

Is it time to go beyond Λ CDM universe?

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Concordance Λ CDM universe is the simplest model that is consistent with a large variety of cosmological observations till date. But few recent observations indicate inconsistencies in Λ CDM model. In this paper, we consider the combination of recent SnIa+Bao+Cmb+Growth+ $H(z)$ + H_0 measurements to revisit the constraints on the dark energy evolution using the widely studied CPL parametrisation for the dark energy equation of state. Although the reconstructed behaviour for the dark energy equation of state confirms the inconsistency of Λ CDM at 95% confidence level, the reconstructed Om diagnostic which is a *null test* for Λ CDM, still allows the concordance Λ CDM behaviour with a lower range of Ω_{m0} than that obtained by Planck-2015. *This confirms that Λ CDM is still the best choice for the dark energy model.* We also measure the parameter $S = \sigma_8 \sqrt{\Omega_{m0}/0.3} = 0.728 \pm 0.023$ which is consistent with its recent measurement by KiDS survey. The confidence contour in the $\Omega_{m0} - \sigma_8$ parameter plane is also fully consistent with KiDS survey measurement.

For nearly two decades, the most pressing problem in cosmology is to explain the late time acceleration of the universe. Almost all the present and future cosmology missions are dedicated to address this issue. After the Planck-2015 data [1] for anisotropy in the cosmic microwave background radiation (CMBR) together with other cosmological data from Supernova type-Ia (SNIa) [2], Baryon Acoustic Oscillations (BAO) observation [3] in the large scale structures in the universe as well as the HST measurement of the Hubble parameter [4], the concordance Λ CDM universe is shown to be consistent with this combined datasets. Being the simplest model in explaining the late time acceleration, this makes

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Λ CDM universe the clear winner among various dark energy models, although the theoretical issues such as the fine tuning and cosmic coincidence problems are still far from being solved and will keep the theoreticians busy in near future (see [5] for nice review on dark energy).

Despite the success of Λ CDM universe to explain a large variety of cosmological observations, there are recent evidences to contradict this success [6]. Recently the latest model independent measurement of H_0 (hereafter Riess16) [7], has more than 3σ deviation from the Planck-2015 measurement of the same for a Λ CDM universe. More recently, the KiDS survey [8] has found a discrepancy in growth measurement at the level of 2.5σ compared to the measurement by Planck-2015 for Λ CDM model. Just recently, Valentini et al [9] have shown that the Λ CDM model is inconsistent at 95% confidence level with the Planck-2015 + Riess16 dataset. This clearly motivates people to revisit the constraint on dark energy behaviour.

Going beyond the Λ CDM universe where the dark energy density is constant throughout the evolution of the universe, there are several approaches to model the dark energy evolution. One can assume simply a constant negative equation of state for the dark energy (like those arise from a network of strings or domain walls [10]). The other choice is to consider a minimally coupled canonical scalar field slowly rolling over a sufficiently flat potential to mimic a negative equation of state. This is similar to the inflaton field in the early universe with the difference that in this case the scalar field evolves at a much lower energy scales. In literature these are called *quintessence* field [11]. Subsequently Caldwell and Linder [12] showed that such quintessence fields can be further divided into categories called *freezing* and *thawing* quintessence. The freezing model has an initial fast roll phase where they mimic the background radiation or matter behaviour; later on, the equation of state decreases and asymptotically approaches $w = -1$ to initiate the late time acceleration. For the thawing models, the field is initially frozen at the flat part of the potential due to large Hubble friction and behaves like $w = -1$ cosmological constant; subsequently the Hubble friction decreases and the field starts rolling and the equation of state starts increasing with time. Generalization of these models to non-canonical [13], non-minimal [14] as well as phantom ($w < -1$) [15] cases have also been studied extensively.

