Astron. Astrophys. 268, 726–735 (1993)

ASTRONOMY AND ASTROPHYSICS

Diffusion and drift of very high energy cosmic rays in galactic magnetic fields

V.S. Ptuskin [1], S.I. Rogovaya [1], V.N. Zirakashvili [1], L.G. Chuvilgin [1], G.B. Khristiansen [2], E.G. Klepach [1], and G.V. Kulikov [2]

[1] Institute of Terrestrial Magnetism, Ionosphere and Radio Wave Propagation, Russian Academy of Sciences, Troitsk, Moscow Region 142092, Russia
[2] Institute of Nuclear Physics, Moscow State University, Moscow 119899, Russia

Received April 6, accepted May 19, 1992

Abstract. We consider diffusive propagation of galactic cosmic rays with energies up to 10^{17} eV. It is shown that drift (Hall diffusion) of particles in the global regular magnetic field of the Galaxy may be responsible for the observed "knee" in the cosmic ray energy spectrum at 3 10^{15} eV. The model implies weak energy dependence of cosmic ray diffusion along the magnetic field lines $D \propto E^m$, $m = 0.15$–0.20, for a unique power law in the entire energy range 10^9–10^{17} eV.

Key words: cosmic rays – interstellar medium: magnetic fields – Galaxy: halo

1. Introduction

The "knee" (the point at which the power law spectrum gets sharply steeper) at $E_k = 3\ 10^{15}$ eV is a well known feature of the cosmic ray spectrum (Kulikov & Khristiansen 1959; Nagano et al. 1984; Fomin et al. 1991). The differential cosmic-ray energy spectrum $N(E) \propto E^{-\gamma}$ has a power law exponent $\gamma = 2.6$–2.7 at $10^{10}\,\mathrm{eV} \leqq E \leqq 3\ 10^{15}\,\mathrm{eV}$ and exponent $\gamma = 3$–3.1 at $3\ 10^{15}\,\mathrm{eV} \leqq E \leqq 10^{18}\,\mathrm{eV}$. Actually, some flattening of the spectrum before the steepening is not excluded (with $\gamma = 2.5$ at 3 10^{13}–3 10^{15} eV), see Fichtel & Linsley (1986), Khristiansen (1987), and Kifune (1990) for review.

The steepening of the spectrum is usually explained by changes in the conditions of cosmic-ray leakage from the Galaxy (Syrovatsky 1971; Wdowczyk & Wolfendale 1984). Changes of the accelerated-particle spectrum at the cosmic ray sources proper (Peters 1961; Karakula et al. 1990; see also Hillas, 1979) or superposition of two populations of sources with different energy spectra (Fichtel & Linsley 1986; Jokipii & Morfill 1986) are also possible. Another version assumes a two-step acceleration process, namely diffusive shock acceleration up to 10^{15}–10^{16} eV at the outer shock front of the supernova remnant and the subsequent re-acceleration in the inner part of the pulsar-driven remnant (Bell 1991) or in the volume of the Galaxy as a whole (Axford 1991).

In the present work, we consider the propagation of cosmic rays in galactic magnetic fields. The random field in the galactic halo is assumed to exist simultaneously with the regular magnetic field, which is mainly toroidal and whose value is comparable with the value of the field in the gas disk. Particle diffusion in the magnetically-active medium is well-known to have a tensor character. The Hall diffusion (determined by the antisymmetrical part of the cosmic-ray diffusion tensor) is not effective at small energies, but it may determined the leakage of cosmic rays out of the Galaxy at $E > E_k = 3\ 10^{15}$ eV. It is then possible to obtain the knee in the cosmic-ray power law spectrum without making any assumptions concerning the occurrence of a feature in the source spectrum or changes in the energy dependence of the diffusion coefficient along the magnetic field lines. Particle transport by Hall diffusion can also be described as a particle drift in a large-scale inhomogeneous magnetic field, see e.g. the paper by Isenberg & Jokipii (1979), where cosmic ray diffusion in interplanetary magnetic fields is discussed.

Send offprint requests to: V.S. Ptuskin

2. Cosmic ray transport in galactic magnetic fields

Cosmic ray propagation in the Galaxy is usually described in terms of diffusion, see Berezinskii et al. (1990). Semiempirical diffusion models make it possible to systematize and explain the set of experimental data on cosmic ray composition and anisotropy, at least in the energy range 10^8–10^{14} eV and perhaps for higher energies. This model is confirmed by the "microscopic" theory of motion for relativistic charged particles in a magnetic field.

The steady-state diffusion equation for cosmic ray density $N(r)$ is (we neglect here by nuclei fragmentation and any energy losses)

$$-\nabla_i D_{ij}(\boldsymbol{r}) \nabla_j N(\boldsymbol{r}) = Q(\boldsymbol{r}). \tag{1}$$

Here $Q(\boldsymbol{r})$ is a source term, and $D_{ij}(\boldsymbol{r})$ is the cosmic ray diffusion tensor. This tensor may be presented as

$$D_{ij} = (D_{ll} - D_{\perp})\, b_i b_j + D_{\perp} \delta_{ij} + D_{\mathrm{A}}\, e_{ijn} b_n . \tag{2}$$

Here $b_i = H_{0i}/H_0$ is a unit vector along the regular galactic magnetic field; D_{ll} $D_{\perp}$, D_{A} are parallel (field-aligned), perpendicular (transverse), and antisymmetric (Hall) diffusion coefficients respectively, e_{ijn} is the absolute antisymmetric tensor.

The cosmic ray flux $\boldsymbol{j}$ and the cosmic ray anisotropy δ are given by the expressions

$$j_s = -D_{sm} \nabla_m N, \qquad \delta_s = 3 j_s/(vN). \tag{3}$$

Hall diffusion gives rise to a particle flux perpendicular to the cosmic ray gradient and to the regular field direction.

Coefficients D_{ll}, $D_\perp$ and D_A are determined by the level of turbulence in the interstellar medium. The random magnetic field with the scale $k^{-1} \leqq r_H$ (k is the wave number of the random field; r_H is the particle gyroradius) provides resonant scattering and local diffusion of cosmic rays. Long-wave wandering of the magnetic field lines with $k^{-1} \gg r_H$ give rise to isotropization of the highly anisotropic local diffusion. On the scales larger then the basic scale of the interstellar turbulence $L \cong 100$ pc, the combined action of random magnetic fields with extended scale spectrum results in a global diffusion with diffusion tensor D_{ij}. The function $N(\boldsymbol{r})$ in Eq. (1) is the cosmic ray density averaged over regions with dimentions larger then L.

Assuming a power-law spectrum for the random magnetic field

$$\delta H^2(>k) = A^2 H_0^2 (kL)^{-1+m}, \qquad kL \geqq 1, \qquad m = \text{const}, \tag{4}$$

with constant $A^2 \leqq 1$ characterizing a relative value of the random field on the characteristic scale L, one can use the approximate expressions

$$D_{ll} = lv/3, \qquad D_\perp = gA^4 lv/3, \qquad D_A = -r_{H_0} v/3. \tag{5}$$

The numerical coefficients here are calculated for $A^2 \ll 1$; $v \cong c$ is the particle velocity; the constant $g \cong 0.1$–0.5 is not well-determined in the modern theory; the mean free path is estimated as

$$l = A^{-2} L (r_H/L)^m, \qquad r_H \leqq L; \tag{6}$$

$$l = A^{-2} L^{-1} r_{H_0}^2, \qquad r_{H_0} > L. \tag{7}$$

The sign of D_A in Eq. (5) corresponds to the diffusion of positively-charged particles $Z>0$. It changes sign for $Z<0$. Global diffusion $D_\perp$ perpendicular to the mean (regular) field is mainly determined by some projection of the local field-aligned diffusion coefficient. Detailed calculations of the diffusion tensor D_{ij} are given by Beresinskii et al. (1990) and by Chuvilgin & Ptuskin (1990).

The observational data, supported by theoretical considerations, show that the real spectrum of the galactic random magnetic field is possibly a power-law spectrum (4) with exponent $m = 0.3 \pm 0.3$, see Armstrong et al. (1981) and Ruzmaikin et al. (1988). The value $m = 1/3$ corresponds to a Kolmogorov spectrum, $m = 1/2$ to a Kraichnan (1965) hydromagnetic spectrum, and $m = 0$ to an ensemble of shocks and discontinuities are given, for example, by the model of interstellar turbulence by Bykov & Toptygin (1987).

