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THE CONFINEMENT OF GALACTIC COSMIC RAYS BY ALFVÉN WAVES

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SUMMARY

A leaky-box model of cosmic ray confinement is investigated in an axially-symmetric model of the Galaxy. The cosmic rays are confined by the Alfvén waves which they excite in the magnetic field. The effect of non-linear damping of the Alfvén waves is discussed, and the residence time and path length of cosmic rays in the Galaxy are calculated as functions of rigidity. Observations of the electron spectrum and of pulsar scintillation are discussed in terms of this model.

1. INTRODUCTION

It was shown in Paper I (Holmes 1974) that the energy-dependence of the average amount of interstellar material traversed by galactic cosmic rays (see, for example, Cesarsky & Audouze 1974) may be explained in terms of the interaction between cosmic rays and Alfvén waves in the galactic magnetic field. When cosmic rays stream at speeds greater than the Alfvén velocity, hydromagnetic waves with wavelength of the order of the cosmic ray gyro-radius form in the magnetic field. These waves in turn scatter the cosmic rays of corresponding gyro-radius, thereby reducing their streaming speed (Wentzel 1969a; Kulsrud & Pearce 1969). But the waves formed by this process have to compete against damping processes, the dominant linear damping mechanism being the effect of collisions between the charged particles moving with the wave and the neutral particles of the interstellar medium (Kulsrud & Pearce 1969). The waves can only begin to form when the linear damping rate due to this mechanism becomes smaller than their growth rate, at heights above the galactic plane where the neutral hydrogen density is low enough. Since the growth rate of these waves varies as the density of resonant cosmic rays, it decreases as the cosmic ray energy increases. Therefore high-energy cosmic rays can only encounter resonant waves at greater heights above the galactic plane, where the density of neutral particles is smaller. The onset of the waves at their respective heights above the galactic plane produces a reflecting boundary which prevents free diffusion out of the Galaxy. The streaming speed of the cosmic rays across the boundary increases with energy, so that high-energy cosmic rays can escape from the Galaxy more readily than those of lower energy.

In Section 2 of this paper, we extend the one-dimensional treatment outlined in Paper I to apply to an axially-symmetric model of the Galaxy. The location of the boundary and the cosmic ray residence time and path length in the Galaxy are determined as functions of cosmic ray rigidity. In Section 3 we investigate the effects of non-linear wave damping on these results, using relations obtained by

Wentzel. Skilling's analysis (1971), upon which Paper I and Section 2 of this paper are based, is shown to give the same results as Wentzel's analysis in the absence of non-linear damping. In Section 4 we investigate the effect of this process on the spectrum of cosmic ray electrons, and in Section 5 we discuss the phenomenon of pulsar scintillation in the light of this model.

2. LOCATION OF THE BOUNDARY IN AN AXIALLY-SYMMETRIC MODEL OF THE GALAXY

The steady-state distribution function $f(x, p)$ of cosmic rays in the presence of waves obeys the equation

$$\boldsymbol{V}_{\mathrm{A}}.\nabla f = \frac{1}{3}(\nabla.\boldsymbol{V}_{\mathrm{A}})\, p\,\frac{\partial f}{\partial p} - \frac{1}{p^3}\nabla.\left(\frac{\Gamma_{\mathrm{D}} B_0{}^2\hat{\boldsymbol{n}}}{4\pi^3 M_{\mathrm{H}}\Omega_{\mathrm{H}} V_{\mathrm{A}}}\right) \tag{1}$$

(Skilling 1971). $B_0\hat{\boldsymbol{n}}$ represents the background magnetic field, p is the cosmic ray momentum, $V_{\mathrm{A}} = B_0(4\pi M_{\mathrm{H}} n_{\mathrm{i}})^{-1/2}$ is the Alfvén velocity, M_{H} is the proton rest mass, and Ω_{H} is its non-relativistic gyro-frequency. Γ_{D} is the damping rate of the waves, which varies as the density of neutral hydrogen, $\Gamma_{\mathrm{D}} = G n_{\mathrm{H}}$. Let the density of the interstellar medium and the strength of the galactic magnetic field vary exponentially with height z above the galactic plane,