On the other hand, given the large number models in the literature, it is rather difficult to confront all individual models to the observational data. The more economical way is to construct parametrization of the dark energy equation of state w as a function of redshift or

scale factor containing a minimal set of parameters that describes a wide set of dark energy models and then confront such parametrization to the observational data. One such widely used parametrization is the Chevallier-Polarski-Linder (CPL) parametrization [16] which was been widely used by all the recent cosmological observations including Planck-2015 to put constrain on the cosmological parameters.

In this paper, we use this CPL parametrization to constrain the dark energy behaviour using the current observations from CMBR, SNIa, BAO, the growth of the matter fluctuations and the measurement of the Hubble parameter at different redshifts including Riess16 data.

To start with, we first describe the CPL parametrization for the dark energy equation of state which has the following form:

$$w(a) = w_0 + w_a(1 - a) = w_0 + w_a \frac{z}{1 + z}, \quad (1)$$

where w_0 and w_a are the two parameters of the model describing the equation of state at present ($a = 1$) and the variation of the equation of state at present respectively. From the infinite past ($a = 0$) till the present ($a = 1$), the equation of state varies between $w_0 + w_a$ and w_0 . Using this form for the equation of state into the energy conservation equation for the dark energy, one can easily get the variation of the dark energy density as

$$\rho_{de} \propto a^{-3(1+w_0+w_a)} e^{-3w_a(1-a)}. \quad (2)$$

Interestingly this simple form of the equation of state fits a wide range of scalar field dark energy behaviours including the supergravity motivated SUGRA model for dark energy [17]. It is easy to check that for $w_0 > -1$ and $w_a > 0$, the dark energy remains non-phantom ($w(a) > -1$) throughout the cosmological evolution. Otherwise it behaves like phantom ($w(a) < -1$) at some point in time in the cosmological evolution. As discussed earlier, one of the most studied example of dark energy is quintessence model with a time dependent scalar field. Depending on the initial conditions and the nature of the potential for the scalar field, quintessence field can either have freezing behaviour or thawing behaviour. In an interesting paper, Caldwell and Linder [12] have obtained two restricted regions in the $w_0 - w_a$ parameter space, where the quintessence field behaves either as freezing model or as thawing model.

With this, the 0–0 component of the Einstein equation for a spatially flat FRW universe that contains matter and dark energy is given by

$$H^2(a) = H_0^2 (\Omega_{m0} a^{-3} + \Omega_{r0} a^{-4} + (1 - \Omega_{m0} - \Omega_{r0}) a^{-3(1+w_0+w_a)} e^{-3w_a(1-a)}). \quad (3)$$

Here Ω_{m0} and Ω_{r0} are the present day density parameter for matter (that includes baryons and cold dark matter) and radiation respectively and H_0 ($100h \text{ Km/sec/Mpc}$) is the present day Hubble parameter. This expression for the Hubble parameter $H(a)$ with five parameters h, w_0, w_a, Ω_{m0} and Ω_{r0} is sufficient to calculate all the observable quantities related to the background cosmology.

To distinguish different dark energy models among themselves and also with the Λ CDM model, there is a very useful diagnostic, called *Om diagnostic* proposed by Sahni et al. [18]. It is defined as

$$Om(z) = \frac{(\frac{H(z)}{H_0})^2 - 1}{(1+z)^3 - 1}. \quad (4)$$

It is not difficult to see that for Λ CDM model, $Om(z)$ is constant throughout the evolution of the universe. It provides powerful *null test* for Λ CDM model whereby a evolving $Om(z)$ confirms a non- Λ CDM model.

To study the growth of matter fluctuations on sub-horizon scales where dark energy behaves as a smooth component, we take the linearised equation for growth of matter density contrast under Newtonian approximation as:

$$\ddot{\delta}_m + 2H\dot{\delta}_m - 4\pi G\delta_m = 0, \quad (5)$$

where δ_m is the matter density contrast and “*overdot*” represents the derivative with respect to the cosmological time t . H is the Hubble parameter given by equation (3). We also define linear growth rate as:

$$f = \frac{d \log \delta_m}{d \log a}. \quad (6)$$

The quantity $f(z)\sigma_8(z)$, where $\sigma_8(z)$ is the rms fluctuation of linear density field δ_m within a box size of $8h^{-1}$ Mpc, is a model independent estimator of the observed growth history in the universe. On sub-horizon scales where dark energy behaves like a smooth component, one can write [19]

$$f(z)\sigma_8(z) = \sigma_8 \frac{\delta'_m}{\delta_m(z=0)}, \quad (7)$$

where “*prime*” denotes differentiation with respect of $\log(a)$ and $\sigma_8 = \sigma_8(z=0)$.

