OG 10-1

MAXIMUM LIKELIHOOD DERIVATION OF SPECTRAL SLOPES

J Linsley
IFCAI-CNR, Via M Stabile 172, 90139 Palermo, Italy

A A Watson
Department of Physics, Leeds University, Leeds LS2 9JT, UK

Abstract

Novel applications of the maximum likelihood technique have been used to obtain best estimates of spectral slopes in situations where there may be a flattening of or a cut-off in the spectrum. The technique has been compared with the least-squares method by Monte Carlo simulations. The validity of the latter approach in certain situations is questioned.

The techniques developed are applied to results obtained from the Haverah Park experiment (Lawrence et al OG 5.1-17)

OG 10-2

A NEW FORMULA FOR CALCULATING COSMIC RAY CROSS SECTIONS IN HYDROGEN TARGETS

W. R. Webber
Space Science Center, University of New Hampshire
Durham, NH 03824 USA

Abstract

Over the last five years we have measured the cross sections of ten different beam nuclei ranging from ^{12}C to ^{58}Ni in CH_2 and C targets at the BEVALAC. These cross sections have been measured at a large number of energies between $\sim$ 300 and 2000 MeV/nuc - as many as six energies for ^{56}Fe nuclei, for example. The cross sections for secondary production have been determined in Hydrogen for several hundred charge changing and several hundred isotopic cross sections at $\sim$ 600 MeV/nuc and other energies. From this systematic study we have been able to develope a new and rather simple cross section formula that applies to the production of secondary nuclei with Z = 4-28 and A = 7-60 and for energies $\gtrsim$ 150 MeV/nuc. It predicts the cross sections in Hydrogen at 600 MeV/nuc to an accuracy $\sim$ 10% or better, an improvement by a factor $\sim$ 3 over the Tsao and Silberberg formula.

1. Introduction. The semi-empirical cross section formula developed by Tsao and Silberberg, 1979, for calculation of cross sections in Hydrogen has been a valuable tool for the study of cosmic ray propagation in the galaxy. The overall estimated accuracy of this formula of ±35% for unmeasured cross sections is now clearly inadequate for the higher precision cosmic ray data that is available. This level is understandable when one realizes the limited cross section data that has been available previously on which to base the systematics of such a formula. Over the last five years we have measured the cross sections for secondary production from ten nuclei ranging from ^{12}C to ^{58}Ni at the BEVALAC (e.g. Webber and Brautigam, 1982.) These nuclei comprise > 90% of all cosmic ray nuclei in the source and $\sim$ 70% of all nuclei arriving at the earth. For each of these ten charges we have a complete set of isotopic cross sections at $\sim$ 600 MeV/nuc for secondary nuclei with $(Z_i-Z_f)/Z_i \lesssim 0.4$ - a total of > 250 isotopic cross sections in all. Charge changing and total cross sections are also measured for a wider range of secondary nuclei at 600 MeV/nuc and also at a large number of other energies between 300 and 2000 MeV/nuc. From a systematic study of these cross sections we have been able to develope a new and rather simple cross section formula. For nuclei with Z = 4-28 and A =

7-60 and for energies $\gtrsim$ 150 MeV/nuc it predicts the measured cross sections in Hydrogen to an accuracy $\sim$ 10% or better.

2. The Cross Section Formula. Several new systematics related to secondary cross sections have led to a great simplification over previous formulations. Our procedure was to first fit the charge and isotopic cross sections at 600 MeV/nuc and then extend the formulation to other energies. This allows the general cross section formula to be written as a product of three essentially independent terms as follows:

$$\sigma(Z_f,A_f,E) = \sigma_0(Z_f,Z_i,A_i) \cdot f_1(Z_f,A_f,Z_i,A_i) \cdot f_2(E,Z_f,Z_i) ,$$

The total σ for a particular final charge Z_f, and mass A_f, depends on: (1) A term, σ_0 that is dependent on the initial charge Z_i and final charge Z_f only, to first order. (2) A term, f_1, that defines the centroid and half width of the mass distribution for a particular Z_f. The centroid is a function of Z_i and A_i and the half-width, δ_{zf}, of the mass distribution is a function of Z_f only to first order. (3) A term, f_2, which describes the overall energy dependence of σ. This is a function of Z_i and $Z_i-Z_f = \Delta Z_f$.

We now consider these terms in more detail.

Term (1):

$$\sigma_0(Z_f,Z_i) = \sigma_{zf} \ \exp \frac{-(Z_i-Z_f)}{\Delta_{zf}} \cdot \exp \frac{-|N_{zi} - N_{ozf}|}{6} .$$

The first term on the RHS represents the fact that the overall elemental cross sections for a particular Z_f can be well represented by a simple exponential function of the difference Z_i-Z_f. The characteristic of this exponential fall off, which is different for each Z_f, is given by Δ_{zf} and the value for $(Z_i-Z_f) = 0$ is given by σ_{zf}. Examples of this exponential behavior are illustrated in Figures 1a and b. This very significant finding permits an enormous simplification in the overall cross section calculation. (A similar behavior has been observed for UH interactions, Waddington, 1986). The values of σ_{zf} and Δ_{zf} are presented in tables for each Z_f between 4 and 28.

