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PROPAGATION OF INJECTED COSMIC RAYS UNDER DISTRIBUTED REACCELERATION

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ABSTRACT

In the stochastic cosmic-ray reacceleration model it is assumed that the particles obtain their energy not from discrete cosmic-ray sources but from subsequent reaccelerations in the Galaxy by encountering blast waves from supernova remnants. This model has difficulty in explaining the observed decrease with increasing energy of the secondary-to-primary flux ratios.

We present here a calculation in which we assume cosmic-ray sources but also allow for reacceleration of propagating particles. It is shown that such a model can be accommodated by the data, but it requires a modification of the path-length distribution. It predicts a weaker energy dependence of the mean traversed matter, $\lambda_{esc}(E)$, which affects the interpretations of the cosmic-ray storage time and the shape of cosmic-ray energy spectra. The statistical accuracy of the present observations and the covered energy ranges are not yet sufficient to determine whether cosmic rays encounter distributed reaccelerations. The shapes of the individual cosmic-ray spectra, however, give hints that reacceleration works.

Subject headings: cosmic rays: abundances — particle acceleration

I. INTRODUCTION

Basic goals of research in cosmic-ray physics are to understand the process of acceleration and the conditions of propagation through interstellar space. For this purpose one usually compares experimental observations of the elemental and isotopic composition of Galactic cosmic rays and their energy spectra with results from model calculations. This is commonly done in the framework of the "leaky box" model. This model treats the Galactic cosmic rays as a gas which fills the Galaxy and which is in spatial and temporal equilibrium. It is assumed that acceleration is coupled with discrete sources, thus separating the acceleration process from propagation.

These assumptions are contradictory to a current model of cosmic-ray acceleration in which the particles gain energy from stochastic processes in interstellar space during propagation by encountering shocks which are caused by supernova remnants (Drury 1983; Blandford and Ostriker 1978; Völk 1983). The problem with this model is that the particles gain energy slowly with time. Accordingly, the amount of matter traversed increases with energy. This leads to a logarithmic increase in the secondary-to-primary ratios with energy, which is inconsistent with observations (Cowsik 1980; Hayakawa 1969; Fransson and Epstein 1980; Eichler 1980). The observed decrease of the secondary-to-primary ratios may therefore suggest that a considerable part of the acceleration has taken place rapidly at the sources. Astronomical observations, on the other hand, indicate that shocks from supernovae do exist in the Galaxy, and observations in interplanetary space show clear evidence that shock acceleration works in principle (Richter and Keppler 1977). Therefore, we present here and discuss a concept in which we combine both ideas: (*a*) cosmic rays are preaccelerated at their sources and are injected into space with a power-law spectrum in rigidity, and (*b*) during their propagation the particles are moderately reaccelerated.

Such a scenario has been mentioned by Silberberg *et al.* (1983). They took the simple approach, however, assuming that all secondaries are produced by primaries in a one-step process at an energy one-fifth of the observed energy. In our approach the individual particles after injection can gain energy by subsequent reaccelerations according to the actual residence time. This then, by taking the exponential age distribution into account, leads to quite different results, and the effect on the abundance ratios is affected not only by the energy-dependent fragmentation cross section as stressed by Silberberg *et al.* (1983) but also by the injected cosmic-ray power-law spectrum, by the reacceleration conditions, and by the energy-dependent path-length distribution. This calculation was performed by means of a Monte Carlo method. We present the results and compare them with the data.

II. ASTROPHYSICAL ASSUMPTIONS

Our distributed reacceleration model is based on the following astrophysical assumptions:

1. Cosmic rays are injected in a defined composition by discrete sources. All injected nuclei show a power-law spectrum in rigidity, providing the same spectral index.
2. Particles are confined to the Galaxy by the structure of the magnetic fields and are scattered at their irregularities, leading to an exponential age or path-length distribution.
3. Particles are in temporal and spatial equilibrium. Loss and production in the Galaxy are balanced.
4. During propagation particles undergo nuclear collisions leading to the production of secondary spallation products. These processes are controlled by the total inelastic and fragmentation cross sections.
5. The Galaxy is filled with expanding supernova remnants in different stages of development. Early stages are accompanied by high-speed shocks, whereas the more developed remnants have lower speeds. Cosmic-ray particles, secondaries and primaries, can gain energy when they encounter these remnants during their propagation. It is not easy to specify the exact frequency of these encounters and the amount of energy the particles may gain per encounter. This depends on the supernova rate, the expanding process of the explosion, and the time a cosmic-ray particle spends in the vicinity of the expanding shock. None of these quan-

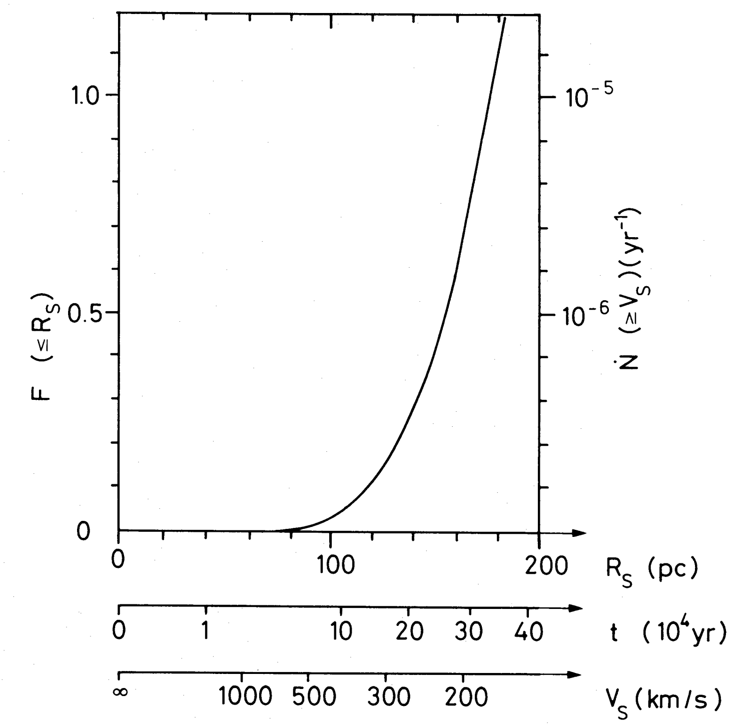

FIG. 1.—Rate at which shock waves per year [$N(\geq v_s)$] with speed $\geq v_s$ pass a given point and the fraction of space in the Galaxy [$F(R < R_s)$] as a function of shock radius R_s, age t_4, and shock speed v_s. This figure was extracted from a figure given in a paper by Axford (1981).

