THE NESTED LEAKY-BOX MODEL FOR GALACTIC COSMIC RAYS

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We present further developments of our model originally proposed to explain observations that (1) the spectra of primary cosmic-ray nuclei in the energy range 1-50 GeV/nucleon become progressively flatter with increasing Z and that (2) the ratios of the fluxes of secondary nuclei to that of their parents are decreasing functions of energy over the same range. The "inner box" of cosmic ray confinement, corresponding to the region immediately surrounding the source, is assumed to have energy-dependent life time. On the other hand subsequent propagation in the "outer-box" corresponding to the interstellar medium, is assumed to be independent of the energy of the cosmic rays. Here, (a) we explicitly calculate the distribution of path-lengths implied by the model, (b) discuss the propagation of superheavy nuclei, (c) derive the spectra of electrons and positrons and (d) show that this model implies well defined gamma-ray fluxes from the cosmic-ray sources.

1. The Model. Careful experimental work by several groups has established that the relative abundances of cosmic-ray nuclei is dependent on energy in the range 3-50 GeV/nucleon (Webber et al 1973, Ormes and Balasubrahmanyan 1973, Smith et al 1973, Juliusson 1974, and Garcia-Munoz et al 1975). The main observational features are (1) the energy spectra of primary nuclei like, H, He, C, O, Si, Fe, etc. become progressively flatter with increasing nuclear charge Z and (2) the ratios of the fluxes of secondary nuclei to that of their progenitors are decreasing function of energy. We have argued that the most likely explanation of these features is that cosmic rays are stored briefly around the sources before they are released into the interstellar medium (Cowsik and Wilson 1973, referred to hereafter as paper I). The mean residence time of the cosmic rays in the source region (the inner-box) is a decreasing function of their energy and the spallation of the nuclei during this time leads to the observed properties. Nuclei at lower energies and higher charges are spalled more effectively because of longer storage time and larger nuclear cross-sections; consequently the spectra of heavier nuclei become flatter and there is a more copious production of secondaries of lower energies i.e. steep spectra for the secondaries. Subsequent to their injection into the interstellar medium the cosmic rays propagate in a manner not dependent on their energy and are lost into the intergalactic medium. The parametrization of the energy dependent transport near the sources can be made in terms of the mean grammage traversed by the cosmic rays

$$\tau_s(E) \equiv \beta c \rho_s m_H T_s(E) \equiv \tau_0 e^{-aE} \cong \beta c m_H \rho_s T_0 e^{-aE} \approx 1.9 \exp[-E/7.85 \text{ GeV/nucl}] \text{ g cm}^{-2}$$

This parametrization is by no means unique but is chosen as an arbitrary form which yields good fit to the data. The transport in the interstellar medium is characterised by

$$\tau_G \equiv \beta c \rho_G m_H T_G \approx 1.9 \text{ g cm}^{-2}.$$

Here τ_s is the grammage in g cm^{-2} traversed by cosmic rays on the average during a mean storage time T_s seconds near the sources, m_H = mass of a hydrogen atom, ρ_s the number density of equivalent hydrogen targets in the source region and the symbols with subscript 6, refer to corresponding values of the parameters in the interstellar medium.

In Paper-I we had shown that the propagation equations describing the effects of leakage and the nuclear interactions on the cosmic-ray densities could be solved easily by matrix methods. Using such a technique detailed calculations of the densities of all the nucleides upto Fe was made showing that the model could adequately explain all the features of the nuclear spectra and the relative abundances, still retaining intact the attractive cannonical assumptions that all the nuclei are accelerated to identical spectral term $\sim E^{-2.6}$ by the cosmic ray sources and that their relative intensity is roughly correlated with the socalled universal abundances.

In this paper we derive distribution of pathlengths of cosmic ray nuclei in the 'nested leaky box model' and discuss the fluxes of superheavy nuclei, electrons and positrons. We also show that this model predicts well defined fluxes of γ-rays from the cosmic ray sources. Finally we point out certain observational tests to the model. In view of need for brevity and readability, the presentation here uses only approximate formulae and a more critical discussion of the results will be provided elsewhere (Cowsik and Wilson 1975).

