# CALCULATIONS OF NUCLEUS-NUCLEUS CROSS SECTIONS, AND THE ATTENUATION OF COMPLEX COSMIC-RAY NUCLEI IN THE ATMOSPHERE

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The partial cross sections for nuclei heavier than oxygen, when they collide with nuclei heavier than helium, are practically unknown. We have developed tentative semi-empirical techniques for calculating the cross sections that apply not only to attenuation in air, but also to interactions in detector materials like plastic track detectors and scintillating materials. There is a major difference in the isotopic (and also in elemental) composition between secondary nuclei produced in interstellar space and in the atmosphere: the atmospheric ones have short-lived isotopes present. A comparison of the calculated atmospheric and galactic secondary nuclei is presented. Calculated fragmentation parameters are compared with existing experimental ones.

1. Introduction. Since the construction of the semiempirical equations of Silberberg and Tsao (1973a and b) for calculating the partial cross sections of p-nucleus reactions, we have recognized the need to have a reasonable means of calculating nucleus-nucleus cross sections in order to account for the secondary particles produced in air and in many of the detector materials. Even in interstellar space, about 20% of the cosmic-ray collisions are with helium nuclei. Compared to the number of p-nucleus cross sections measured, the data on nucleus-nucleus cross sections are very scanty. Cumming et al. (1974) have measured several spallation cross sections of Cu induced by nitrogen ions at a total kinetic energy of 3.9 GeV ($\sim$280 MeV/u). To a first approximation, the partial cross sections of $^{14}N$ with Cu are proportional to those of p-Cu reactions at 3.9 GeV. A marked deviation from this proportionality is that light fragments like $^{3}H$ and $^{7}Be$ are produced at a higher rate. Moreover, the yields of neutron-deficient products with mass numbers $A > 35$ gave indications of being enhanced. There more recent data (Cumming et al. 1976) on carbon targets show similar characteristics in the enhancement of the lighter products—with $A \leqslant 0.5$ A (target), but there is no enhancement of the neutron-deficient products.

Karol (1974) has studied the collisions of helium nuclei at 720 MeV (180 MeV/u) with copper. He found the proportionality factor, i.e., the scaling factor, to be about 1.9. However, the neutron-deficient products with $A > 35$ are found to be suppressed in the helium-induced reactions, unlike those of the nitrogen-induced reactions.

Lindstrom et al. (1975) have measured the partial cross sections from the breakup of C and O nuclei in collisions with H, Be, C, Al, Cu, Ag and Pb. Again there is a general proportionality between the C-nucleus and C-p reactions. The deviations from complete proportionality have been explained in terms of (1) nuclear transparency (i.e., the energy deposition is less in p-nucleus interactions) so that as $\Delta A$ increases, the relative yields of p-nucleus reactions diminish progressively, and (2) giant dipole resonances, as a result of which single-nucleon stripping in nucleus-nucleus reactions is enhanced. More recently Lindstrom et al. (1977) have measured the reaction cross sections of $^{56}$Fe on targets ranging from H to U. The target effects appear to follow simple geometrical arguments.

2. Semiempirical Equations. As a starting point for calculating the nucleus-nucleus cross sections, we take the p-nucleus cross sections, and determine the appropriate scaling factors and other correction terms.

We denote by $N_1$ the nuclide whose breakup mode is being explored; it may be a stationary "target", or it may be a beam of particles. In the former case one determines the yields of radioactive products in the target, while in the latter, one identifies the products from the fragmented beam particles. The "collision partner" with which $N_1$ collides is denoted by $N_2$.

The cross sections of $N_1$-$N_2$ reactions can be calculated from those of $N_1$-p interactions, $\sigma$ ($N_1$-p) with the equation:

$$\sigma(N_1-N_2) = \sigma\,(N_1-p)\; S_c\; \epsilon_n\; \epsilon_L\; \epsilon_1\; \epsilon_\Delta \qquad (1)$$

The scaling factor $S_c$, described as a function of nuclear skin thickness by Lindstrom et al.(1975), is fitted empirically. The other factors $\epsilon_n$, $\epsilon_L$, $\epsilon_1$, and $\epsilon_\Delta$ respectively represent correction terms for neutron-deficient products, light products, single-nucleon stripping, and for reactions with large $\Delta A$.

