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# A dip in the UHECR spectrum and the transition from galactic to extragalactic cosmic rays

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The dip is a feature in the diffuse spectrum of ultra-high energy (UHE) protons in the energy range  $1 \times 10^{18} - 4 \times 10^{19}$  eV, which is caused by electron-positron pair production on the cosmic microwave background (CMB) radiation. For a power-law generation spectrum  $E^{-2.7}$ , the calculated position and shape of the dip is confirmed with high accuracy by the spectra observed by the Akeno-AGASA, HiRes, Yakutsk and Fly's Eye detectors. When the particle energies, measured in these detectors, are calibrated by the dip, their fluxes agree with a remarkable accuracy. The predicted shape of the dip is quite robust: it is modified very weakly when the discreteness and inhomogeneities in the source distribution are taken into account, and for different regimes of propagation (from rectilinear to diffusive). The cosmological evolution of the sources, with parameters inspired by observations of Active Galactic Nuclei (AGN), also results in the same shape of the dip. The dip is modified strongly when the fraction of nuclei heavier than protons is high at injection, which imposes some restrictions on the mechanisms of acceleration operating in UHECR sources. The existence of the dip, confirmed by observations, implies that the transition from galactic to extragalactic cosmic rays occurs at  $E \leq 1 \times 10^{18}$  eV. We show that at energies lower than a characteristic value  $E_{\rm cr} \approx 1 \times 10^{18}$ eV, determined by the equality between the rate of energy losses due to pair production and adiabatic losses, the spectrum of extragalactic cosmic rays flattens in all cases of interest, and it provides a natural transition to a steeper galactic cosmic ray spectrum. This transition occurs at some energy below  $E_{\rm cr}$ , corresponding to the position of the so-called second knee. We discuss extensively the constraints on this model imposed by current knowledge of acceleration processes and sources of UHECR and compare it with the traditional model of transition at the ankle.

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# I. INTRODUCTION

A very important step toward unveiling the origin of the sources of UHECR is to identify the range of energies where cosmic rays become mainly of extragalactic origin. For such extragalactic cosmic rays, in the hypothesis of a proton dominated spectrum, the propagation in the intergalactic medium induces three features in the spectrum observed at the Earth: 1) the GZK feature [1], a suppression of the flux at energies in excess of ~  $10^{20}$  eV, due to the photopion production interactions of cosmic rays off the CMB photons; 2) a bump ([2] - [3]), due to the accumulation of particles below the kinematic threshold for photopion production. As was shown in [4], while the bump is present in the calculated spectra of single sources, it almost disappears in the diffuse spectrum, because the position of the bump depends upon the source redshift; 3) A dip ([2] - [5]), generated due to pair production,  $p + \gamma_{\rm CMB} \rightarrow p + e^+ + e^-$ , where the target is provided by the CMB photons.

The detection of these features would be a definitive test of the extragalactic origin of UHECR and of the fact that they are mainly protons. Since the detection of the GZK feature requires very large statistics of events and, as stressed above, the bump is almost absent in the diffuse spectrum, at present the spectral feature that can be detected more easily is the dip. As we show here (see also [5] - [6]), the dip (see Fig. 1) is a quite robust prediction of

The traditional explanation of the transition from galactic to extragalactic cosmic rays invokes the intersection between a steep  $(E^{-3.1})$  galactic spectrum and a flat  $(E^{-\alpha}$  with  $\alpha = 2 - 2.3)$  extragalactic spectrum, at the *ankle*, located at an energy  $E_a \approx 10^{19}$  eV and identified as a flattening of the spectrum in the data of AGASA, HiRes and Yakutsk detectors (see Fig. 2 and [13] for a general discussion of the transition).

It is important to stress that in the dip scenario the predicted spectrum flattens below and above the dip location (see Fig 1). The high energy flattening, at  $E_a \approx 1 \times 10^{19}$  eV, reproduces perfectly well the observed *ankle*. The low energy flattening, at  $E_{\rm cr} \approx 1 \times 10^{18}$  eV, obtained for both cases of rectilinear [6] and diffusive propagation [14, 15, 16], provides the transition to a steeper galactic component. Note that this property, the intersection of steep and flat spectral components, is the same for both models of transition.