tities are well known, but in order to make estimates we adapted a calculation from Axford (1981) (see Fig. 1). It provides the rate of shock waves N ($\geq v_s$) (yr^{-1}) and the fraction of space F ($\leq R_s$) in the Galaxy covered by shock waves with radii $\leq R_s$. A supernova rate of 1/30 per year has been assumed in this calculation by Axford (1981).

Since our goal was to study the principles of the reacceleration process, we did not incorporate in detail the distribution of Figure 1. We assumed that the reacceleration is predominantly controlled by supernova remnants in a developed stage. Encounters with expanding supernova remnants in their early stages with high shock speed are very unlikely, since these fill only a very small portion of the volume of the Galaxy. The reacceleration by old supernova remnants, on the other hand, is also limited, since the shock speed becomes too small to account for significant reacceleration. Thus we assumed a reacceleration around every 10^6 years to be a realistic estimate for our calculation. This, then, refers to shock velocities of 200–250 km s^{-1}.

According to shock acceleration theory (Drury 1983), the amount of rigidity gain, $R = R_0 \exp(t/t_{acc})$, is determined by the efficiency of acceleration t_{acc} and the time t the particle spends in this region, where R_0 is the rigidity before the encounter. The acceleration efficiency t_{acc}, on the other hand, depends on the diffusion coefficient K and the shock speed v_s: $t_{acc} = 4K/v_s^2$. Using values for K between 10^{27} and 10^{28} cm^2 s^{-1} and $v_s = 200$ km s^{-1}, the acceleration time is roughly $t_{acc} \sim 5 \times 10^{13}$ s. We further assume that the cosmic rays encounter a number of these expanding shells when they travel through the Galaxy; thus the time they spend in the vicinity of each of these shells should be much smaller than the time between two subsequent encounters. Under this constraint we allow in our calculation a rigidity gain of typically 10%–30%.

III. THE MONTE CARLO PROPAGATION PROGRAM

Following our concept of subsequent reacceleration, particles can increase their energy several times, according to the time they actually spend in the Galaxy. This time is determined by the energy-dependent age or path-length distribution, $\bar{\lambda}_{esc}(E)$. So a particle on its way through interstellar space can change its mean path length $\bar{\lambda}_{esc}(E)$ several times. Also, the probability of fragmentation into different spallation products may change, since the fragmentation cross sections are energy-dependent below 1 GeV nucleon^{-1}. In order to describe this complex situation mathematically, we developed a Monte Carlo propagation program. It is capable of following each particle through space, considering interaction, escape, and reacceleration. We treat the reacceleration as an instantaneous increase in rigidity every time a reacceleration occurs. The frequency of reacceleration and the rigidity gain are parameters in this program. The primary particles can leave the cosmic-ray sources any time. The mean escape length $\bar{\lambda}_{esc}(E)$ and the fragmentation cross sections of primaries and secondaries were adjusted according to their actual energy. The individual path length, the position of interaction, and the fragmentation channels were treated as random variables with appropriate distribution functions.

Each individual primary particle and also the produced secondaries are followed along their paths, and the numbers of arriving particles are finally stored in different energy, mass, and charge bins. The various input parameters and considerations which describe the propagation and which are needed in order to calculate quantitatively the arriving composition are listed in the Appendix.

The program in its current version does not include ionization losses. This neglect, especially at low energies and for heavy particles, leads to overestimation of the secondary-to-primary ratio. At about 1 GeV nucleon^{-1} this effect can reach a level of typically 10%, as pointed out by Garcia-Munoz *et al.* (1981). But this is a minor effect compared with the increase of the secondary-to-primary ratio caused by reacceleration.

IV. RESULTS

In a first approach we tested this Monte Carlo propagation program under the assumptions of the standard leaky box model. For this test it was very helpful that P. S. Freier (1979, 1981) some years ago organized a cross-check on existing propagation programs between different groups. With the input parameters Freier distributed, we calculated the arriving composition and compared it with results from other groups which used analytic solutions of the propagation equation. Our calculated abundances for the different isotopes are in excellent agreement with those from the analytic calculations. The improvement we made in comparison with our results from earlier years was due solely to a refinement of the cross sections.

In the next step we used the $\bar{\lambda}_{esc}$ values as derived from observations by various groups under the leaky box approximation and as compiled by Protheroe, Ormes, and Comstock (1981) and adapted the following fit to these data (see also Fig. 2):

$$\bar{\lambda}_{esc} = x_0 \text{ (g cm}^{-2}) \quad \text{with } x_0 = 8.0 \quad \text{for } R \le 6 \text{ (GV)},$$

$$\bar{\lambda}_{esc} = x_0(R/6)^{-\alpha} \text{ (g cm}^{-2}) \quad \text{with } x_0 = 8.0 \text{ and } \alpha = 0.5 \quad \text{for } R > 6 \text{ (GV)}. \quad (1)$$

In the literature the values for x_0 and α differ slightly among the various groups (Ormes and Protheroe 1983; Simpson 1982; Garcia-Monoz *et al.* 1984), but these differences are small and not of principal importance for the purpose of our calculation. We then applied this $\bar{\lambda}_{esc}(E)$ dependence as given in equation (1) and allowed for reacceleration.