Table 1. Parameters of Equation (1) for $4 \le Z_1 \le 30$ and $Z = 2$

$$S_c = \begin{cases} 1.6 & \text{for } Z_1 \le 8 \text{ and } \Delta A \le 2 \\ 1.8 & \text{otherwise} \end{cases}$$

$$\epsilon_n = \begin{cases} 0.85 + 0.15(A-A_z) \ge 0.3 & \text{for } 35 < A < A_z-1 \\ 1 & \text{otherwise} \end{cases}$$

$$A_z = [S - (S^2 - 4TZ)^{1/2}]/2T - M + 1$$

$$\epsilon_L = \begin{cases} 1 + 0.4\,[1 + 0.02(Z_m/Z)^2]\,(1 - 1.5\,Z/Z_m) & \text{for } 3 \le Z \le 5 \\ 1 & \text{otherwise} \end{cases}$$

$$Z_m = \text{smaller of } (Z_1, 8)$$

$$\epsilon_1 = 1$$

$$\epsilon_\Delta = 1$$

Table 2. Parameters of Equation (1)
For $4 \le Z_1 \le 47$ and $3 \le Z_2 \le 9$

$$S_c = 2\,(1 - 0.02\,A_1^{2/3})$$

$$\epsilon_n = 1$$

$$\epsilon_L = \begin{cases} \epsilon_Z & \text{for } 3 \le Z \le 5, \text{ and } Z_1 \le 10 \\ \epsilon_Z/(1 - 0.02\,A_1^{2/3}) & \text{for } 3 \le Z \le 5, \text{ and } Z_1 > 10 \\ 1 & \text{otherwise} \end{cases}$$

$$\epsilon_Z = 1 + 0.4\,[1 + 0.02\,(Z_1/Z)^2](1 - 1.5\,Z/Z_1)$$

$$\epsilon_\Delta = \begin{cases} 3\exp(-2\,A/A_1) & \text{for } A < 0.5\,A_1, \text{ and } Z > 5 \\ 1 & \text{otherwise} \end{cases}$$

$$\epsilon_1 = 1$$

Table 3. Parameters of Equation (1)
For $4 \le Z_1 \le 30$ and $10 \le Z_2 \le 82$

$$S_c = (1.6 + 0.07\,A_2^{2/3})\,(1 - 0.02\,A_1^{2/3})$$

$$\epsilon_n = 1$$

$$\epsilon_L = 1^*$$

$$\epsilon_1 = \begin{cases} 1 + 0.004\,(Z_2 - 10) & \text{for } \Delta A = 1 \\ 1 & \text{for } \Delta A > 1 \end{cases}$$

$$\epsilon_\Delta = \exp[0.02(A_1 - A)]$$

*For the breakup of nuclei with $Z_1 > 8$, $\epsilon_L$ has not been measured; temporarily we assign $\epsilon_L = 1$.

Table 1 prescribes the appropriate parameters for collisions with $^4$He. The expressions S, T and M are described in the paper by Silberberg and Tsao (1973a) to which we have referred earlier. For collisions with $^4$He, equation (1) may be used for energies down to 800 MeV (200 MeV/u), using the value of $\sigma(N_1+p)$ calculated at proton energy equal to that of the total kinetic energy of the nucleus. The restriction to $Z_1 > 8$ in the expression $\epsilon_L$ in Table 1 is based on the work of Raisbeck and Yiou (1976). Tables 2 and 3 are applicable to high energies only (E/u > 600 MeV/u); the cross sections are calculated with the asymptotic high-energy values.

3. Collisions in Air. Equation (1) was used to calculate all the values of the required cross sections needed to account for the secondary particles produced in collisions with air. In addition we have assumed a charge spectrum like that observed at low geomagnetic latitudes (4 GV cut-off) (Shapiro and Silberberg, 1975). For the isotopic composition we have used the results of our earlier calculation on the propagation of cosmic rays in space (Tsao et al. 1973).

Table 4 shows the variation of particle flux for each element as a function of depth. In general there is no simple way of correcting for air absorption and secondary production.

Table 4. Variation of Particle Flux as a Function of Air Depth (g/cm$^2$)

| | 2 | 4 | 8 g/cm$^2$ | | 2 | 4 | 8 g/cm$^2$ |
|---|---|---|---|---|---|---|---|
| Li | 1.02 | 1.04 | 1.05 | P | 1.00 | 0.99 | 0.93 |
| Be | 1.01 | 1.02 | 1.00 | S | 0.89 | 0.79 | 0.63 |
| B | 0.97 | 0.94 | 0.87 | Cl | 1.02 | 1.01 | 0.97 |
| C | 0.93 | 0.86 | 0.74 | Ar | 0.91 | 0.83 | 0.68 |
| N | 0.95 | 0.90 | 0.80 | K | 0.93 | 0.85 | 0.72 |
| O | 0.91 | 0.82 | 0.68 | Ca | 0.88 | 0.77 | 0.60 |
| F | 1.04 | 1.06 | 1.05 | Sc | 1.01 | 0.99 | 0.91 |
| Ne | 0.90 | 0.82 | 0.67 | Ti | 0.90 | 0.82 | 0.66 |
| Na | 0.97 | 0.94 | 0.85 | V | 0.92 | 0.85 | 0.70 |
| Mg | 0.89 | 0.79 | 0.62 | Cr | 0.91 | 0.82 | 0.66 |
| Al | 0.90 | 0.82 | 0.67 | Mn | 0.91 | 0.83 | 0.67 |
| Si | 0.87 | 0.76 | 0.59 | Fe | 0.82 | 0.67 | 0.46 |

