CROSS SECTIONS FOR ATMOSPHERIC CORRECTIONS

J.P. Meyer, M. Cassé
DPh/EP, Centre d'Etudes Nucléaires de Saclay (France)
N. Westergaard
Danish Space Research Institute (Denmark)

A set of cross sections for spallation of relativistic nuclei is proposed, base on (i) the best available proton cross sections, (ii) an extrapolation to heavier nuclei of the dependance on the number of nucleons lost of the "target factor" observed for C^{12} and O^{16} by Lindstrom et al.(1975), in analogy with Rudstam's formalism,and (iii) on a normalization of all cross sections to the total cross sections for production of fragments with $A_f \geqslant 6$. The obtained cross sections for peripheral interactions are not inconsistent with simple geometrical considerations.

1. Introduction. The recent Bevatron observations of spallation of relativistic C^{12} and O^{16} nuclei on heavy targets has motivated us to calculate atmospheric corrections. This requires extrapolating the C and O observations up to Nickel. This can certainly not be reliably done in a detailed way, but hopefully a reasonable rough approximation can be obtained, valid at least for the most important cross sections, i.e. those for production of fragments comparatively close to the incident nucleus (larger cross sections and parent nuclei abundances).

2. Total reaction cross sections.

2.1. Proton-nucleus reaction cross sections.

Total proton induced reaction cross sections σ_{react} have been widely studied, and are usually found to follow approximately a smooth $A_i^{2/3}$ law, where A_i is the mass of the nucleus involved.

The most recent experimental studies are due to Renberg et al. (1972) on proton-nucleus reactions and to Heckmann et al. (1973),complemented by Lindstrom et al. (1975) for the loss of neutrons only, for nucleus-proton collisions. Renberg et al. (*) find a smooth variation of σ_{react} with A_i over a wide range of masses A_i, that Juliusson et al. (1975) have fitted by

$$\sigma_{react} = 54\ (A_i^{1/3}-0.20)^2 \text{ mb.}$$

The nucleus-proton observations are restricted to ^{12}C and ^{16}O. While the C point is within 5 % of Renberg's systematics, the O point is higher by 18 % , i.e. 50 mb. It is difficult to understand why two alpha - structure nuclei (C^{12} and O^{16}) could behave so differently with respect to the $A^{2/3}$ law(**). Hence we tentatively give the preference to the Remberg's data.

2.2. Nucleus-nucleus reaction cross-sections

The total nucleus-nucleus reaction cross section will be fitted by a Bradt and Peters (1950) type formula.

$$\sigma_{react} = K(A_i^{1/3} + A_t^{1/3} - \delta)^2 \text{ mb}$$

(*) Remberg's points are at 570 MeV. But the energy variation of the total reaction cross section between 570 MeV and the GeV range is very small.

(**) The ratio of the O to the C cross section is observed to be 1.24 by Remberg and 1.38 by Heckman. Barshay et al (1975) calculate 1.27.

for an incident nucleus i fragmenting on a target t.

Until very recently the coefficients K and δ were determined only from emulsion work, in which reactions of neutron loss only (in both i and t nuclei) could not be observed. Most observations were made before 1959. They are summarized by Powell et al. (1959) p.605, and suggest K = 66 mb, δ = 1.17. The new observations by Heckmann (1973) in the GeV/n range, complemented by those of Lindstrom et al. (1975) for the loss of neutrons only by the incoming nucleus, yield genuine total reaction cross sections.

We are interested in fitting the data only up to Z 30. In this range, Heckman's study gives us four points : C and O beams on Carbon and Copper Targets. When trying to fit these data with a Bradt and Peters type formula, it is again found that the C and O points on the Carbon Target cannot be simultaneously fitted together with the Copper points. In view of the doubt on the cross section obtained with the Oxygen beam on Hydrogen we choose to fit the C point, and get :

$$\sigma_{react} = 60.5\,(A_i^{1/3} + A_t^{1/3} - 0.93)^2 \text{ mb } ^{(*)}.$$

We shall adopt this form, which is actually fairly close to that resulting from the emulsion work.