Since the B/C flux ratio is pretty well measured, we considered this ratio to be appropriate for comparison with our calculations. The result is shown in Figure 3. The standard leaky box approximation (*open circles*) is in good agreement with the data, confirming the $\bar{\lambda}_{esc}$ values in Figure 2. However, if allowance is made for reacceleration, the calculated B/C flux ratio increases considerably and falls well above the experimental data. The same effect can also be seen in Figure 4, where we compare the calculations with the relative abundance of the heavy secondaries, $17 \le Z \le 25$/Fe. From the two figures one realizes that the ratios at lower energies are more affected owing to an increasing residence time with decreasing energy. Roughly above 80 GeV nucleon^{-1} the influence of reacceleration has diminished. Particles above this energy do not stay long enough in the Galaxy to be effectively reaccelerated in this model.

It is surprising that this moderate reacceleration leads to a quite remarkable increase in the secondary-to-primary ratios. The explanation for this increase is somewhat complex and has different reasons. The energy dependence of the fragmentation cross sections, for instance, can cause such an effect, as pointed out by Silberberg *et al.* (1983). For a cross section which increases with decreasing energy, more secondaries at low energies are produced, which then after reacceleration show up at higher energies. But the cross sections do not always

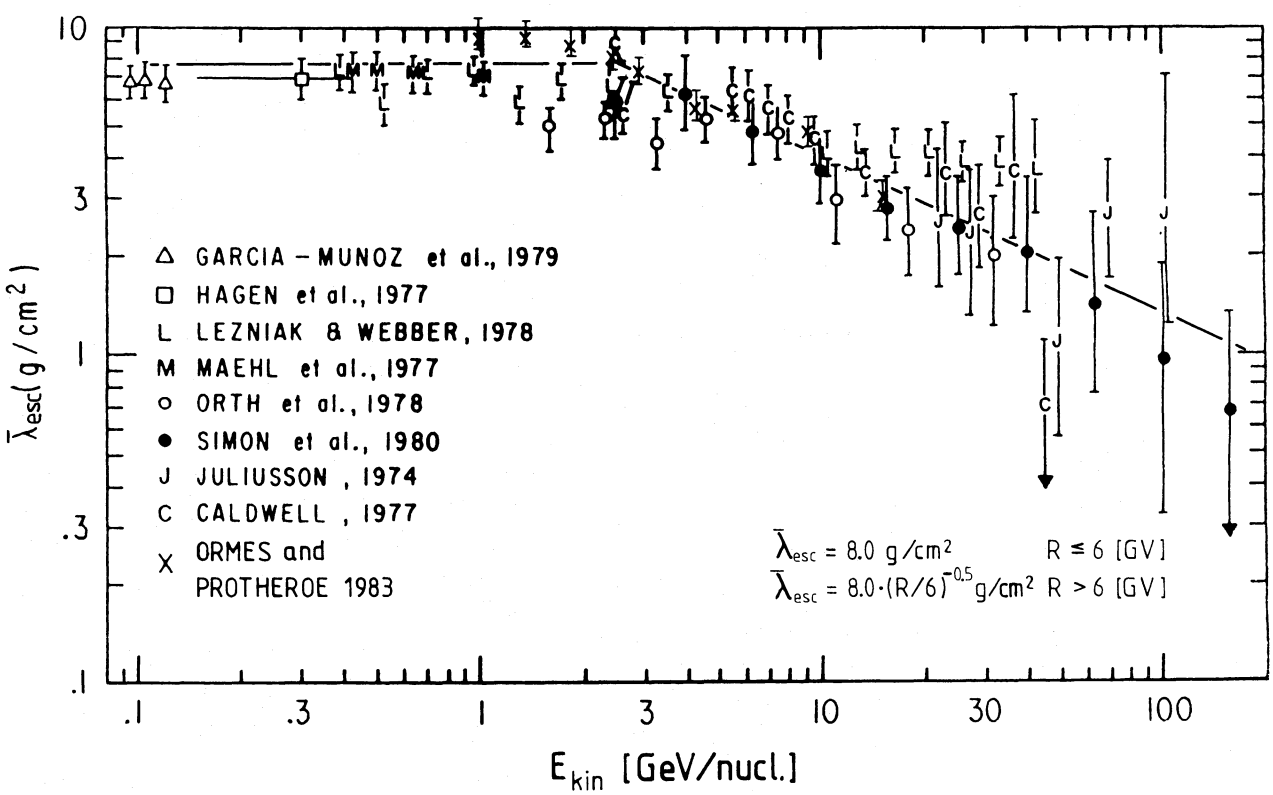

FIG. 2.—Energy dependence of the mean escape length, $\bar{\lambda}_{esc}(E)$, as derived in the framework of the standard leaky box model. The collection of data was taken from Protheroe, Ormes, and Comstock (1981).