With this, we use the latest observational data to constrain the parameters in our model and subsequently constrain the behaviour of the dark energy. We use the following observational data for our analysis:

- We use the measurements of the luminosity distance of SNIa from the “*Joint Light Curve Analysis (JLA)*” taken from SDSS and SNLS catalogue [21].
- We use the combined BAO/CMB constraints on the angular scales of the BAO oscillations in the matter power spectra as compiled by Giotri et al [22].
- We use the measurement of $f(z)\sigma_8(z)$ by various galaxy surveys as compiled by Basikalos et al [23].
- We use the acoustic scale and CMB shift parameters as measured by Planck-2015 observations [24].
- We use the measurement of Hubble parameter as a function of redshift as compiled by Farooq et al [25].
- We also use the latest measurement of H_0 by Reiss et al [7] (Riess16).

With these set of cosmological data, we use publicly available Markov Chain Monte Carlo (MCMC) package “*emcee*” [26] to put constrain on the parameters in our model. To obtain the covariance matrices from the MCMC sample chains, we use the publicly available python package “*GetDist*” [27].

In table 1, we quote the 1-D marginalised 68% confidence interval for our model parameters. As one can see, the constrain on σ_8 is significantly lower than the Planck-2015 measured value of σ_8 for Λ CDM model. This is due to the inclusion of growth data which are already in tension with Planck-2015 measurement. This measurement is in tension with the σ_8 measurement by CFHTLenS tomographic weak lensing survey at 1.8σ which is not significant. We also measure the parameter $S = \sigma_8\sqrt{\Omega/0.3}$ which is 0.728 ± 0.023 . This is consistent with that measured by Kilo Degree Survey (KiDS). Our measured h is also

Ω_{m0}	w_0	w_a	h	Ω_{r0}	σ_8
0.283 ± 0.009	-0.91 ± 0.11	$-0.70^{+0.5}_{-0.42}$	0.702 ± 0.0074	$(4.68 \pm 0.20) \times 10^{-5}$	0.749 ± 0.023

TABLE I: The 1D marginalised 68% confidence intervals for the model parameters.

consistent with independently measured value for h by HoLiCOW project [28] using three strong lenses system.

In figure 1, we show the confidence contours in the $w_0 - w_a$ parameter space. In the same contour we also show the regions obtained by Caldwell and Linder, where the scalar field quintessence models behaves either as freezing or as thawing model. As one can see, without the data for the Hubble parameter, the cosmological constant ($w_0 = -1$ & $w_a = 0$) is perfectly consistent. But once we add the $H(z)$ and H_0 data, the cosmological constant model sits at the edge of the 95% confidence region. Moreover the freezing models are now ruled out at 95% confidence level whereas the thawing models are just marginally allowed at 95% level. Given that the minimally coupled, canonical scalar quintessence field can have either thawing or freezing behaviour, the figure 1 shows that these kind of scalar field models are practically ruled out at 95% confidence level except a very tiny region for thawing models that is allowed at 95% confidence interval.

In figure 2, we show the confidence contour in the $\Omega_{m0} - \sigma_8$ plane for the SNIa+BAO+CMB+Growth+H(z)+ H_0 data. In this plot we also show the same confidence contour as obtained by KiDS tomographic weak lensing survey. For this, we use cosmic shear measurements from the Kilo-Degree Survey (Kuijken et al. 2015 [29], Hildebrandt et al. 2017 [8], Fenech Conti et al. 2016 [30])(KiDS). The KiDS data are processed by THELI (Erben et al. 2013 [31]) and Astro-WISE (Begeman et al. 2013 [32], de Jong et al. 2015 [33]). Shears are measured using lensfit (Miller et al. 2013 [34]), and photometric redshifts are obtained from PSF-matched photometry and calibrated using external overlapping spectroscopic surveys (see Hildebrandt et al. 2016 [35]). As one can see, our measurement is fully in agreement with the KiDS result.