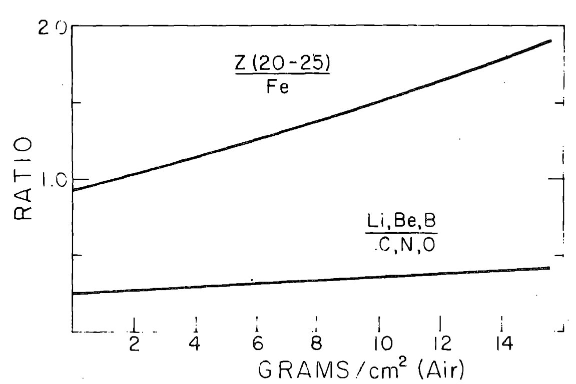


Fig. 1 shows the variation of the L/M ratio and that of the sub-iron elements to iron. The latter shows a more pronounced increase as a function of depth in air.

In general the isotopic composition for most elements does not vary a great deal with depth in air except when the isotopes produced are radioactive. Since many of the isotopes in our calculation are spallation products of collisions with interstellar matter, those produced in air have a similar composition. However, there are no short lived radioactive components in the primaries because of decay in space. Those produced in air, on the other hand, will persist. Table 5 shows the variation of the isotopic composition for the two elements Cl and Sc as a function of depth in air.

Table 5. Variation of the Isotopic Composition as a Function of Depth in Air

| | 0 | 2 | 4 | 8 g/cm$^2$ |
|---|---|---|---|---|
| Cl 35 | 80% | 74% | 65% | 56% |
| Cl 36* | 0 | 4 | 14 | 22 |
| Cl 37 | 20 | 22 | 21 | 22 |
| Sc 44* | 0 | 5 | 8 | 13 |
| Sc 45 | 100 | 91 | 86 | 77 |
| Sc 46* | 100 | 4 | 6 | 10 |

*Radioactive

Table 6 compares the fragmentation parameters in various media for several groups of elements. These can be used for approximate corrections in many practical applications. The experimental parameters for interactions involving products that stay in the same group of elements, e.g., $H_1 \rightarrow H_1$ probably suffer from a low detection efficiency for such events, especially in emulsions.

Table 6. Fragmentation Parameters in Air, Lexan and Emulsion

| | | Air | Lexan | Emulsion |
|---|---|---|---|---|
| $H_1$(Z=20-26) → | $H_1$ | 0.24 (0.17±.02)* | 0.32 | 0.31 (0.11±.01) |
| | $H_2$ | 0.11 (0.20±.02) | 0.14 | 0.13 (0.15±.01) |
| | $H_3$ | 0.11 (0.22±.03) | 0.13 | 0.14 (0.18±.01) |
| | M | 0.07 (0.17±.02) | 0.08 | 0.06 (0.15±.01) |
| | L | 0.15 (0.24±.03) | 0.11 | 0.20 (0.25±.01) |
| $H_2$(Z=16-19) → | $H_2$ | 0.07 (0.06±.03) | 0.09 | 0.09 (0.06±.01) |
| | $H_3$ | 0.20 (0.42±.09) | 0.24 | 0.22 (0.30±.02) |
| | M | 0.09 (0.20±.06) | 0.10 | 0.09 (0.21±.04) |
| | L | 0.15 (0.17±.05) | 0.12 | 0.15 (0.21±.04) |
| $H_3$(Z=10-15) → | $H_3$ | 0.11 (0.16±.04) | 0.13 | 0.12 (0.16±.03) |
| | M | 0.17 (0.39±.06) | 0.21 | 0.16 (0.26±.04) |
| | L | 0.13 (0.21±.04) | 0.14 | 0.15 (0.15±.03) |
| M (Z= 6- 9) → | M | 0.16 (0.17±.02) | 0.17 | 0.19 (0.09±.02) |
| | L | 0.15 (0.24±.03) | 0.19 | 0.17 (0.15±.02) |
| L (Z= 3- 5) | L | 0.18 (0.11±.04) | 0.15 | 0.12 (0.11±.03) |

*Values inside the parentheses are experimental ones given by Freier and Waddington (1975).

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