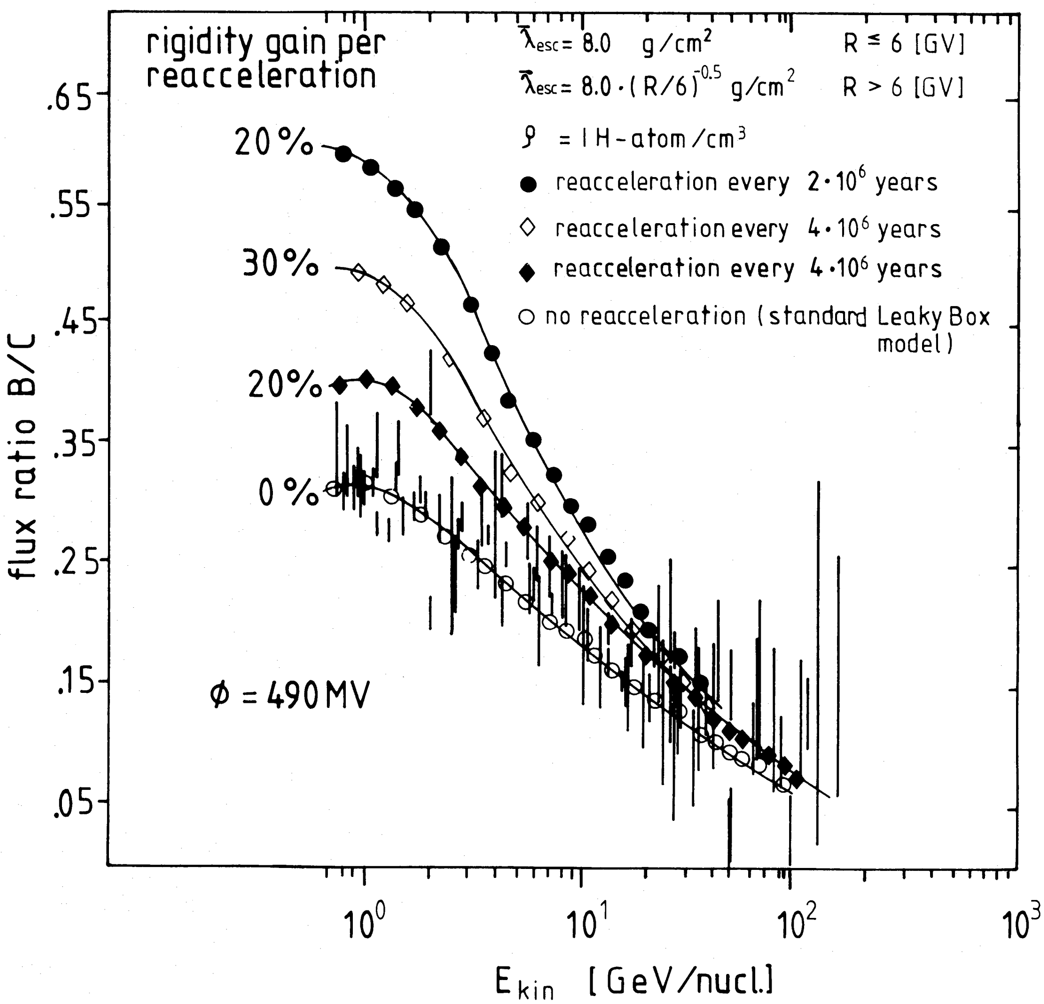

FIG. 3.—Calculated B/C flux ratio under different reacceleration conditions. The collection of data was taken from Garcia-Munoz *et al.* (1984).

increase with decreasing energy. They may also decrease, so that the cross section dependence alone does not necessarily lead to an enhanced production of secondaries.

There is, however, another effect which always leads to an increased production of secondaries and which works the following way. Reacceleration always moves particles of lower energies to higher energies, and the longer a particle stays in the Galaxy, the higher is the probability of its obtaining more energy. There are certainly always only a few particles with long residence times, since the age distribution decreases exponentially. But these few particles, which contribute after reacceleration to higher energies, have to be weighted by the steep-falling injection spectrum, by which their contribution then becomes quite considerable. To illustrate this general effect more clearly, we present calculated numbers for the two isotopes ^{12}C and ^{11}B in Table 1. These numbers result from a detailed propagation calculation which was performed by allowing a reacceleration every 2×10^6 years with a rigidity gain of 20%. The arriving energy in this calculation was 1.83 GeV nucleon^{-1}. We adapted $\lambda_{esc}(E)$ as given in Figure 2. As can be seen, only 18% of the 1.83 GeV nucleon^{-1} ^{12}C particles stem directly from the cosmic-ray sources. The majority of the ^{12}C particles have encountered reaccelerations. There are even some particles which encountered nine reaccelerations before they were pushed from 126 MeV nucleon^{-1} starting energy to 1.83 GeV nucleon^{-1} arriving energy. For the secondary ^{11}B isotope the effect is even more pronounced. Only a small contribution originates from the fragmentation of nonreaccelerated 1.83 GeV nucleon^{-1} primary particles. The majority of the ^{11}B particles have either encountered reacceleration by themselves or have been produced from primaries which have been reaccelerated before they interacted. Primary particles which have encountered a number of reaccelerations, on the other hand, are those out of the tail of the exponential pathlength distribution. A larger amount of traversed matter translates to a higher amount of produced secondaries. As a result the corresponding ^{11}B/^{12}C flux ratio is quite high for these particles, as can be seen in column (4) of Table 1. Although these larger ratios have to be weighted by the fraction they finally contribute to the 1.83 GeV nucleon^{-1} arriving energy, this effect always results in an enhanced secondary-to-primary ratio.

V. DISCUSSION

Moderate reacceleration causes a considerable enhancement of the production of secondary particles. The calculated secondary-to-primary ratios under reacceleration conditions are much too high in comparison with the data. One can try to