We demonstrate here (see also [16]) that  $E_{\rm cr}$  is connected with the energy scale  $E_{\rm eq} = 2.3 \times 10^{18}$  eV, where the rates of pair production and adiabatic energy losses are equal. The visible transition from galactic to extragalactic cosmic rays occurs at  $E_{\rm tr} < E_{\rm cr}$ , and this energy coincides with the position of the *second knee* (Akeno -  $6 \times 10^{17}$  eV, Fly's Eye -  $4 \times 10^{17}$  eV, HiRes -  $7 \times 10^{17}$  eV and Yakutsk -  $8 \times 10^{17}$  eV). The transition at the second knee was also proposed as a consequence of the study of the propagation of galactic cosmic rays ([17] - [19]).

The energy region around  $E_{\rm cr} \approx 10^{18}$  is also expected to correspond to a change in the chemical composition, from a heavy galactic component to a proton-dominated composition of UHECR. While HiRes [20], HiRes-MIA [21] and Yakutsk [22] data support this prediction and Haverah-Park [23] data do not contradict it at  $E \gtrsim (1-2) \times 10^{18}$  eV, the Akeno [24] and Fly's Eye [8] data favor a mixed composition, dominated by heavy nuclei (for a review see [11] and [25]).

In this paper we shall use the number density of UHECR sources estimated from the small-scale clustering in the angular distribution of the arriving particles [26]. In [27, 28] it was found that the observed small-scale clustering implies, in the case of rectilinear propagation of particles, a spatial density of UHECR  $n_s \approx (1-3) \times 10^{-5} \text{ Mpc}^{-3}$ . Approximately the same space density was found from a study of the small-scale clustering for propagation in magnetic fields [29]. This density is of the same order of magnitude as the density of powerful AGN. It is however worth stressing that the simulated AGASA spectra with this source density are incompatible with the observed AGASA spectrum at the  $5\sigma$  level [30]. The potential of future detectors such as the Pierre Auger Observatory to measure the source density from small scale anisotropies has been discussed in [31]. From the phenomenological point of view, there are some observational indications of AGN as the sources of UHECR [32], although the subject is still matter of much debate.

The paper is organized as follows: in Section II we discuss the physics behind the formation of the dip, and compare our predictions with the data of several experiments. We discuss also the robustness of the prediction of the dip and some physical phenomena which modify the shape of the dip. In Section III we address the more specific issue of the transition from galactic to extragalactic cosmic rays, stressing the differences between the *dip scenario* and the *ankle scenario*. In the Appendix we discuss the problems of acceleration relevant for the dip scenario. We conclude in Section IV.

# II. THE DIP

In this section we describe in detail the physical arguments that explain the formation of the dip in the spectrum.

In order to do this, we use the formalism of the *modification factor*, first introduced in [4] and defined as the ratio of the spectrum  $J_p(E)$  with all energy losses taken into account, and the unmodified spectrum  $J_p^{\text{unm}}$ , where only adiabatic energy losses (red shift) are included:  $\eta(E) = J_p(E)/J_p^{\text{unm}}(E)$ . The spectrum  $J_p(E)$  can be calculated from the conservation of the number density of particles as

$$n_p(E, t_0)dE = \int_{t_{min}}^{t_0} dt Q_{\text{gen}}(E_g, t)dE_g,$$
(1)

where  $n_p(E, t_0)$  is the space density of UHE protons at the present time,  $t_0$ ,  $Q_{\text{gen}}(E_g, t)$  is the generation rate per comoving volume at cosmological time t, and  $E_g(E, t)$  is the generation energy at time t for a proton with energy E at  $t = t_0$ . This energy is found from the loss equation dE/dt = -b(E, t), where b(E, t) is the rate of energy losses at epoch t. The spectrum, Eq. (1), calculated for a power-law generation spectrum  $\propto E^{-\gamma_g}$  and for a homogeneous distribution of sources, is called *universal spectrum*. The important feature of the universal spectrum is its independence of the