$$n_{\mathrm{H}} = n_{\mathrm{H}}(r)\exp(-z/z_{\mathrm{H}})$$
$$n_{\mathrm{i}} = n_{\mathrm{i}}(r)\exp(-z/z_{\mathrm{i}}) = n_{\mathrm{i}}(r)\exp(-xz/z_{\mathrm{H}})$$
$$B_0 = B_0(r)\exp(-z/z_{\mathrm{B}}) = B_0(r)\exp(-wz/z_{\mathrm{H}})$$

where

$$x = z_{\mathrm{H}}/z_{\mathrm{i}} \quad \text{and} \quad w = z_{\mathrm{H}}/z_{\mathrm{B}}.$$

Neglecting radial density gradients compared to z-gradients, equation (1) becomes

$$z_{\mathrm{H}}\frac{\partial f}{\partial z} - \frac{x}{6}p\frac{\partial f}{\partial p} = \frac{1}{p^3}gb(r)\left(1+\frac{x}{2}-w\right)\exp\left[-(1+x-w)\frac{z}{z_{\mathrm{H}}}\right] \tag{2}$$

where

$$g = \frac{GcM_{\mathrm{H}}}{\pi^2 e}$$

and

$$b(r) = \frac{n_{\mathrm{i}}(r)\,n_{\mathrm{H}}(r)}{B_0(r)}.$$

Equation (2) has a solution

$$f(r,z,p) = f_0(r)\,p^{-\delta}\exp\left[\frac{-(x/6)\,z\delta}{z_{\mathrm{H}}}\right] - gb(r)\,p^{-3}\exp\left[-(1+x-w)\frac{z}{z_{\mathrm{H}}}\right]. \tag{3}$$

In the free zone, where the waves cannot form, the cosmic rays are free to stream along the field, so that $\hat{\boldsymbol{n}}.\nabla f = 0$. This condition must hold at the boundary of the wave zone, at $z = z(r, p)$. Setting $\partial f/\partial z = 0$ at $z = z(r, p)$ in equation (3) yields an expression for the height $z(r, p)$ at which the waves which scatter cosmic ray protons of momentum p begin to form,

$$\exp\left[(1+x-w) - \frac{x}{6}\frac{z(r,p)\delta}{z_{\mathrm{H}}}\right] = \frac{1+x-w}{x\delta/6}\frac{gb(r)}{f_0(r)}p^{\delta-3} \tag{4}$$

At the boundary, the cosmic ray distribution function must be equal to that below the boundary in the free zone,

$$f(r, z = z(r, p), p) = F(r)\, p^{-\gamma}. \tag{5}$$

Equations (3), (4) and (5) together yield a relation between γ and δ,

$$\gamma - \delta = \frac{\delta(\delta-3)\,x}{6(1+x-w)-\delta x}. \tag{6}$$

It is evident from equation (3) that the density spectrum $N(p)\,dp$ of cosmic rays in the wave zone, far enough above the boundary to render the second term on the right-hand side negligible, is a power-law with differential gradient $-(\delta-2)$. Since the gradient in the free zone is $-(\gamma-2)$, a change in spectral gradient of $\gamma-\delta$, given by equation (6), is expected around $p_0(r, z)$, the momentum of the protons which encounter a boundary at coordinates (r, z). This relation has been derived independently by McIvor & Skilling (1974) for the case when $x = 1$, $w = 0$.

Such a bend has been observed around a rigidity of 8 GV (Durney *et al.* 1964), and they discovered that the extent and position of the bend did not vary over the solar cycle, which suggests that it is unlikely to be a result of solar modulation. Taking their values of $\gamma = 4{\cdot}5$, $\delta = 4{\cdot}2$, equation (6) requires

$$5x + 2w = 2.$$

An important constraint on this model is that, in the wave zone, V_A must increase with z, otherwise the cosmic rays would be streaming *up* their own density gradient (Skilling 1971). Therefore this model is only valid if $z_B > 2z_i$. The condition that $0 \leqslant w < \frac{1}{2}x$ then requires that

$$\tfrac{1}{3} < x \leqslant \tfrac{2}{5}$$

$$0 \leqslant w < \tfrac{1}{6}.$$

Observations of pulsar dispersion measures and 21-cm hydrogen emission suggest that z_i may be greater than z_H by a factor of 2 or 3 (Bridle & Venugopal 1969), and Falgarone & Lequeux (1973) state that this factor is at least 2. The above condition on x is not in conflict with these observational inferences that $x \leqslant \frac{1}{2}$. If the neutral particle scale height is 130 pc, then the condition on w requires that $z_B > 780$ pc. Ilovaisky & Lequeux (1972) suggest that the thickness of the galactic magnetic field in the vicinity of the Sun is about 2 kpc. For simplicity we shall set our parameter values at $x = 0{\cdot}4$, $w = 0$.