3. Nucleus-nucleus individual reaction cross sections in the GeV/n range.

The available observations. There are only three groups of observations of nucleus-nucleus reactions between nuclei both heavier than He^4.

3.1. The Bevatron data (Heckmann et al. 1972, Lindstrom et al. 1975) They refer to the spallation of 1.05 and 2.10 GeV/n C^{12}, N^{14} and O^{16} nuclei on various targets ranging from H to Pb. They show that the relative yields of the various fragments (at least above Li^6) are about equal for all targets between Be and Pb, i.e. for targets whose size is at least comparable to that of the incident C^{12} or O^{16} nuclei.

To relate the cross sections on heavy targets to those on hydrogen, we define as "enhancement factor"

$$\Gamma_T(f) = \frac{\sigma_T\,(f)}{\sigma_H\,(f)}$$

the ratio of the cross section for production of a particular fragment f on a target T to that on an Hydrogen target$^{(**)}$. Γ_T has been plotted as a function of the fragment mass A_f in fig. 1, for a few couples of incident and Target nuclei. The corresponding ratio for the total reaction cross section Γ_T (react) have also been indicated. Fig. 1 calls for the following remarks :

(i) Compared to the Hydrogen cross sections, the emission of lighter fragments, i.e. the deeper destruction of the incident C^{12} and O^{16} nuclei, is slightly favoured on heavy targets. Note that Γ_T varies slowly and quite smoothly with A_f (at least for $A_f \geqslant 6$ and for reactions with $\sigma > 2$ mb). Hence Rudstam's formalism should be applicable to spallation on heavy targets. This variation has been fitted by Lindstrom et al. as

$$\Gamma_T(A_f) \propto (0.66 + \frac{A_f}{36})^{-1} \qquad (1)$$

valid for all targets between Be and Pb, and for both C and O incident nuclei (but only for these nuclei !). To permit a later generalization, we prefer to rewrite this fit as a function of the mass difference $\Delta A = A_i - A_f$ between the incident nucleus and the fragment. With $A_i \approx 14$ in Lindstrom's experiments, the fit (1) becomes approximately :

$$\Gamma_T\,(\Delta A) \;\underset{\sim}{\propto}\; (1 - \frac{\Delta A}{36})^{-1}. \qquad (2)$$

(*) Giving an equal weight to the C and O points on Carbon would lead to K = 55.6 and δ = 0.69.

(**) Γ_T differs from Lindstrom's "target factor" γ_T insofar as Lindstrom et al. do not directly normalize their cross sections to the Hydrogen cross sections.

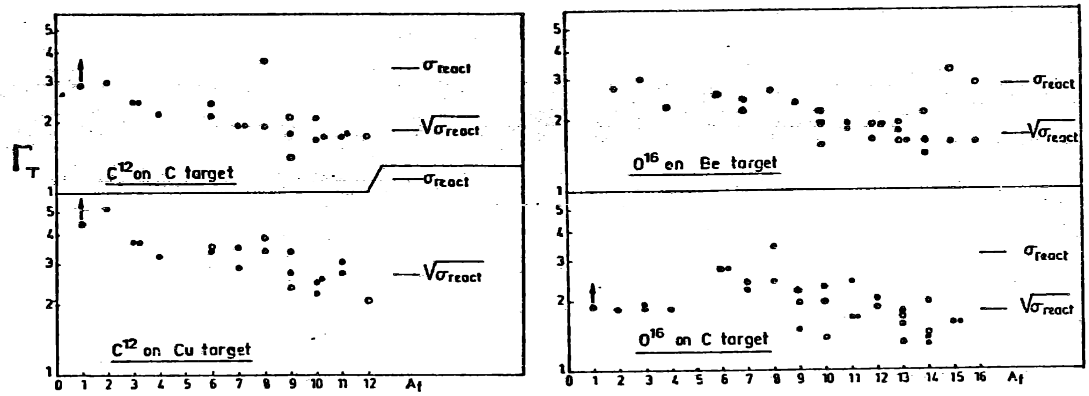

Fig. 1 : Enhancement factors $\Gamma_T(f) = \sigma_T(f)/\sigma_H(f)$ deduced from some of the observations of Lindstrom et al. (1975), as a function of the fragment mass A_f, and for the total reaction cross section and its square root. Full points : $\sigma > 2$ mb ; open points $\sigma < 2$ mb.