We should mention that the two parameters w_0 and w_a are related to the dark energy equation of state at present ($z = 0$). Hence the confidence contour shown in figure 1, strictly represents dark energy property at present. To constrain the evolution of the dark energy equation of state $w(z)$, we need to reconstruct $w(z)$ using the error propagation technique.

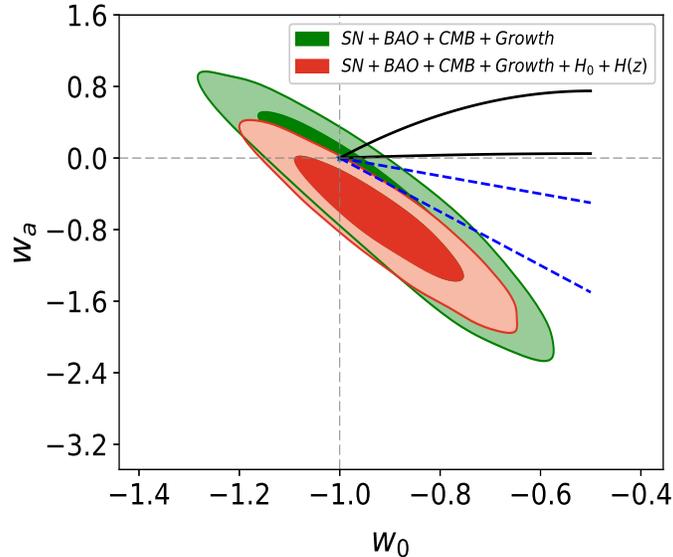


FIG. 1: 68% and 95% confidence contours in the $w_0 - w_a$ parameter plane using two different set of observational data. The region inside the dashed line, the quintessence field behaves like thawing model whereas the region inside the solid lines, the quintessence field behaves as freezing model (see [12]).

In figure 3, we show the constrained $w(z)$ at 68% and 95% confidence level. As one can see, the $w = -1$, cosmological constant behaviour is not always within the 95% confidence region. Within the redshift range $0.5 \leq z \leq 1.25$, the 2σ confidence region does not allow the cosmological constant behaviour. This confirms the inconsistency of the cosmological constant model at 95% confidence level. Moreover, the figure also confirms that models which are always non-phantom ($w > -1$) are ruled out at 95% confidence level. This practically ruled out all canonical, minimally coupled scalar field models. Models which are always phantom ($w < -1$) or model where a phantom to non-phantom crossing happens are still consistent. This opens up the possibility for more exotic dark energy models.

We also plot the reconstructed deceleration parameter $q(z)$ in figure 4. At 95% confidence level, the universe starts accelerating approximately between $z = 0.6$ and $z = 0.8$. The reconstructed $H(z)$ is also shown in figure 4.

To check further whether the Λ CDM model is indeed inconsistent at 95% confidence level, we further study the reconstructed behaviour of the $Om(z)$ parameter that we describe in equation (4). As we mention above, for Λ CDM model the Om parameter is constant

