OG H.2-4

ISOTOPE ANALYSIS USING THE GEOMAGNETIC METHOD

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1. Introduction

The data recorded by the C2 instrument on board the HEAO3 satellite should improve our understanding of isotopic composition of heavy relativistic cosmic ray nuclei. The C2 counters and hodoscope combination is suitable for an accurate determination of momentum and arrival direction of individual cosmic rays (ref. 3 and 4) . The next step in data analysis is to assign a cut-off rigidity value to each arrival direction with the help of a magnetic field model and to build the average transmission function for each element. The transmission function method has been reported in ref. 1 and 2 and mean mass estimates have been reported.

This method is very useful for organizing data collected in a finite cut-off rigidity range, but in spite of its conceptual simplicity it does not allow the reader to easily figure out how systematic and statistical errors propagate. Indeed the geomagnetic method does not separate isotopes in the usual sense : it is based on a comparison of the shape of the transmission to that of a reference element.

In this paper we describe an analysis of the observed transmission based on its comparison to expectation. For this we have selected oxygen nuclei with directions where the rigidity cut-off is $\gtrsim$ 7 GV and can be considered sharp. We describe this selection and the cut-off assignment in section II. In section III we present the observed transmission and compare it to that calculated under the assumption of sharp cut-offs. This calculation takes into account the finite resolution of the counters. This allows to estimate the level of systematic errors. It is also shown that under sharp cut-off selection the transmission function for a single isotope is weakly dependent on charge and does not depend on the cut-off distribution, thus ensuring an unambiguous comparison to the reference element. In conclusion we use this technique to derive a tentative abundance of Neon isotopes.

2. The cut-off assignment and sharp cut-off selection

2.1 The trajectory calculation : We compute the penumbra pattern for each incoming direction. This is done by computing the trajectories in a model magnetic field (IGRF, 1979) for a sequence of equally spaced decreasing rigidities. The spacing is chosen equal to 2%. This value was considered as an acceptable trade-off between computing time and field model uncertainties. In addition each identified transition rigidity value from allowed to forbidden is calculated with higher precision through an iterative process (fig. 1).

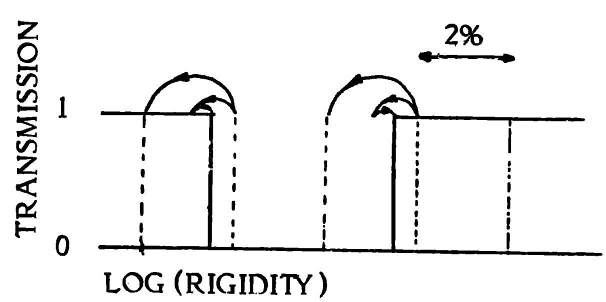

Fig.1 Sketch of the procedure used for trajectory calculation.

2.2 The selection criteria and cut-off assignment : There is not a unique way to assign a cut-off value to each incoming direction for isotope analysis. The combination of selection criteria and cut-off assignment is aimed at keeping the sharp cut-off edge and at rejecting as much background as possible below cut-off. For this we select directions with at most two allowed bands (fig.2) and such that the width of the forbidden bands is smaller than the counter resolution. At cut-off rigidities above ~ 7 GV these criteria discard a moderate amount of otherwise good events.

The cut-off value is defined as that of the lowest allowed rigidity (fig.2). The resulting average transmission looks like curve a (fig.3). This selection relies on trajectory calculation results. We tentatively estimate the accuracy of these results as follows : in the rigidity range around the cut-off value the trajectories are often very complex and their fate is sharply dependent on the value of the shooting rigidity.

Examination of these trajectories reveals that near the earth cosmic rays almost behave like trapped particles : they bounce from one hemisphere to the other while drifting in longitude. This drift pattern is moderately changing in a ~ 10% rigidity range above cut-off. It seems reasonable to assume that the departure between the calculated trajectory and the actual one increases with the integrated tracklength, or with the total drift in the earth field and that the calculated cut-off departs accordingly from the actual one.

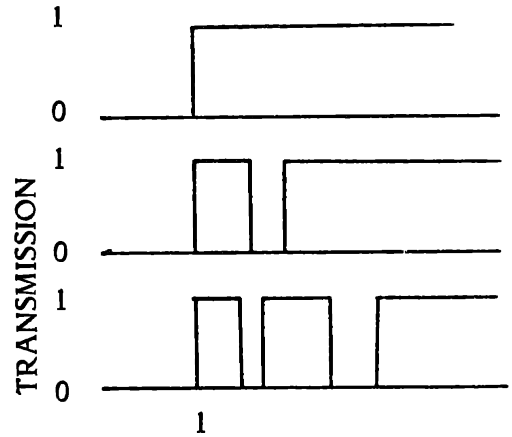

Fig.2 Cut-off assignment for 3 different penumbra patterns.