FIG. 1: Modification factor for a power-law generation spectrum with slope  $\gamma_g = 2.0$  and 2.7. The horizontal line  $\eta = 1$  corresponds to adiabatic energy losses only. The curves  $\eta_{ee}$  and  $\eta_{tot}$  correspond respectively to the modification factor for adiabatic and pair production energy losses and the modification factor where all losses are taken into account.

mode of propagation: it is the same for rectilinear propagation and propagation in arbitrary magnetic fields. This property of the universal spectrum is guaranteed by the propagation theorem [14], according to which the spectra do not depend on the propagation mode if the distance between sources is less than any propagation length, e.g. energy attenuation or diffusion length. For homogeneous distribution of the sources with vanishing distance between them the propagation theorem is obviously fulfilled.

The generation rate  $Q_{\text{gen}}(E, t)$  might include the cosmological evolution of the sources. In the results presented in this section, we shall not include it in the calculations for two reasons: (i) The evolution involves at least two free parameters, m and  $z_{\text{max}}$ , where m is the exponent in the evolution rate  $(1 + z)^m$ , and  $z_{\text{max}}$  is the maximum redshift up to which evolution takes place. This makes the fit to the data more arbitrary. (ii) Evolution is a very model-dependent phenomenon, and as such we will discuss it later in Section II A 3, regarding it as an uncertainty in the predictions.

Since the injection spectrum  $E^{-\gamma_g}$  enters both the numerator and the denominator of  $\eta(E)$ , one may expect that the modification factor depends weakly on  $\gamma_g$  and numerical calculations confirm it.

In Fig. 1 we plot the modification factor as a function of energy for two slopes of the injection spectrum,  $\gamma_g = 2.0$  and  $\gamma_g = 2.7$ . As expected, the differences are quite small.

In Fig. 2 we show the comparison of the modification factor calculated for  $\gamma_g = 2.7$  with the observational data of AGASA, HiRes, Yakutsk and Auger. The dip, i.e. the modification factor  $\eta_{ee}(E)$ , is well confirmed by the data at energy below  $E \approx 4 \times 10^{19}$  eV, above which the photopion production dominates (see Fig. 2). Fly's Eye data, not shown here, confirm the dip equally well. Auger spectrum does not contradict the high energy part of the dip, but needs continuation of the spectrum to lower energies to test the dip as a whole.

At energy  $E \ge 1 \times 10^{19}$  eV the dip shows a flattening, which explains the ankle, seen in the data in Fig. 2 at this energy.

By definition the modification factor must be less than unity. At energy  $E < 1 \times 10^{18}$  eV the modification factors of AGASA-Akeno and HiRes exceed this bound. This signals the appearance of another component, which is most probably given by galactic cosmic rays. This is the first indication in favor of a transition from extragalactic to galactic cosmic rays at  $E \sim 1 \times 10^{18}$  eV.

The best fit to the data provided by analytical calculations corresponds to  $\gamma_g = 2.7$ , though  $2.6 \leq \gamma_g \leq 2.8$  provide an acceptable description of the data. The detailed Monte-Carlo simulations of the spectra at  $E \geq 3 \times 10^{18}$  eV, accounting for statistical errors in the energy determination of the events, lead to a best fit injection spectrum with slope  $\gamma_g = 2.6$  [34], in rather good agreement with the results of analytical calculations. In addition to the statistical errors, the simulations in [34] may also account for a systematic error in the energy determination. For most currently operating experiments such error is of order 20% and sometimes in excess of this. The Monte-Carlo simulations of [34] lead to the conclusion that the alleged discrepancy between the AGASA and HiRes experiments could be explained





FIG. 2: Predicted dip in comparison with AGASA, HiRes, Yakutsk and Auger[33] data.

in terms of a combination of statistical, systematic errors in the energy of the events, and statistically limited number of events at energies above  $\sim 10^{20}$  eV. This conclusion was strengthened in [35] where the same authors showed that the realizations of the simulated propagation of UHECR that are found to have 11 or more events at energy above  $10^{20}$  eV resemble very closely the AGASA data, although the *average* spectrum has a pronounced GZK feature. In the same paper, the authors also make an attempt to extract events at random directly from the AGASA data and calculate the probability of obtaining the HiRes spectrum by chance. In both cases the alleged discrepancy between the two experiments is found to be statistically not very significant.