Equations (6), (7) show that diffusion retainment of cosmic rays in the Galaxy may be effective up to a gyroradius $r_H \leqq L$, or up to a maximum energy of about

$$E_{max} \cong 2\ 10^{17} Z\ (H/2\ 10^{-6}\,\text{Oe})\ (L/100\,\text{pc})\ \text{eV} \tag{8}$$

at which the gyroradius becomes equel to the characteristic scale of the random field. At $E > E_{max}$ ($r_H > L$), the scattering effectiveness decreases as E^{-2}. However, the value $L = 100$ pc used above has been inferred from the observational data relating to the vicinity of about 500 pc from the Galactic plane (where the rotation measure for distant radio sources is mainly accumulated). The development of convective and Parker instabilities may give rise to the occurrence of random fields on scales of 1–3 kpc in the galactic halo. Therefore, even if the field intensity decrease at high latitudes is allowed for, the diffusion with mean free path (6) may be assumed to take place in the Galaxy as the whole up to energies of about $E_{max} \cong 1$–$3\ 10^{18}$ eV.

The global structure of the regular magnetic field in the Galaxy is very important, since it determinates the orientation of the diffusion tensor (2). We have reliable information on the regular field in the region of the galactic disk (see e.g. Heiles 1987, Ruzmaikin et al. 1988). The regular field in the disk is predominantly toroidal. Thus, H_φ is the main component of the regular field in galactic cylindrical coordinates (subscript 0 marking the regular field is omitted here). The field may have the form of a bisymmetric spiral with typical ratios of radial and perpendicular components to the azimuthal component on the order of

$$H_r/H_\varphi \cong 0.1\text{–}0.3; \; H_z/H_\varphi \cong 10^{-3}\text{–}3\ 10^{-2}. \tag{9}$$

Actually, observations are in agreement with the assumption that the field lines of the regular field are closed circles and that the field $H_\varphi(r)$ changes sign with radial distance r after each 3 kpc (Rand & Kulkarni 1989). The regular magnetic field has the same direction above and below the galactic plane.

We have scant information on the magnetic field beyond the galactic disk, i.e. at distances $|z| > 500$ pc. The Galaxy posseses an extended magnetic corona of size 3–10 kpc (Sofue et al. 1986; Broadbent et al. 1990; Beck 1990). The orientation of the regular field in the Galactic halo is not known, but the toroidal component H_φ dominates to all appearances.

The galactic wind model (Jokipii & Morfill 1986; Breitschwerdt et al. 1987) predicts a structure of the regular galactic field at large distances from the galactic centre that is simular to the Parker spiral in interplanetary space. The field has asymptotic behavior $H_\varphi \propto 1/r$ in this case. The field is blown up from the galactic disk to the halo, and perhaps it conserves structure with interchanging of sign of H_φ. We do not take into account possible advection of cosmic rays by galactic wind flow in this paper.

A regular magnetic field in the halo could come into existence independently of the disk field (Sokoloff & Shukurov 1990). A dynamo mechanism of field generation works if there are differential rotation and turbulent gas motions with non-zero helicity in the halo. The predominance of magnetic fields parallel to the disk is inherent to dynamo generation in spherical bodies (Krause & Radler 1980). An important feature of this field is its nonparity $H_\varphi(z) = -H_\varphi(-z)$, i.e. the regular field in the halo has opposite directions above and below the galactic disk. The expected ratio of poloidal to toroidal components of the field is $H_p/H_\varphi \cong 0.1$ on the average. (It is determined by the small ratio of the characteristic scale of turbulent motions to the height of the halo).

Under the assumption of azimuthal symmetry of the system $(\frac{\partial}{\partial \Phi} = 0)$, one can write Eq. (1) as

$$\begin{aligned} \Big\{ &-\frac{\partial}{\partial r} r (D_{ll} b_r^2 + (1-b_r^2) D_r) \frac{\partial}{\partial r} \\ &- \frac{\partial}{\partial z} (b_z^2 D_{ll} + (1-b_z^2) D_\perp) \frac{\partial}{\partial z} - \frac{1}{r} \frac{\partial}{\partial r} r b_r b_z (D_{ll} - D_\perp) \frac{\partial}{\partial z} \\ &- \frac{\partial}{\partial z} b_z b_r (D_{ll} - D_\perp) \frac{\partial}{\partial r} - \frac{\partial}{\partial z} (D_A b_\varphi) \frac{\partial}{\partial r} \\ &+ \frac{1}{r} \frac{\partial}{\partial r} (r D_A b_\varphi) \frac{\partial}{\partial_z} \Big\} N(r,z) = Q(r,z). \end{aligned} \tag{10}$$

Further simplification is achieved if one takes into account the dominance of the toroidal field component ($b_z/b_\varphi < 0.1$ and $b_r/b_\varphi < 0.1$) and uses expressions (5) for the diffusion coefficients D_{II} and $D_\perp$ ($D_\perp/D_{II} > 3\ 10^{-2}$). Then one can set $b_\varphi = 1$, $b_r = b_z = 0$, and Eq. (10) reduces to:

$$\left\{-\frac{1}{r}\frac{\partial}{\partial r} r D_\perp \frac{\partial}{\partial r} - \frac{\partial}{\partial z} D_\perp \frac{\partial}{\partial z} - \frac{\partial}{\partial z}(D_A)\frac{\partial}{\partial r} + \frac{1}{r}\frac{\partial}{\partial r}(rD_A)\frac{\partial}{\partial z}\right\} \cdot N(r,z) = Q(r,z). \tag{11}$$

We use Eq. (11) to study cosmic ray diffusion in the Galaxy. The components of the cosmic ray anisotropy vector (3) are given now by:

$$\delta_s = 3 j_s / vN, \quad j_r = -D_\perp \frac{\partial N}{\partial r} + D_A \frac{\partial N}{\partial z},$$

$$j_z = -D_A \frac{\partial N}{\partial r} - D_\perp \frac{\partial N}{\partial z}, \quad j_\varphi = 0. \tag{12}$$

As mentioned in the Introduction, Hall diffusion may be treated as a particle drift. Let us separate the symmetric and antisymmetric parts in the diffusion tensor (2)

$$D_{ij} = D_{Sij} + D_A e_{ijn} b_n, \tag{13}$$

where $D_{Sij} = (D_{II} - D_\perp)\, b_i b_j + D_\perp \delta_{ij}$,

and write the basic equation in the form

$$-\nabla_i D_{Sij} \nabla_j N + u_i \nabla_i N = Q. \tag{14}$$

Here

$$u_j = -\nabla_i (D_A e_{ijn} b_n) = \frac{1}{3}\frac{pcv}{Ze}\,\mathrm{rot}_j\left(\frac{H}{H^2}\right) \tag{15}$$

is called the drift velocity. A general discussion of the concept of drift for the description of cosmic ray propagation is given by Burger et al. (1985).

Thus, cosmic ray diffusion with the diffusion tensor (2) may be described as diffusion with a symmetrical tensor D_{Sij} (13) and advective transport with drift velocity u_i (15).

The simplified Eq. (11) also contains drift terms, determining the drift in radius and in the z-coordinate with velocities

$$u_r = -\frac{\partial}{\partial z} D_A, \; u_z = \frac{1}{r}\frac{\partial}{\partial r}(r D_A). \tag{16}$$

It is worth noting that one should use the concept of cosmic ray advection with drift velocity u_i with some care. The expressions (3), (12) for cosmic ray flux do not have a term of type $u_i N$, as would be the case for particle transport by the flow of a real medium moving with velocity u_i.

3. Flat halo model

We assume that the propagation region of the cosmic rays in the Galaxy has the form of a cylinder of radius R and height $2h$. The cosmic ray sources are distributed in a thin disc of thickness $2h_s$. Supposing that $h_s \ll h$, we use the delta approximation $Q(r,z) = 2h_s q(r)\delta(z)$ for the source distribution in the z-coordinate. At the boundaries of the halo (on the surface Σ). The particles can escape freely into intergalactic space, where the density of cosmic rays is negligibly small, i.e. $N|_\Sigma = 0$.