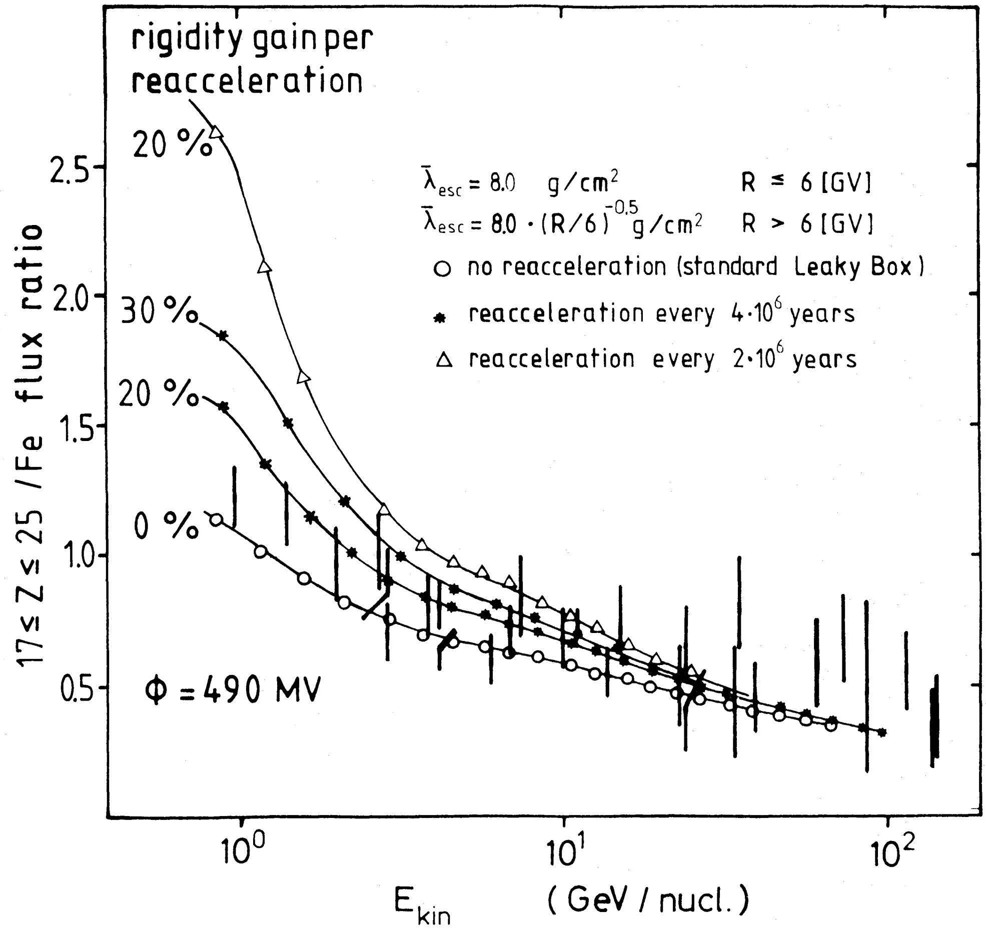

FIG. 4.—Calculated $17 \leq Z \leq 25$/Fe flux ratio under different reacceleration conditions. The collection of data was taken from Protheroe, Ormes, and Comstock (1981).

TABLE 1

FRACTION OF PARTICLES ARRIVING AT EARTH WITH A CERTAIN ENERGY THAT STARTED FROM THEIR SOURCES WITH LOWER ENERGIES

Starting Energy (GeV nucleon^{-1})	Fraction of Particles (%) Starting at Lower Energies before Reacceleration to Arriving Energy: ^{12}C	^{11}B	Ratio ^{11}B/^{12}C
1.83	18.0	2.6	0.07
1.42	27.5	10.8	0.2
1.1	16.6	18.7	0.56
0.835	10.8	12.6	0.59
0.628	7.7	9.1	0.60
0.466	6.7	13.9	1.03
0.342	5.3	10.9	1.03
0.248	3.0	8.7	1.45
0.178	2.8	7.0	1.3
0.126	1.6	5.6	1.75

NOTE.—The arriving energy in this calculation was 1.83 GeV nucleon^{-1}. The arriving fluxes (normalized to ^{12}C = 100) were ^{12}C = 100; ^{11}B = 50. This calculation was performed under the following reacceleration conditions: reacceleration every 2×10^6 years with a rigidity gain of 20% per reacceleration.

compensate for the more effective production of secondaries in this model by reducing the mean thickness of traversed matter. Results of a calculation under these modified conditions are shown in Figures 5 and 6. This calculation has been done under the assumption that every 2×10^6 years a reacceleration with a rigidity gain of 20% occurs. As one can see, reacceleration can in principle be accommodated with these data, but it requires a modification of the path-length distribution. It is found that the B/C and the sub–Fe/Fe ratios can be reproduced simultaneously by the modified path-length distribution which is given in Figures 5 and 6.

Compared with the path-length distribution derived under the assumptions of the standard leaky box model, one finds two main differences. First, at lower energies where the reacceleration works quite efficiently, the required path length has to be reduced. Second, in order to fit the data also at higher energies, the energy dependence of $\bar{\lambda}_{esc}(E)$ is weaker. For this special case of reacceleration we found that the mean traversed matter around 1 GeV nucleon^{-1} is closer to 4 g cm^{-2} instead of 8 g cm^{-2}, and the energy dependence of $\bar{\lambda}_{esc}(E)$ is more like $\bar{\lambda}_{esc} \sim R^{-0.3}$ instead of $\bar{\lambda}_{esc} \sim R^{-0.5}$. It is evident that different reacceleration conditions will require different path-length distributions to be consistent with the data, but in all cases, when reacceleration is allowed, the calculations show the following trends. The stronger the reacceleration is (more frequent and/or higher rigidity gain), the less traversed matter with an even flatter energy dependence is required in order to fit the