The shape of the dip can be used for the energy calibration of the detectors. Shifting the energies by a factor  $\lambda$  for each detector in the energy interval of the dip  $((1 - 40) \times 10^{18} \text{ eV})$  one can determine the minimum  $\chi^2$  and from it deduce the value of  $\lambda$  that best fits the data. This procedure leads to  $\lambda_{\text{Ag}} = 0.9$ ,  $\lambda_{\text{Hi}} = 1.2$  and  $\lambda_{\text{Ya}} = 0.75$  for AGASA, HiRes and Yakutsk detector, respectively. It is worth noticing that the required correction factor is less than unity for ground arrays and exceeds unity for fluorescence experiments. It is impressive that after this shift (calibration by the dip) the spectra of these three detectors agree very well (see Fig. 3). The results shown in Fig. 3 have already been presented by some of us at different conferences starting from 2004 (see e.g. [36]). Recently AGASA collaboration [37] has reduced the energies by about 10% as we predicted.



FIG. 3: The spectra as measured by Akeno-AGASA, HiRes and Yakutsk arrays (left panel) and after energy calibration by the dip (right panel).

### A. Robustness and caveats

The prediction that a dip is present in the spectrum of extragalactic cosmic rays has been obtained using what we called the universal spectrum. Several assumptions have been used in the calculation (see below) and we need to assess what could be their role in changing the basic predictions of the dip scenario. We will prove that within some reasonable limitations, the universality of the spectrum is not substantially changed, in particular in the region of energy around the dip,  $1 \times 10^{18}$  eV  $\leq E \leq 4 \times 10^{19}$  eV. The GZK feature, located at higher energies is expected to exhibit noticeable deviations from the universal spectrum due to possible local inhomogeneities in the source distribution (local overdensity or deficit of the sources [6, 38]), due to a possible acceleration related cutoff and discreteness in the source distribution [6].

We start with listing the phenomena which may modify the universal spectrum:

- 1) Discreteness in the source distribution for the case of propagation in magnetic fields;
- 2) Inhomogeneous source distribution;
- 3) Cosmological evolution of the sources;
- 4) Energetics corresponding to the best fit injection spectrum;
- 5) Chemical composition.

### 1. Discreteness in the source distribution for propagation in magnetic fields

Here we discuss how the discreteness in the source distribution affects the universal spectrum. We distinguish the weak magnetic field case, when protons propagate with moderate deflections, and the case of strong magnetic field when propagation is diffusive.

We shall start with the unrealistic case of a rectilinear propagation of protons in the dip energy region, which is formally valid in the absence of magnetic field. In Fig. 4 the spectra are calculated in the case of source distributions with different distances between them, as indicated in the plot. One can see that while the shape of the GZK steepening is noticeably modified as the distance d between sources changes, the dip is very weakly affected by it, and the spectrum remains universal (the curve with d=1 Mpc in Fig. 4). This result directly follows from the propagation theorem, because the only propagation length scale in this problem, the energy attenuation length, exceeds 1000 Mpc in the region of the dip, being thus considerably larger than the distance between the sources used in the calculations.



FIG. 4: UHE proton spectra for rectilinear propagation from discrete sources, located in the vertexes of the cubic grid with spacing d=60, 40, 20, 10, 5 and 1 Mpc. The calculations are performed for  $z_{\text{max}} = 4$ ,  $E_{\text{max}} = 1 \times 10^{22}$  eV and  $\gamma_g = 2.7$ .