We consider models with strongly flattened halo $h \ll R$ in this section. One can neglect radial diffusion as compared with perpendicular diffusion in this case. Thus, we can omit the first term in Eq. (11) and obtain the transport equation in the form

$$\left(-\frac{\partial}{\partial z} D_\perp \frac{\partial}{\partial z} - \left(\frac{\partial D_A}{\partial z}\right)\frac{\partial}{\partial r} + \frac{1}{r}\frac{\partial (rD_A)}{\partial r}\frac{\partial}{\partial z}\right) \cdot N(r,z) = 2h_s\, q(r)\, \delta(z) \tag{17}$$

For further simplification of the solution, we introduce the special radial dependence

$$D_A(r) = D_{A0}(z)\frac{r}{r_0}, \; D_\perp(r,z) = \mathrm{const}, \tag{18}$$

where $D_{A0}(z)$ and r_0 do not depend on radius r. The perpendicular component of the drift velocity (16) u_z does not depend on r in this case.

Consider now two realizations of the flat halo model with different structures of the regular magnetic field.

3.1. Case A

The model with symmetric field $H_\varphi(z) = H_\varphi(-z)$.

Suppose that the regular field H_φ has the same sign in the entire system, see Fig. 1. We set $D_{A0}(z) = \mathrm{const}$ in the formula (18). Then

$$u_r = 0, \; u_z = 2D_{A0}/r_0 = \mathrm{const}, \tag{19}$$

and the Eq. (17) is reduced to the following

$$-D_\perp \frac{\partial^2 N}{\partial z^2} + \frac{2}{r_0} D_{A0} \frac{\partial N}{\partial z} = 2h_s\, q(r)\, \delta(z). \tag{20}$$

The solution of this equation with the boundary condition $N(|z| = h) = 0$ is

$$N(r,z) = \frac{h_s r_0 q(r)}{2D_{A0}} \frac{\exp(2hD_{A0}/r_0 D_\perp) - 1}{\mathrm{sh}\,(2hD_{A0}/r_0 D_\perp)} (1 - \exp(2(z-h)\, D_{A0}/r_0 D_\perp)), \; z \geqq 0;$$

$$N(r,z) = \frac{h_s r_0 q(r)}{2D_{A0}\,\mathrm{sh}(2hD_{A0}/r_0 D_\perp)} \cdot (\exp(-2hD_{A0}/r_0 D_\perp) - 1) (1 - \exp(2(z+h)\, D_{A0}/r_0 D_\perp)), \quad z < 0. \tag{21}$$

In accordance with the definition (2), $D_{A0} > 0$ for $ZH_\varphi < 0$, and $D_{A0} < 0$ for $ZH_\varphi > 0$ (Z is particle charge).

The spatial distribution $N(z)$, described by Eq. (21), is shown in Fig. 2. The presence of drift leads to the obvious modification of the usual linear fall of cosmic ray density towards the boundaries of the halo, inherent to diffusion without drift.

For the observer at the central plane $z = 0$, Eq. (21) gives

$$N(r, z=0) = \frac{q(r)\, h_s r_0}{D_{A0}}\,\mathrm{th}\left(\frac{hD_{A0}}{r_0 D_\perp}\right). \tag{22}$$

For a power law energy spectrum at the sources $q \propto E^{-\gamma_s}$, and for power law dependencies $D_\perp \propto E^m$, $D_A \propto E$ (see Eqs. (5), (6)), we find from Eq. (22) the asymptotic behavior $N(E) \propto E^{-\gamma_s - m}$ for $hD_{A0}/r_0 D_\perp \ll 1$ and $N(E) \propto E^{-\gamma_s - 1}$ for $hD_{A0}/r_0 D_\perp \gg 1$. The break in the spectrum $\Delta\gamma = 1 - m$ occurs at the energy E_k, corresponding to the fulfillment of the condition $hD_{A0}/r_0 D_A \cong 1$ (see Fig. 3). For smaller energies ($E \ll E_k$), cosmic ray leakage out of the Galaxy is determined by the usual diffusion $D_\perp$. For higher energies ($E \gg E_k$) leakage is determined by the Hall diffusion D_A (by drift).

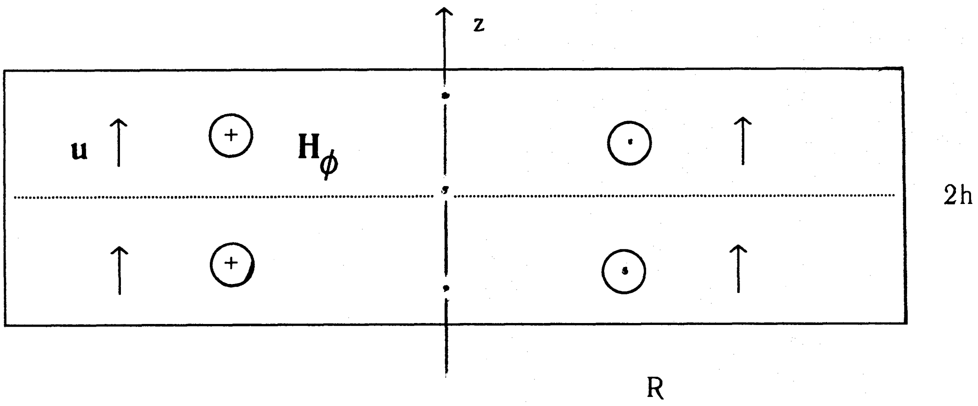

Fig. 1. Propagation region of cosmic rays in the flat halo model with symmetric field $H_\varphi(z)=H_\varphi(-z)$. The directions of magnetic field and drift velocity are shown

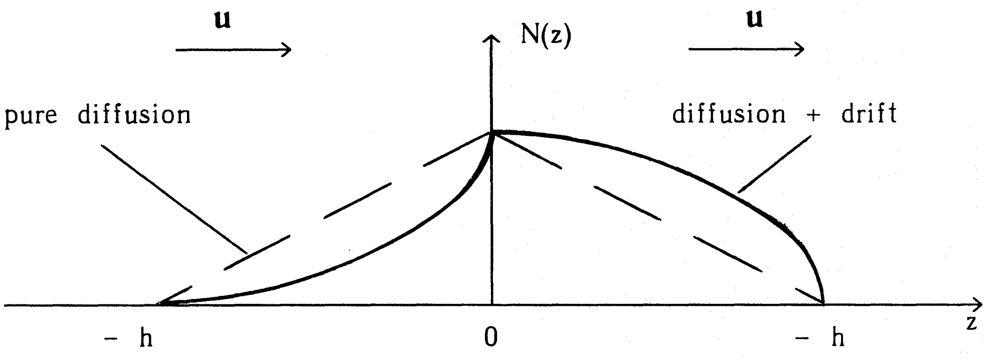

Fig. 2. Spatial distribution of cosmic ray density in the flat halo model with symmetric magnetic field

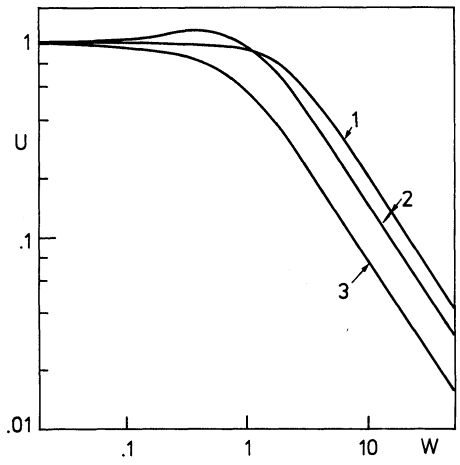

Fig. 3. Normalized differential energy spectrum of particles $U=w^\gamma N_0(w)$ (where $\gamma=\gamma_s+m$) as a function of parameter $w=2h|D_{A0}|/(r_0 D_\perp)$ in the flat halo models with symmetric magnetic field (the curve 1) and antisymmetric field (the curve 2 for $ZH_\varphi(z>0)<0$, and the curve 3 for $ZH_\varphi(z>0)>0$)

3.2. Case B

The model with antisymmetric field $H_\varphi(z)=-H_\varphi(-z)$.