The second term on the RHS is related to the fact that the production of a secondary nucleus Z_f,A_f, from a primary nucleus Z_i,A_i, is a maximum when the primary and secondary nucleus both have A_i and A_f near the location of the mass stability line. The quantity N_o (N is the neutron excess, defined as A-2Z) defines this line of β stability for Z_f. The half width of $\sim$ 6 AMU for this function is determined from the cross section data from primaries of different neutron excess,eg. ^{40}Ar and ^{40}Ca.

Term (2):

$$f(A_f,A_i) = \frac{1}{\delta_{zf}\sqrt{2\pi}} \ \exp \frac{-(N-N_{zf})^2}{2\delta_{zf}} ,$$

where $\delta_{zf} = 0.32 \cdot Z_f^{0.39}$ except for $Z_f = 4$

The quantities N_{zf} are defined separately for Z_f (even) and Z_f (odd) by fitting the extensive set of isotopic cross section data. Space does not permit us to describe this term in more detail here.

Term (3) consists of series of four terms to describe the energy dependence as determined from the fragmentation of ^{56}Fe at six energies between 300-1700 MeV/nuc, ^{24}Mg at four energies and other incident charges at several energies.

$$f(E,Z_i,Z_i-Z_f) = [\ 1 + m(\Delta Z)\ g(Z_i)\ \exp \frac{-(E-E_m)^2}{\Delta E_m} \quad + \ . \ . \ .$$

The first term (m term), illustrated here, dominates the energy range from 500-2000 MeV/nuc; the coefficients of this term, m(ΔZ), depend on the difference $Z_i - Z_f$. Other terms include a term effective above 2000 MeV/nuc, a term effective between ∿ 200-500 MeV/nuc and a term effective below ∿ 200 MeV/nuc. The factors g(z) are of the form $(Z_i/28)^m$ and are present in each term. They take into account the observation that the magnitude of the energy dependence of the cross sections increases with increasing Z. Space does not permit us to describe these terms in more detail here.

Finally, there are specialized terms for proton and neutron stripping as derived from the measurements.

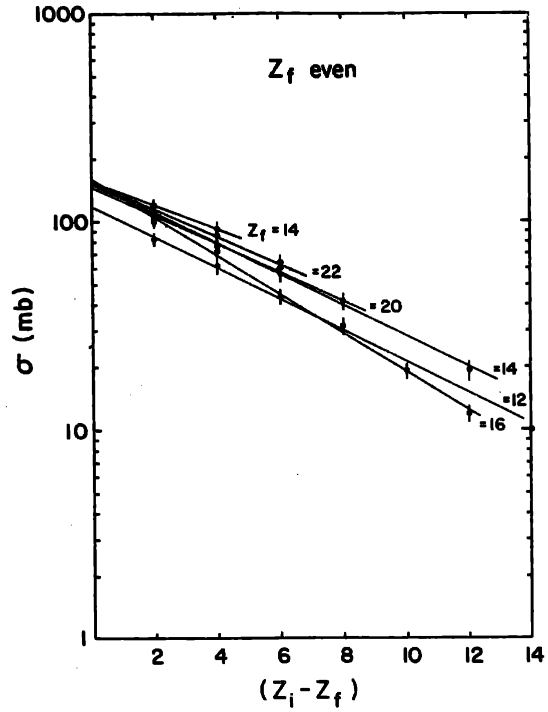

Fig.1a. Dependence of charge changing cross sections on $(Z_i - Z_f)$ for Z_f even