Since for C^{12} and O^{16} incident nuclei $\Delta A \ll 36$, the above linear fit cannot be distinguished from the exponential fit

$$\Gamma_T(\Delta A) \mathrel{\underset{\sim}{\propto}} e^{+\frac{\Delta A}{36}} \quad . \tag{3}$$

We choose the latter form in view of its similarity with Rudstam's formalism.

(ii) The observed Γ_T's for the production of H and He isotopes appear unreasonnably low. In particular the production of He^4 would not be more enhanced in heavy targets than that of Li, which is puzzeling. In the case of O^{16} on C and Cu targets f.i., the observed Γ_T is even significantly lower for He^4 than for Li formation. Further Lindstrom et al observe only < 70 % of the outcoming fragments with $A_f \lesssim 4$ in their 12.5 mrad angle of observation. For all these isotopes but protons, they restitue the emission yield by extrapolation to larger angles.

(iii) Almost all the Γ_T (f)'s for the individual reactions yielding the various fragments are smaller than the corresponding Γ_T (react), for the total reaction cross sections(*). Hence there exists a "missing cross section", to be found somewhere. Lindstrom et al (1975) note that they miss many ejected protons . However it is difficult to believe that a significant part of the "missing cross section" can be due to complete destruction into nucleons only.

(iv) For peripheral reactions fig. 1 is consistent with

$$\Gamma_T \text{ (periph. f)} \mathrel{\underset{\sim}{\propto}} \sqrt{\Gamma_T \text{ (react)}}$$

which can easily be understood in terms of simple geometry if the cross sections for the loss of few nucleons indeed corresponds to grazing collisions. But recall that the observations are limited to C, N, O incident nuclei.

Hence, in view of the "missing" cross section, of the low enhancement factors Γ_T observed for $A_f \lesssim 4$, and of the reduced efficiency in detecting fragments for $A_f \lesssim 4$, we shall not use Lindstrom's data for $A_f \leq 4$.

(*) In contradiction with the statement made in Heckmann et al. (1972)

TABLE 1

Bevatron results : (Lindstrom et al. 1975)
2.1 GeV/n

Incident carbon

Target	$\sigma_{\geq 6}$	σ_{react}	$\sigma_{\geq 6}/\sigma_{react}$	$\sigma_{\leq 4}/\sigma_{react}$
H	123	236	.52	.48
Be	232	663	.35	.65
C	232	754	.31	.69
Cu	361	1709	.21	.79

Incident Oxygen

Target	$\sigma_{\geq 6}$	σ_{react}	$\sigma_{\geq 6}/\sigma_{react}$	$\sigma_{\leq 4}/\sigma_{react}$
H	233	291	.80	.20
Be	425	763	.56	.44
C	420	860	.49	.51
Cu	660	1868	.35	.65

Table 1 shows the cross sections $\sigma_{\geq 6}$ for production of fragments with $A_f \geq 6$ and $\sigma_{\leq 4}$ for production of only fragments with $A_f \leq 4$, compared to the total reaction cross sections.

3.2. The Princeton data (Cumming et al. 1974)

They refer to the bombardment of Cu by 3.9 GeV protons and 3.9 GeV = 280 MeV/n N^{14} nuclei. The Copper Target is radiochemically analyzed for many radioactive isotopes. Absolute cross sections are obtained with protons but only relative yields with N^{14}.

These data show beautifully that the charge yield for isobars is very independant of the circumstances of the primary interaction. It depends essentially on the stability of the various isobars (their fig. 7), in very good agreement with Rudstam's formalism for the charge dispersion curves.