The presence of a bend in the proton spectrum at 8 GeV suggests that protons of higher energy experience a free zone in the vicinity of the Sun ($r = 10$ kpc, $z = 0$). Using this assumption to normalize the damping factor in equation (4), and simplifying the exponent by use of equation (6), we obtain a relation for the position of the boundary $z(r, p)$,

$$\exp\left[z(r,p)/z_H\right] = \left(\frac{p}{p_0(r, z=0)}\right)^{6(\gamma-\delta)/x\delta}, \tag{7}$$

where

$$p_0(r, z=0) = 8\left[\frac{f_0(r)}{f_0(r=10)}\,\frac{b(r=10)}{b(r)}\right]^{1/\delta-3} \text{GeV/c}$$

which is the maximum momentum of the protons which encounter waves in the galactic plane at radius r. The boundary height is plotted in Fig. 1 for a selection of proton momenta. For this purpose, $n_i(r)$ and $n_H(r)$ are taken to vary as the density of giant H II regions and the surface density of atomic hydrogen respectively (from Lequeux 1973), normalized at r = 10 kpc to n_i = 0·025 cm^{-3} and n_H = 0·16 cm^{-3}, the accepted densities of the local intercloud medium. The cosmic ray distribution function in the galactic plane, $f_0(r)$, is taken to vary as that inferred from observations of the diffuse gamma radiation (Puget & Stecker 1974). The galactic field strength is taken to vary from 3 μG at r = 10 kpc to a value of 7 μG in the region $r < 7$ kpc (Piddington 1970).

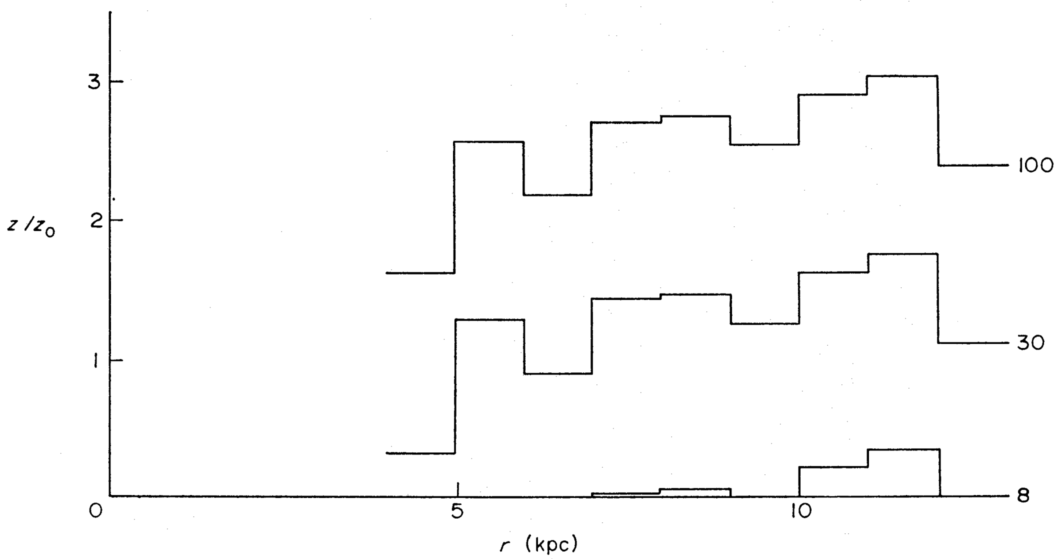

FIG. 1. *The location of the wave zone boundaries in the Galaxy for particles of rigidity* 8 *GV*, 30 *GV and* 100 *GV. The height above the galactic plane is represented as multiples of the neutral hydrogen scale height. The* 8 *GV boundary passes by the Sun.*

Cosmic rays of equal rigidity resonate with the same waves, so that the above treatment for protons may be extended to cosmic rays of other species by replacing momentum p with rigidity R.

