rate, even if it is initially much smaller, and shortly after that the full thermal equilibrium will be achieved. Depending on whether $m_0 > \Gamma_d$ or $m_0 < \Gamma_d$, different situations will arise which we discuss separately ⁹⁴.

- $m_0 > \Gamma_d$: In this case the inflaton decays after the flat direction oscillations start. The inflaton oscillations, which give rise to the equations of state close to non-relativistic matter, dominate the energy density of the universe for $H(t) > \Gamma_d$. This implies that $a(t) \propto H^{-2/3}(t)$, and $|\varphi|$ is redshifted $\propto H(t)$ in this period. We therefore have, $\varphi_d \sim (\Gamma_d/m_0)\varphi_0$, where φ_d denotes the amplitude of the flat direction oscillations at the time of the inflaton decay $H(t) \simeq \Gamma_d$. By using the total energy density of

⁹⁴ If two or more flat directions with non-zero VEVs induce mass to the gauge bosons, then $|\varphi|$ denotes the largest VEV.

the plasma; $\rho \approx 3(\Gamma_d M_P)^2$, and note that $|\varphi|$ and n are both redshifted $\propto a^{-3}(t)$ for $H < \Gamma_d$, after using Eq. (428), we find that complete thermalization occurs when the Hubble expansion rate is [121]

$$H_{\text{thr}} \sim 10\alpha^2 \left(\frac{M_P}{\varphi_0}\right)^2 \frac{m_0^2}{m_\phi}. \quad (429)$$

- $m_0 < \Gamma_d$: In this case the flat direction starts oscillating after the completion of inflaton decay. The universe is dominated by the relativistic inflaton decay products for $H(t) < \Gamma_d$, implying that $a(t) \propto H(t)^{-1/2}$. The number density of particles in the plasma is redshifted $\propto H(t)^{3/2}$ until $H(t) = m_0 \sim \mathcal{O}(100)$ GeV. Note that $n, |\varphi|^2 \propto a(t)^{-3}$, and hence Γ_{thr} remains constant, for $H(t) < m_0$. The reheat plasma then thermalizes when the Hubble expansion rate is [121]

$$H_{\text{thr}} \sim 10\alpha^2 \left(\frac{\Gamma_d}{m_0}\right)^{1/2} \left(\frac{M_P}{\varphi_0}\right)^2 \frac{m_0^2}{m_\phi}. \quad (430)$$

Since the kinetic equilibrium is built through $2 \rightarrow 2$ scattering diagrams as in Fig. (A), which have one interaction vertex less than those in $2 \rightarrow 3$. Therefore the rate for establishment of kinetic equilibrium will be $\Gamma_{\text{kin}} \sim \alpha^{-1}\Gamma_{\text{thr}}$. In SUSY case, the relevant time scales have an hierarchy [121]:

$$\Gamma_d \gg \Gamma_{\text{kin}} > \Gamma_{\text{thr}}. \quad (431)$$

The relative chemical equilibrium among different degrees of freedom is built through $2 \rightarrow 2$ annihilations in the s -channel with a rate $\sim \alpha^2 n/E^2 \ll \Gamma_{\text{thr}}$. Hence composition of the reheat plasma will not change until full thermal equilibrium is achieved. This implies that the universe enters a long period of quasi-adiabatic evolution after the inflaton decay has completed. During this phase, the comoving number density and (average) energy of particles remain constant ⁹⁵.

⁹⁵ The decay of flat directions and their interactions with the reheat plasma are negligible before the universe fully thermalizes.

5. Reheat temperature of the universe

The temperature of the universe after it reaches full thermal equilibrium is referred to as the reheat temperature T_R . In the case of MSSM, we therefore have [121, 122]:

$$T_R \simeq (H_{\text{thr}} M_P)^{1/2}, \quad (432)$$

where, depending on the details, H_{thr} is given by Eqs. (429) and (430). Since $H_{\text{thr}} \ll \Gamma_d$, the reheat temperature is generically much smaller in MSSM (or in a generic theory with gauge invariant flat directions) than the standard expression $T_R \simeq (\Gamma_d M_P)^{1/2}$, which is often used in the literature with an assumption that immediate thermalization occurs after the inflaton decay. Note that the reheat temperature depends very weakly on the inflaton decay rate, for instance Eq. (430) implies that $T_R \propto \Gamma_d^{1/4}$, while T_R is totally independent of Γ_d in Eq. (429). Regardless of how fast the inflaton decays, the universe will not thermalize until the $2 \rightarrow 3$ scatterings become efficient. A larger φ_0 results in slower thermalization and a lower reheat temperature, see the following table for some sample examples ⁹⁶.

VEV (in GeV)	$T_R(\Gamma_d = 10 \text{ GeV} < m_0)$	$T_R(\Gamma_d = 10^4 \text{ GeV} > m_0)$
$\varphi_0 \sim M_P$	3×10^3	7×10^4
$\varphi_0 = 10^{-2} M_P$	3×10^5	7×10^6
$\varphi_0 = 10^{-4} M_P$	3×10^7	7×10^8
$\varphi_0 \leq 10^{-6} M_P$	3×10^9	7×10^{10}

Table 1: The reheat temperature of the universe for the inflaton mass, $m_\phi = 10^{13}$ GeV, and two values of the inflaton decay rate, $\Gamma_d = 10, 10^4$ GeV (if the inflaton decays gravitationally, we have $\Gamma_d \sim 10$ GeV). The flat direction mass is $m_0 \sim 1$ TeV. The rows show the values of T_R for flat direction VEVs. Note that when the VEV of a flat direction is $< 10^{12}$ GeV, the flat direction can no longer delay the thermalization, and the reheat temperature, $T_R \simeq (\Gamma_d M_P)^{1/2}$, remains a good approximation ⁹⁷.

⁹⁶ If $H_{\text{thr}} \geq \Gamma_d$, the reheat temperature in such cases follows the standard expression: $T_R \simeq (\Gamma_d M_P)^{1/2}$.

This will be the case if the flat direction VEV is sufficiently small at the time of the inflaton decay, and/or if the reheat plasma is not very dilute.

⁹⁷ Majority of MSSM flat directions can take a VEV $\geq 10^{14}$ GeV, which are lifted by more than dimensional 6 operators, for instance $udd, LLe, \text{etc.}$

E. Quasi-adiabatic thermal evolution of the universe

Before complete thermalization is reached the universe remains out of equilibrium⁹⁸. The deviation from full equilibrium can be quantified by the parameter " \mathcal{A} " [121, 731], where

$$\mathcal{A} \equiv \frac{3\rho}{T^4} \sim 10^4 \left(\frac{\Gamma_d M_{\text{P}}}{m_\phi^2} \right)^2. \quad (433)$$

Here we define $T \approx \langle E \rangle / 3$, in accordance with full equilibrium. Note that in full equilibrium, see Eq. (375), we have $\mathcal{A} \approx g_*$ ($= 228.75$ in the MSSM).

One can also associate parameter $\mathcal{A}_i \equiv 3\rho_i/T^4$ to the i -th degree of freedom with the energy density ρ_i (all particles have the same energy E , and hence T , as they are produced in one-particle decay of the inflaton). Note that $\mathcal{A} = \sum_i \mathcal{A}_i$, and in full equilibrium we have $\mathcal{A}_i \simeq 1$. Note that \mathcal{A} depends on the total decay rate of the inflaton Γ_d and its mass m_ϕ through Eq. (433). While, \mathcal{A}_i are determined by the branching ratio for the inflaton decay to the i -th degree of freedom. The composition of the reheat plasma is therefore model-dependent before its complete thermalization. However, some general statements can be made based on symmetry arguments.

During the quasi-adiabatic evolution of the reheated plasma, i.e., for $H_{\text{thr}} < H < \Gamma_d$, we have⁹⁹ [121]

$$\rho_i = \mathcal{A}_i \frac{3}{\pi^2} T^4, \quad n_i = \mathcal{A}_i \frac{1}{\pi^2} T^4, \quad H \simeq \mathcal{A}^{1/2} \left(\frac{T^2}{3M_{\text{P}}} \right). \quad (434)$$

In this epoch T varies in a range $T_{\text{min}} \leq T \leq T_{\text{max}}$, where $T_{\text{max}} \approx m_\phi/3$ is reached right after the inflaton decay. Because of complete thermalization, T sharply drops from T_{min} to T_{R} at H_{thr} , where the conservation of energy implies that

$$T_{\text{R}} = \left(\frac{\mathcal{A}}{228.75} \right)^{1/4} T_{\text{min}}, \quad \implies \quad T_{\text{R}} \leq T_{\text{min}}. \quad (435)$$

⁹⁸ Right after the inflaton decay has completed the energy density of the universe is given by; $\rho \approx 3(\Gamma_d M_{\text{P}})^2$, and the average energy of particles is $\langle E \rangle \simeq m_\phi$. For example, in a two-body decay of the inflaton, we have exactly $E = m_\phi/2$.

⁹⁹ One can express \mathcal{A}_i in terms of a *negative* chemical potential μ_i , where $\mathcal{A}_i = \exp(\mu_i/T)$. Note that for a large negative chemical potential, i.e., in a dilute plasma, the Bose and Fermi distributions are reduced to the Maxwell-Boltzmann distribution and give essentially the same result. The assignment of a chemical potential merely reflects the fact that the number of particles remains constant until the number-violating reactions become efficient. It does not appear as a result of a conserved quantity (such as baryon number) which is due to some symmetry. Indeed, assuming that inflaton decay does not break such symmetries, the same chemical potential to particles and anti-particles are the same.

1. Particle creation in a quasi-thermal phase

In this section, we wish to create weakly interacting χ particles from the scatterings of the MSSM particles during the quasi-thermal phase of the universe. Recall the Boltzmann equation governing the number density of χ particles, which is given by [95, 334]:

$$\dot{n}_\chi + 3Hn_\chi = \sum_{i \leq j} \langle v_{\text{rel}} \sigma_{ij \rightarrow \chi} \rangle n_i n_j. \quad (436)$$

Here n_i and n_j are the number densities of the i -th and j -th particles, $\sigma_{ij \rightarrow \chi}$ is the cross-section for producing χ from scatterings of i and j , and the sum is taken over all fields which participate in χ production. Also $\langle \dots \rangle$ denotes averaging over the distribution. Since the production of χ quanta will be Boltzmann suppressed if $T < m_\chi/3$. Therefore, in order to obtain the total number of χ quanta produced from scatterings, it will be sufficient to integrate the RH side of Eq. (436) from the highest temperature down to T_{min} . The relic abundance of χ , normalized by the entropy density, s , is given by [121]

$$\frac{n_\chi}{s} \sim 10^{-5} \left(\frac{228.75}{\mathcal{A}} \right)^{5/4} \sum_{i,j} \left[\int_{T_{\text{min}}}^{T_{\text{max}}} \mathcal{A}_i \mathcal{A}_j \langle v_{\text{rel}} \sigma_{ij \rightarrow \chi} \rangle M_{\text{P}} dT \right], \quad (437)$$

where we have used Eq. (435).

2. Gravitino production

For flat direction(s) $\text{VEV} \geq 10^{12}$ GeV, slow thermalization results in a low reheat temperature, i.e. $T_{\text{R}} \leq 10^9$ GeV, which is compatible with the BBN bounds on thermal gravitino production. However gravitinos are also produced during the quasi-thermal phase prior to a complete thermalization of the reheat plasma. Generically gravitinos are produced from the scatterings of gauge, gaugino, fermion and sfermion quanta with a cross-section $\propto 1/M_{\text{P}}^2$.