Let us assume that regular field H_φ changes sign at the plane $z=0$, i.e. the field has opposite directions above and below the plane, see Fig. 4. The Hall diffusion coefficient has the form $D_A=D_{A0}\dfrac{r}{r_0}\,\mathrm{sign}(z)$. Then the drift velocity is

$$u_r=-D_{A0}\frac{2r}{r_0}\,\delta(z),\qquad u_z=D_{A0}\frac{2}{r_0}\,\mathrm{sign}(z). \tag{23}$$

The radial drift velocity u_r (23) does not vanish only at the central plane $z=0$. The cosmic ray transport Eq. (17) now has the form

$$-D_\perp\frac{\partial^2 N}{\partial z^2}-D_{A0}\frac{2r}{r_0}\,\delta(z)\,\frac{\partial N}{\partial r}+D_{A0}\frac{2}{r_0}\,\mathrm{sign}(z)\,\frac{\partial N}{\partial z}=2h_s\,q(r)\,\delta(z). \tag{24}$$

This may be equivalently presented as the equation for cosmic ray density in the region $0<z<<h$ above the central plane

$$-D_\perp\frac{\partial^2 N}{\partial z^2}+D_{A0}\frac{2}{r_0}\,\mathrm{sign}(z)\,\frac{\partial N}{\partial z}=0, \tag{25}$$

supplemented with the following condition at $z=0$

$$\left.\left(-D_\perp\frac{\partial N}{\partial z}-D_{A0}\frac{r}{r_0}\frac{\partial N}{\partial r}\right)\right\}_{z=+0}=h_s\,q(r), \tag{26}$$

and with condition $N(z)=N(-z)$, which determines the cosmic ray density at $-h<z<0$ below the central plane.

The solution of Eq. (24) [or (25)–(26)] is

$$N(r,z)=\frac{h_s r_0}{D_{A0}}\,(1-\exp(-w(1-|z|/h)))\,(1-e^{-w})^{-1}\int_1^{R/r}\mathrm{d}y\,q(yr)\,y^{-(1+2(e^w-1)^{-1})},$$

for $D_{A0}\geqq 0$ $(ZH_\varphi(z>0)<0)$;

$$N(r,z)=\frac{h_s r_0}{|D_{A0}|}\,(1-\exp(w(1-|z|/h)))\,(1-e^{w})^{-1}\int_0^{1}\mathrm{d}y\;q(yr)\;y^{-1+2(1-e^{-w})^{-1}}, \tag{27}$$

for $D_{A0}<0$ $(ZH_\varphi(z>0)>0)$,

where $w=2h|D_{A0}|/(r_0 D_\perp)$.

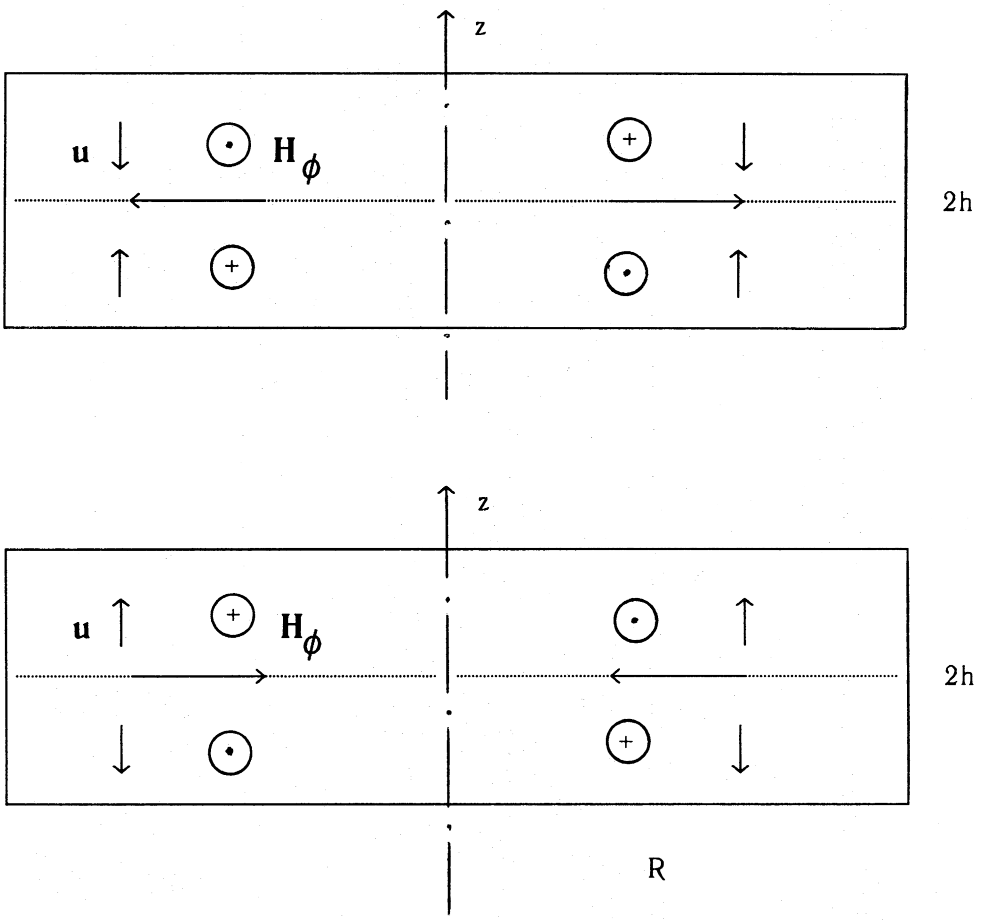

Fig. 4. Propagation region of cosmic rays in the flat halo model with antisymmetric field $H_\varphi(z) = -H_\varphi(-z)$

The expressions (27) are illustrated by Figs. 3 and 5. It shows the steepening of the energy spectrum at energy E_k, where parameter $D_{A0}\,h/D_\perp r_0 \cong 0.2$. Curve 2, corresponding to $H_\varphi(z>0)<0$ in Fig. 3, displays some flattening of the energy spectrum before the steepening at E_k. This effect is not strong (for $s \leqq 2$), but it is in qualitative agreement with the observational data, see e.g. Fichtel & Linsley (1986).

Cosmic rays contain nuclei with different charges Z. The predicted form of the spectrum for each kind of nucleus is universal, if all source spectra have the same exponent γ_s. (Remember that we do not account for nuclear fragmentation and possible energy losses.) This universal spectrum depends on the particle magnetic rigidity, or on the energy per unit charge E/Z for ultra-relativistic particles. Superposition of the spectra on the total energy E for the entire set of cosmic-ray nuclei shows the steepening also, but not so sharply as for a single type of nucleus. The total cosmic ray spectrum expected in the model under consideration is shown in Fig. 6. We adopt the chemical composition at the sources given by Meyer (1985) and do not take into account possible influence of nuclear fragmentation for very high energy cosmic rays.

Let us now estimate the value of the cosmic ray diffusion coefficient. We choose curve 2 of Fig. 3 as describing the form of the energy spectrum for one kind of nucleus.

The measurements of the abundances of secondary nuclei in cosmic rays at energies of a few GeV per nucleon gives the thickness of matter $x_l \cong 10\,\mathrm{g\,cm^{-2}}$ traversed by particles in the interstellar gas (see e.g. Engelmann et al. 1990). Hall diffusion is not effective at such low energies. The relation of the value of x_l with the parameters of the flat halo diffusion model is (Berezinskii et al. 1990).

$$x_l = \varrho v h_g h/D_\perp, \tag{28}$$

where ϱ is the mass density of the gas in the galactic gas disk and h_g the height of the gas disk. The gas density in the halo is set to zero. For the number density of the gas $n = 1\,\mathrm{cm^{-3}}$ and for the height $h_g = 100\,\mathrm{pc}$, one can find from Eq. (28)

$$\begin{aligned} D_\perp &= \frac{\varrho v h_g h}{x_l} \\ &= 1.4\ 10^{28}\ (E/3 10^9\,\mathrm{eV})^m\ (h/3\,\mathrm{kpc})\,\mathrm{cm^2\,s^{-1}}. \end{aligned} \tag{29}$$

for a typical halo size $h = 3\,\mathrm{kpc}$.