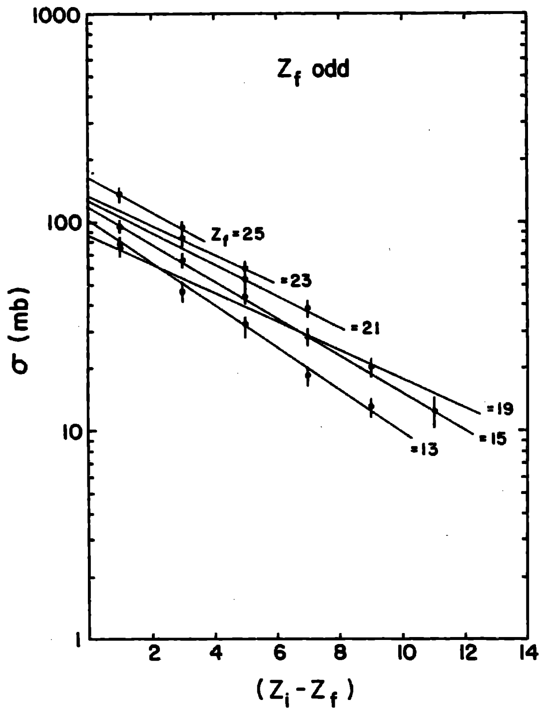

Fig.1b. Same as 1a except for Z_f odd

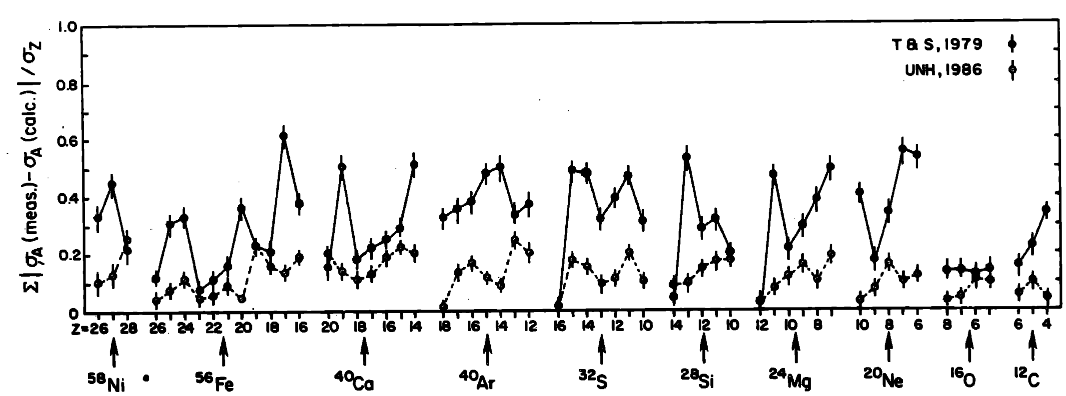

Fig.2. Differences in measured and predicted cross sections obtained using different cross section formulas

3. Comparison of Predicted and Measured Cross Sections. There are several ways to make this comparison. Here we have taken our complete list of >250 isotopic cross sections measured at ∿ 600 MeV/nuc as a base. For each secondary charge for which all isotopic cross sections have been measured, we have determined $\Sigma\,|\sigma_A\,(\text{meas}) - \sigma_A\,(\text{calc})|$ in mb. A ratio of this difference to the respective total charge changing σ is shown in Figure 2 for all ten of the beam charges we have measured. The average difference between the calculated and measured σ is 10.2% for our new cross section formula and 31.6% when the Tsao and Silberberg, 1979, formula is used to calculate the isotopic . Similar differences between calculations and our measurements are found at other energies, and also when comparisons are made with other available measurements. A major difference in the two cross section formula is in the energy dependence of the cross sections from heavier nuclei - particularly secondaries from ^{56}Fe.

4. Effect of New Cross Sections on Interstellar Cosmic Ray Propagation Calculations. The new cross section measurements and those predicted using the new cross section formula have been incorporated into the SACLAY interstellar propagation program (Koch et.al., 1981). The difference between the predictions using the new set of cross sections and those using the Tsao and Silberberg, 1979, cross sections are far-reaching and encompass almost all aspects of the galactic propagation problem as it is related to heavier nuclei. These results are being discussed in several papers by the UNH and SACLAY groups; here we briefly summarize some of the initial findings.

a) The path length traversed by cosmic rays in interstellar space is increased by ∿ 20%. This is due to the fact that the total production into Boron at ∿ 1 GeV/nuc is ∿ 8% less than previously calculated.

b) The total production of Nitrogen isotopes is 8-15% greater than previously calculated at ∿ 1 GeV/nuc, which along with differences in Boron production goes a long way towards explaining previous discrepencies in B/C and N/O abundance calculations, (Meyer, 1985.)

c) Numerous changes in cosmic ray source abundances - of both charges and isotopes are evident. For example the N/O or $^{14}N/O$ source abundances are now deduced to be ∿ 4% or less.

d) The interstellar production of the heavy isotopes of Ne, Mg, and Si is increased considerably, thus reducing the overabundance of these isotopes inferred in the cosmic ray source (Wiedenbeck, 1984).

e) The situation regarding path length truncation at short path lengths, which arises because of predictions of the B/C and Fe sec/Fe ratios is greatly modified.

5. Acknowledgements. This work was supported by a HEAO Guest Investigator Grant NAG8-606 and NASA Grant NGR-30-002-052.

References

1. Koch L. et al., Proc. 17th ICRC, 2, 18, 1981.
2. Meyer J.P., Proc. 19th ICRC, 9, 141, 1985.
3. Tsao C.H. & Silberberg R., Proc. 16th ICRC, 2, 202, 1979.
4. Waddington C.J., Private communication, 1986.
5. Webber W.R. and Brautigam D.A., Ap. J., 260, 894, 1982.
6. Wiedenbeck M.E., Adv. Space Res., 4, 15, 1984.