During the quasi-thermal phase, the gauge and gaugino quanta have large masses $\sim \alpha^{1/2} \varphi_{\text{d}}$ (induced by the flat direction VEV) at a time most relevant for the gravitino production, i.e., when $H \simeq \Gamma_{\text{d}}$, therefore, they decay to lighter fermions and sfermions at a rate $\sim \alpha^{3/2} \varphi_{\text{d}}^2 / m_\phi$, where $\alpha^{3/2} \varphi_{\text{d}}$ is the decay width at the rest frame of gauge/gaugino quanta, and $\varphi_{\text{d}} / m_\phi$ is the time-dilation factor. The decay rate is $\gg \Gamma_{\text{d}}$, thus gauge and gaugino quanta decay almost instantly upon production, and they will not participate in the gravitino production. As a consequence, production of the helicity $\pm 1/2$ states will not

be enhanced in a quasi-thermal phase as scatterings with a gauge-gaugino-gravitino vertex will be absent ¹⁰⁰.

The following channels contribute to the gravitino production [302]: (a) *fermion + anti-sfermion* \rightarrow *gravitino + gauge field*, (b) *sfermion + anti-fermion* \rightarrow *gravitino + gauge field*, (c) *fermion + anti-fermion* \rightarrow *gravitino + gaugino*, (d) *sfermion + anti-sfermion* \rightarrow *gravitino + gaugino*.

The total cross-section involves cross-sections for multiplets comprising the LH (s)quarks Q , RH up-type (s)quarks u , RH down-type (s)quarks d , LH (s)leptons L , RH (s)leptons e and the two Higgs/Higgsino doublets H_u, H_d . Since particles and anti-particles associated to the bosonic and fermionic components of the multiplets which belong to an irreducible representation of a gauge group have the same parameter \mathcal{A}_i . This implies that [121]

$$\begin{aligned} \Sigma_{\text{tot}} \equiv & \sum_{i,j=1}^3 \sum_{a,b=1}^2 \sum_{\alpha,\beta=1}^3 \mathcal{A}_{i,a,\alpha} \mathcal{A}_{j,b,\beta} \langle \sigma_{\text{tot}} v_{\text{rel}} \rangle = \\ & \frac{1}{32M_{\text{P}}^2} \sum [6\alpha_3(2\mathcal{A}_Q^2 + \mathcal{A}_u^2 + \mathcal{A}_d^2) + \frac{9}{4}\alpha_2(3\mathcal{A}_Q^2 + \mathcal{A}_L^2 + \mathcal{A}_H^2) \\ & + \frac{1}{4}\alpha_1(\mathcal{A}_Q^2 + 8\mathcal{A}_u^2 + 2\mathcal{A}_d^2 + 3\mathcal{A}_L^2 + 6\mathcal{A}_e^2 + 3\mathcal{A}_H^2)], \end{aligned} \quad (438)$$

where $1 \leq i, j \leq 3$, $a, b = 1, 2$ and $1 \leq \alpha, \beta \leq 3$ are the flavor, weak-isospin and color indices of scattering degrees of freedom respectively. Also α_3 , α_2 and α_1 are the gauge fine structure constants related to the $SU(3)_C$, $SU(2)_W$ and $U(1)_Y$ groups respectively. The sum is taken over the three flavors of Q, u, d, L, e and the two Higgs doublets. After replacing Σ_{tot} in Eq. (437), and recalling that $T_{\text{max}} \approx m_\phi/3$, we obtain [121]

$$\frac{n_{3/2}}{s} \simeq (10^{-1} M_{\text{P}}^2 \Sigma_{\text{tot}}) \left(\frac{228.75}{\mathcal{A}} \right)^{5/4} \left(\frac{T_{\text{max}}}{10^{10} \text{ GeV}} \right) 10^{-12}. \quad (439)$$

Note that in full thermal equilibrium, $\Sigma_{\text{tot}} = (4\pi/M_{\text{P}}^2) \times (16\alpha_3 + 6\alpha_2 + 2\alpha_1) \simeq (10^{-1}/M_{\text{P}}^2)$ (up to logarithmic corrections which are due to renormalization group evolution of gauge couplings).

Let us consider a simplistic scenario when the inflaton primarily decays into one flavor of LH (s)quarks. In this case $\mathcal{A}_Q = 1/24$ for the relevant flavor ¹⁰¹, while $\mathcal{A} = 0$

¹⁰⁰ Otherwise gauge and/or gaugino quanta in the initial state (particularly scattering of two gluons) have the largest production cross-section [302-305].

¹⁰¹ The total number of degrees of freedom in one flavor of LH (s)quarks is 2 (particle – antiparticle) \times 2 (fermion – boson) \times 2 (weak – isospin) \times 3 (color).

for the rest of the degrees of freedom. This results in a gravitino abundance: $n_{3/2}/s \simeq (\mathcal{A}/228.75)^{3/4} (T_{\max}/10^{10} \text{ GeV}) 10^{-12}$, where \mathcal{A} is given by Eq. (433) and note that $T_{\max} \gg T_R$. The largest value for $T_{\max} \simeq 10^{12} \text{ GeV}$, therefore the the tightest bound for unstable gravitinos come from BBN ($n_{3/2}/s \leq 10^{-16}$ (arising for $m_{3/2} \simeq 1 \text{ TeV}$ and a hadronic branching ratio $\simeq 1$) is satisfied if $\mathcal{A} \leq 10^{-6}$. Much weaker bounds on \mathcal{A} are found for a radiative decay. For example, $\mathcal{A} \leq 10^{-3}$ (1) if $m_{3/2} \simeq 100 \text{ GeV}$ (1 TeV).

VII. GENERATING PERTURBATIONS WITH THE CURVATON

A. What is the curvaton ?

The curvaton scenario is an interesting possibility [124, 125, 127–130, 802], see also [126], where the perturbations of more than one light scalar fields play important role during inflation. In many realistic examples of particle physics having more than one light scalar fields is quite natural, especially in the case of MSSM [90]. However, as we shall argue below that within MSSM there are only handful of such good candidates for a curvaton [132–134]¹⁰².

In the original curvaton paradigm it was assumed that the curvaton is responsible for generating the entire curvature perturbations, and the perturbations arising from the inflaton component are subdominant [124, 127–129]. However, there are variants where both inflaton and curvaton can contribute to the curvature perturbations, see for instance [807, 808]. Irrespective of their origins the curvaton must possess the following properties:

- Lightness of the curvaton:

During inflation the Hubble expansion rate is: $H_{inf} \gg m_\varphi$, where m_φ is the effective mass of the curvaton. Since it does not cost anything in energy, therefore the quantum fluctuations are free to accumulate along a curvaton direction and form a condensate with a large VEV, φ_0 . During inflation, $V(\phi) \gg V(\varphi)$, where ϕ is the inflaton here.

¹⁰² Sneutrino as a curvaton have been considered in Refs. [803–805], With an extension of MSSM, where right handed sneutrinos are introduced as SM gauge singlets. However sneutrino alone is not a D -flat direction, one must consider the D -term contribution of the potential also. There are also models of curvaton where the field is a pNGB [806], however, it is uncertain how such a field would excite dominantly the SM degrees of freedom.

After inflation, $H \propto t^{-1}$, and the curvaton stays at a relatively large VEV due to large Hubble friction term until $H \simeq m_\varphi$, when the curvaton starts oscillating around the origin with an initial amplitude $\sim \varphi_0$. From then on $|\varphi|$ is redshifted by the Hubble expansion $\propto H(t)$ for matter dominated and $\sim H^{3/4}(t)$ for radiation dominated Universe. The energy of the oscillating flat direction may eventually start to dominate over the inflaton decay products.

- Longevity of the curvaton and its dominance:

Furthermore, the curvaton must not evaporate due to thermal interactions from the plasma already created by the inflaton decay products [134, 809]. The curvaton and the inflaton decay products can be weakly coupled, i.e. via non-renormalizable interactions. In such a case the curvaton can survive long enough and its oscillations can possibly dominate the energy density while decaying. It has been shown that MSSM flat direction can remain long lived [402, 403]. However, if the curvaton does not dominate while decaying, and the curvaton decay products do not thermalize with the inflaton decay products, then this will lead to potentially large isocurvature fluctuations [129, 810–813]. Note that the non-Gaussianity parameter, f_{NL} , is also constrained by the allowed isocurvature perturbations [131, 814, 815]. The curvaton can easily dominate the energy density if the inflaton decay products are dumped outside our own Hubble patch, which may be realizable in certain brane-world models with warped extra dimensions [816–818].

- Thermalization:

When the curvaton energy density does not dominate while decaying, then the curvaton decay products must thermalize with that of the inflaton decay products, otherwise, there will be large remnant isocurvature perturbations. It is desirable that both inflaton and curvaton excites *solely* the SM and/or MSSM degrees of freedom, which requires non-trivial construction on both the sectors [134].

Curvaton paradigm with more than 2 light fields were also constructed in [819, 820], but they bear more uncertainties, especially when the curvatons belong to the hidden sector. There are various challenges to the curvaton paradigm, which we discuss below.

- An absolute gauge singlet or a hidden sector curvaton:

An absolute gauge singlet curvaton’s origin might lie within string theory [127, 128], or within a hidden sector [309, 806, 818, 821–823]. The modulus will generically couple to all other sectors presumably with a *universal coupling*, i.e. gravitationally suppressed interaction. Note that dumping entropy into non-SM-like sectors from the curvaton decay would lead to large isocurvature perturbations, if the curvaton is subdominant, and these non-SM like degrees of freedom do not thermalize with that of the inflaton decay products before BBN [810].

If the inflaton is also a modulus or a hidden sector field, then it is important to make sure that the inflaton and the curvaton are extremely weakly coupled, otherwise, inflaton decay into curvaton can destroy the curvaton condensate prematurely before its dominance.

- A SM gauge invariant curvaton:

MSSM provides gauge invariant curvaton candidates [132–134, 803–805, 809, 816, 817, 824–827], which can decay into MSSM degrees of freedom directly. In this case the curvaton must ensure its longevity, and its dominance at the time of decay. If the inflaton decay products are of MSSM/SM like quanta, then they can interact with the MSSM curvaton and can possibly destroy its coherence and longevity [134, 809]. The advantage of an MSSM curvaton is that the couplings are that of the SM, therefore, predictions are robust.

B. Cosmological constraints on a curvaton scenario

The main observable constraints for a curvaton scenario are: (1) isocurvature perturbations, and (2) primordial non-Gaussianity. The isocurvature perturbations are created when the curvaton fail to dominate the energy density while decaying and the curvaton decay products fail to thermalize with that of the inflaton decay products. There are potentially well motivated isocurvature perturbations one can create, CDM [129, 224–226, 813, 828], baryon isocurvature perturbations [126, 129, 829–834], neutrino isocurvature perturbations were also considered in [129]¹⁰³.

¹⁰³ There is a way to avoid large isocurvature component provided the inflaton perturbations are subdominant during inflation, i.e. $(\delta\rho/\rho)_{inf} \ll 10^{-5}$. This can be achieved very well in many inflationary models [807],

For a conserved number density $\delta n_i = 0$, where i corresponds to baryons or CDM, the entropy perturbation is given by [129, 246]

$$\mathcal{S}_i = 3(\zeta_i - \zeta), \quad (440)$$

where ζ is the total curvature perturbations, and ζ_i is the curvature perturbations in the i component. Recall that in a curvaton scenario the total curvature perturbations is given by; $\zeta = r\zeta_\varphi$, where $r = \rho_\varphi/\rho_r$ at the time of curvaton decay, and ρ_r is the radiation energy density due to the inflaton decay products.