The exponent m may be determined empirically from the dependence of the matter thickness $x_l(E)$ on energy. The interpretation of corresponding data depends on the effectiveness of cosmic ray reacceleration in the interstellar medium (Simon et al. 1986; Cesarsky 1987). This leads to the wide range of possible values of $m = 0.3 \pm 0.3$. Direct measurements of secondary B-nuclei at the highest available energies $\cong 1\,\mathrm{TeV}/n$ (Swordy et al. 1990) give $x_l(1\,\mathrm{TeV}) \cong 1$–$2\,\mathrm{g\,cm^{-2}}$, which implies $m \cong 0.2$–0.3 without reacceleration taken into account (reacceleration decreases the value of m). Swordy et al. (1990) suggest $m = 0.6$ as compatible with all observations.

The value $m = 0.3 \pm 0.3$ does not contradict the theory of cosmic ray scattering in a random magnetic field, combined with

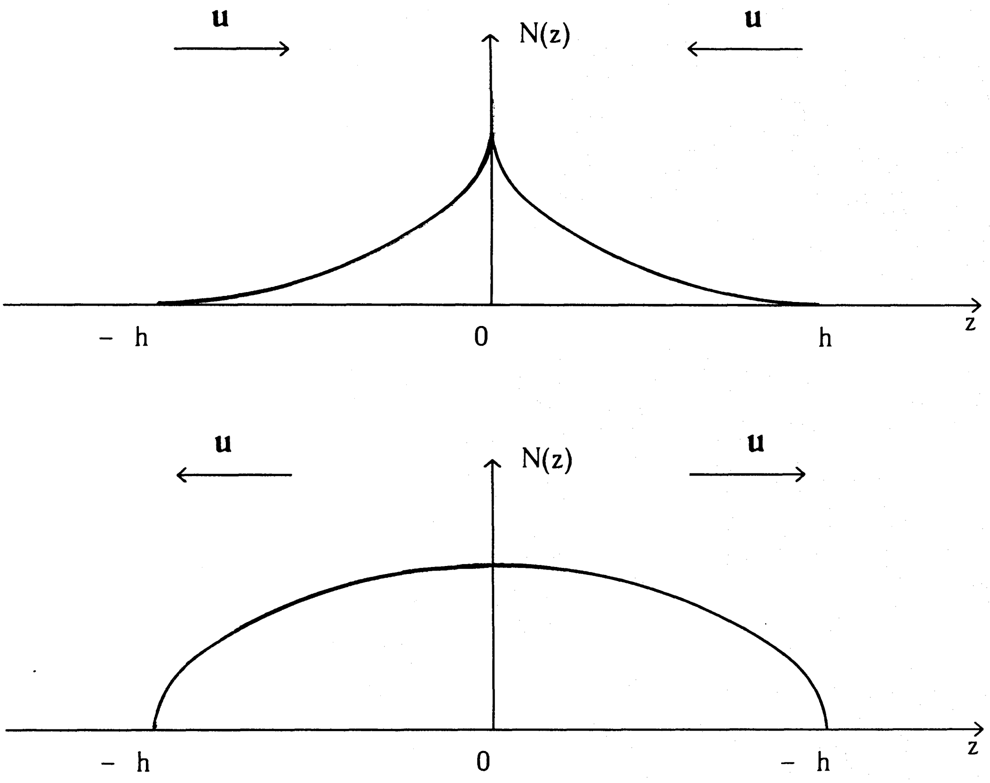

Fig. 5. Spatial distributions of cosmic ray density in the flat halo models with antisymmetric magnetic field

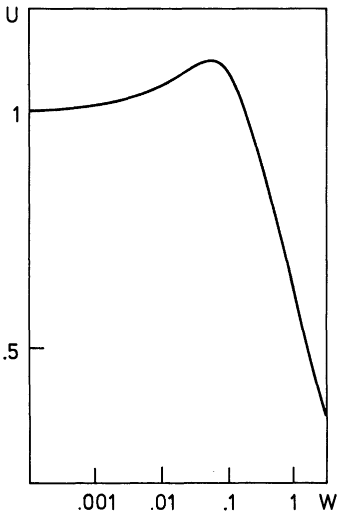

Fig. 6. The total cosmic ray spectrum corresponding to the individual nucleus spectrum 2 in Fig. 3, and with chemical composition at the sources given by Meyer (1985)

the data on the turbulence spectrum in the interstellar medium, see the Eq. (6) and subsequent discussion.

Having in mind the large spread of values allowed for m over the range of energies 10^9 eV–10^{17} eV, we take m as a free parameter in formula (8).

The value of the Hall diffusion coefficient is

$$D_A = \frac{r_{H0} V}{3} = 8.8\ 10^{22}\ (E/3\ 10^9\ \mathrm{eV})\ (10^{-6}\ \mathrm{Oe}/\mathrm{H}_0)\ \mathrm{cm}^2\,\mathrm{s}^{-1}. \quad (30)$$

if the magnetic field in the galactic halo is $H_0 = 10^{-6}$ Oe.

Calculations, illustrated by curve 2 in Fig. 3, show the position of the knee at $D_{A0}\, h/D_{\perp}\, r_0 \cong 0.2$. Then, supposing that the knee corresponds to the energy $E_k = 3\ 10^{15}$ eV and choosing $r_0 = 10$ kpc, we find $m = 0.16$ from Eqs. (29), (30). The exponent m increases with decreasing H_0 in Eq. (30), or with decreasing value of $\varrho\, h_g$ in Eq. (29).

The break of the energy spectrum at the knee is now $\Delta\gamma = 1 - m \cong 0.8$ for one kind of nuclei, and $\Delta\gamma \cong 0.4$ for the total spectrum including all kinds of nuclei.

According to Eqs. (5), (6), the diffusion coefficient (29) with $m = 0.16$, $L = 100$ pc implies $gA^2 \cong 0.08$. At $A^2 = 1$, this gives

$$D_{II} = D_{\perp}/g A^4$$
$$= 1.8\ 10^{29}\ (E/3\ 10^9\ \mathrm{eV})^{0.16}\ (h/3\ \mathrm{kpc})\ \mathrm{cm}^2\,\mathrm{s}^{-1} \quad (31)$$

for the diffusion coefficient along the mean magnetic field.

The relatively small value of the exponent $m = 0.16$ results in the weak energy dependence of the grammage $x_l \propto E^{-0.16}$ ($E \gg Mc^2$). This dependence is not in contradiction with the data on the secondary nuclei if one assumes the weak reacceleration of cosmic rays in the interstellar medium. It implies the source energy spectrum of high energy cosmic rays close to $E^{-2.5}$.

4. The model with large halo

Some radio astronomical data indicate that our Galaxy has a rather extended halo, with $h \cong 10$ kpc (Broadbent et al. 1990). We consider here cosmic ray diffusion and drift in the model with halo size h comparable with the galactic radius $R = 20$ kpc. Let cosmic ray sources be distributed in the disk with thickness $2h_s = 400$ pc. Cosmic ray transport is described by Eq. (11). We suppose that the regular magnetic field H_φ has constant sign in the region $h_m \leqq z \leqq h$ and the opposite sign for $-h \leqq z \leqq h_m$, $h_m = 500$ pc (see