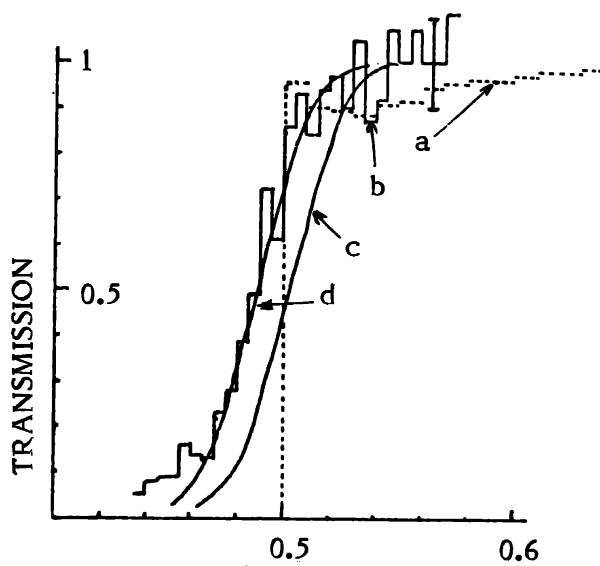

Fig.3 Transmissions in the cut-off rigidity range 6.8-8.5 GV :
Curve a : from trajectory calculation.
Curve b : observed with 0 nuclei.
Curve c : calculated from (4).
Curve d : same as c shifted 3% downwards in P/R.

When we compare the transmission curves built from groupings of the directions with similar drifts, we find an upper limit forthe observed shift of ~ 1%, suggesting that this is about the systematic error in the cut-off value.

3. Results and interpretation

3.1 The observed transmission : We have built the average transmission for oxygen events selected in the cut-off rigidity range 6.8 to 8.5 GV and with arrival directions fulfilling the above criteria. This rigidity range matches the most sensitive momentum range of the aerogel block counter in the C2 instrument and is thus suitable for a detailed study of the transmission method. The transmission curve was built as a function of P/R (Momentum/Cut-off) as explained in ref. 2 and is thus an average over the rigidity range. About 4000 oxygen nuclei with $P/R < 0.6$ were available. This observed transmission curve is displayed on fig.3 (curve b).

3.2 The calculated transmission : In order to estimate the level of residual errors it may be useful to compare the observed transmission to the expected one with a single isotope with $A = 2Z$. For this we compute two distributions at P/R : the observed one : OB(P/R) and the one which would have been observed in the absence of magnetic screening (the unfiltered spectrum in ref. 2 MS (P/R). This is a reference distribution taking into account the observed momentum spectrum and the time exposure at each rigidyty cut-off. We may write :

$$OB(P/R) = \int_{R_1}^{R_2} \psi(R).R.dR . \int_{x=P_o}^{+\infty} t(x,R).\Phi(x)\, EXP(-(P-x)^2/2\sigma^2)/\sqrt{2\pi}.dx \quad (1)$$

$$MS(P/R) = \int_{R_1}^{R_2} \psi(R).R.dR. \int_{-\infty}^{+\infty} \Phi(x).EXP(-(x-P)^2/2\sigma^2)/\sqrt{2\pi}.dx \quad (2)$$

where the integrals must be calculated at constant P/R and the average transmission at P/R is defined as $T_{2Z} = OB/MS$. $\Phi(x)$ is the "true" momentum spectrum. p is the observed momentum derived from the Cerenkov signal and x is the "true" one. t(x,R) is the transmission at cut-off R. This is an average over all directions with cut-off R. $\psi(R)$ is the cut-off distribution and p_o is some momentum value (lying around 0.4 * R) below which all directions are forbidden. σ is the momentum resolution. From (2) we may write :

$$MS(P/R) = \int_{R_1}^{R_2} \psi(R).R.\varphi(p)\, dR \quad (3)$$

where $\varphi(p)$ is the observed momentum spectrum in the instrument. If $\varphi(p) \propto P^{-\gamma}$ we see that $MS(P/R) \propto (P/R)^{-\gamma}$ and the reference spectrum has the same shape as the observed one in the instrument.