Let us now imagine, when the curvaton decays either into baryons (need not be the SM baryons), or non-relativistic CDM, then $\zeta_i = \zeta_\varphi = (1/r)\zeta$. Combining this fact, the isocurvature perturbations yield:

$$\mathcal{S}_i = 3\left(\frac{1-r}{r}\right)\zeta. \quad (441)$$

However, if the curvaton does not decay into the baryons or CDM at all, then $\zeta_i = 0$ and $\mathcal{S}_i = -3\zeta$, the isocurvature mode becomes three times larger than the curvature perturbations, which is ruled out by the current CMB and LSS data [223–225], for axion isocurvature perturbations, see [226].

In terms of a constrainable parameter, $\sqrt{\alpha} \equiv S_i/\zeta$, the constrains on r read as:

$$0.98 < r \approx 1 - \frac{\sqrt{\alpha}}{3} < 1.0, \quad (442)$$

for $\alpha < 0.0037$ at 95% c.l. [12, 269]. The fraction of energy densities, $r = \rho_\varphi/\rho_r$, also governs the non-Gaussianity generated by the curvaton. Thus the constraint on non-Gaussianity parameter f_{NL} yields

$$-1.21 > f_{NL} \equiv \frac{5}{4r} > -1.25, \quad (443)$$

which is 1σ away from the central value of the quoted value from WMAP [12], which is $-9 < f_{NL} < 111$ (at 95% c.l.) In view of these bounds, it seems that the curvaton decay products cannot generate large CDM, or baryon isocurvature perturbations, or they must not decouple from the thermal plasma created before by that of the inflaton. This could be

but the curvaton perturbations can create the required level of perturbations even if they do not dominate the energy density while decaying. This can be possible for a curvaton potential discussed in Eq. (456) for specific choices of (n, m_0) , see Ref. [807].

achieved simply if the curvaton and the inflaton decay products excite the relevant degrees of freedom required for BBN. Otherwise, the curvaton must dominate while decaying and create all the SM or MSSM degrees of freedom. There are mixed curvaton-inflaton scenarios, where the curvature perturbations are contributed by both inflaton and curvaton [808]. In such models larger f_{NL} is expected.

C. Curvaton candidates

Only handful of the curvaton candidates exist which belong to the observable sector, i.e. charged under the SM gauge group [132–134, 803, 804, 809, 816, 817, 824, 827]. The SM Higgs can act as a potential curvaton, since Higgs is light compared to any high scale of inflation, it would induce fluctuations $\sim H_{inf}/2\pi\langle h \rangle \sim 10^{-5}$, where $\langle h \rangle$ is the Higgs VEV during inflation. Validity of the SM Higgs potential beyond the electroweak scale makes it actually less attractive candidate, besides it cannot dominate the energy density of the universe. Within MSSM, with two Higgses, it is possible to realize a curvaton scenario, where the inflaton energy density is dumped out of our own observable world, as a consequence the Higgses can dominate the energy density and create all the matter fields [816, 817]. However this scenario will work *only* if the inflaton does not couple to the MSSM sector at all, which is very unlikely.

1. Supersymmetric curvaton

An important constraint arises from CMB temperature anisotropy involving the ratio of the perturbation and the background VEV of the curvaton, since this ratio is related to the curvature perturbations [125, 127–129, 246]. Provided the perturbations do not damp during its evolution, strictly speaking for a quadratic potential, the final curvature perturbation is given by ¹⁰⁴:

$$\delta = \frac{\delta\varphi}{\varphi} = \frac{H_{inf}}{2\pi\varphi_{inf}} \sim 10^{-5}. \quad (444)$$

For an MSSM flat direction curvaton, it is important to keep in mind that they carry SM gauge couplings, therefore, if the inflaton decay products create a plasma which has

¹⁰⁴ A detailed analysis of a curvaton scenario for a non-quadratic potential can be found in Refs. [807, 835].

MSSM degrees of freedom then they would interact with the curvaton rendering thermal corrections to the curvaton potential inevitable. There are three issues which have to be taken into account.

- Curvaton must not have a renormalizable coupling to the inflaton, otherwise curvaton cannot obtain large VEV during inflation. For instance, neither $H_u H_d$ nor LH_u are good curvaton candidates, because a gauge singlet inflaton can couple to these flat directions through renormalizable interactions [121, 122, 134]. The inflaton couplings to LH_u or $H_u H_d$ ought to be very weak in order for them to be a curvaton candidate.
- Curvaton must not induce a mass $\geq m_\phi/2$ to the inflaton decay products, otherwise, the two-body inflaton decay into MSSM quanta will be kinematically blocked. The inflaton decay will be delayed until the relevant flat direction has started its oscillation and its VEV has been redshifted to sufficiently small values [121, 122, 134].
- The flat direction VEV of a curvaton must not break all of the SM gauge symmetry. This will affect the inflaton decay products to thermalize quickly, both kinetic and chemical equilibrium of MSSM degrees of freedom will be delayed, and furthermore, the curvaton oscillations will not be able to dominate the energy density [121, 122, 134].

Let us denote the flat direction superfield by φ . In the case of $u_i d_j d_k$ (for $i, j, k = 1, 2, 3, j \neq k$) as a flat direction, φ would represent the VEV. The MSSM superpotential can be rewritten as:

$$W_{\text{MSSM}} \supset \lambda_1 H_u \varphi \varphi_1 + \lambda_2 H_d \varphi \varphi_2 + \lambda_3 L \varphi \varphi_3, \quad (445)$$

where $\varphi_{1,2,3}$ are MSSM superfields, see Eq. (103), L is denoted by L_1, L_2 or L_3 , and $\lambda_1, \lambda_2, \lambda_3$ are the Yukawas. In general $m_{inf} \leq H_{inf}$, which is true for high scale models of inflation, and $\varphi_{inf} \sim 10^5 H_{inf}$. Note that the VEV of the flat direction induces VEV dependent SUSY preserving masses to the MSSM particles, $\sim \lambda_{1,2,3} \langle \varphi_{inf} \rangle$. Therefore, for the inflaton decay into MSSM quanta to be kinematically allowed, we require:

$$\lambda_1, \lambda_2 \leq 10^{-5}, \quad (446)$$

if the $\underline{\Phi H_u H_d}$ coupling is allowed by R-parity, and

$$\lambda_1, \lambda_3 \leq 10^{-5}, \quad (447)$$

if the $\Phi H_u L$ coupling is allowed, where Φ is the inflaton superfield.

The above conditions considerably restrict the curvaton candidates within the MSSM, as only the first generations of (s)quarks and (s)leptons have Yukawa coupling $\lesssim 10^{-5}$, while first two generations have Yukawas $\geq 10^{-3}$.

Given these constraints an acceptable flat directions should include *only* one Q or one u . The reason is that D - and F -flatness conditions for directions which involve two or more Q and/or u require them to be of different flavors (for details, see [399]). The only flat directions with $\lambda_1 \leq 10^{-5}$ are as follows:

- udd : This monomial represents a subspace of complex dimension 6 [399]. D -flatness requires that the two d are from different generations (hence at least one of them will be from the second or third generation). This implies that $\lambda_2 \geq 10^{-3}$, see Eq. (445). As a consequence the two-body inflaton decay via the superpotential term; $\Phi H_u H_d$ term will be kinematically forbidden, but $\Phi H_u L$ term will led to the inflaton decay into MSSM degrees of freedom. The VEV of udd direction keeps $SU(2)_W$ unbroken, so that the decay products of the inflaton with $SU(2)_W$ degrees of freedom can completely thermalize before the curvaton, udd , has a chance to dominate and decay.
- QLd : This monomial represents a subspace of complex dimension 19 [399]. F -flatness requires that Q and d belong to different generations. Then, since Q and d are both coupled to H_d , Eq. (445) implies that $\lambda_2 \geq 10^{-3}$. The two-body inflaton decay via superpotential $\Phi H_u H_d$ term will be kinematically forbidden, but the other superpotential term is kinematically allowed, i.e. $\Phi H_u L$. The VEV of QLd directions completely break the $SU(2)_W \times U(1)_Y$, but leave a $SU(2)$ subgroup of the $SU(3)_C$ unbroken. Therefore the associated color degrees of freedom can thermalize quickly to SM degrees of freedom, before QLd could dominate the energy density.
- LLe This monomial represents a subspace of complex dimension three [399]. D -flatness requires that the two L s are from different generations, while F -flatness requires that e belongs to the third generations (therefore all the three lepton generations will be involved). A feasible curvaton candidate will be $L_2 L_3 e_1$ direction. For this flat direction we have $\lambda_1 = 0$ and $\lambda_3 \sim 10^{-5}$ (see Eq. (445)). This implies that two-body inflaton decay will proceed via the $\Phi H_u L_1$ term without trouble. The flat direction

VEV breaks the electroweak symmetry $SU(2)_W \times U(1)_Y$, while not affecting $SU(3)_C$.

- *LLddd*: This monomial represents a subspace of complex dimension three [399]. D -flatness requires that the two L s and the three d s are all from different generations. This implies that that $\lambda_2 \simeq 10^{-2}$ and $\lambda_1 = \lambda_3 = 0$ (see Eq. (445)). This direction breaks all of the SM gauge group. This results in late thermalization of the Universe [121, 731] and the absence of thermal effects which, as a consequence, does not yield early oscillations of the flat direction.

Thus within MSSM the potential curvaton candidates could be the $u_1 d_2 d_3$ and $Q_1 L_3 d_2$ (and possibly $L_2 L_3 e_1$) flat directions. Because in all these two cases the inflaton can decay and produce a thermal bath, which will enable the curvaton oscillations to dominate early during its oscillations, and the curvaton can receive finite temperature thermal corrections. Without the latter the curvaton can never lead to dominate the energy density of the universe while decaying.

2. The A -term curvaton and a false vacuum

Let us consider an MSSM flat direction potential:

$$V = m_0^2 |\varphi|^2 + \lambda_n^2 \frac{|\varphi|^{2(n-1)}}{M_P^{2(n-3)}} + \left(A \lambda_n \frac{\varphi^n}{M_P^{n-3}} + h.c. \right), \quad (448)$$

where $\lambda_n \sim \mathcal{O}(1)$, $n \geq 4$, and $A \sim m_0 \sim \mathcal{O}(100 \text{ GeV} - 1 \text{ TeV})$, and depends on a phase. Note that we have not added the Hubble-induced corrections to the mass and the A -term. They can be avoided for a simple choice of no-scale type Kähler potential, either motivated from R-symmetry [318, 319], shift symmetry or Heisenberg symmetry [408, 416].

The radial and angular direction of the potential remains flat during inflation, therefore they obtain random fluctuations. There will be equally populated domains of Hubble patches, where the phase of the A -term is either positive or negative. In either case, during inflation, the flat direction VEV is given by:

$$\varphi_{inf} \sim (m_0 M_P^{n-3})^{1/n-2}. \quad (449)$$

However there is a distinction between a positive and a negative phase of the A -terms. The difference in dynamics arises after the end of inflation. In the case of a positive A -term

the flat direction starts rolling immediately, but in the case of a negative A -term, the flat direction remains in a **false vacuum** at a VEV which is given by Eq. (449).

The mass of the flat direction around this false minimum is very small compared to the Hubble expansion rate during inflation, i.e. $(3n^2 - 9n + 8)m_0^2 \ll H_{inf}^2$ where $n > 3$. During inflation the flat direction obtains *quantum fluctuations* whose amplitude is given by Eq. (444). The flat direction can exit such a metastable minimum only if *thermal corrections* are taken into account.

3. Thermal corrections to the curvaton

The flat direction VEV naturally induces a mass $\sim \lambda\varphi_{inf}$ to the field which are coupled to it, where λ is a gauge or Yukawa coupling. Note that φ_{inf} is the initial VEV of the curvaton, after the end of inflation slides down gradually. If there is a thermal bath with a temperature T then there is a thermal corrections to the flat direction depending on whether $\lambda\varphi_{inf} \leq T$ or $\lambda\varphi_{inf} > T$, different situations arise.