In the free zone, cosmic rays undergo three-dimensional diffusion between magnetic field irregularities separated by about 30 pc (Skilling, McIvor & Holmes 1974), so that in their lifetime of $\sim 10^6$ yr they travel on average about 3 kpc in the galactic plane. Therefore as long as $z(r, R)$ does not vary appreciably over a distance of $\sim$3 kpc in the vicinity of the Sun, the residence time of cosmic rays in the free zone in our part of the Galaxy may be determined from their rate of escape across the boundary, $(\gamma/3)(4/3)\, V_z = 2\, V_z$ (Skilling 1971). V_z is the z-component of the Alfvén velocity at the boundary. Once across the boundary, they either convect out of the Galaxy along the field, if the galactic field is connected to the intergalactic field as Piddington (1973) suggests, otherwise they could escape by inflating regions of weaker field (Parker 1965). In the latter case, the streaming velocity at the boundary would regulate the rate at which the ' bubble ' is inflated.

From reasoning similar to that in Section 3 of Paper I, the residence time is found to be

$$\tau(R) = 5 \times 10^6 \ln\left(\frac{R_{GV}}{8}\right)\left(\frac{R_{GV}}{8}\right)^{-0\cdot214} \text{yr}, \qquad (8)$$

where the z-component of the galactic magnetic field is taken to be constant at 10^{-6} G. Assuming the average density of the interstellar medium decreases exponentially with a scale height of 130 pc from a value of 1 atom cm^{-3} in the plane, the amount of matter traversed in the above lifetime is

$$l(R) = 7{\cdot}2 \left[1 - \left(\frac{R_{\mathrm{GV}}}{8}\right)^{-1{\cdot}07}\right] \left(\frac{R_{\mathrm{GV}}}{8}\right)^{-0{\cdot}214} \mathrm{g\ cm^{-2}}. \tag{9}$$

These relations become invalid, of course, when the variation in the boundary height over 3 kpc in the galactic plane is of the order of the boundary height itself, when $R < 30$ GV. However, the lifetime at $R = 8$ GV may be determined inside the region shown in Fig. 1, in which the cosmic ray density may be taken to be constant. In this region $\tau \approx 10^6$ yr, so that $l \approx 1{\cdot}5$ g cm^{-2}.

$\tau(R)$ and $l(R)$ are directly proportional to z_{H} and inversely proportional to B_z. As these two quantities are not known accurately, the lifetime and path length may be multiplied by a numerical factor (of order unity) to normalize them to observations. The *shape* of these functions offers a crucial test to the theory, not their absolute values. It is evident from equation (9) that, above $R \sim 30$ GV, $l(R)$ decreases as a power-law $l(R) \propto R^{-0{\cdot}214}$. This is too shallow to be consistent with the results of Cesarsky & Audouze (1974), who obtained a significantly steeper variation, $l(R) \propto R^{-1}$ approximately, from measurements of the ratio of the light to the medium elements in the galactic cosmic radiation. In the following section we show that a steeper decrease of $l(R)$ with R may be obtained by taking into account the effects of non-linear damping of the Alfvén waves.

3. THE EFFECTS OF NON-LINEAR DAMPING OF ALFVÉN WAVES

Ion-neutral particle collisions are the dominant form of linear damping of Alfvén waves in the interstellar medium (Kulsrud & Pearce 1969), but once the waves are allowed to grow to a finite amplitude they can be degraded by non-linear wave–wave interactions. A forward-travelling Alfvén wave can transform into a backward-travelling Alfvén wave and a sound wave. The backward-travelling Alfvén wave can subsequently transform in a similar manner into another forward-travelling one, resulting in a ' cascade ' to longer wavelengths (Chin & Wentzel 1972). Alternatively the backward-travelling Alfvén wave can combine with a forward-travelling one to produce a magnetosonic wave. The rate of these reactions depends upon the wave amplitude, and consequently the method adopted in Section 2 is unable to accommodate this form of damping. Skilling's approach, namely that *either* waves exist and cosmic rays stream at the Alfvén speed *or* the waves are damped out and the cosmic rays are free to stream at the velocity of light, does not require knowledge of the wave amplitude. This approach is criticized by Wentzel (1974), who states that Skilling's expansion of the scattering equation in inverse powers of the scattering frequency ignores the regime of weak scattering, where the cosmic rays stream at intermediate velocities. In order to accommodate the effects of non-linear damping, we shall adopt the approach of Wentzel, as outlined in his paper of 1969a and reviewed in his article of 1974. But before proceeding in this way, we shall first show that these two methods of approach are equivalent in that they both yield the same results in the absence of non-linear damping.