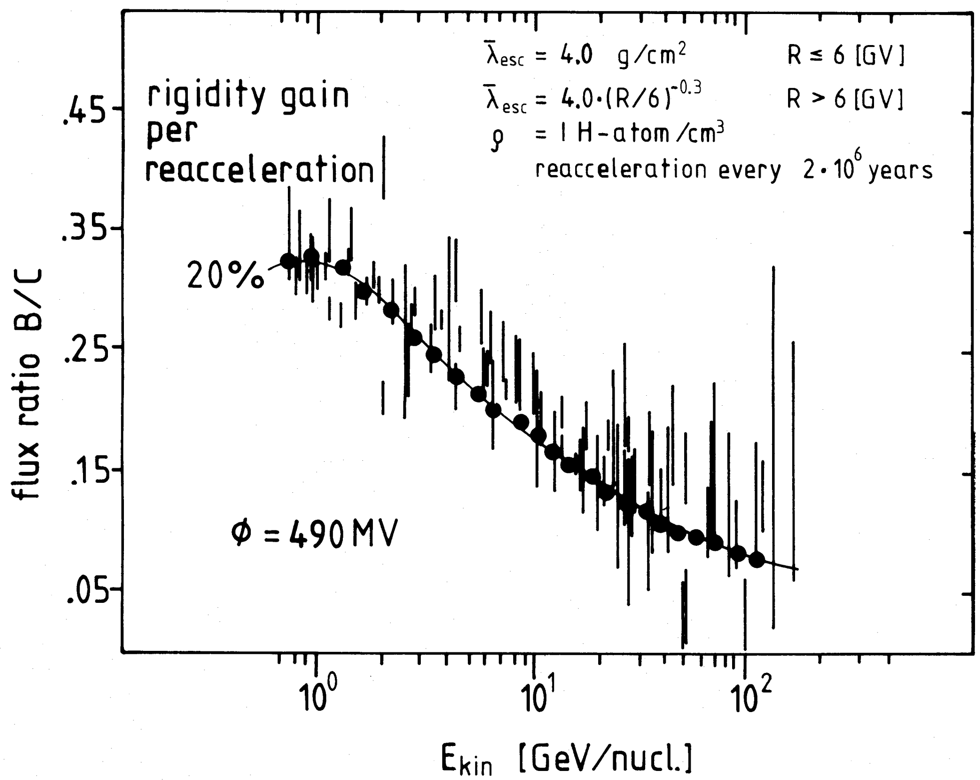

FIG. 5.—B/C flux ratio under reacceleration conditions. Calculated with a modified path-length distribution, so that the data can be fitted. The collection of data was taken from Garcia-Munoz *et al.* (1984).

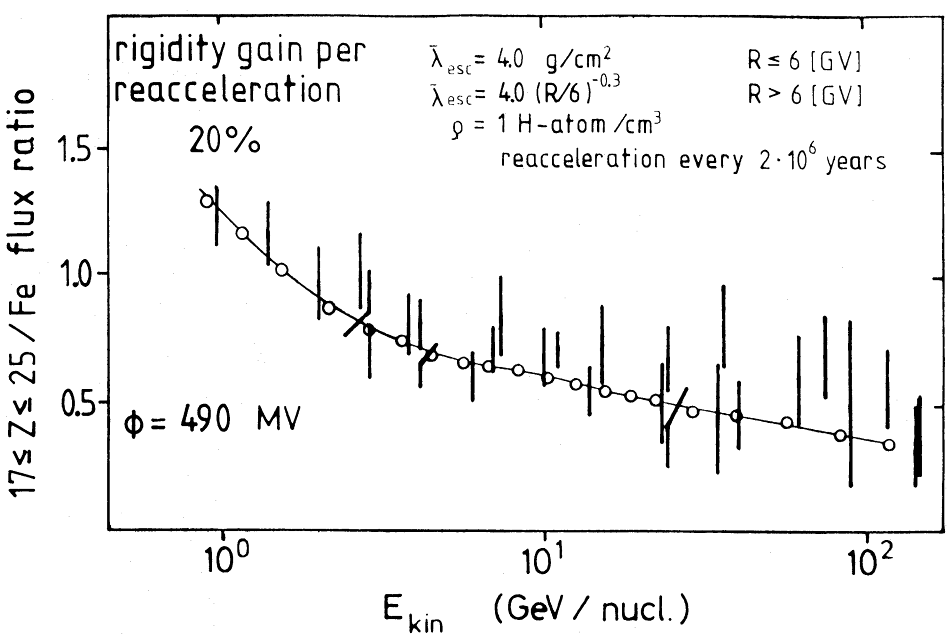

FIG. 6.—The $17 \leq Z \leq 25$/Fe flux ratio under reacceleration conditions. Calculated with the same modified path-length distribution that fits the B/C data. The collection of data was taken from Protheroe, Ormes, and Comstock (1981).

data. The weaker the reacceleration is, the more closely it reflects the standard leaky box situation. This prediction may be used as a check on this model. Measurements on the energy-dependent decrease of the secondary-to-primary ratios beyond 100 GeV nucleon^{-1} may place limits on the validity of reacceleration processes if one assumes that the weaker energy dependence of $\bar{\lambda}_{esc}(E)$ extends to higher energies. Figure 7 illustrates this schematically. At this time no conclusions can be drawn, since the data are scarce and uncertain. There is, however, another possible way to check on the validity of reacceleration. Since the arriving probability of a primary cosmic ray depends on the various energy-changing losses of cosmic rays due to escape and interaction (Webber 1982), the reacceleration process in addition to the required modified path-length distribution should shape the cosmic-ray spectra. We calculated the shape of the oxygen and iron spectra under various conditions and compared them with the data. This is shown in Figure 8. The reacceleration parameters and the corresponding path-length distributions are also given in Figure 8. These path-length distributions were adjusted in such a way that the secondary-to-primary ratios could be fitted by a method similar to that used in the case shown in Figures 5 and 6. The absolute fluxes for the calculated curves were adapted to the data by means of a χ^2 minimum fit. The resulting χ^2 values are also given in Figure 8. We calculated similar curves for different reacceleration conditions, and in all cases we found that the data show more evidence for reacceleration than for the standard leaky box model. This result certainly provides more a trend than a proof. With future measurements improved in both resolution and statistical accuracy, however, particularly at high energies, one has a tool to check on reacceleration more conclusively.

VI. CONCLUSIONS

It has been shown that the experimental observations in cosmic rays can be understood in the framework of a leaky box model under reacceleration conditions. Reacceleration requires a modification of the path-length distribution. It is difficult to specify this modified path-length distribution, since the reacceleration parameters are not well known yet and the calculations were based only on reasonable estimates. But if reacceleration works at all, the mean traversed matter which cosmic rays penetrate before they leave the Galaxy has to be reduced and the energy dependence of the mean escape length $\bar{\lambda}_{esc}(E)$ is flatter than that derived in the framework of the standard leaky box model. Tests on this reacceleration model can be performed by measuring the decrease of the secondary-to-primary ratios with energy above 100 GeV nucleon^{-1}. A flatter dependence than $E^{-0.5}$ would favor reacceleration. An additional test is given by the shape of individual cosmic-ray spectra. The available data from these spectra show evidence for reacceleration, but the data are too scarce and not precise enough to make conclusive statements yet.