- $\lambda\varphi_{inf} \leq T$:

Fields in the plasma which have a mass smaller than temperature are kinematically accessible to the thermal bath. They will reach full equilibrium and result in a thermal correction V_{th} to the flat direction potential

$$V_{th} \sim +\lambda^2 T^2 |\varphi|^2. \quad (450)$$

The flat direction then starts oscillating, provided that $\lambda T > H$ [121, 122, 134, 731].

- $\lambda\varphi_{inf} > T$:

Fields which have a mass larger than temperature will not be in equilibrium with the thermal bath. For this reason they are also decoupled from the running of gauge couplings (at finite temperature). This shows up as a correction to the free energy of gauge fields, which is equivalent to a logarithmic correction to the flat direction potential [134]:

$$V_{th} \sim \pm\alpha T^4 \ln(|\varphi|^2) , \quad (451)$$

where α is a gauge fine structure constant. Decoupling of gauge fields (and gauginos) results in a positive correction, while decoupling of matter fields (and their super-

partners) results in a negative sign. The overall sign then depends on the relative contribution of decoupled fields. As an example, let us consider the $H_u H_d$ flat direction. This direction induces large masses for the top (s)quarks which decouples them from the thermal bath, while not affecting gluons and gluinos. Therefore this leads to a positive contribution from the free energy of the gluons.

Obviously only corrections with a positive sign can lead to flat direction oscillations around the origin. Oscillations begin when the second derivative of the potential exceeds the Hubble rate-squared which, from Eq. (451), reads $(\alpha T^2/\varphi_{inf}) > H$.

Note that thermal effects of the first type require that fields which have Yukawa couplings to the flat direction are in full equilibrium, while those of the second type require the gauge fields (and gauginos) be in full equilibrium. There are two main possibilities:

- $u_1 d_2 d_3$: $SU(2)_W$ remains unbroken in this case. This implies that the corresponding gauge fields and gauginos, H_u and H_d (plus the Higgsinos) and the LH (s)leptons reach full thermal equilibrium. The back-reaction of H_u results in a thermal correction $\lambda_1^2 T^2 |\varphi|^2$, see Eq. (445), with $\lambda_1 \sim 10^{-5}$. The free energy of the $SU(2)_W$ gauge fields result in a thermal correction $\sim +\alpha_W T^4 \ln(|\varphi|^2)$. Note that the sign is positive since the flat direction induces a mass which is $> T$ (through the d) for the LH (s)quarks but not the $SU(2)_W$ gauge fields and gauginos.
- $Q_1 L d$: An $SU(2)$ subgroup of $SU(3)_C$ is unbroken in this case. Therefore only the corresponding gauge fields and gauginos plus some of the (s)quark fields reach full thermal equilibrium. Then the back-reaction of u_1 and d_1 results in a thermal correction $(\lambda_1^2 + \lambda_2^2) T^2 |\varphi|^2$ according to Eq.(445), where $\lambda_1 \sim \lambda_2 \sim 10^{-5}$. Note that decoupling of a number of gluons (and gluinos) from the running of strong gauge coupling results in a negative contribution to the free energy of the unbroken part of $SU(3)_C$.

Therefore, within MSSM and from the point of view of thermal effects, the $u_1 d_2 d_3$ flat direction is the most suitable curvaton candidate. The $L_2 L_3 e_1$ flat direction can obtain a large VEV. The $SU(3)_C$ part of the SM gauge symmetry remains unbroken for this flat direction, and hence gluons, gluinos and (s)quarks will reach full equilibrium. We note that neither of L and e are coupled to the color degrees of freedom. This implies that there

will be no $T^2\varphi^2$ or logarithmic correction to the flat direction potential in this case. This excludes the $L_2L_3e_1$ flat direction from being a successful curvaton candidate.

4. $u_1d_2d_3$ as the MSSM curvaton

From the discussion in the previous section, it follows that thermal effects will lead to a following potential:

$$V_{\text{th}} \sim \lambda_1^2 T^2 |\varphi|^2 + \alpha_W T^4 \ln(|\varphi|^2), \quad (452)$$

where $\lambda_1 \sim 10^{-5}$ and $\alpha_W \sim 10^{-2}$. Typically at initial (larger) temperatures the first term dominates and at later (lower) temperatures the second term dominates. Note that the curvaton is in a metastable vacuum, only the temperature corrections can lift the curvaton, therefore we need $V_{\text{th}} > m_0^2 \varphi_{\text{inf}}^2$, so that the thermal effects will overcome the potential barrier. This yields:

$$\alpha_W T^4 > m_0^2 \varphi_{\text{inf}}^2. \quad (453)$$

The thermal mass should be sufficient to trigger curvaton oscillations, otherwise, the curvaton will remain in a metastable vacuum. In order for the flat direction oscillations to start we must have $d^2V_{\text{th}}/d|\varphi|^2 > H^2$. This leads to:

$$\alpha_W^{1/2} \frac{T^2}{\varphi_{\text{inf}}} > H(T), \quad (454)$$

which always holds in a radiation-dominated phase where $H \simeq T^2/M_{\text{P}}$ (note that $\varphi_{\text{inf}} \ll M_{\text{P}}$). This implies that the $u_1d_2d_3$ direction starts oscillating once the condition given in Eq. (453) is satisfied. This happens, when temperature of the Universe is given by

$$T_{\text{osc}} \sim \left(\frac{\varphi_{\text{inf}}}{10^{14} \text{ GeV}} \right)^{1/2} \times 10^9 \text{ GeV}. \quad (455)$$

Although, its a very high temperature, the flat direction curvaton does not evaporate due to the presence of a thermal bath. Thermal scatterings are governed by $\lambda_1^2 \varphi^2 \chi^2$, with φ being the flat direction and, χ collectively denotes the fields in thermal equilibrium. The rate for evaporation is proportional to $\lambda_1^4 \sim (10^{-5})^4$, much feeble to destroy the flat direction by evaporation.

The flat direction curvaton, $u_1d_2d_3$, is lifted by $n = 6$ superpotential level, therefore $\varphi_{\text{inf}} \sim 3 \times 10^{14} \text{ GeV}$, see Eq. (449). In order to be a curvaton, the perturbations should match the observed limit, therefore $H_{\text{inf}} \sim 2\pi\varphi_{\text{inf}} \times 10^{-5} \approx 10^{10} \text{ GeV}$.

The final reheat temperature is determined by the radial oscillations of the curvaton. The curvaton oscillations dominate the energy density as the ambient temperature of the plasma redshifts below the T_{osc} very rapidly due to the expansion of the universe. This is the *only* example where the finite temperature effects can render the curvaton oscillations dominating while decaying [134]. The decay of $u_1 d_2 d_3$ curvaton will take place via instant preheating discussed in section V D. The decay happens via SM gauge and Yukawa couplings and the decay products are the MSSM degrees of freedom. There is no residual isocurvature fluctuations and the LSP dark matter can be created from the thermal bath.

Since $u_1 d_2 d_3$ as a curvaton dominates the energy density while decaying, there will be no significant non-Gaussianity. However as we shall show in the next section, there are MSSM flat directions which do not dominate while decaying, they have a possibility to generate large non-Gaussianity, see for example [825, 826], where the imaginary part of the MSSM flat direction curvaton is responsible for generating the perturbations. In these models the treatment for the inflaton decay products is not quite complete, it is not clear in which sector the inflaton decays. Thermal corrections are important for the MSSM flat direction potential as we have seen in the above discussion.

5. Models without A -term

There are many viable candidates of curvaton within MSSM which do not use A -term. These are flat directions which are lifted by hybrid operators, such as $W \sim \Phi^{n-1} \Psi / M^{n-3}$, for such operators the A -term becomes dynamically negligible. A generic curvaton potential is given by [132, 133, 816, 817]:

$$V(\varphi) = m_0^2 \varphi^2 + \lambda_n^2 \frac{|\varphi|^{2(n-1)}}{M_{\text{P}}^{2(n-3)}}. \quad (456)$$

The flat direction candidate, i.e $QuQuQue$ (lifted by $QuQuQuH_d ee$) by the superpotential term $n = 9$ is the only viable term which can dominate the energy density of the universe at the time of decay [132, 133]. Note that the superpotential term which lifts the direction is a hybrid type and therefore the A -term vanishes, see the discussion after Eq. (315). In these models, it was assumed that prior to the curvaton domination the inflaton decays primarily into the hidden sector radiation, which is not a satisfactory assumption.

For $n = 7$ the flat direction is $LLddd$ (lifted by $H_u LLLddd$), and for $n = 6$ the flat

direction QdL (lifted by $QdLudd$) are allowed to be a curvaton candidate, provided the background equation of state is that of a stiff fluid at the time of curvaton oscillations. The flat directions which are lifted by $n = 4$ or by $n = 3$ can never dominate the energy density of the universe. This is due to the fact that their initial amplitude of the oscillations is very low.

There is of course a way to cure flat directions which are lifted by $n = 4$ by the renormalizable superpotential, such as $H_u H_d$ (lifted by $(H_u H_d)^2$), and for $n = 3$, the flat direction $NH_u L$, in the extension of the SM gauge group by $U(1)_{B-L}$, where N is the right handed neutrino. They can dominate the energy density of the universe during oscillations, provided the inflaton energy never gets dumped into the observable world [817].

To summarize, when the curvaton does not dominate while decaying, then they can generate significant non-Gaussian perturbations and isocurvature fluctuations, however, the results are model dependent, as it depends on the nature of the inflaton decay products and the choice of the flat direction.

6. Curvaton and non-Gaussianity

One of the prime observable signature of a curvaton mechanism is to generate non-Gaussian perturbations of order $f_{NL} \sim \mathcal{O}(10 - 100)$. However as we have seen earlier, in most of the models of inflaton and curvaton, the non-Gaussian perturbations are constrained by the isocurvature perturbations. Models where the curvaton dominates while decaying will generate no large non-Gaussianity. For a realistic A -term curvaton model, $u_1 d_2 d_3$ will never generate large non-Gaussianity. The flat direction decays in couple of oscillations and its energy density dominate while decaying.

On the other hand, models without A -term curvaton can potentially generate large non-Gaussianity, as many of the directions, $n = 3, 4, 6, 7$, do not seem to dominate the energy density while decaying. There are however non-trivial constraints on such models in order to avoid large isocurvature perturbations.

- Inflaton decay products must create the MSSM degrees of freedom. There should not be any trace of hidden degrees of freedom which do not thermalize with that of the MSSM. Otherwise the inflaton perturbations must be sub-dominant, i.e. $(\delta\rho/\rho)_{inf} \ll 10^{-5}$ during inflation [807, 835].

- Provided the curvaton perturbations and the inflaton perturbations are of the same order, the curvaton must not create *large* baryon asymmetry and/or dark matter through its decay. In principle an *absolute gauge singlet* moduli curvaton can decay into gravitino or axino LSP. In such a case a stringent constraint on reheat temperature applies [306, 308, 794–796].