Wentzel obtained a relation between the streaming velocity V_{s} of the cosmic rays and the growth rate Γ_G of the Alfvén waves which they excite,

$$\Gamma_G = \frac{e}{c(M_H n_i)^{1/2}} N(>p)\left(V_s - \frac{\gamma V_A}{3}\right). \tag{10}$$

As before, we shall call the damping rate due to ion-neutral collisions $\Gamma_D = G n_H$. Inserting the values for n_i, n_H and B_z which were used in Section 2, and describing the cosmic ray spectrum as

$$N(>p) = 2{\cdot}5 \times 10^{-10} p^{-1{\cdot}5}\ \text{cm}^{-3} \quad (p \text{ measured in GeV/c})$$

the streaming speed in the z-direction is obtained by equating Γ_D and Γ_G,

$$\frac{V_s}{c} = 7 \times 10^{-5} \exp\left[\frac{(x/2)\,z}{z_H}\right] + G\,2{\cdot}73 \times 10^5 p^{1{\cdot}5} \exp\left[-(1+x/2)\frac{z}{z_H}\right]. \tag{11}$$

Clearly the net loss rate out of the Galaxy is governed by the minimum streaming speed along the field lines. Calling $z(p)$ the height at which this occurs, the waves cannot exist in the region of decreasing streaming speed, where $z < z(p)$, otherwise the cosmic rays would be streaming up their own density gradient. Consequently the region $z < z(p)$ corresponds to the free zone in Skilling's approach. Minimizing equation (11) we obtain

$$\exp\left[\frac{z(p)}{z_H}\right] = \left(\frac{p}{p_0}\right)^{1{\cdot}5/1+x}, \tag{12}$$

where

$$p_0 = \left[\frac{x/2}{1+x/2}\,\frac{7\times 10^{-5}}{2{\cdot}73 \times 10^5}\,\frac{1}{G}\right]^{1/1{\cdot}5}$$

which is the momentum of the cosmic ray protons which experience their minimum streaming velocity at $z = 0$. By application of equation (6), it can be verified that equation (12) is identical to equation (7) which was obtained from Skilling's approach.

The rate at which the protons stream across the boundary at $z = z(p)$ is obtained by substituting equation (12) into equation (11),

$$\left.\frac{V_s}{c}\right|_z = \left(1+\frac{x/2}{1+x/2}\right)\frac{\gamma}{3}\left.\frac{V_A}{c}\right|_z = \left(1+\frac{x/2}{1+x/2}\right) 7 \times 10^{-5} \exp\left[\frac{(x/2)\,z}{z_H}\right]. \tag{13}$$

For $0 \leqslant x \leqslant 1$, V_s lies between $(\gamma/3)\,V_A|_z$ and $(4/3)(\gamma/3)\,V_A|_z$, which agrees with Skilling's estimate of the streaming velocity at the boundary.

Now that we have reconciled the approaches of Skilling and Wentzel, we shall use Wentzel's method to include the effects of non-linear damping. When non-linear damping dominates over ion-neutral collisions, Wentzel (1974) showed that, for $B = 3 \times 10^{-6}$ G,

$$V_s = \frac{\gamma}{3} V_A + 56\left(\frac{100\ \text{pc}}{L}\right)^{1/2}\left(\frac{p}{Mc}\right)^{0{\cdot}75}\ \text{km s}^{-1},$$

where L is the cosmic ray scale height in the presence of the waves. When we include the ion-neutral damping term, we obtain an expression for the streaming velocity in the z-direction, in a uniform z-field of 10^{-6} G,

$$\left.\frac{V_S}{c}\right|_z = 7\times10^{-5}\exp\left[\frac{(x/2)\,z}{z_H}\right] + G\,2{\cdot}73\times10^5\,p^{1{\cdot}5}\exp\left[-\left(1+\frac{x}{2}\right)\frac{z}{z_H}\right] + 2{\cdot}9\times10^{-5}\,p^{0{\cdot}75}, \quad (14)$$

where we have used a scale height $L = 6\,Z_H/(4{\cdot}2\times0{\cdot}4) \approx 460$ pc as suggested by equation (3).