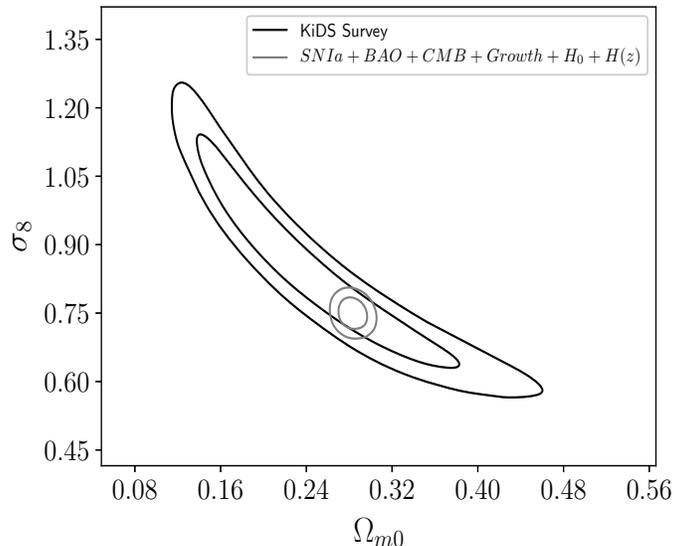


FIG. 2: 68% and 95% confidence contours in the $\Omega_{m0} - \sigma_8$ parameter plane. The KiDS survey plot is based on data products from observations made with ESO Telescopes at the La Silla Paranal Observatory under programme IDs 177.A-3016, 177.A-3017 and 177.A-3018 (see text for detail references).

throughout the evolution of the universe. In figure 5, we show the reconstructed behaviour of Om parameter as a function of redshift. It is not difficult to see that this reconstructed evolution of Om parameter as a function of redshift does allow a constant behaviour at 95% confidence level making the Λ CDM model consistent at 95% confidence level. We should mention that unlike the equation of state of the dark energy, Om does not depend explicitly on the current value of the Ω_{m0} . The information on $H(z)$ is sufficient to reconstruct the Om . Although a determination of H from a single observable can suffer systematic uncertainties, but in our investigation, we use the determination of H from SNIa, BAO, CMB acoustic scale, growth of matter fluctuations, as well independent measurement of H from local experiments, to reconstruct the $Om(z)$. Hence this is a robust estimate for the $Om(z)$ parameter which clearly shows the consistency of the Λ CDM model with presently available observational data. One can also find out that for Λ CDM to be consistent, the 95% bound on Ω_{m0} for Λ CDM model is [0.251, 0.292] which is marginally lower than the Planck-2015 measurement of Ω_{m0} for Λ CDM model.

To conclude, we have revisited the constraint on the evolution of the dark energy with

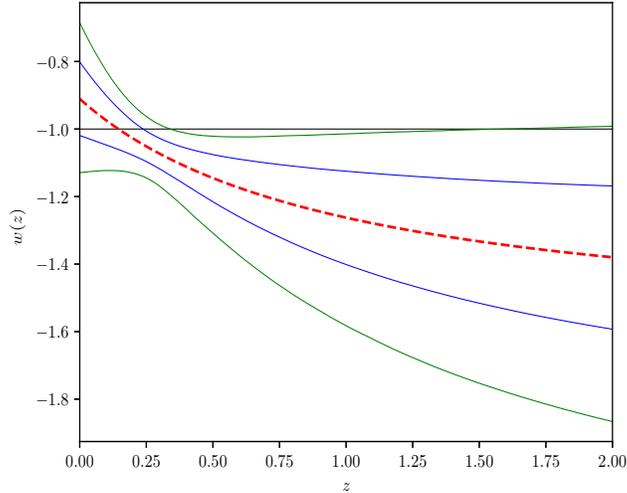


FIG. 3: 68% and 95% confidence region for the reconstructed $w(z)$ with SNIa+BAO+CMB+Growth+ $H(z)$ + H_0 data. The dashed line represent the best fit behaviour. The horizontal solid line represent the $w = -1$ cosmological constant behaviour.