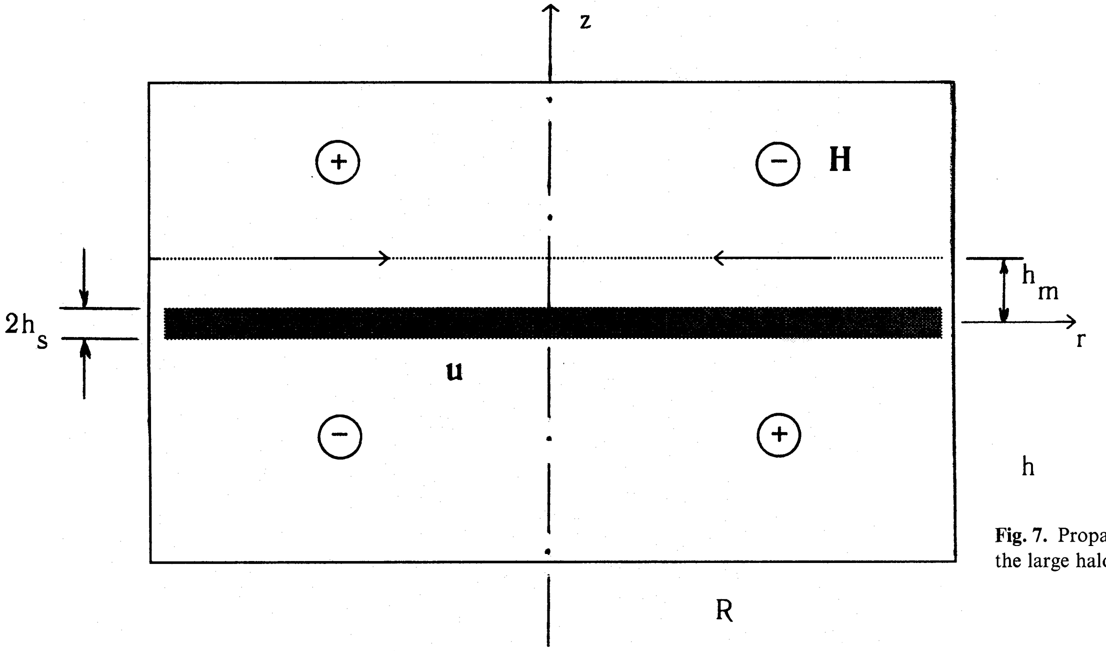

Fig. 7. Propagation region of cosmic rays in the large halo model

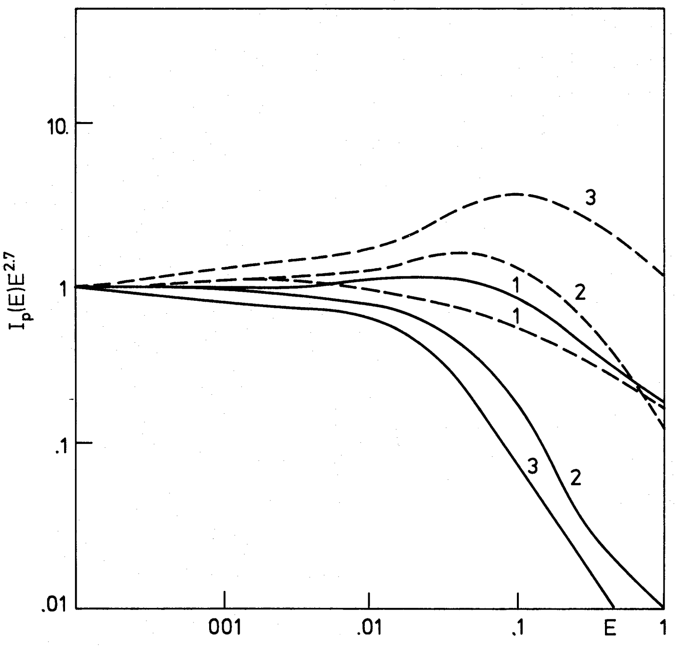

Fig. 8. Proton spectra near the knee in the large halo model for $H_\varphi(z=0)>0$ (solid curves) and $H_\varphi(z=0)<0$ (dashed curves). Energy E is given in arbitrary units. The following radial distributions of the sources are chosen: (1) $q(r)=\text{const}$, (2) $q(r)\propto\delta(r-6\,\text{kpc})$, (3) $q(r)\propto\delta(r-3\,\text{kpc})$

Fig. 7). This simplified configuration corresponds roughly to dynamo generation of an antisymmetric regular field H_φ in a large halo and independent generation of the symmetric field H_φ in the disk $|z|\leqq h_m$, see Sokoloff & Shukurov (1990). We assume that the spatial dependence of diffusion is

$$D_\perp \propto |D_A| \propto \exp\left(\frac{z^2}{z_0^2}+\frac{r^2}{r_0^2}\right) \tag{32}$$

with constants z_0 and r_0.

The cosmic ray transport equation (11) has now the form

$$\begin{aligned}&-\frac{\partial^2 N}{\partial r^2}-\frac{\partial^2 N}{\partial z^2}\\ &+\left[\frac{1}{r}+\frac{2r}{r_0}+\frac{2za}{z_0^2}\,\text{sign}(z-h_m)+2a\,\delta(z-h_m)\right]\frac{\partial N}{\partial r}\\ &+\left[-\frac{2z}{z_0^2}+\frac{a}{r}\,\text{sign}(z-h_m)+\frac{2ra}{r_0^2}\,\text{sign}(z-h_m)\right]\frac{\partial N}{\partial z}\\ &=\frac{q(r)\,\vartheta(h_s-|z|)}{D_{\perp 0}},\end{aligned} \tag{33}$$

where $a=D_A(z>h_m)/D_\perp$, $D_{\perp 0}=D_\perp(r=0,z=0)$, and $\vartheta(z)$ is the step function.

The results of numerical solution of the Eq. (33) for $h=10$ kpc, $z_0=5$ kpc, $r_0=10$ kpc are shown in Fig. 8, 9. The Crank-Nicolson approximation was used to discretize the corresponding equations. The resulting system of difference equations was solved by the Liebmann method. Figure 8 illustrates the dependence of the form of particle spectrum on the spatial distribution of cosmic ray sources in the source disk. The position of the observer is at $r=10$ kpc, $z=0$. Figure 9 shows the fit of the model to the real measurements of the cosmic ray spectrum in the vicinity of the knee.

Now we find the diffusion coefficient $D_\perp$ in the large halo model using the data on the matter thickness x_l. Taking into account the inequalities $h_g \ll h$, $h_s \ll h$, the source distribution on z-coordinate may be described approximately the delta function. Then, the transport diffusion equation for cosmic ray nuclei in the GeV energy region (where $D_A \cong 0$) is

$$\left[-\frac{1}{r}\frac{\partial}{\partial r}rD_\perp\frac{\partial}{\partial r}-\frac{\partial}{\partial z}D_\perp\frac{\partial}{\partial z}+2nv\sigma h_s\delta(z)\right] \cdot N(r,z)=2h_s q(r)\,\delta(z). \tag{34}$$

Here σ is the fragmentation cross section of cosmic ray nuclei in the interstellar gas.

After integration of Eq. (34) with respect to z in the vicinity of the plane $z=0$

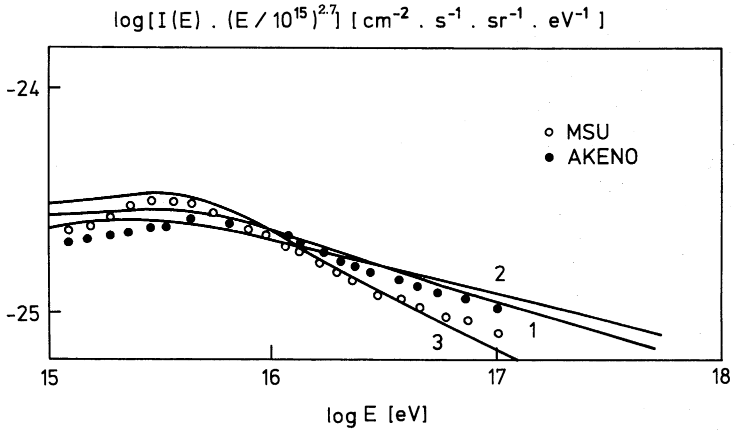

Fig. 9. Cosmic ray spectra with mixed chemical composition at the sources. Curve 1: the flat halo model as in the Fig. 6. Curve 2: the large halo model with $H_\varphi(z=0)<0$, $q(r)\propto\delta(r-6\,\mathrm{kpc})$ for all kinds of nuclei. Curve 3: the large halo model with $H_\varphi(z=0)>0$, $q(r)=\mathrm{const}$ for protons and $q(r)\propto\delta(r-6\mathrm{kpc})$ for heavy nuclei. Akeno and Moscow State University data are shown according to Fomin et al. (1991)

$$\lim_{\varepsilon\Rightarrow 0}\int_{-\varepsilon}^{\varepsilon} dz(\ldots), \tag{35}$$

we find the relation at $z=0$

$$-D_{\perp 0}\frac{\partial N_0}{\partial z}+n v\sigma h_{\mathrm{g}} N_0 = h_{\mathrm{s}} q(r), \tag{36}$$

where $N_0=N(r;z=0)$; $\frac{\partial N_0}{\partial z}=\left.\frac{\partial N}{\partial z}\right|_{z=+0}$, $D_{\perp 0}=D_\perp(r;z=0)$.