2.1 Distribution of Vacuum Pathlengths. This distribution consists of two parts, a) the distribution in the source region characterised by τ_s and b) that in the interstellar medium characterised by τ_G. With the assumption that the probability of leakage of cosmic rays is independent of time, the 'vacuum pathlength distribution' (Cowsik et al 1967) in each of the regions has an exponential dependence on the path-length. The probability that the path length is X_s in the source is given by

$$P_s(x_s) = \frac{1}{\tau_s} e^{-x_s/\tau_s}$$

and the corresponding expression for the interstellar medium is

$$P_G(x_G) = \frac{1}{\tau_G} e^{-x_G/\tau_G}$$

Now, the probability that the total path length is $X = X_s + X_G$ is given by

$$P(x) = \frac{1}{\tau_s \tau_G} \int_0^x e^{-x_s/\tau_s} \cdot e^{-(x-x_s)/\tau_G} \, dx_s$$

$$= \frac{e^{-x/\tau_G}}{\tau_G \tau_s} \left\{ \frac{\tau_G \tau_s}{\tau_G - \tau_s} \left(1 - e^{-\frac{\tau_G - \tau_s}{\tau_G \tau_s} x} \right) \right\}$$

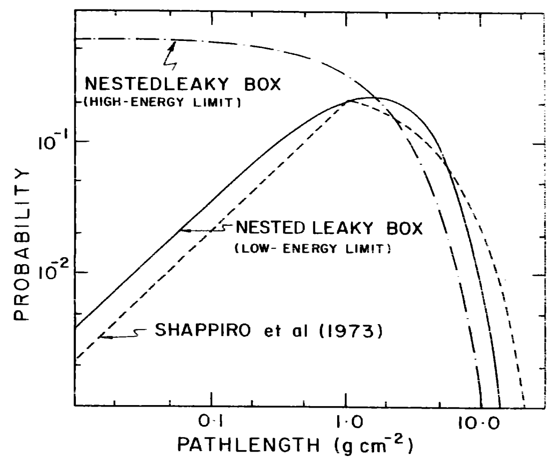
τ_s has the form $\tau_0 e^{-aE}$ and tends to $\tau_0 \approx \tau_G \approx \tau$ at low energies and to zero at high energies. The nett path length distribution $P(x)$ has the corresponding limits

$$P_L(x) = \frac{x e^{-x/\tau}}{\tau^2} \qquad \text{at low energies}$$

and $$P_H(x) = \frac{e^{-x/\tau}}{\tau} \qquad \text{at high energies}$$

These limits are shown in Fig.1. In the same figure we have also shown the distribution used by Shapiro et al (1973) for estimating the cosmic ray fluxes at the sources. The low energy limit $P_L(x)$ which is relevant in this context, is strikingly similar to the empirical distribution postulated by Shapiro et al. The nested leaky box model thus provides a physical basis for the path length distribution used to fit the cosmic ray abundances so well. The convolution of the two exponential distributions one in the source region and the other in the interstellar medium produces the distribution which increases roughly linearly at small pathlengths and decays exponentially at large values.

Fig.1. The distribution in pathlengths predicted by the nested leaky box model is compared with the empirical distribution obtained by Shapiro et al.



2.2 Fluxes of Heaviest Nuclei at Injection. The relative injection rates ("source abundances") of the various nuclei have been calculated by several authors (for example, Shapiro et al 1973, paper I). The main feature of the relative intensities is that their ratio with respect to the socalled 'universal abundances' has a simple dependence on Z, increasing by a factor of about ten from $Z \approx 1$ to $Z \gtrsim 27$. This enhancement has been interpreted in various ways by different authors. In estimating the "source abundance" of the very heavy nuclei ($Z \gtrsim 50$) whose mean free path for break up is ~ 1 g.cm^{-2}, it is the distribution of path lengths at small values of X (~ 1 g.cm^{-2}) that is most relevant; identical path length distributions will lead to essentially identical 'source abundances'. Therefore the nested leaky-box model will predict similar source abundances as Shapiro et al (1973). In fact preliminary calculations grouping the nuclei beyond Fe five at a time substantiates this equivalence. It should be noted however that in the discussion of the fluxes of radioactive nuclei, not merely is the path length distribution but also the densities of target atoms in the various regions of propagation are relevant. A detailed discussion of the radioactive nuclei is provided elsewhere (Cowsik and Wilson 1975).

3. Electrons and Positrons in the Nested Leaky Box.

3.1 Electrons. The spectrum of electrons in the galactic disc has a spectral term different from that of the nuclear component in that the logarithmic slope of the energy spectrum is -1.6 below ~ 3 GeV and is -2.6 above (Meyer 1971). An energy dependent transport in the immediate vicinity of the cosmic ray accelerator might lead to such a flatter spectrum at low energies (Cowsik 1971). However, in this note we restrict ourselves mainly to a discussion of the further transformations suffered by the electron spectrum subsequent to its release into the source region (the inner box). The spectrum of electrons in the inner box is controlled by leakage with a time constant $T_s(E)$ and by energy losses in processes such as the synchrotron radiation (which can be parametrised in the form $\frac{dE}{dt} = -bE^2$; $b = \frac{2}{3}c\left(\frac{E}{mc^2}\right)^2 B^2 \left(\frac{E}{mc^2}\right)^2 \approx 3.8\times10^{-6}$ B^2 GeV Gauss^2/GeV^2 Sec.). With $A_e(E,t)$ being the spectrum of electrons coming out of the accelerator, the density of electrons in the source region $N_s(E,t)$ satisfies the differential equation

$$\frac{\partial N_s(E,t)}{\partial t} = A_e(E,t) - \frac{\partial}{\partial E}\left[-bE^2 N_s(E,t)\right] - \frac{N_s(E,t)}{T_s}$$

with the integral

$$N_s(E,t) = \int_{1/bE}^{t} A_e\left(\frac{E}{1-bEt'}, t'\right)\cdot(1-bEt')^2 \cdot Exp\left[-\int_{t'}^{t}\frac{dt''}{T_s(E/1-bEt'')}\right]$$