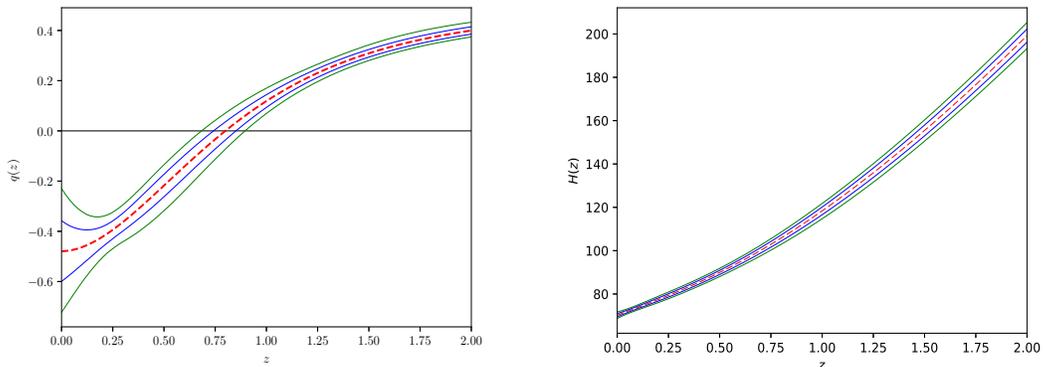


FIG. 4: 68% and 95% confidence region for the reconstructed deceleration parameter $q(z)$ and $H(z)$ for the same data set as in figure 2. The dashed line represent the best fit behaviour.

a wide variety of presently available observational data. We use the CPL parametrisation to model the dark energy evolution and use the combination of SNIa, BAO, CMB shift and acoustic scale measurements, growth measurement as well as the measurement of $H(z)$ including the recent measurement of H_0 . Recently with a combination of Planck2015+Riess16

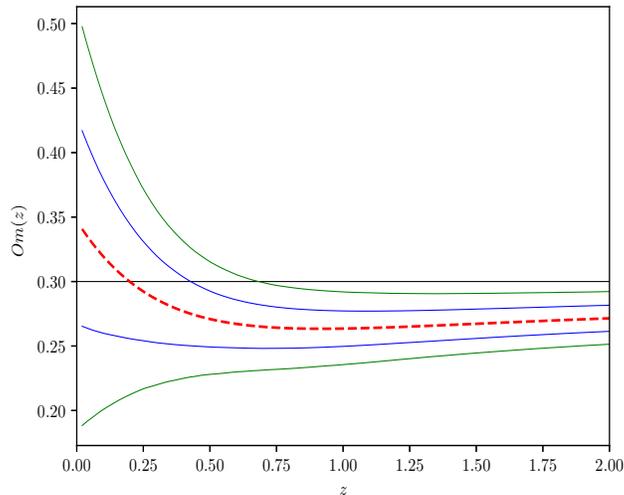


FIG. 5: 68% and 95% confidence region for the reconstructed $Om(z)$. The dashed line represent the best fit behaviour.

dataset, Valentino et al [9] have shown that the Λ CDM model is inconsistent at 95% confidence interval using the same parametrisation. We confirm this result using the reconstructed the equation of state of the dark energy using a different combination of datasets. Moreover the confidence contour in the $w_0 - w_a$ parameters space also confirms the inconsistency of the freezing models at 95% confidence level whereas the thawing model is marginally allowed at 95% confidence level. But with our reconstructed $Om(z)$ diagnostic, we show that Λ CDM is indeed consistent with the current set of observational data with a slightly lower range of Ω_{m0} from that obtained by Planck-2015. We have also shown that measured value of the parameter $S = \sigma_8 \sqrt{\Omega/0.3}$ as well as the confidence contour in the $\Omega_{m0} - \sigma_8$ plane is fully in agreement with the recent measurement by KiDS Survey.

In most of the investigations related to the constraints on dark energy equation of state, it is a standard practice to use the dark energy equation of state to obtain the constrain. But one has to be careful in this regard, because knowing the evolution of $H(z)$ from different data set, reconstruction of $w(z)$ for dark energy needs the information about Ω_{m0} . The Om diagnostic is useful in this sense as the information about Ω_{m0} is not necessary for its reconstruction. Hence this diagnostic is a more powerful *null test* for Λ CDM. Using this Om diagnostic, we show that the Λ CDM model is still very much consistent with the presently

available data. We need to wait for some more time to see whether the dark energy is indeed Λ or something else. Till that time, Λ is still the best choice for the dark energy.

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