Hence

$$N_0 = \frac{q(r)\,h_{\mathrm{s}} h_{\mathrm{eff}}}{D_{\perp 0}}\,(1+\sigma n v h_{\mathrm{g}} h_{\mathrm{eff}}/D_{\perp 0})^{-1}, \tag{37}$$

where

$$h_{\mathrm{eff}}^{-1} = -\frac{1}{N_0}\frac{\partial N_0}{\partial z}. \tag{38}$$

The expression (37) has the same form as that for cosmic ray density in the leaky-box model (T_l is the leakage time)

$$N=Q T_l(1+\sigma x_l)^{-1}, \tag{39}$$

see Berezinskii et al. (1990) for discussion.

Thus, for calculations of fragmentation of the stable nuclei, our model is equivalent to the leaky box with the mean matter thickness traversed by cosmic rays

$$x_l=\varrho v h_{\mathrm{g}} h_{\mathrm{eff}}/D_{\perp 0} \tag{40}$$

(compare with Eq. (28)).

The value of h_{eff} describing the cosmic-ray gradient near the galactic plane is determined numerically by a solution of Eq. (33). The diffusion coefficient $D_{\perp 0}$ in the GeV energy range may then be found from Eq. (40).

For the case of uniform source distribution $q(r)=\mathrm{const}$, the diffusion coefficient $D_\perp$ turns out to be

$$D_{\perp 0}(r=10\,\mathrm{kpc})=1.8\;10^{28}\;(E/3\;10^9\,\mathrm{eV})^{0.18}\,\mathrm{cm}^2\,\mathrm{s}^{-1} \tag{41}$$

($a>0$, $H_0=10^{-6}$ Oe).

This value is approximately equal to the diffusion coefficient (29) found for the flat halo model with constant $D_\perp$ and with halo size $h=4$ kpc. The corresponding value of the mean field-aligned diffusion coefficient is [for $L=100$ pc, $A^2=1$; see Eqs. (5), (6)]

$$D_{\parallel 0}=2.0\;10^{29}\;(E/3\;10^9\,\mathrm{eV})^{0.18}\,\mathrm{cm}^2\,\mathrm{s}^{-1}, \tag{42}$$

5. Local anisotropy of cosmic rays

The complicated nature of cosmic ray propagation at energies $E\lesseqq 10^{17}$ eV may be roughly presented simply as the superposition of the diffusion of magnetized particles mainly along the magnetic field lines and of the random wandering of the magnetic field lines themselves. The transport equation (1) describes the global diffusion. The quantities $N(r)$ and $D_{ij}(r)$ which figure in Eq. (1) are defined as the mean cosmic-ray density and the mean diffusion tensor, averaged over scales greater than the characteristic scale of the random field $L\cong 100$ pc. Spatial variations of cosmic-ray density about the mean value are not larger, and local measurements at the Earth may be considered as representative of the mean density of cosmic rays in the vicinity of about few hundreds parsecs, but the yield of local galactic cosmic-ray sources which are closer than a few hundreds parsecs must be calculated individually. However, local cosmic ray anisotropy may deviate greatly from the mean anisotropy $\delta_i(r)$ defined by Eq. (3). The deviation is due to the large difference between the mean and local diffusion tensors.

Large-scale wandering of magnetic field lines results in isotropization of the local diffusion tensor. For approximate calculation of the local anistropy, one can suppose that local diffusion is one-dimensional (along the local magnetic field) with the mean free path given by Eq. (6). The local cosmic ray density and the local cosmic ray gradient may be set equal to the averaged values N and $G_i=N^{-1}\nabla_i N$, respectively. Thus, cosmic ray anisotropy a_i measured at the Earth is calculated according to the formula

$$a_i=-l e_i e_j G_j, \tag{43}$$

where e_i is the unit vector directed along the local magnetic field.

The absolute value of the anisotropy vector (43) is

$$a=|-l e_j G_j| = l|e_z G_z + e_r G_r|, \tag{44}$$

($G_\varphi=0$ in our model). The anisotropy is directed along the local magnetic field.

The maximal possible value of anisotropy $a_{\max}=lG$ at each energy E is achieved when the cosmic ray gradient $G_i(E)$ is parallel to the local field direction e_i. Corresponding values are presented in Table 1.

The direction of the magnetic field on a scale of the order of 100 pc from the Sun is given by observations of polarization of the optical starlight. The data by Mathewson and Ford (1970)

Table 1. Maximal local anisotropy of cosmic rays a_{max} ($z = \pm 50$ pc is the distance of an observer above or below the galactic midplane)

Energy (eV)	Flat halo model	Large halo model $z = +50$ pc	Large halo model $z = -50$ pc
$3\ 10^{12}$	$4.7\ 10^{-3}$	$5.1\ 10^{-3}$	$5.1\ 10^{-3}$
$3\ 10^{15}$	$1.1\ 10^{-2}$	$1.5\ 10^{-2}$	$9.8\ 10^{-3}$
10^{17}	$1.1\ 10^{-4}$	$1.1\ 10^{-1}$	$5.3\ 10^{-3}$

indicate the direction $l^{II} = 50°$, $|b^{II}| \leqq 10°$. The uncertainty in the determination of the latitude b is essential. We calculate the cosmic ray anisotropy for the two cases: $b^{II} = \pm 10°$ and $b^{II} = 0°$. Anisotropy is determined for the position of the observer at $|z| = 50$ pc, above or below the midplane.

Table 2 illustrates the results of the calculations under the assumption of uniform source distribution $q(r) = \text{const}$. Versions of the flat halo model and of the model with large halo which provide the flattening of the cosmic ray spectrum before the knee are only taking into account in Tables 1, 2. The anisotropy in Table 2 is considerably smaller then a_{max} (Table 1), since the vector G_i is directed predominantly perpendicular to the galactic plane and the local magnetic field is supposed to lie primarily in the galactic plane.

We are not trying to give here a complete interpretation of measurements of cosmic ray anisotropy. This should include effects of local galactic cosmic ray sources (Dorman et al. 1985). Detailed structure of the magnetic field must be considered for calculation of the second angular harmonic (Kota 1975; Dorman et al. 1983; Ptuskin 1984; Nagashima et al. 1989). This problem will be discussed in a separate paper. Here it is sufficient to say that Tables 2 show the amplitudes of anisotropy that does not exceed the observational values, see Kifune (1990) for review. Relevant observational data give $a(10^{12}\text{–}10^{13}\,\text{eV}) \cong 5\ 10^{-4}\text{–}10^{-3}$ and $a(10^{17}\,\text{eV}) \cong 3\ 10^{-2}$.

6. Conclusion

A global toroidal magnetic field disturbs the random walk of cosmic ray particles in stochastic magnetic fields. This process may be described in terms of Hall diffusion D_A (or as systematic drift) superimposed on the usual diffusion D. At an energy of 10^9 eV, the Hall diffusion is insignificant and plays a minor role in cosmic ray leakage from the Galaxy. However, the Hall diffusion coefficient increases rapidly and directly with energy, and at energy $\cong 3\ 10^{15}$ eV may begin dominating the slowly-increasing usual diffusion. That is why the knee occurs in our model. A power-law generation spectrum with constant exponent is assumed for cosmic ray sources.

The calculations of the cosmic ray energy spectrum, matter thickness traversed by low-energy cosmic rays, and cosmic ray anisotropy have confirmed the realism of the model, but imply that the energy dependence of the diffusion should not be strong ($D \propto E^m$, $m = 0.15\text{–}0.20$) in the range of energies $E = 10^9\text{–}10^{17}$ eV on the whole. Analytical calculations were made for the case of a flat halo and numerical calculations were done for the case of a large halo. It is shown that an empirical diffusion model is in agreement with the "microscopic" theory of cosmic ray propagation in random and regular magnetic fields. This model does not contradict our knowledge of the structure of galactic magnetic fields. The highest proton energy at which the diffusion approximation is applicable in the Galaxy is $10^{17}\text{–}10^{18}$ eV. (Consideration of cosmic ray propagation at higher energies requires trajectory calculations, see Berezinskii et al. (1990), Smith & Clay (1990), Berenzinsky et al. (1991), Giler et al. (1991)).
The model exhibits another interesting feature. If the toroidal regular field in the halo is directed oppositely in the northern and southern galactic hemispheres, the spectrum may get flatter before the knee, seemingly in qualitative agreement with observations. It should be noted that this is the magnetic field a geometry to be expected if the regular field in quasi-spherical halo is generated by a dynamo mechanism, irrespective of the field in the disk.