It is clear that with a weak magnetic field the spectra remain universal, like for rectilinear propagation, if particles are not deflected by large angles, when propagating from a source to the observer. It is less trivial that the spectra shown in Fig. 4 should coincide with the spectra obtained for the tangled trajectories (including diffusion) for the same d as in the case of rectilinear propagation. This follows again from the propagation theorem which states that for diffusive propagation the spectrum remains universal provided that the attenuation length and the diffusion length are larger than the mean separation d between sources.

As pointed out in [16], at energy  $\sim 1 \times 10^{18}$  eV the maximum distance  $R_{\max}(E)$  from which particles can reach the observer suffers a sharp increase (the so-called antiGZK effect [16]). The energy at which this takes place was found to be independent of the choice of the diffusion coefficient [16]. This is illustrated in Fig. 5 where it is visible that for different choices of the diffusion coefficient (Kolmogorov, Bohm and  $D(E) \propto E^2$ ) the curves depart from the universal spectrum at the same energy  $E_{\rm cr} = 1 \times 10^{18}$  eV. For rectilinear propagation (Fig. 4) one may see the same value of  $E_{\rm cr}$ .

This universality of  $E_{\rm cr}$  value may be explained recalling the proximity of  $E_{\rm cr}$  to the energy  $E_{\rm eq}$ , where pairproduction energy losses are as fast as the adiabatic energy losses. In terms of evolution of the generation energy with the look-back time, when the proton energy reaches  $E_{\rm eq}$ , its energy increases fast, the diffusion length grows even faster and  $R_{\rm max}(E_{\rm cr})$  becomes large in an almost discontinuous way (see [16] for analytical and numerical calculations). In a semi-quantitative way, the connection between  $E_{\rm cr}$  and  $E_{\rm eq}$  can be expressed as  $E_{\rm cr} = E_{\rm eq}/(1 + z_{\rm eff})^2$ , where  $z_{\rm eff}$ is an effective redshift of the sources contributing to the flux of cosmic rays at energy ~  $E_{\rm cr}$ . A simplified analytical estimate for  $\gamma_q = 2.6 - 2.8$  gives  $1 + z_{\rm eff} \approx 1.5$  and hence  $E_{\rm cr} \approx 1 \times 10^{18}$  eV.

While the position  $E_{\rm cr}$  and the high energy behavior of the 'cutoff' (in fact in terms of J(E) this appears as a flattening of the spectrum) are determined by an increase of energy losses, there is a more evident spectral steepening and cutoff at lower energies, caused by the magnetic horizon. In a very rough approximation, particles cannot reach the observer from distances larger than  $R_{\rm hor}(E) \sim \sqrt{D(E)t_0}$ . When E is small enough, so that  $R_{\rm hor}(E)$  is smaller than the distance d to the nearby sources, the spectrum develops an exponential cutoff. At low energies, where energy losses of protons are only adiabatic and diffusion coefficient is  $D(E) \propto E^{\alpha}$ , the magnetic horizon can be calculated (see [16]) as

$$R_{\rm hor}(E) = 2 \left(\frac{D(E)}{\alpha H_0}\right)^{1/2} \left(e^{\alpha H_0 t_0} - 1\right)^{1/2},\tag{2}$$

where  $H_0$  is the Hubble parameter and  $t_0$  is the age of the universe. For  $B_c = 1$  nG and  $l_c = 1$  Mpc one finds the energy of the magnetic horizon cutoff from the condition  $R_{\text{hor}}(E_{\text{cut}}) = d$ , where d is the distance between sources, as  $E_{\text{cut}} = 2.3 \times 10^{15}$  eV for d = 30 Mpc and  $E_{\text{cut}} = 4.8 \times 10^{16}$  eV for d = 50 Mpc, both for the Kolmogorov diffusion. For the case of Bohm diffusion these energies are larger.