Since the non-linear damping term is independent of gas density, and therefore of height, the height at which V_S is a minimum is not altered by its inclusion. Therefore the boundary height remains as described by equation (12). The streaming speed across the boundary is now

$$\left.\frac{V_S}{c}\right|_z = 8{\cdot}2\times10^{-5}\left(\frac{p}{p_0}\right)^{0{\cdot}214} + 2{\cdot}9\times10^{-5}\,p^{0{\cdot}75}. \quad (15)$$

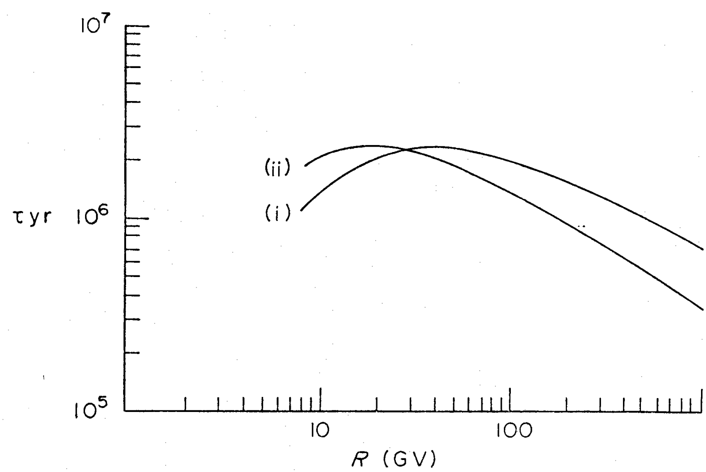

FIG. 2. *The residence time of cosmic rays in the free zone as a function of their rigidity, taking into account non-linear damping effects. Curve (i) assumes an exponential density distribution, while curve (ii) assumes a gaussian one.*

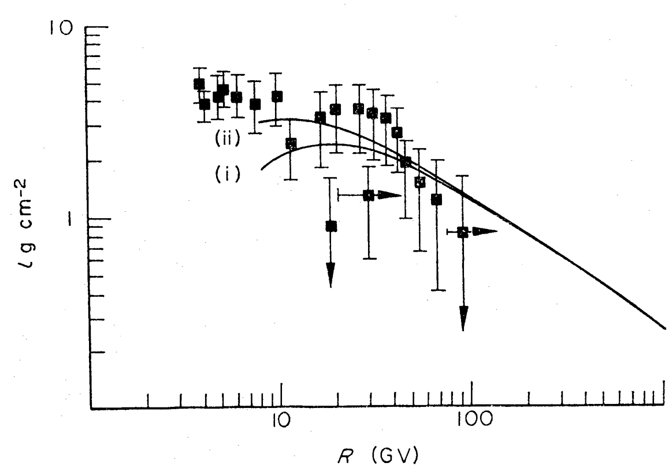

FIG. 3. *The average path length of cosmic rays in the free zone as a function of their rigidity, taking into account non-linear damping effects. Curve (i) assumes an exponential density distribution, while curve (ii) assumes a gaussian one. The results are normalized to fit the values obtained by Cesarsky & Audouze (1974).*

Non-linear damping therefore causes the streaming speed across the boundary to increase steeply with rigidity, and will therefore make the decrease of lifetime and path length of galactic cosmic rays with increasing rigidity more pronounced.