The mass yields for the loss of up to 27 nucleons upon interaction with H and N^{14} nuclei are shown in their fig. 6. It shows that protons tend to produce a more through destruction of Cu than N^{14}. Such a behaviour would be extremely difficult to believe for Cu bombardment by p and N^{14} beams of the same energy/nucleon (or for Cu incident with the same energy on H and N^{14} targets). We conclude that the mass yields observed by Cumming et al. for 280 MeV/n N^{14} nuclei is widely different from that which would have been obtained with 3.9 GeV/n N^{14} nuclei. Since we are interested in comparing the break up of Cu nuclei in the GeV/n range on H and N^{14} nuclei, we cannot make use of the mass yields observed by Cumming et al.(*)

Their fig. 3 shows the observed Γ_T's to within an unknown constant, since only relative yields are obtained for N^{14}. The decrease of Γ_T between mass 64 and ≈ 40 (mainly for n-poor isotopes) corresponds to the difference of slope of both mass yields, which we have just discussed.

This figure also shows that Γ_T increases strongly below $A_f \approx 15$. This increase corresponds probably mainly to fragments accompanying a "principal"

(*) In particular the larger production of neutron-deficient fragments in the spallation of Cu by 280 MeV/n N^{14} nuclei than by 3.9 GeV protons could well be due to a comparatively less important cascade and more important evaporation induced by the lower velocity N^{14} nuclei.

heavier fragment (fission-like processes ; evaporation of comparatively heavy fragments). This feature should be qualitatively conserved for higher energy N^{14}'s.

3.3. Older emulsion data (Waddington 1960 a,b ; Friedlander et al. 1963) They usually give the fragmentation parameters for nuclear interactions of GeV/n cosmic ray nuclei, occuring, either mainly on the CNO nuclei of the emulsion itself, or in Teflon (CF_2) or graphite targets. These fragmentation parameters were obtained with low statistics and poor charge resolution. However they give a good idea of the probability of complete destruction into $A_f \leqslant 4$ fragments only ($\sigma_{\leqslant 4}$). A detailed inspection of the data yields the following probabilities for the formation of fragments with $A_f \geqslant 6$:

A_i of incident nuclei	$\langle A_i \rangle$	$\frac{\sigma_{\geqslant 6}}{\sigma_{react}}$
40–58	54	.90
19–40	26	.75
12–16	14	.36
6–11	9	.13

The check with $\sigma_{\geqslant 6}/\sigma_{react}$ = .31 and .49 for C^{12} and O^{16} on a Carbon Target obtained in Table 1 from the data of Lindstrom et al (1975) support the validity of these older emulsion data.

4. Procedure for the estimate of the nucleus-nucleus, individual reaction cross sections up to Nickel. Our purpose is to extrapolate the Bevatron observations for C^{12} and O^{16} nuclei up to Nickel. We wish to get a rough approximation, good at least for the cross sections most important to us, i.e. those for production of fragments comparatively close to the incident nucleus. In this context, we shall not try to reproduce the characteristics raise of Γ_T for very large ΔA's observed by Cumming et al.

We first extimate $\sigma_{\geqslant 6}$, the total cross section for production of fragments with $A_f \geqslant 6$ on air . In fig. 2 we combine the values deduced from the Bevatron data (Table 1) with the emulsion data on the ratio $\sigma_{\geqslant 6}/\sigma_{react}$, and fit them by a smooth curve as a function of A_i. We then multiply by σ_{react}, and get $\sigma_{\geqslant 6}$.

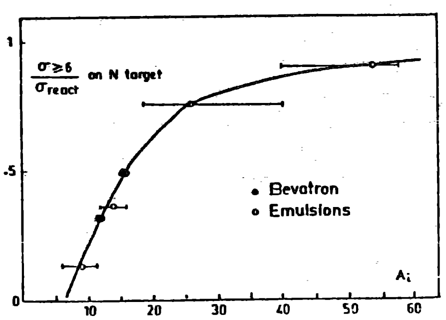

Fig. 2. Ratio of the cross section for production of fragments with $A \geqslant 6$ to the total reaction cross section on a Nitrogen target, as a function of the incident nucleus mass A_i.