We did not discuss the impact on the surviving fraction of radioactive cosmic-ray nuclei under these reacceleration conditions. Abundance measurements of radioactive nuclei such as ^{10}Be, ^{16}Al, and ^{36}Cl in principle can be used to provide information on the Galactic confinement time. Combined with the path-length distribution determined by stable secondaries, one can also probe the density and homogeneity of the material traversed (Wiedenbeck 1983; Simon, Scherzer, and Enge 1979). In the framework of the standard leaky box model, it is a common procedure, since it is assumed that the surviving fraction is solely a function of the density of the interstellar gas. If we allow for distributed reacceleration, the situation becomes more complicated. The time dilatation is not constant for a particle during propagation. The mean Lorentz factor is less than that determined by the arriving energy. This leads to a more frequent decay and causes underestimation of the gas density when the data are interpreted in the framework of the standard leaky box model.

There is another problem when reacceleration is allowed.

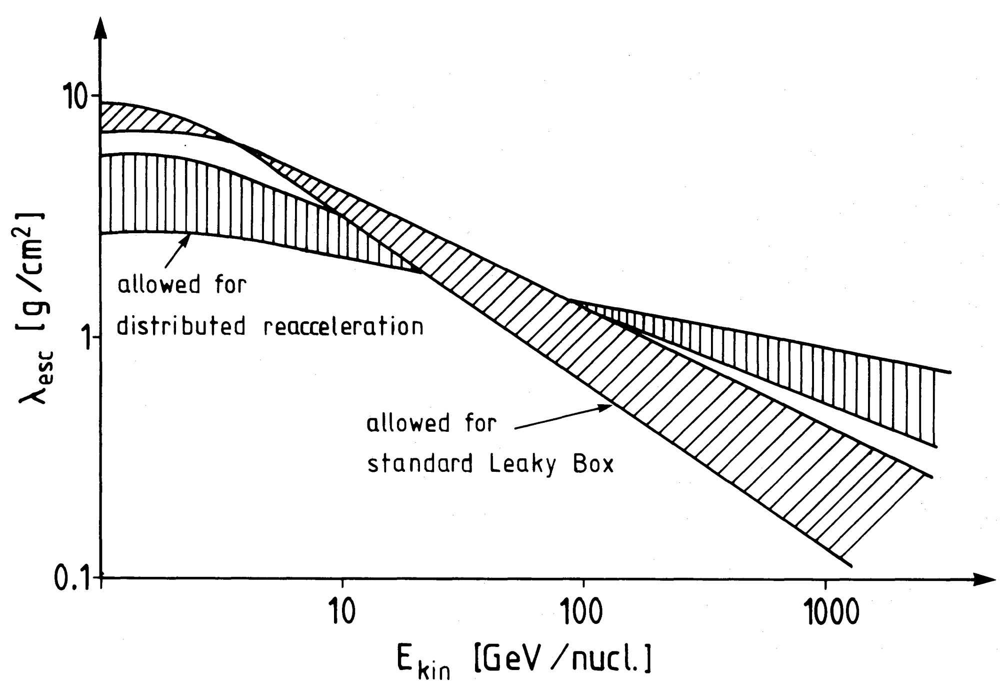

FIG. 7.—Schematic drawing of the energy dependence of $\bar{\lambda}_{esc}(E)$ as predicted by the standard leaky box model and the distributed reacceleration model