In presence of non-renormalizable potential [814, 836], the f_{NL} and g_{NL} parameters are given by:

$$f_{NL} = \frac{5}{4r} \left(1 + \frac{\varphi_0 \varphi_0''}{\varphi_0^2} \right) - \frac{5}{3} - \frac{5r}{6}. \quad (457)$$

$$g_{NL} = \frac{25}{54} \left[\frac{9}{4r^2} \left(\frac{\varphi_0^2 \varphi_0'''}{\varphi_0^3} + \frac{3\varphi_0 \varphi_0''}{\varphi_0^2} \right) - \frac{9}{r} \left(1 + \frac{\varphi_0 \varphi_0''}{\varphi_0^2} \right) + \frac{1}{2} \left(1 - \frac{9\varphi_0 \varphi_0''}{\varphi_0^2} \right) + 10r + 3r^2 \right], \quad (458)$$

where φ_0 is the initial amplitude of the curvaton field. For $r \ll 1$, which is the ratio of curvaton and ambient energy densities, the above equations simplify to:

$$f_{NL} \simeq \frac{5}{4r} \left(1 + \frac{\varphi_0 \varphi_0''}{\varphi_0^2} \right), \quad g_{NL} \simeq \frac{25}{54} \left[\frac{9}{4r^2} \left(\frac{\varphi_0^2 \varphi_0'''}{\varphi_0^3} + 3 \frac{\varphi_0 \varphi_0''}{\varphi_0^2} \right) \right]. \quad (459)$$

For a quadratic potential, $f_{NL} \sim (5/4r)$ and $g_{NL} \sim -(10/3r)$, holds true and $g_{NL} \simeq -(10/3)f_{NL}$. In Ref. [835, 836], the authors have studied the departure from quadratic potential for a curvaton and obtained, $g_{NL} \sim \mathcal{O}(-10^4) - \mathcal{O}(-10^5)$ for $r \sim 0.01$. These values also depend on the non-renormalizable operator, n . The constraints on isocurvature perturbations are extremely model dependent, as it depends on details of thermal history of the universe and the assumptions behind inflaton and curvaton sectors.

D. Inhomogeneous reheating scenarios

Inhomogeneous reheating scenarios were considered in Refs. [248–251, 685]. This idea is similar to the curvaton paradigm, the main difference here is that the isocurvature perturbations are now converted into curvature perturbations during the inflaton decay. Here we present a simple example within MSSM. For a SM gauge singlet inflaton, Φ , the only renormalizable coupling to the MSSM is either $W \sim g\Phi H_u H_d$, or $g\Phi L H_u$ where $g \leq \mathcal{O}(1)$. There are other non-renormalizable couplings which appear in the combinations with three superfields are [122]:

$$W \supset \frac{\Phi}{M_*} H_u Q u + \frac{\Phi}{M_*} H_d Q d + \frac{\Phi}{M_*} H_d L e + \frac{\Phi}{M_*} Q L d + \frac{\Phi}{M_*} u d d + \frac{\Phi}{M_*} L L e. \quad (460)$$

Where M_* is a cut-off scale. Since Φ has a large VEV during inflation, therefore its direct couplings to $H_u H_d$ or LH_u for a reasonable range of coupling strength g will render them heavy compared to the Hubble expansion rate, $g\langle\phi\rangle \geq H_{inf}$. These fields will not obtain large quantum fluctuations, and will dynamically stay close to their respective minimum, $\langle H_u \rangle, \langle H_d \rangle, \langle L \rangle \approx 0$. Note here that it is assumed that inflation is driven by the large VEV. Therefore, fields coupled to either H_u , H_d or L will remain massless by virtue of their zero VEV, and subject to large quantum fluctuations of order $H_{inf}/2\pi$ during inflation. It is possible to convert these fluctuations when the inflaton decays to the MSSM quanta at the time of reheating.

The effective coupling for the inflaton to decay either via $H_u H_d$ or LH_u is given by:

$$g = g \left(1 + \frac{\langle S \rangle}{M_*} + \dots \right), \quad (461)$$

where S is the VEV of the field which couples to either H_u , H_d , L , then the inflaton decay will generate a quasi-thermal bath. The initial decay width of the inflaton will generate a plasma with a temperature $T \propto \Gamma_d^{1/2} \propto g$. Therefore, fluctuations in temperature induced by $\langle S \rangle \sim H_{inf}/2\pi$, would lead to

$$\frac{\delta T}{T} \sim \frac{H_{inf}}{2\pi S} \Big|_{decay}, \quad \text{or} \quad \frac{\delta T}{T} \sim \frac{H_{inf}}{2\pi M_*} \Big|_{decay}, \quad (462)$$

depending on whether the inflaton decays via non-renormalizable or renormalizable operators. In order to match the seed perturbations for CMB, $\delta T/T \sim 10^{-5}$, either the VEV of S or the scale of new physics should be around $10^5 H_{inf}$.

VIII. STRING THEORY MODELS OF INFLATION

One of the best motivated framework of quantum gravity is the string theory. Therefore it is natural to seek whether string theory can shed some light on inflation. There are many reviews dedicated to stringy inflation [8, 45–53]. Since, there are many models of inflation with large VEVs close to the Planck scale, which are particularly sensitive to the UV properties of the field theory, it is possible that string theory can provide some insight into the shape and stability of the potential. String theory also involves many degrees of freedom with equally large number of *physical* solutions, which makes it vulnerable when it comes to making concrete predictions, such as selecting the right vacuum at low energies. It

is hoped that cosmological observations along with particle phenomenology beyond the SM would help us constructing inflationary models ¹⁰⁵.

There are important avenues where string theory can actually help us in our understanding the inflation dynamics and various cosmological consequences.

- UV completion:

The UV completion of an inflationary potential is related to the fact that whether the potential can be kept flat enough during the interval of slow-roll inflation or not at VEVs close to the cut-off of the theory. For instance, the slow-roll parameters, i.e. $\epsilon \leq 1, \eta \leq 1$, ought to be maintained during sufficiently large e-foldings of slow-roll inflation. One particular example is the realization of chaotic inflation [4] in string theory, but below the Planckian VEV, with the help of *assisted inflation* [152]. In string theory it is possible to realize N number of string axions arising from partners of Kähler moduli, which can collectively drive inflation below the $4d$ Planck scale [280]. These axions have shift symmetry from $10d$ gauge invariance, which is broken non-perturbatively, and avoids the SUGRA η problem, thus keeping the potential sufficiently flat enough for the success of inflation [282, 546].

- Initial conditions:

String theory provides multiple vacua with all possible values of the cosmological constant [55, 57, 846, 847], for reviews, see [44, 56]. There has been a new revelation in string theory after the advent of a KKLT scenario [54], where the moduli are stabilized in an anti de Sitter (AdS) space with a negative cosmological constant. The initial configuration is stabilized with the help of various flux vacua [42]. Various non-perturbative effects such as gaugino condensation and/or warped brane instantons lift the vacuum from AdS to de Sitter (dS) with a meta stable minimum with a life time

¹⁰⁵ With the discovery of branes [837, 838], string theory has influenced string phenomenology and cosmology in a radical way, see [46, 51]. As a consequence, not all interactions see the same number of space-time dimensions. For instance, gravitons are allowed to propagate in the entire space-time dimensions, while the gauge fields including photons are localized on the brane position where the open strings must end [837, 838]. This has lead to a number of paradigms such as; **a low string scale**: the string scale could be as low as TeV, the large extra dimensions could be as large as the micron scale [524, 839–841], or at intermediate scales [842], and **cosmological constant problem**: a possible solution emerges where it is possible to decouple $4d$ vacuum energy from the $4d$ curvature [843–845].

greater than the present universe. These numerous vacua eliminate the problem of initial conditions for inflation [57, 848, 849]. Our patch of the universe could emerge from one such initial vacuum with a large cosmological constant. It is also quite plausible that string theory would provide some hints on the nature of trans-Planckian problem [286–290].

- Low primordial tensor to scalar ratio:

One of the bold predictions of string theory models of inflation is that the tensor to scalar ratio will be generically small, i.e. $r < 0.001$. In order to obtain large tensor to scalar ratio, i.e. $r \sim 0.1$, one requires field values to be in the range, where $\Delta\phi \gg M_{\text{P}}$ [206]. In a stringy setup the scale of inflation is *always* below the 4d Planck scale, M_{P} . Note that the *chaotic type inflation* is now driven with the help of *assisted inflation* [282, 546], where the largest tensor to scalar ratio could be detectable by the future experiments if $r < 0.13$. Similar arguments hold for brane inflation models, where the brane-anti-brane separation acts as an inflaton, but the tensor to scalar ratio remains quite small [850, 851].

- Cosmic (super)strings, localized gravity waves:

In brane-anti-brane case, inflation ends via annihilation of the branes. This process is likely to generate cosmic strings [451–453, 852], see Sec. III G 2. However their longevity is a model dependent issue [452]. Brane-anti-brane annihilation would also lead to exciting gravity waves on sub-Hubble scales with a peak frequency governed by the string scale and a distinct sharp spectrum [709]. If the string scale is close to TeV, then there is a possibility of detecting them in future gravitational wave observatories.

Stringy models of inflation also bring various challenges, whose roots are tied to the origin of particle phenomenology. One of the issues is pertaining to exciting the SM degrees of freedom, baryons and cold dark matter. Reheating and thermalization of the SM degrees of freedom in stringy models of inflation are poorly understood [853–857].

- Embedding MSSM in a stringy setup:

There are tremendous progress in embedding MSSM in a stringy setup, with the help of intersecting branes in Type IIA/B theories. Typically these theories have quantized fluxes in presence of sources which lead to a warped geometry. Embedding MSSM in

a realistic warped geometry is not well understood yet. Most of the constructions are done in a simple background geometry [858–864]. Furthermore, besides the SM gauge group there are extra $U(1)$'s which appear in the spectrum, which can be advantageous for cosmology, such as a natural embedding for neutrino masses, leptogenesis, for a review see [51], or can bring numerous uncertainties in thermal history of the universe depending on what scales they are broken.

- Inflationary sector:

It has been proven hard to embed inflationary sector within an observational sector, in most of the examples inflation and (MS)SM sectors are two distinct sectors. For all practical purposes inflaton remains in a hidden sector, whose couplings to the SM world remains unknown a priori. In a warped geometry, with multi-throats it is not clear why only the SM or MSSM throat will be dominantly excited after the end of inflation, as required for the success of BBN. Furthermore, it has been argued that during inflation the effect of back reaction would alter the compactified geometry and all the throats below the inflationary scale [854].

- Graceful exit from a string landscape:

In a string landscape scenario, if the universe were to tunnel out of the false vacuum, then the universe would be devoid of any entropy as the nucleated bubble would keep expanding forever with a negative spatial curvature [55, 279, 847]. Such a universe would have no place in a real world. Therefore, it is important that a bubble with an MSSM like vacuum must undergo the last phase of slow-roll inflation, in order to gracefully exit the string landscape. Inflationary epoch should explain the observed temperature anisotropy and also the right degrees of freedom upon reheating [279].

A. Moduli driven inflation

Moduli are abundantly in large numbers in $4d$ effective theory, which can be used to construct inflation models. The potential along the moduli remains massless in a SUSY limit. However, SUSY breaking, non-renormalizable superpotential terms, along with non-perturbative effects lift their flatness. These corrections are important to compute in order to keep the inflationary potential sufficiently flat. Some of these corrections are understood

recently in the context of a KKLT scenario [54], which helps understanding the stabilization of all the complex structure moduli with the help of flux compactification [42], leaving a single volume (Kähler) modulus which can be stabilized via non-perturbative effects.