For $T_s \sim T_0\, e^{-aE}$ the above integral cannot easily be written in terms of elementary functions; but for slow time dependence of $A(E,t)$, the integral can be approximated as

$$N_s(E) \simeq \frac{A_e(E)\, T_s(E)}{1+(\gamma-1)\, bE\, T_s(E} = \frac{A_e(E)\, T_0\, e^{-aE}}{1+(\gamma-1)\, bE\, T_0\, e^{-aE}}$$

where $\gamma \approx d\ln N_s(E)/dE$ is the spectral slope of the electrons in the relevant energy region.

Now, since the probability of leakage per unit time is $\sim 1/T_s$ the spectrum of electrons injected into the interstellar medium is

$$f(E) = \frac{N_s(E)}{T_s(E)} = \frac{A_e(E)}{1+(\gamma-1)\, bE\, T_0\, e^{-aE}}$$

Experiment demands that f(E) has the form $\sim E^{-1.6}$ below ~ 3 GeV and $\sim E^{-2.6}$ above 3 GeV. Assuming that A(E) has itself such simple power law behaviour the parameters of the innerbox (source region) have to jointly satisfy the condition
$(\gamma-1)\, bE\, T_0\, e^{-aE} \ll 1$ or equivalently $10^{-5} B^2\, T_0 / \beta \ll 1$
Quite a wide range of parameters representative of young supernova remnants (should one choose them as the inner boxes) can easily satisfy this restriction. We have chosen B^2 To $\approx 3 \times 10^4$ Gauss^2.sec. (B 10^{-3} Gauss and To $\approx 3 \times 10^{10}$ Sec say) and have shown the ratios $N_s(E)/A(E)$ and $f(E)/A(E)$ indicating the spectral form inside the sources and that injected into the intergalactic medium in Fig.2. This choise of parameters is not unique and a more detailed discussion of the spectra of electrons will be presented elsewhere. In the same figure we also show the typical frequency at which electrons of the various energies will radiate through the synchrotron process in the magnetic fields present in the sources.

We may note that (1) even intense fields as $\sim 10^{-3}$ Gauss in the source region do not produce drastic changes in the spectrum injected into the interstellar medium; (2) the spectrum of electrons inside the sources themselves are cut off around 10 GeV because of energy dependent leakage. However this corresponds to synchrotron radio-frequencies of $\sim$ 400 GHz. Thus the observation of a flat synchrotron spectra upto $\sim$10 GHz of supernova remnants by Milne and Hill (1969) does not pose any special problem in the nested leaky box model. A(E) can be chosen with the cannonical for $\sim E^{-1.6}$ upto 3 GeV and $\sim E^{-2.6}$ beyond to explain their observations adequately.

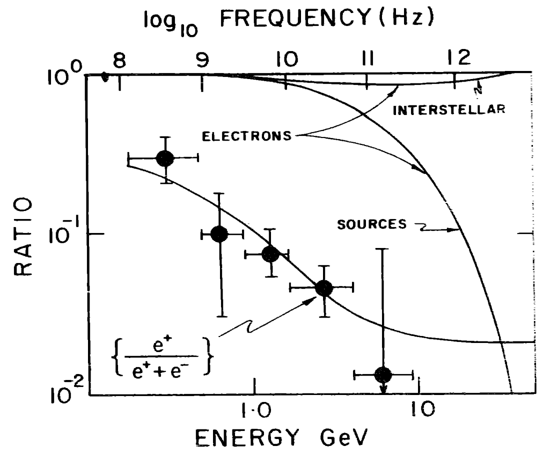


Fig.2. The spectrum of electrons inside the sources and that in the interstellar space are shown as ratios with respect to the intrinsic spectrum generated by the cosmic-ray accelerators. The frequency scale refers to the mean frequency of synchrotron emission by the electrons in the magnetic fields of the sources. The positron ratio calculated in the model is compared with the experimental data (Fanselow et al 1969).