We are conscious that most observations have been made with α - structure nuclei (C^{12}, O^{16} of Lindstrom et al, most abundant nuclei in cosmic rays used in emulsion work), which may break up into α's more easily than other nuclei. Our estimate may therefore be biased towards lower $\sigma_{\geqslant 6}$. However the very abundance of these α - structure nuclei in cosmic rays makes them the principal progenitors of secondary nuclei in the atmosphere. Hence our bias may be considered acceptable(*).

We then treat the relative yields of the various products in air. We start with the best abailable proton data, and modify them by smooth enhancement factors Γ_T. The observations of Lindstrom et al. and of Cumming et al.convince us that the formalism of Rudstam remains roughly valid for heavy targets. This

(*) We considered treating C^{12} as a special case, not to be fitted by the general smooth law, in view of the channel $C^{12}(X,X\alpha)\ Be^8 \rightarrow 2\alpha$. However Lindstrom's C^{12} fits smoothly onto neighbouring measurements in fig. 2 .

formalism describes the mass yield (except for peripheral and very thorough interactions) on Hydrogen target nuclei by : $\sigma(\Delta A) \propto e^{-\Delta A/q}$

where $q \approx 6.6$ around 2 GeV/n, independant of the incident nucleus (q levels off at a few GeV/n) (Reeves 1970).

We have fitted Lindstrom's enhancement factor Γ_T for C and O incident nuclei by $\Gamma_T \propto e^{+A/36}$ valid for any target with a size at least comparable with the size of the incident nucleus, and for peripheral as well as non peripheral interactions. It is therefore tempting to extend Rudstam's formula for C and O on heavy targets :

$$\sigma(\Delta A) \propto e^{-\Delta A\left(\frac{1}{6.6}-\frac{1}{36}\right)} = e^{-\frac{\Delta A}{8.1}}$$

In analogy with the independance of q on the size of the incident nucleus for the break up on Hydrogen, we now assume that the q for break up on heavy targets obtained for C and O is valid for all incident nuclei, as long as the size of the target is at least comparable with that of the incident nucleus (which is the case of air target nuclei for cosmic ray nuclei up to nickel). In other words, we extrapolate to all nuclei up to nickel incident on air the enhancement factors Γ_T observed by Lindstrom et al for C^{12} and O^{16} in the form :

$$\Gamma_T \propto e^{+\frac{\Delta A}{36}} .$$

This enhancement factor will be used for both peripheral and non peripheral interactions.

We calculate the yields for all individual reactions, on an arbitrary scale $\sigma_{air}(f) \propto \sigma_H(f) \cdot e^{+\frac{\Delta A}{36}}$

and normalize to $\sigma_{\geqslant 6}$ to get the absolute cross sections :

$$\sigma_{air}(f) = \frac{\sigma_H(f) \cdot e^{+\Delta A/36}}{\sum_{\Delta A=1}^{\Delta A=A-6} \sigma_H(f) \cdot e^{+\Delta A/36}} \cdot \sigma_{\geqslant 6}$$

5. Resulting atmospheric corrections. Based on proton nucleus cross sections at 2.5 GeV/n kindly provided by M. Meneguzzi (based on the latest experimental data and on Silberberg and Tsao's procedure) we have constructed a table of atmospheric cross sections , which is available from the authors.

Of course for ^{12}C and ^{16}O incident nuclei we have directly interpolated Lindstrom's data for an "air" target with mass 14.4. Similar tables can be obtained for isotope to isotope cross sections, and for other energies .

It is of interest to note that the cross sections obtained for peripheral interactions do not deviate from a$\sqrt{\sigma_{react}}$ law suggested by simple geometrical argument by more than a factor 1.3. This gives us some confidence in the cross sections yielded by our procedure (normalization to $\sigma_{\geqslant 6}$ after weighting the proton cross sections by Γ_T (ΔA)).

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/ ERRATUM /

Fig. 1 : ^{16}O on C target : the points for $A_f \leq 4$ have been erroneously quoted from Lindstrom et al. (1975).

Remove from text the sentence "in the case of ^{16}O on C and Cu targets, f. i., the observed Γ_T is even significantly lower for He than for Li formation."