1. Basic setup

The basic setup for any string theory is the gravitational action in $10d$, which can be reduced to an effective $4d$ with a metric ansatz:

$$ds^2 = e^{2A(x^m)} \eta_{\mu\nu} dx^\mu dx^\nu + g_{mn} dx^m dx^n, \quad (463)$$

where μ, ν runs for the non-compact space-time, $4d$, while m, n runs for the compact $6d$. Demanding that in $4d$ we arrive at $N = 1$ SUSY, g_{mn} is required to be *Calabi Yau* manifold [39–41], a Ricci-flat metric with $SU(3)$ holonomy. The compactification scale of $6d$ is given by; $M_c \ll M_s$, where M_c is the compactification scale and M_s is the string scale. The gravitational part of the action is given by:

$$S = \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-g_{10}} e^{-2\phi} R_{10} = \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-g_4} e^{2A} e^{6u} e^{-2\phi} R_4 \quad (464)$$

where e^{6u} is the six dimensional volume of the compact metric and where $2\kappa_{10}^2 = (2\pi)^7 \alpha'^4$ [41]). Eq. (464) does not give the usual Einstein-Hilbert term for $g_{\mu\nu}$; therefore, a following transformation; $g_{\mu\nu} = e^{6u} e^{-2\phi} g_{\mu\nu}$, leads to decouple the dilaton and the overall volume of the compactification. The string coupling is determined by $g_s = e^\phi$. Then the effective action is the usual

$$S = \frac{1}{2\kappa_4^2} \int d^4x \sqrt{-g_E} R_E, \quad \frac{1}{\kappa_4^2} = \frac{V_6}{\kappa_{10}^2}, \quad V_6 = \int d^6x \sqrt{\det g_{mn}} e^{2A} e^{-6u}. \quad (465)$$

Note that now there can be a large hierarchy between $10d$ and $4d$ Planck constants, due to a large warp factor, $\propto e^A$, see [865–868].

For the rest of the discussion, an important ingredient will be the $10d$ type IIB SUGRA, which arises as a low energy limit of type IIB string theory. The fields of IIB SUGRA are the metric, a complex scalar, two 3-form field strengths, and a self-dual 5-form field strength. The dilaton-axion scalar, $\tau = C_0 + ie^{-\phi}$, combines the RR scalar with the string coupling. It is convenient to combine the R-R ($F_3 = dC_2$) and NS-NS ($H_3 = dB_2$) 3-forms into a single

complex field, $F_3 = F_3 - CH_3$. The 5-form, $\tilde{F}_5 = dC_4 - C_2H_3$, is neutral under self-duality, $\star\tilde{F}_5 = \tilde{F}_5$, which is imposed as an equation of motion [41], for a review see [43, 868].

There are other non-perturbative stringy objects such as D branes, a Dp brane is an extended hypersurface over p flat spatial dimensions in $p + 1$ dimensional space-time, where open strings are free to move [837, 838]. Open strings do not propagate in the bulk, while the closed strings do. In type IIB, the allowed number of p -branes are $p = 1, 3, 5, 7, 9$. The two D -branes interact by exchanging gravitons, dilatons and antisymmetric tensors, provided their separation is large compared to the string scale.

In a compact manifold the D -branes can modify the dynamics. They do so via gravitational field they create, which gives rise to the warping of the metric as shown in Eq. (463). The presence of anti-symmetric tensor fields, for which the branes act as sources also lead to quantization of three-form fluxes similar to the argument for monopoles [42], i.e.

$$\frac{1}{2\pi\alpha'} \int_c H_3 \propto n_1, \quad \frac{1}{2\pi\alpha'} \int_c F_3 \propto n_2, \quad (466)$$

where C is a 3-cycle, n_1, n_2 are integers. The presence of fluxes induces scalar potential to the moduli, which can be used for fixing them. In this respect fluxes can break some or all the residual SUSY in $4d$. Finally the branes can also wrap around a non-contractable surface, in particularly $D7$ branes, which fill 7 spatial dimensions also extends into 4 of the compact 6 dimensions can wrap around 4-cycle in the internal dimensions. The brane tension provides potential for the moduli, depending on how many times it wraps. There are also negative tension branes, known as orientifold planes, i.e. $O7$, required to cancel to total charge in the internal dimensions created by the warping of $D7$ branes. All these contributions lead to fix all the moduli, which are known as complex structure moduli, within type IIB compactifications [42].

2. *KKLT scenario*

In KKLT, flux compactification lead to fixing all the moduli with a constant superpotential term, W_0 , while the volume modulus, T , is assumed to be fixed by the non-perturbative superpotential term in $4d$ [54]:

$$W(T) = W_0 + Ae^{-aT}, \quad K = -3\ln(T + \bar{T}). \quad (467)$$

where $a = 2\pi/N$, A are constants, K is the Kähler potential, which is assumed to be given at the tree level arising from the compactification volume, $K = -2 \ln(M_s^6 V_6)$, where $M_s^6 V_6 \propto (T + \bar{T})^{3/2}$. The origin of non-perturbative superpotential term arises due to instanton contribution or the presence of $D7$ brane wrapping certain cycles of the internal dimensions carrying asymptotically free, $SU(N)$, gauge group. Then the gaugino condensation along such a gauge group will induce a non-perturbative potential [869, 870]. Note that usually the Kähler potential also obtains non-perturbative corrections [621–623], however, for a choice of W_0 it is possible to neglect them ¹⁰⁶. With the help of the above superpotential and the Kähler potential, the resulting minimum is found to be the SUSY one (for a pedagogical discussion, see [46]):

$$D_T W(T_m) = -aAe^{aT_m} - \frac{3(W_0 + Ae^{-aT_m})}{T + \bar{T}_m} = 0, \quad (468)$$

$$V(T_m, \bar{T}_m) = -\frac{3(W_0 + Ae^{T_m})^2}{(T_m + \bar{T}_m)^3} = -\frac{(aAe^{-aT_m})^2}{3(T_m + \bar{T}_m)} < 0 \quad (469)$$

where T_m denotes the minimum. Note that the potential is adS in the minimum, and requires uplifting to get a dS universe. As suggested by KKLT, this can be achieved by adding SUSY breaking contributions, such as anti-D3 brane ($\overline{D3}$), although breaks explicitly, its effect can be made small by the choice of brane tension, or in other words placing the $\overline{D3}$ brane in a warped geometry [54], whose contribution turns out to be:

$$V_{\overline{D3}} = \frac{C}{(T + \bar{T})^2}, \quad (470)$$

where $C > 0$, given in terms of the brane tension and the six dimensional volume. The addition of $\overline{D3}$ lifts the potential keeping the minimum almost intact, $T \simeq T_m$, for which either the total potential vanishes or become dS. For $|T| \rightarrow \infty$, potential $V \rightarrow 0$ leads to decompactification to $10d$. Thus, the potential is separated by the barrier which can keep the metastable minimum stable enough for the life time of the universe [54, 871].

¹⁰⁶ There are two kinds of string corrections, string loop corrections in powers of $g_s \sim e^\phi$ and $\alpha' \sim 1/M_s^2$ type corrections. It is known that non-renormalization theorem forbids holomorphic superpotential to obtain corrections of either type [620]. However, the Kähler potential is not similarly protected.

3. *N*-flation

Amongst various realizations of assisted inflation, *N*-flation is perhaps the most interesting one¹⁰⁷. The idea is to have $N \sim 300(M_{\text{P}}/f) \sim 10^4$ number of axions, where f is the axion decay constant, of order $f \sim 0.1M_{\text{P}}^{-1}$. These axions can drive inflation simultaneously with a leading order potential [282]:

$$V = V_0 + \sum_i \Lambda_i^4 \cos(\phi_i/f_i) + \dots \quad (471)$$

where ϕ_i correspond to the partners of Kähler moduli. The ellipses contain higher order contributions. The advantage of this potential is that there are *no* $H^2\phi_i^2$ type contributions at the lowest order provided there is a shift symmetry, therefore the SUGRA η problem can be evaded. The axions have shift symmetry, which are only broken at non-perturbative level.

In string theory case, the assumption behind the potential is following. In a compactified framework, it is assumed that all the moduli are heavy and thus stabilized by prior dynamics, including that of the volume modulus. Only the axions of $T_i = \phi_i/f_i + iM_s^2 R_i^2$ are light [282]. The assumption of decoupling the dynamics of Kähler modulus from the axions is still a debatable issue, see the discussion in [47]. These axions can then obtain an exponential superpotential, $W \sim \sum_i W_i e^{2\pi T_i}$, correction similar to the KKLT setup, in addition to a constant superpotential piece, W_0 . The shift symmetry is now protected by the choice of Kähler potential [47]

$$K = -\ln \left[i \frac{C_{ijk}}{3!} (T_i - \bar{T}_i)(T_j - \bar{T}_j)(T_k - \bar{T}_k) \right], \quad (472)$$

where C_{ijk} is the Calab-Yau intersection number. After rearranging the potential for the axions, $V \approx \sum V_i \approx \sum_i |\partial_i W|^2$, expanding them around their minima and for a canonical choice of the kinetic terms, the Lagrangian simplifies to the lowest order in expansion:

$$\mathcal{L} = \frac{1}{2} \partial_\mu \phi_i \partial^\mu \phi_j - \frac{1}{2} m_i^2 \phi_i^2 + \dots \quad (473)$$

¹⁰⁷ There are also realizations of assisted inflation via branes [547, 548] in a type IIB setting, with a large number of Kaluza Klein modes [280], exponential potentials arising from a generic string compactification [537], and in a M-theory setting with the help of large number of NS-5 branes in [549], etc.

The exact calculation of m_i is hard, assuming all of the mass terms to be the same $m_i \sim 10^{13}$ GeV, and $N > (M_{\text{P}}/f)^2$, it is possible to match the current observations with a tilt in the spectrum, $n \sim 0.97$, and *large* tensor to scalar ratio: $r \sim 8/N_Q \sim 0.13$ for $N_Q \sim 60$.

4. Inflation due to Kähler modulus

Various realizations of modular inflation have been studied in the context of volume modulus, out of which the racetrack models [872–874], and the Kähler moduli inflation are the most popular ones [875, 876]. In the simplest version of racetrack inflation, the Kähler and the superpotential are given by:

$$K = -3 \ln(T + \bar{T}), \quad W = W_0 + Ae^{-aT} + Be^{-bT}, \quad (474)$$

where A, B, a, b are calculable constant, and T is a complex Kähler modulus. Similar to the KKLT scenario, it is also assumed that there is an uplifting of the minimum of the Kähler potential by $\overline{D3}$ brane, see Eq. (470). The potential is spanned in two real scalar directions with a complicated profile with many dS minima. For a certain choice of fine tuned parameters, it is possible to obtain sufficient slow-roll inflation near the saddle point spanned in two real directions. The model can produce the spectral tilt $n_s \approx 0.95$.

A better racetrack model has been constructed with the help of two Kähler moduli [877], the Kähler and the superpotential are given by [873]:

$$K = -2 \ln \left[\frac{1}{36} \left((T_2 + \bar{T}_2)^{3/2} - (T_1 + \bar{T}_1)^{3/2} \right) \right], \quad (475)$$

$$W = W_0 + Ae^{-aT_1} + Be^{-bT_2}, \quad (476)$$

where there are 4 real scalars involved, i.e. $T_{1,2} = X_{1,2} + iY_{1,2}$. The uplifting of the potential from adS to dS is given by: $\delta V \sim D/(X_2^{3/2} - X_1^{3/2})^2$.

In all these models the choice of W_0 is such that the corrections of the Kähler potential can be neglected. This is due to the fact that W_0 is chosen small in order to explain the current cosmological constant. However W_0 need not be small and in which case α' corrections to kähler potential cannot be neglected [875, 876]. One such toy model with three moduli were constructed where the Kähler and superpotential are given by:

$$K = -2 \ln \left[(T_1 + \bar{T}_1)^{3/2} - k_2(T_2 + \bar{T}_2)^{3/2} - k_3(T_3 + \bar{T}_3)^{3/2} + \frac{\xi}{2} \right], \quad (477)$$

$$W = W_0 + \sum_{i=1}^3 A_i e^{a_i T_i}, \quad (478)$$

where A_i, a_i are constants. Of course, now there are more parameters and the full potential is hard to analyze, however, it is possible to imagine that only one of the modulus is driving inflation, say T_3 , and rest of them are decoupled from the dynamics. The potential along the lightest modulus is then given by:

$$V \approx V_0 - C(T_3 + \bar{T}_3)^{4/3} \exp[-c(T_3 + \bar{T}_3)^{4/3}] , \quad (479)$$

for some positive constants c and C . In all these class of models the spectral tilt comes out to be close to the observed limit, $n_s \sim 0.96$, with no significant gravity waves and no cosmic string strings are produced aftermath of inflation [875, 876].