3.2 Spectrum of Positrons. Positrons in the cosmic rays are the result of the $\pi^{\pm} \to \mu^{+} \to e^{+}$ decay chain of the pions produced by the interactions of the nuclear component of the cosmic rays with the matter in the source regions and in the interstellar space. Because of the smallness of the ratio of the energy of the positron to the energy of the nucleonic parent, $\eta \approx 0.09$, the positron fluxes are controlled by the fluxes of nucleons at much higher energy. The cosmic ray nucleonic density in the sources is given by

$$N_n(E) \approx A_n(E)\, T_s(E)$$

In the energy range over which η is roughly constant the density of positrons in the source regions is

$$N_{e^+}(E) = \frac{A_n(E/\eta)\, T_s(E/\eta)}{\eta}\, \sigma_{e^+}\, c\, \rho_s\, T_s(E)$$

and the rate at which these leak into the interstellar medium is

$$f_{e^+}(E) = \frac{A_n(E/\eta)\, T_s(E/\eta)}{\eta}\, \sigma_{e^+}\, c\, \rho_s$$

Note here that f_{e^+} has a very sharp cut-off at high energies because of the term $T_s(E/\eta) \sim T_0 \mathrm{Exp}(-aE/\eta)$. Assuming $A_n(E) \sim A_{no}\, E^{-2.6}$ the above expression can be evaluated. Besides these positrons are also created in the interstellar medium at a rate $g_{e^+}(E) = (N_{G,n}(E/\eta) \cdot \sigma_{e^+} \cdot c \cdot \rho_G)/\eta$ where $N_{G,n}$ is the density of cosmic-ray nucleons in the interstellar space. In Fig.2 we show the ratio of the densities $e^+/(e^+ + e^-) \approx (f_e + g_{e^+})/(f_{e^+} + g_{e^+} + f_e)$ and compare our calculations with the observations of Fanselow et al (1969). The steep fall of f_{e^+} would in principle be a very good test for the model. Unfortunately the statistical confidence and the energy-range of observation of the positron fluxes are hardly adequate at present to lend firm support to the model.

4. The Gamma-ray Flux from a Cosmic Ray Source. Cosmic ray sources would be strong sources of γ-rays which are produced through the interactions of cosmic rays with the ambient matter and radiation fields. Perhaps the most important process for the production of high energy γ-rays (~ 100 MeV) is the nuclear interaction creating π^0-mesons (Pinkau 1971, Meneguzzi 1973). The number of γ-rays $E_\gamma \geq 100$ MeV) produced per second in a cosmic ray source is given by

$$G_s \approx \int_{E_\gamma = 100\,\mathrm{MeV}}^{\infty} \int_{E_{th}}^{\infty} N_n(E)\, c \cdot \beta \cdot \sigma_{n,\pi^0}(E, E_{\pi^0} \approx 2E_\gamma)\, \rho_s\, dE\, dE_\gamma$$

τ_0 being approximately same as τ_s and the threshold for the π^0- production being small compared with $1/a$, there is roughly equal production of γ-rays in the sources and in the interstellar medium. If ν be the number of sources then

$$G_s \approx \frac{N_{s,n}\, c\, \sigma\, \rho_s V_s}{\nu} \cdot \frac{\tau_0}{\tau_s} \approx 10^{41}/\nu \qquad \gamma\text{-rays/sec}$$

Here we have made use of Stecker's (1969) calculations for the product $N_{s,n}\,\sigma c$
Now, suppose the ν sources are uniformly distributed over a disc of galactic radius $r_G \approx 4.5 \times 10^{22}$ cm the mean distance to a typical source $\langle d \rangle \approx \sqrt{\frac{\pi r_G^2}{\nu}}$. The flux of γ-rays from a typical source turns out to be

$$\langle f_\gamma \rangle \approx \frac{G_s}{4\pi \langle d \rangle^2} \approx \frac{10^{41}}{4\pi^2 r_G^2} \approx 10^{-6}\ \gamma/\text{cm}^2.\text{sec.ster.}$$

The flux is just under the upper limits placed on several likely galactic sources of γ-rays by the SAS-II experiments (Fichtel et al 1975). It is quite likely that the next generation of γ-ray telescopes will be able to detect these sources whose angular diameters are expected to be much smaller than 1°; i.e. essentially there will be point sources of γ-rays.

5. Discussion and Summary. Among the various models attempting to explain the energy dependent relative intensities of cosmic ray nuclei the "nested leaky-box" model has been generally favoured (Juliusson 1974, Daniel and Stephens 1975). However, as expressed by the later authors, at first glance at the model one might fear that it might predict too high a source abundance of the super heavy nuclei and too steep a radio spectrum for the cosmic ray sources. Explicit calculations of the path length distributions and electron spectra presented here allay these fears. Also the fluxes of γ-rays and positrons calculated here provide further tests to the model. A rigorous treatment of the various points presented here will be given elsewhere.

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