The calculation of OB(P/R) is not simple in general. For the resolution to be as high as possible sharp cut-off directions must be selected. In that case $t(x,R) = 1$ for $p_o > 0.5 * R$ and $t = 0$ otherwise. OB(P/R) can thus be calculated with the help of an educated guess on the change of counter resolution with momentum. As an example this calculation has been performed for constant resolution σ/p. As a result the transmission is independent of the cut-off distribution and may be expressed as :

$$T_{2Z}(P/R) = 0.5 \times (2xP/R)^{\gamma}(1-ERF(v/\sqrt{2}) + \varepsilon_1 + \varepsilon_2) \text{ for } P/R < 0.5$$

$$T_{2Z}(P/R) = 0.5 \times (1 + ERF(v/\sqrt{2}) - \varepsilon_2) \text{ for } P/R > 0.5 \quad (4)$$

where ERF is the error function and $v = (1-2P/R)/(\sigma/p)$. ε_1 and ε_2 are terms of order σ/p with maximum value around $P/R = 0.5$. Keeping only terms of first order in σ/p we may write :

$$\varepsilon_1 \sim \gamma\sigma/p \times V(1-ERF(V/\sqrt{2}))$$ and $$\varepsilon_2 \sim \gamma\sigma/p \times \exp(-V^2/2)/\sqrt{2\pi}$$

For an isotope with atomic mass A the transmission is $T_A(P/R) = T_{2Z}(AP/2ZR)$.

In practice one may wish to compare transmissions of elements with widely different charges within similar resolution ranges and thus in charge dependent cut-off rigidity ranges. The above result shows that a sharp cut-off selection ensures an unambiguous comparison between elements.

In any case it may be interesting to compare the observed transmission to expectation (fig.3, curves b and c). The observed 2-3% shift between these curves may come from various sources : the P/R edge value depends on the adopted cut-off definition and this is taken care of through the comparison to a reference element. However, for sharp cut-off this value should be equal to Z/A = 0.5.

We feel that there are still some errors in our momentum assignment and that they may be charge dependent, especially outside the charge range Z = 6 to Z = 12.

4. Conclusion

The observed transmission built with oxygen nuclei recorded in the cut-off rigidity range∼7-8.5 GV has been compared to prediction. We feel that the systematic shift between these curves is mostly due to yet uncorrected momentum errors. As an example we wish to do the same comparison for Neon. This is done on fig.4 where the observed transmission built with ~ 400 Neon particles with P/R <.6 in the above cut-off rigidity range is plotted, together with those expected from mixtures of the three stable Neon isotopes and calculated under the assumption of sharp cut-offs. From this the best estimate of the ^{22}Ne abundance is ~ 30 ± 5%.

This result should be insensitive to residual errors on cut-off calculation. We hope in the future to improve the momentum assignment and increase the statistical significance of our results.

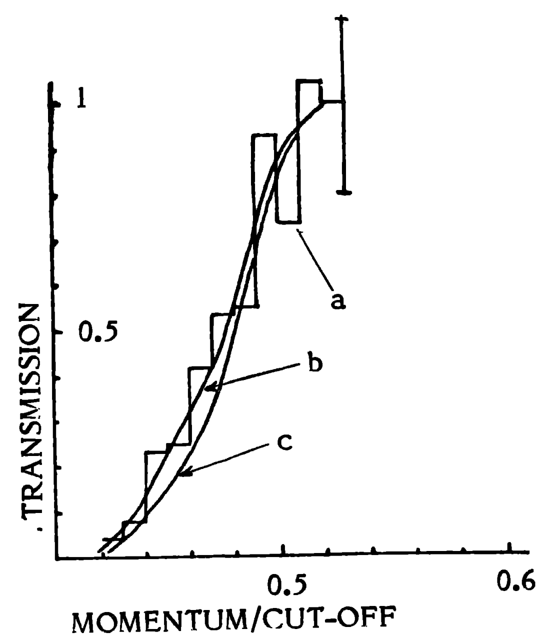

Fig.4 Transmission for Neon
curve a) observed
curve b) and c : calculated shifted 3% downwards in P/R. Isotopic abundances of Neon 20, 21, 22 : curve b (60, 10, 30) ; curve c (70, 10,20).

REFERENCES

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2. Juliusson E. et al., 1981, this Conf. 9, 101.
3. Lund N. et al., 1981, this conf. 8, 63.
4. Rotenberg M. et al., 1981, this conf. 8, 112.