B. Brane inflation

The position of various branes along with their motion can lead to inflation. Let us imagine that there is a gas of Dp branes in $p + 1$ space-time dimensions. Their stress energy tensor leads to an equation of state [878]

$$p_p = \left[\frac{p+1}{d} v^2 - \frac{p}{d} \right] \rho_d , \quad (480)$$

where p_p is the pressure and ρ_p is the energy density of the gas of p branes. In a relativistic limit, $v^2 \rightarrow 1$, a gas of branes behaves as a relativistic fluid with an equation of state, $w \equiv p_p/\rho_p = 1/d$, while in the non-relativistic case, $v^2 \rightarrow 0$, we obtain $w = -p/d$, a negative pressure which could lead to a power law inflation with a scale factor $a \propto t^{2/(d-p)}$ for $d = p+1$. This would inflate the entire $p + 1$ dimensional bulk. A stringy realization would require that the dilaton be fixed at all times. A bound state of fractionally charged branes in $10d$ universe can also lead to a high entropy state, with an initial correlation length larger than the string scale, as discussed in [879]. These scenarios are helpful in setting up the initial conditions for the universe. First of all they provide a homogeneous Hubble patch with a large causal horizon (bigger than the string scale), where subsequent phases of inflation can take place.

Motion of much fewer branes can also lead to inflation, first realized in [50, 547, 630, 648, 652–654, 662, 663, 880–885], for a review see [48]. Consider a system of $Dp - \overline{Dp}$ branes, where they interact via closed string exchanges between the branes, i.e. the attractive gravitational (NS-NS), and the massive (R-R) interactions, see [838], yields,

$V(y) \approx -\kappa_{10}^2 T_p^2 \Gamma((7-p)/2) (1/\pi^{(9-p)/2} y^{7-p})$, where $T_p = (2\pi\alpha')^{(p+1)/2}$ is the Dp brane tension, and y is the inter-brane separation. For $p < 7$, the potential vanishes for large y . At very short distances close to the string scale, there develops a tachyon in the spectrum, $\alpha' m_{tachyon}^2 = (y^2/4\pi^2\alpha') - 1/2$, which leads to annihilation of the branes, and a graceful end of branes driven inflation.

In a more realistic scenario, the branes have to be placed in a warped geometry. As a consequence of flux compactification, any mass scale, M , in the bulk becomes $h_A M$, where $h_A \ll 1$, near the bottom of the warped throat. Thus the warping affects the overall normalization of the potential. It is assumed that a $D3$ brane is slowly falling into the attractive potential of an $\overline{D3}$ brane placed at the bottom of the throat. The sum total potential for a $D3 - \overline{D3}$ brane potential is given by [48]:

$$V(\phi) = \frac{1}{2}\beta H^2 \phi^2 + 2T_3 h_A^4 \left(1 - \frac{1}{N_A} \frac{\phi_A^4}{\phi^4}\right) + \dots, \quad (481)$$

where $\phi = \sqrt{T_3} y$, the value of warping depends on the throat geometry, typically $h_A \sim 10^{-2}$, $N_A \gg 1$ is the $D3$ charge on the throat, and $\beta \sim \mathcal{O}(1)$ arises due to the kähler potential, which obtains contributions from the brane positions. The first term is reminiscent to the SUGRA η problem, which plagues the brane inflation paradigm in general. A successful inflation would require $\beta \ll 1$, the inflationary predictions are very similar to that of the hybrid model of inflation.

There are some drawbacks of this scenario, the flatness of the potential is hard to obtain naturally, one can try to modify the situation with dual formulation where instead of brane separation, one uses branes at angles [881], assisted inflation [547, 548], or $D3$ brane falling towards $D7$ branes [648, 662, 663, 882]. The issue of initial condition is crucial for the brane inflation scenario to work, the position of a $D3$ brane has to be away from the bottom of the throat, but there exists no stringy mechanism to do so. In a recent study [886–888] an argument has been provided where it is possible to realize a slow-roll motion for a $D3$ brane where $D7$ brane is also extended in the bottom of the throat. In all these examples inflation happens near the *point of inflection*, which was first studied in the context of MSSM inflation [86, 88, 89].

Another variant of brane inflation has been discussed in the context of DBI (Dirac Born-Infeld) action [234, 889], where a $D3$ brane rolls fast with almost a relativistic velocity, $v = 16/27$, for a particular case of KS-throat [890]. Inside a warped throat, the $4d$ metric is

given by:

$$ds^2 = h^2(y)(-dt^2 + a(t)^2 dx^2) + h^{-2}(y)g_{mn}(y)dy^m dy^n, \quad (482)$$

The DBI action for a $D3$ brane is given by the brane position, $\phi(t)$, from the bottom of the throat:

$$S = - \int d^4x a^3(t) \left[T(\phi) \sqrt{1 - \frac{\dot{\phi}^2}{T(\phi)}} + V(\phi) - T(\phi) \right], \quad (483)$$

where $T(\phi) = T_3 h^4(\phi)$ and $h(\phi)$ is the warp factor depending on the brane position. For a slow-roll inflation, the action yields the standard kinetic term $\approx \dot{\phi}^2/2$. The potential is given by a phenomenological mass term, and the coulomb potential between $D3$ and $\overline{D3}$ brane:

$$V(\phi) \approx \frac{m^2}{2} \phi^2 + V_0 \left(1 - \frac{V_0}{4\pi^2 v} \frac{1}{\phi^4} \right), \quad (484)$$

where $V_0 = 2T_3 h_A^4$ is the brane tension at the bottom of the throat, where $h_A \sim 0.2 - 10^{-3}$. The DBI inflation is quite similar to the K-inflation picture [150], where inflation is driven by a non-canonical kinetic term, a simple analogy can also be made with a fluid dynamical picture, where an equation of state can be determined via:

$$p(\phi, \mathcal{K}) = -T_3 h^4(\phi) \sqrt{1 - 2\mathcal{K}/h(\phi)^4} + T_3 h^4(\phi) - V(\phi), \quad \rho(\phi, \mathcal{K}) = 2\mathcal{K}p_{,\mathcal{K}} - p, \quad (485)$$

where $\mathcal{K} = (\dot{\phi}^2/2T_3)$. The speed of sound is given by:

$$c_s^2 = \frac{p_{,\mathcal{K}}}{\rho_{,\mathcal{K}}} = \frac{p_{,\mathcal{K}}}{p_{,\mathcal{K}} + 2\mathcal{K}p_{,\mathcal{K}\mathcal{K}}} = 1 - 2\mathcal{K}/h^4 = \frac{1}{\Gamma^2}, \quad (486)$$

where Γ is a relativistic factor. Besides matching the amplitude of the perturbations and the scalar tilt, $n_s \sim 0.98$, there is a possibility of generating large non-Gaussianity towards the end of inflation. The value of f_{NL} is determined by the relativistic motion of the brane, when $\Gamma \gg 1$, $|f_{NL}| \approx 0.32\Gamma^2$. For $|f_{NL}| < 300$, gives $\Gamma \leq 32$ [234, 891]. The tensor to scalar ratio depends on the choice of initial conditions and it drops significantly as $\Gamma \gg 1$ [892].

C. Reheating and thermalization

In stringy models of inflation, reheating and thermalization of the SM or MSSM degrees of freedom are poorly understood. This is mainly due to the fact that inflation happens in a hidden sector as far the MSSM sector is concerned. One problem for all these models is

that they are bound to excite *possibly light non-SM* like degrees of freedom, which can pose several challenges for a successful BBN ¹⁰⁸.

There are also some observational virtues of stringy reheating. For instance, inflation driven by $D3 - \overline{D3}$ case has interesting consequences. Their annihilation leads to the production of $D1$ branes and fundamental $F1$ strings, which can be understood via tachyon condensation [893–896]. The tachyon couples to $U(1) \times U(1)$ gauge theory associated with each brane, as it develops a VEV it breaks the gauge group spontaneously, which results in formation of $D1$ strings via Kibble mechanism and a confining flux tubes which become the fundamental closed strings [897]. In Type IIB setup domain walls and monopoles are not excited, which correspond to $D0$ and $D2$ branes. The cosmic string tension, μ , can be within $10^{-13} < G\mu < 10^{-6}$ [453, 884]. Brane-anti-brane annihilation also excited gravity waves, whose peak frequency is determined by the string scale [709]. A string scale greater than TeV makes it impossible for such gravity waves to be detected in future.

Reheating in a warped geometry is a complicated process [854, 855, 898]. Especially, in a multiple throat picture, where inflation happens in one throat and the SM is in another, there are many possibilities to reheat not only the SM throat, but also the other throats.

- Reheating various non-(MS)SM throats:

First of all, by no means it is guaranteed that the (MS)SM throat will be the only recipient of the inflaton energy density, there are other throats with similar cosmological constant, which are also reheated simultaneously depending on the relative warping between the SM throat and the other throats.

- Stable KK modes and gravitons:

$D3 - \overline{D3}$ annihilation also excites massive KK modes and gravitons. The heavy KK modes can decay into the lightest KK mode and gravitons. The lightest KK mode can be absolutely stable at the bottom of the throat due to conserved angular momentum [855]. The self annihilation of these KK modes is gravitationally suppressed, $\sigma \sim (L/R)^6 (1/M_{\text{P}} h^2)$, where L is linear size of the $6d$ compactification volume and R is the size of the throat, h is the relative warping [855, 856], therefore once produced copiously during brane annihilation, these KK modes can overclose the universe.

¹⁰⁸ One possibility would be to dilute all of them via late inflation, and create the MSSM degrees of freedom afresh within an observable sector [279].

- Breakdown of an effective description of a (MS)SM throat:

Due to the hierarchy between inflation and (MS)SM throat, during inflation the (MS)SM throat will also be sensitive to inflation-induced vacuum fluctuations. The curvature scalar of the (MS)SM throat will obtain corrections of order, $R \sim 12H_{inf}^2 e^{-2A_{SM}}$, with $e^{-2A_{SM}} \sim (M_P/M_{SM})^2 \gg 1$ during inflation [854], which will render perturbative description of the (MS)SM throat ineffective. The (MS)SM throat will now be uplifted by warping given by $e^{A_{SM}} \sim H_{inf}/M_P$ [854]. Once inflation ends, the geometry of (MS)SM throat will start relaxing gradually before settling down to its original value. The process of relaxation could give rise to a violent particle production and excitations of open and closed strings [854].

- Reheating via tunneling:

The massive KK modes can decay into the (MS)SM throat via quantum tunneling [899, 900]. There is a large uncertainty in the tunneling rate, for a range of parameters given by the RR flux, $n_R \sim 10-100$, $6d$ compactified volume, $e^{4u}/g_s \sim 1-10^3$ and $g_s \sim 1/10$, the rate is roughly given by: $\Gamma_t \sim \mathcal{O}(10-10^{10})(H_{inf}/M_P)^{3/2}H_{inf}$ [854]. With the above tunneling rate the reheat temperature of the (MS)SM throat, $T_R \sim \sqrt{\Gamma_t M_P}$, will exceed the fundamental scale M_{SM} . This would lead to excitations of the KK modes and gravitons in the (MS)SM throat. Again the universe would be dominated a gas of non-relativistic KK particles.

In all the cases, the reheating temperature is very close to the string scale, $T_R \sim M_{SM}$, in the (MS)SM throat. This gives rise to thermal excitations of long open, and closed strings, a phase reminiscence to the Hagedorn phase [901]. Note, a very similar picture would unfold in any neighboring throats, (MS)SM throat is not a unique one.