Their lifetime and path length in a free zone of height $z(R)$, from which the streaming velocity is $V_S|_z$, are plotted as functions of rigidity in Figs 2 and 3, respectively. Curve (i) assumes an exponential density distribution, as used in the above calculations, while curve (ii) is the result of equivalent calculations assuming a gaussian density distribution with the same surface density. In the case of curve (ii) the Sun is assumed to be situated 10 pc above the galactic plane, where the z-gradients begin to predominate over radial density gradients. The path length has been normalized to fit the results of Cesarsky & Audouze (1974), and the lifetime subsequently normalized to produce that path length in a medium of density 1 atom cm^{-3} at $z = 0$.

4. THE COSMIC RAY ELECTRON SPECTRUM

Since the decrease in lifetime with rigidity is now much steeper than that predicted in Paper I, the conclusions stated in that paper regarding the electron spectrum must now be modified.

The lifetime of cosmic ray electrons in the Galaxy is the same as that of protons of the same rigidity, and hence the same relativistic energy, but during this time they lose energy continually by synchrotron emission and the inverse Compton effect on microwave photons. This energy loss produces characteristic changes in the electron spectrum (see for example Daniel & Stephens 1970). In the case of a constant lifetime, the spectrum steepens by an additional factor 1 in the spectral index around the energy where the characteristic energy-loss time $E/(dE/dt)$ is equal to the containment lifetime τ. But if the containment lifetime decreases with energy, the situation is altered significantly (Silverberg & Ramaty 1973). When the containment lifetime is a power-law, $\tau(E) = \tau_0(E_0/E)^\delta$, the overall change in spectral index is $1-\delta$ when $0 \leqslant \delta < 1$, and 0 when $\delta \geqslant 1$. Furthermore the energy range over which the index changes appreciably becomes wider as δ increases from zero.

A constant lifetime of $\tau = 10^6$ yr and a magnetic field strength of 3×10^{-6} G would produce a bend in the electron spectrum at an energy of ~ 500 GeV. But from Fig. 2 we see that above 100 GeV the lifetime decreases approximately as a power law with index 0·5–0·6. From the results of Silverberg & Ramaty (1973) it is evident that the change in the index of the electron spectrum up to 10^3 GeV will only be $\lesssim 0{\cdot}2$ in this case. Observations are not consistent with a change in index of 1 at 500 GeV (Webber 1973), although the scatter is still too great to obtain an estimate of the position and extent of any bend. Future measurements of the electron spectrum in this region will be very useful as a means of determining the rate of decrease of lifetime with energy.

5. PULSAR SCINTILLATION

Pulses received from distant pulsars are observed to be considerably broadened at low frequencies (Ables, Komesaroff & Hamilton 1970; Lang 1971). The effect has been explained in terms of fluctuations in the electron density in the line of sight, which scatter the radiation and produce a distribution of optical path lengths between the pulsar and the Earth. Observations require a rms density fluctuation $\langle \Delta n_e{}^2 \rangle^{1/2} \sim 10^{-4}$ cm^{-3} over a length scale $a \sim 10^{11}$ cm. Wentzel (1969b) suggested that this indicates the existence of hydromagnetic waves generated by streaming cosmic rays, although these waves would be resonant only with low-energy cosmic rays ($E \lesssim 1$ MeV).

The Alfvén waves which scatter the cosmic rays do not themselves compress the interstellar plasma. But the magnetosonic and sound waves into which they decay by non-linear interaction are compressive, although they are rapidly Landau-damped. A useful problem for future research would be to investigate the equilibrium amplitude of the compressive waves under the influence of streaming cosmic rays and Landau damping, with non-linear interactions coupling the three modes.

Fluctuations of length scale $a \gg 10^{11}$ cm do not cause scintillation, but they can manifest themselves through the effect of refraction. Shishov (1973) showed that a combination of two fluctuation length scales, when one is much larger than the other, can produce a variation of scintillation effects with frequency over and above that produced by the smaller length-scale fluctuations alone. He concluded that the scintillation of CP 0328 (300 pc distant) indicates the existence of fluctuations with length scale 10^{14}–10^{15} cm, which would be produced by cosmic rays of 'rigidity 10 GV'.