The calculated cosmic ray anisotropy in our model is not in disagreement with observations. Detailed consideration of the problem of anisotropy of very high energy cosmic rays will be given in a separate paper.

Diffusion and drift of different cosmic ray nuclei at ultrarelativistic energies depend on the energy per unit charge E/Z. Thus the position of the knee for each kind of nucleus depends on E/Z. For the composite cosmic ray spectrum on the total energy E, one expects an increase of the fraction of heavy nuclei after the knee (at $E > E_k$). The observations give no definite conclusion on this subject now, but it is not excluded that the fraction of heavy nuclei decreases with energy at $E \geqq E_k$ (see e.g. Fichtel & Linsley 1986. The situation remains highly uncertain, Kifune (1990). To save the model in this case, we could suggest different spatial distributions of cosmic ray sources for protons and heavy nuclei, since the position of the knee and the form of the energy spectrum at $E \cong E_k$ depend on the source distribution $q(r)$, see Fig. 8. For example, the proton sources may be distributed approximately homogeneously and the heavy nuclei (iron) sources may be concentrated at distance $r = 4\text{–}6$ kpc, where the largest concentration of pulsars and supernovae resides.

Table 2. Local cosmic ray anisotropy a for different orientations of the local magnetic field ($l = 50°$; $b = 0°$; $\pm 10°$) and for different positions of an observer ($r = 10$ kpc; $z = \pm 50$ pc)

Energy (eV)	Flat halo model			Large halo model					
	$z = +50$ pc			$z = +50$ pc			$z = -50$ pc		
	$b = 0°$	$b = \pm 10°$	$b = -10°$	$b = 0°$	$b = +10°$	$b = -10°$	$b = 0°$	$b = +10°$	$b = -10°$
3 1012	$< 10^{-5}$	$7.8\ 10^{-4}$	$7.8\ 10^{-4}$	$3.6\ 10^{-4}$	$5.0\ 10^{-4}$	$1.2\ 10^{-3}$	$3.6\ 10^{-4}$	$5.0\ 10^{-4}$	$1.2\ 10^{-3}$
$3\ 10^{15}$	$8.0\ 10^{-4}$	$1.1\ 10^{-3}$	$2.8\ 10^{-3}$	$2.0\ 10^{-3}$	$5.0\ 10^{-4}$	$4.7\ 10^{-3}$	$2.0\ 10^{-3}$	$4.8\ 10^{-4}$	$3.7\ 10^{-3}$
10^{17}	$4.2\ 10^{-6}$	$1.6\ 10^{-5}$	$2.5\ 10^{-5}$	$3.3\ 10^{-3}$	$1.6\ 10^{-2}$	$2.3\ 10^{-2}$	$3.3\ 10^{-3}$	$3.2\ 10^{-3}$	$3.7\ 10^{-3}$

References

Axford W.I., 1991, Astrophys. Aspects of the Most Energetic Cosmic Rays, Nagano M., Takahara F. (ed.), World Scientific, p. 406
Bell A.R., 1991, 22 Intern. Cosmic Ray Conf., Dublin 2, 420
Berezinskii V.S., Bulanov S.V., Dogiel V.A., Ginzburg V.L., Ptuskin V.S., 1990, Astrophysics of Cosmic Rays, North-Holland, Amsterdam
Berezinsky V.S., Grigoryeva S.I., Mikhailov A.A., Rubinstein H., Ruzmaikin A.A., Sokoloff D.D., Shukurov A.M., 1991 (to be published)
Beck R., 1990, The Interstellar Disk-Halo Connection in Galaxies, Bloemen H. (ed.), Kluwer, Dordrecht, p. 267
Breitschwerdt D., McKenzie J.F., Volk H.J., 1987, Interstellar Magnetic Fields, Beck R., Grade R. (ed.). Springer-Verlag, Berlin Heidelberg New York, p. 131
Broadbent A., Haslam C.G.T., Osborne J.L., 1990, 21 Intern. Cosmic Ray Conf., Adelaide 3, 229
Burger R.A., Moraal H., Webb G., 1985, Ap&SS 116, 107
Bykov A.M., Toptygin I.N., 1987, Ap&SS 138, 341
Cesarsky C.J., 1987, 20 Intern. Cosmic Ray Conf., Moscow 8, 87
Chuvilgin L.G., Ptuskin V.S., 1990, Preprint IZMIRAN 49; 22 Intern. Cosmic Ray Conf., Dublin 2, 193
Dorman L.I., Fedorov Yu.I., Katz M.E., 1983, Ap&SS 93, 97
Dorman L.I., Ghosh A., Ptuskin V.S., 1985, Ap&SS 109, 87
Engelmann J.J., Ferrando P., Soutoul A., Garet P., Juliusson E., Koch-Miramond, Lund N., Masse P., Peters B., Petrou N., Rasmussen I.R., 1990, A&A 233, 96
Fichtel C.E., Linsley J., 1986, ApJ 300, 474
Fomin Yu.A., Khristiansen G.B., Kulikov G.V., Pogorely V.G., Solovjeva V.I., Sulakov P.V., Trubitsyn A.V., 1991, 22 Intern. Cosmic Ray Conf., Dublin OG-6.2.5
Giler M., Osborne J.L., Wdowczyk J., Zielinska M., 1991, 22 Intern. Cosmic Ray Conf., Dublin OG-6.3.10
Heiles C., 1987, Interstellar Processes, ed. Hollenbach D.J., Thronson H.A. (ed.), Reidel, Dordrecht, p. 171
Hillas A.M., 1979, 16 Intern. Cosmic Ray Conf., Kyoto 8, 7
Isenberg P.A., Jokipii J.R., 1979, ApJ 234, 746
Jokipii J.R., Morfill G.E., 1986, ApJ 312, 170
Karakula S., Nikolsky S.I., Stamenov J.N., Tkazyk W., 1990, 21 Intern. Cosmic Ray Conf., Adelaide 3, 169
Khristiansen G.B., 1987, 20 Intern. Cosmic Ray Conf., Moscow 8, 54
Kifune T., 1990, 21 Intern. Cosmic Ray Conf., Adelaide 11, 75
Kota J., 1975, J. Phys. A 8, 1349
Kraichnan R.H., 1965, Phys. Fluids 8, 1385
Krauze F., Radler K.-H., 1980, Mean-Field Magnetohydrodynamics and Dynamo Theory, Academic-Verlag
Kulikov G.V., Khristiansen G.B., 1959, Sov. Phys. JETP 35, 639
Mathewson D.C., Ford V.L., 1970, Mem. Roy. Astron. Soc. 74, 139
Meyer J.P., 1985, ApJS, 57, 173
Peters B., 1961, Nuovo Cim. 22, 800
Ptuskin V.S., 1984, Sov. AZh 61, 610
Rand R.J., Kulkarni S.R., 1989, ApJ 343, 760
Ruzmaikin A.A., Sokoloff D.D., Shukurov A.M., 1988, Magnetic Fields of Galaxies. Kluwer, Dordrecht
Simon M., Heinrich W., Mathis K.D., 1983, ApJ 300, 32
Smith A.G.K., Clay R.W., 1990, Aust. J. Phys. 43, 373
Sofue Y., Fujimoto M., Weilebinski R., 1986, ARA&A 24, 459
Sokoloff D.D., Shukurov A.M., 1990, Nature 347, 51
Swordy S.P., Muller D., Meyer P., L'Heureux J., Grunsfeld J.M., 1990, ApJ 349, 625
Syrovatsky S.I., 1971, Comm. Astrophys. Space Phys. 3, 155
Wdowczyk J., Wolfendale A.W., 1984, J. Phys. G. 10, 1453