D. String theory landscape and a graceful exit

The vacuum energy in a string theory landscape can be written as:

$$V = M_P^2 \Lambda = M_P^2 \Lambda_0 + \alpha^{-2} \sum_i c_i n_i^2, \quad (487)$$

where $c_i \lesssim 1$ are constants and n_i are flux quantum numbers (note that Λ has dimension mass² in our notation as it enters the Einstein equation as $\Lambda g_{\mu\nu}$).

All in all, string theory (from the landscape point of view) could have from 10^{500} to even 10^{1000} vacua [44, 55–57], with the vast majority of those having large cosmological constants. In addition, our knowledge of the distribution of gauge groups over the landscape suggests that one out of 10^{10} vacua have the SM spectrum, at least in simple models [58–60]. Even if this fraction is much smaller on the whole landscape, there are so many vacua that it seems likely very many will have a SM-like spectrum. From statistical arguments, most vacua should have badly broken SUSY, with large F -terms in the SUSY breaking sector.

In addition, there exist vacua with large cosmological constant that are “almost SUSY” in the sense of [56, 902]. These vacua have vanishing (or nearly vanishing) F -terms, and their cosmological constant and SUSY breaking are provided by a D -term, such as that created by an anti-brane. Upon decay of the D -term, the remaining vacuum has small Λ and low energy SUSY. The original model of [54] is almost SUSY in this sense because anti-D3-branes provide both the cosmological constant and SUSY breaking. There are also examples of toy “friendly landscape” models [903], in which the dynamics of N scalars create a landscape of vacua. The landscape can also harbor negative Λ [904]. Also, [848] has discussed the importance of negative Λ vacua in possibly separating parts of the landscape from each other.

It is expected that small jumps in Λ with small bubble tension τ to be the most common decays, which can in fact be quite rapid [279]. In Ref. [905] the authors have argued that resonance can also play an important role in tunneling across a landscape of many metastable vacua. Also, the fact that there are many possible decays, as emphasized by [906], the whole landscape, including the MSSM-like vacua, will be populated eventually almost independently of initial conditions.

One interesting way to exit the string landscape is through MSSM inflation. Note that, when MSSM inflation starts, the “bare” cosmological constant (that not associated with the MSSM inflaton) might still be considerably larger than the present value. This means that further instanton decays should take place to reduce the bare cosmological constant, and these decays should occur during MSSM inflation in order to percolate efficiently. Fortunately, MSSM inflation naturally includes a self-reproduction (eternal inflation) regime prior to slow-roll [86–88], see Sec. V E 3.

E. Other stringy paradigms

There are other interesting paradigms, such as string gas cosmology and bouncing cosmology, for a review see [907]. The basic setup relies on stringy ingredients, such as statistical properties of a gas of strings and branes, and some aspects of string field theory. Their primary aim is to explain the seed perturbations for CMB without invoking cosmic inflation, see also [135, 136, 908]. However, all these scenarios suffer from the same symptoms, they require the universe to be exponentially large from the very beginning [909].

1. *String gas cosmology*

In [910], Brandenberger and Vafa (BV) proposed a seemingly very natural initial condition for cosmology in string theory. In BV cosmology, all nine spatial dimensions are compact (and toroidal in the simplest case) and initially at the string radius. The matter content of the universe is provided by a Hagedorn temperature gas of strings. In addition to proposing a very interesting initial condition and analyzing the thermodynamics of string at that point, however, BV argued that string theory in such a background provides a natural mechanism for decompactifying up to three spatial dimensions (that is, allowing three spatial dimensions to become macroscopic). The BV mechanism works because winding strings provide a negative pressure, which causes contraction of the scale factor, as was shown explicitly in [911, 912]. BV then gives a classical argument that long winding strings can only cross each other in three or fewer large spatial dimensions. Therefore, since winding strings freeze out quickly in four or more large spatial dimensions, the winding strings would cause re-collapse of those large dimensions.

The initial paper, [910], has inspired a broad literature. One important generalization has been including branes in the gas of strings [913–916], and other space time topologies have also been considered [917, 918]. In particular, [919] showed that interesting cosmological dynamics happen when the expanding dimensions are still near the string scale.

Importantly, several tests have been made of the BV mechanism for determining the number of macroscopic dimensionality of space, see [920–924]. These tests are all based on the fact that the winding strings will be unable to annihilate efficiently if their interaction rate, Γ , drops below the Hubble parameter for the expanding dimensions. Based on simple

arguments from the low energy equations of motion and string thermodynamics it was demonstrated that the interaction rates of strings are negligible, so the common assumption of thermal equilibrium cannot be applicable [924, 925].

2. Seed perturbations from a string gas

Recently, a new structure formation scenario has been put forward in [139, 926]. It was shown that string thermodynamic fluctuations in a quasi-static primordial Hagedorn phase in $4d$, during which the temperature hovers near its limiting value, namely the Hagedorn temperature, T_H [901], can lead to a scale-invariant spectrum of metric fluctuations. The crucial point of the mechanism is to note that provided three large spatial dimensions are compact, the heat capacity C_V of a gas of strings in thermodynamical equilibrium scales as r^2 with the radius of the box [927–932], then the heat capacity determines the root mean square mass fluctuations, i.e. $\langle(\delta M)^2\rangle = T^2 C_V$. With the help of Poisson equation, $\nabla^2 \Phi = 4\pi G \delta \rho$, and the definition of power spectrum, $\mathcal{P}_\Phi(k) \equiv k^3 |\Phi(k)|^2$, one obtains [139, 926]:

$$\begin{aligned} \mathcal{P}_\Phi(k) &= 16\pi^2 G^2 k^{-1} |\delta\rho(k)|^2 = 16\pi^2 G^2 k^2 (\delta M)^2(r(k)), \\ &\approx 1920\pi^2 c^{-1} G^2 T_H^4 (kr)^2 \frac{T}{T_H(1 - T/T_H)}, \end{aligned} \quad (488)$$

where c is the velocity of light, G is the Newton's constant and T_H is the Hagedorn temperature. The mean squared mass fluctuation $|\delta M|^2(r)$ in a sphere of radius $r(k) = k^{-1}$ is given by $|\delta\rho(k)|^2 = k^3 |\delta M|^2(r(k))$. The tilt in the spectrum is scale invariant and the fine tuning in temperature has to be $\Delta T/T_H \sim 10^{-30}$ for $M_S \sim 10^{-10} M_P$. The tensor mode also requires similar level of fine tuning but with slight tendency towards a blue tilt, which could be a distinguishing feature of this setup [139, 926].

To obtain this result, several criteria for the background cosmology need to be satisfied. First of all, the background equations must indeed admit a quasi-static (loitering) solution. Next, our three large spatial dimensions must be compact. It is under this condition that [927, 928] the heat capacity C_V as a function of radius r scales as r^2 . Thirdly, thermal equilibrium must be present over a scale larger than 1mm during the stage of the early universe when the fluctuations are generated. Since the scale of thermal equilibrium is bounded from above by the Hubble radius, it follows that in order to have thermal equilibrium on the required scale, the background cosmology should have a quasi-static phase. Finally,

the dilaton velocity needs to be negligible during the time interval when fluctuations are generated [140, 933]¹⁰⁹.

3. Example of a non-singular bouncing cosmology

In string theory, higher-derivative corrections to the Einstein-Hilbert action appear already classically (i.e. at the tree level), but we do not preclude theories where such corrections (or strings themselves) appear at the loop level or even non perturbatively. From string field theory [935–941] (either light-cone or covariant) the form of the higher-derivative modifications can be seen to be Gaussian, i.e. there are e^{\square} factors appearing in all vertices (i.e. $(e^{\square}\phi)^3$). These modifications can be moved to kinetic terms by field redefinitions ($\phi \rightarrow e^{-\square}\phi$). The non perturbative gravity actions that we consider here will be inspired by such stringy kinetic terms [942]¹¹⁰.

It was realized that if we wish to have both a ghost free and an asymptotically free theory of gravity¹¹¹, one has little choice but to look into gravity actions that are non-polynomial in derivatives, such as the ones suggested by string theory [942]:

$$S = \int d^4x \sqrt{-g} F(R), \quad (489)$$

where

$$F(R) = R + \sum_{n=0}^{\infty} \frac{c_n}{M_*^{2n}} R \square^n R, \quad (490)$$

and M_* is the scale at which non-perturbative physics becomes important. c_n 's are typically

¹⁰⁹ It is not easy to satisfy all of the conditions required for the mechanism proposed in [139, 926] to work. In the context of a dilaton gravity background, the dynamics of the dilaton is important. If the dilaton has not obtained a large mass and a fixed VEV at a high scale, then it will be rolling towards weak coupling at early times. This will lead to [911, 934] a phase in which the string frame metric is static, and thus the string frame Hubble radius will tend to infinity, i.e. $H \approx 0$.

¹¹⁰ There are various discussions on singular bouncing cosmology in $4d$ in the context of ‘ek-pyrotic’ and cyclic universe [137, 138, 943, 944]. These models are interesting in their own right. However, constructing and stabilizing a background with a singular bouncing cosmology is a non-trivial task, see for some related discussions [945–950].

¹¹¹ While perturbative unitarity requires the theory to be ghost free, in order to be able to address the singularity problem in General Relativity, it may be desirable to make gravity weak at short distances, perhaps even asymptotically free [951].

assumed to be $\sim \mathcal{O}(1)$ coefficients. It is convenient to define a function,

$$\Gamma(\lambda^2) \equiv \left(1 - 6 \sum_0^\infty c_i \left[\frac{\lambda}{M_*} \right]^{2(i+1)} \right). \quad (491)$$

One can roughly think of $p^2\Gamma(-p^2)$ as the modified inverse propagator for gravity (see [942] for details). It was shown that if $\Gamma(\lambda^2)$ does not have any zeroes, then the action is ghost-free, thus free from any classical instabilities, i.e. Ostrogradski instabilities (see [952] for a review).

For homogeneous and isotropic cosmologies, where $a(t)$ is the scale factor, it is sufficient to look at the analogue of the Hubble equation for the modified action (489,490). Just as in ordinary Einstein gravity, here also the Bianchi identities (conservation equation) ensure that for FRW metrics, the field equation satisfies [942, 953]

$$\tilde{G}_{00} = F_0 R_{00} + \frac{F}{2} - F_{0;00} - \square F_0 - 2 \sum_{n=1}^\infty F_n \square^n R - \frac{3}{2} \sum_{n=1}^\infty \dot{F}_n (\square^{n-1} R) = T_{00}, \quad (492)$$

where we have defined

$$F_m \equiv \sum_{n=m}^\infty \square^{n-m} \frac{F}{\square^n R}. \quad (493)$$

It was shown in [942] that Eq. (492) admits exact bouncing solutions of the form

$$a(t) = \cosh \left[\frac{\Lambda t}{\sqrt{2}} \right]. \quad (494)$$

in the presence of radiative matter sources and a non-zero cosmological constant.

A non-singular bouncing cosmology can also lead to a *cyclic inflation* [148, 954], where a *negative cosmological constant* plays a dominant role. The evolution gives rise to every cycle undergoing inflation on average [148]:

$$\langle H \rangle \equiv \frac{\int H dt}{\int dt} = \frac{1}{\tau_n} \ln \left(\frac{a_{n+1}}{a_n} \right) \equiv \frac{\mathcal{N}_n}{\tau_n}, \quad (495)$$

where τ_n is the time period of the n th cycle. Imagining $\tau_n \approx \tau$, every cycle leads to moderate inflation with the scale factor increasing with every cycle; $a_{n+1}/a_n \approx \exp \mathcal{N}$. On average the Hubble expansion rate remains constant over many cycles. The exit of inflation happens when the universe leaves the negative cosmological constant to the positive cosmological constant via a dynamical scalar potential [148].

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