Further support for the hypothesis that pulsar scintillation is produced by hydromagnetic waves comes from the conclusions of Williamson (1974 and earlier references therein). Williamson analysed the pulse shapes expected from various configurations of the scattering region, from thin-screen to total line-of-sight, and concluded that scattering must take place in regions of finite size but which do not occupy the whole line of sight. This can be interpreted in terms of waves forming in regions of enhanced ionization, where the damping rate due to collisions with neutral particles is smaller. Such a situation would produce the correlation between the amount of scattering and the pulsar dispersion measure which Williamson mentions. He also states that there is no apparent relationship between the amount of scattering and the degree of H I absorption. In fact four of the pulsars which exhibit scintillation effects show no H I absorption. This suggests that scattering does not take place in H I clouds, which is in agreement with the consideration that an enhanced concentration of neutral hydrogen would damp the hydromagnetic waves.

6. CONCLUSION

The Alfvén waves which are formed by streaming cosmic rays undergo damping by collisions between ions and neutral particles, and by non-linear interactions. The former damping process determines where the waves can form, and once they have grown to a finite amplitude the non-linear effects increase the streaming velocity of the cosmic rays through the waves. This causes the rate of escape of the cosmic rays from the Galaxy to increase rapidly with rigidity, and the resulting decrease in path length is sufficient to explain the observed energy dependence of the ratio of the light elements to the medium elements in the galactic cosmic radiation. The rapid decrease in residence time with energy also applies to cosmic ray electrons, and has the effect of significantly reducing the overall value of the change in their spectral index expected from energy losses in the Galaxy. Recent developments in the theory of pulsar scintillations support the hypothesis that they are caused by hydromagnetic waves produced by streaming cosmic rays.

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REFERENCES

Ables, J. G., Komesaroff, M. M. & Hamilton, P. A., 1970. *Astrophys. Lett.*, **6**, 147.
Bridle, A. H. & Venugopal, V. R., 1969. *Nature*, **224**, 545.
Cesarsky, C. J. & Audouze, J., 1974. *Astr. Astrophys.*, **30**, 119.
Chin, Y. C. & Wentzel, D. G., 1972. *Astrophys. Space Sci.*, **16**, 465.
Daniel, R. R. & Stephens, S. A., 1970. *Space Sci. Rev.*, **10**, 599.
Durney, A. C., Elliot, H., Hynds, R. J. & Quenby, J. J., 1964. *Proc. R. Soc., A.*, **281**, 553.
Falgarone, E. & Lequeux, J., 1973. *Astr. Astrophys.*, **25**, 253.
Holmes, J. A., 1974. *Mon. Not. R. astr. Soc.*, **166**, 155.
Ilovaisky, S. A. & Lequeux, J., 1972. *Astr. Astrophys.*, **20**, 347.
Kulsrud, R. M. & Pearce, W. P., 1969. *Astrophys. J.*, **156**, 445.
Lang, K. R., 1971. *Astrophys. J.*, **164**, 249.
Lequeux, J., 1973. *Proceedings of the NATO Advanced Study Institute on the Interstellar Medium*, Schliersee, Germany.
McIvor, I. & Skilling, J., 1974. *Mon. Not. R. astr. Soc.*, **167**, 49.
Parker, E. N., 1965. *Astrophys. J.*, **142**, 584.
Piddington, J. H., 1970. *Aust. J. Phys.*, **23**, 731.
Piddington, J. H., 1973. *Mon. Not. R. astr. Soc.*, **162**, 73.
Puget, J. L. & Stecker, F. W., 1974. *NASA GSFC X*-640-74-17.
Shishov, V. I., 1973. *Astr. Zh.*, **50**, 941.
Silverberg, R. F. & Ramaty, R., 1973. *Nature Phys. Sci.*, **243**, 134.
Skilling, J., 1971. *Astrophys. J.*, **170**, 265.
Skilling, J., McIvor, I. & Holmes, J. A., 1974. *Mon. Not. R. astr. Soc.*, **167**, 87P.
Webber, W. R., 1973. Report Paper, 13*th Int. Conf. cosmic rays*, Denver.
Wentzel, D. G., 1969a. *Astrophys. J.*, **156**, 303.
Wentzel, D. G., 1969b. *Astrophys. J. Lett.*, **156**, L91.
Wentzel, D. G., 1974. *A. Rev. Astr. Astrophys.*, **12**, 71
Williamson, I. P., 1974. *Mon. Not. R. astr. Soc.*, **166**, 499.