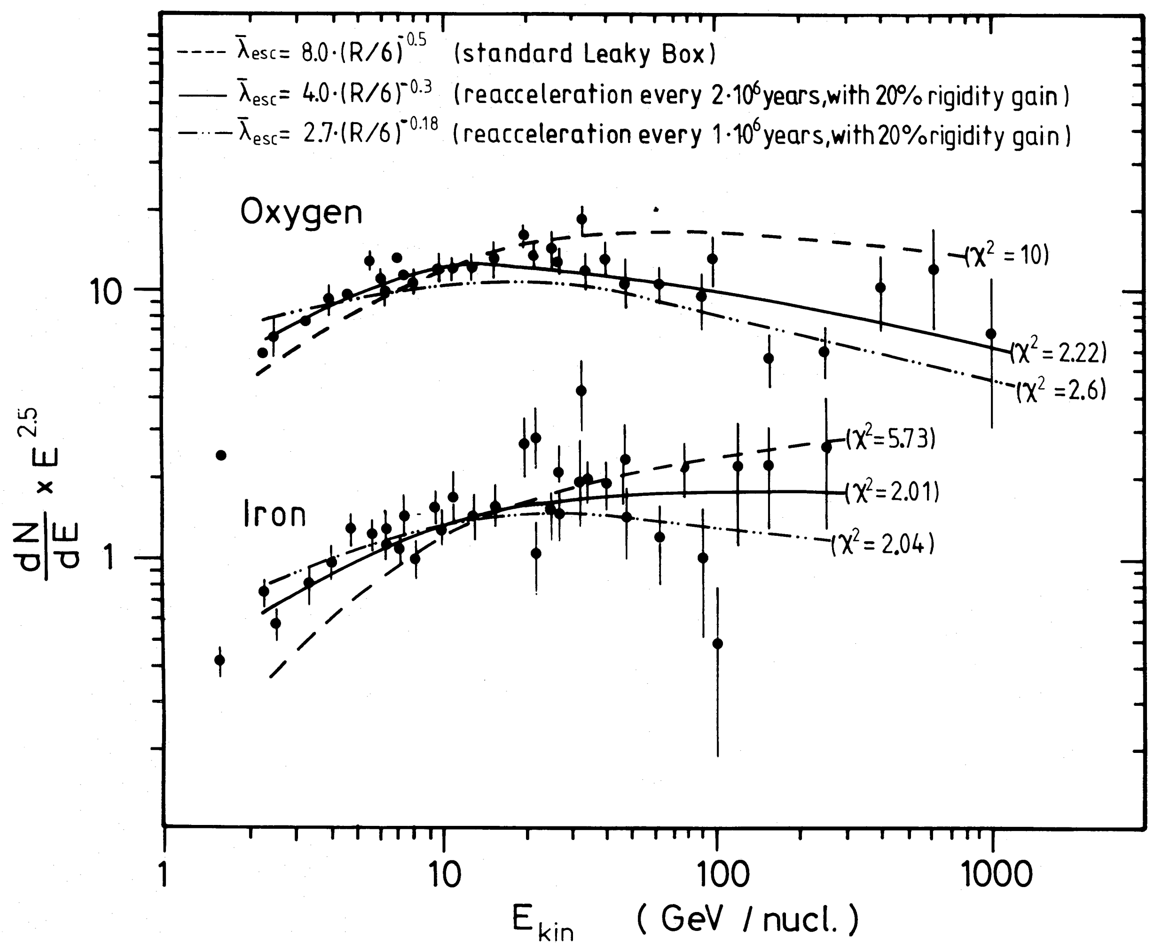

FIG. 8.—Calculated energy spectra under different reacceleration conditions. The data were taken from Simon *et al.* (1980). The absolute flux of the curves is adapted to the data by means of a χ^2 calculation. The path-length distributions which are used in this calculation are such that the secondary-to-primary ratios could be fitted, in a manner similar to that used in Figs. 5 and 6.

Since the surviving fraction cannot be measured directly, one normally measures flux ratios such as $^{10}Be/^{9}Be$. A propagation under distributed reacceleration produces these secondaries according to their energy-dependent production cross sections, so that this ratio is not solely dependent on the decay probability of ^{10}Be, as assumed in the standard leaky box model. All this makes the interpretation of the measured abundances of radioactive nuclei quite complicated when reacceleration works on these particles. Good data on the $^{10}Be/^{9}Be$ flux ratio are available around 100 MeV nucleon^{-1}, but this is an energy range where we are not capable of making meaningful calculations since our computer program in its present state does not take ionization losses into account. We aim to include ionization losses in the near future. But in general reacceleration works toward a shorter cosmic-ray age because of a smaller amount of traversed matter and a smaller mean Lorentz factor. When reacceleration is allowed, we are probably closer to 10^6 years than to 10^7 years, as derived in the framework of the standard leaky box model (Wiedenbeck and Greiner 1980).

It is worthwhile to mention that flux ratios in the energy regime around 100 MeV nucleon^{-1} may be very much affected by cross section variations which occur below 100 MeV nucleon^{-1}. It is observed that some cross sections show resonance-like increases below 100 MeV nucleon^{-1}. Production of ^{10}B and ^{11}B, for instance, from the proton-^{16}O interaction is 4 times as high around 60 MeV nucleon^{-1} as around 200 MeV nucleon^{-1} (Garcia-Munoz *et al.* 1985). Under the reacceleration condition the high production rate of these particles should also influence the result above 100 MeV nucleon^{-1}. The effect may be somewhat washed out by ionization losses, but measurements in this low-energy regime should open new possibilities to check on the validity of reacceleration processes.

We wish to thank Dr. R. Schlickeiser, of the Max-Planck-Institut für Radioastronomie, Bonn, for his helpful and valuable discussions.

APPENDIX

The calculation has been performed with the following input:

1. For the cosmic-ray source we used Cameron's isotopic fraction (Cameron 1973; see also Freier 1981).
2. The total inelastic cross sections for collisions with hydrogen were taken from Protheroe, Ormes, and Comstock (1981).

3. To calculate the spallation cross sections we used the Silberberg and Tsao (1973, 1977) semiempirical formulae down to 100 MeV nucleon^{-1}.

4. We used an energy-dependent exponential path-length distribution with a decrease of the mean traversed matter $\bar{\lambda}_{esc}(R)$ with increasing rigidity R such that

$$\bar{\lambda}_{esc} = x_0 \text{ (g cm}^{-2}) \qquad \text{for } R \le 6 \text{ (GV)},$$
$$\bar{\lambda}_{esc} = x_0(R/6)^{-\alpha} \text{ (g cm}^{-2}) \qquad \text{for } R > 6 \text{ (GV)},$$

where x_0 (g cm^{-2}) and α could be used as parameters.

5. The injection spectra were taken to be proportional to $R^{-\gamma_0}$, where R is rigidity. The exponent γ_0 is linked to the exponent α in the path-length distribution by

$$\gamma_0 = 2.7 - \alpha . \tag{A1}$$

This takes into account that the observed spectrum $N(R)$ of primary particles is related to the injection spectrum by

$$N(R) \sim \left(\frac{1}{\lambda_{int}} + \frac{1}{\lambda_{esc}(R)}\right) R^{-\gamma_0} , \tag{A2}$$

where λ_{int} stands for the interaction mean free path. At high energies $\lambda_{esc}(R)$ alone determines the shape of the spectrum:

$$N(R) \sim \lambda_{esc}(R) R^{-\gamma_0} \sim R^{-(\gamma_0 + \alpha)} . \tag{A3}$$

The proton and helium spectrum is now measured up to 100 TeV nucleon^{-1} (Ryan, Ormes, and Balasubramanyan 1972; Burnett *et al.* 1983), and the exponent is close to 2.7. This justifies equation (A1).

6. The effects of solar modulation have been included in the analysis. In the "force-field" approximation we used a modulation parameter $\phi = 490$ MV (Evenson *et al.* 1983), which is an appropriate representation of the time during which most of the data were measured. This corresponds to an adiabatic deceleration of 245 MeV nucleon^{-1} for nuclei with $A/Z = 2$.

7. The time t between two subsequent reaccelerations was fixed and expressed in years and could be used as a parameter. In order to translate time t into path length x (g cm^{-2}), we used

$$x \text{ (g cm}^{-2}) = \langle m \rangle n c \beta t ,$$

where t is in years; $\langle m \rangle$ means the mass of the interstellar gas, n the number density, and c the speed of light, and the particles velocity v is given by $v = c\beta$. We kept the gas density fixed at $n = 1$ H atom cm^{-3}.

8. When a reacceleration occurred, the rigidity was increased by a certain percentage. This rigidity gain could be used as a parameter.

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