We present in Fig. 5 the spectra calculated for diffusive propagation in relatively strong turbulent magnetic field with basic turbulent length  $l_c = 1$  Mpc, with coherent magnetic field on this scale  $B_c = 1$  nG and for separation between sources d = 50 Mpc. At energies higher than  $E_{cr}$  diffusion proceeds in the regime where  $D(E) \propto E^2$ , (see



FIG. 5: Diffusive energy spectrum for  $B_c = 1$  nG,  $l_c = 1$  Mpc, d = 50 Mpc and  $\gamma_g = 2.7$  for the Bohm, Kolmogorov and  $D(E) \propto E^2$  diffusion. The dash-dotted curve shows the universal spectrum. The data of Akeno-AGASA are also shown.

[16]), while at lower energies we assume the appropriate diffusion regimes with  $D(E) \propto E^{\alpha}$ , as indicated in Fig. 5. One can see that the dip calculated in the diffusive approximation differs very little from the one in the universal spectrum, shown by dash-dot line.

We conclude this section asserting that the presence of magnetic fields in the universe modify quite weakly, within a wide range of parameters, the shape of the dip calculated in the form of a universal spectrum. We provided evidences that a transition from extragalactic to galactic cosmic rays occurs at energy  $E_{\rm cr} = 1 \times 10^{18}$  eV, independently of the mode of propagation (from rectilinear to diffusive). However, for any reasonable extragalactic magnetic fields the propagation of protons and nuclei at  $E < 1 \times 10^{18}$  eV is expected to be diffusive. Even for random magnetic fields with parameters  $l_c = 1$  Mpc and  $B_c = 0.1$  nG, taken as average over the universe, the diffusion length at  $E = 3 \times 10^{17}$  eV, is only ~ 10 Mpc, which is considerably smaller than the size of the region contributing the observed diffuse flux at this energy. This makes the low-energy diffusive cutoff, which provides the transition in our model, most reliable.

### 2. Inhomogeneity in the source distribution

It is often argued that the distribution of matter in the universe is not homogeneous and that the inhomogeneous distribution of the sources of UHECR may have an effect on the observed spectrum. This comment clearly applies to all inhomogeneities on scales smaller than the loss length of particles with given energy. For instance, at extremely high energies (around  $10^{20}$  eV) the loss length is 20 - 100 Mpc. On these scales the universe is indeed inhomogeneous, and this may result in a shape of the GZK feature which reflects this inhomogeneity: a local overdensity (deficit) would make the GZK feature less (more) pronounced.

On the other hand, we know from cosmological observations that the universe is homogeneous and isotropic on the scale of the cosmological horizon, and in fact already on scales of the order of  $l \geq 100$  Mpc. The particles from the dip, i.e. with energies below  $4 \times 10^{19}$  eV, have attenuation length of the order of 1000 Mpc, and thus the dip shape is insensitive to inhomogeneities, provided they occur on spatial scales smaller than ~ 1000 Mpc. We checked this conclusion by including in the calculations some spatial inhomogeneities on scales of 100 Mpc.

# 3. Cosmological evolution of UHECR sources

The cosmological evolution of the sources, namely the increase in the source luminosity or comoving density with red-shift z, is observed for many classes of astrophysical objects. The evolution is reliably observed for the star formation rate in normal galaxies, but this case is irrelevant for most of the cases of our concern, because neither stars nor normal galaxies can be the sources of UHECR due to insufficient cosmic-ray luminosities  $L_p$  and maximum energy at acceleration  $E_{\rm max}$ . An exception to this rule might be represented by the case of Gamma Ray Bursts (GRBs).



FIG. 6: Dip calculated in the models with cosmological evolution. The parameters of evolution used in the calculations for curves 1 and 2 are close to those observed for AGN. The curve 3 is the universal spectrum with m = 0.

Active Galactic Nuclei (AGN), which are most probable candidates for UHECR sources, exhibit evolution in the radio, optical and X-ray bands. X-rays are probably the most relevant tracer for evolution of AGN as the sources of UHECR: X-rays are produced in accretion disks around massive black holes, and the X-ray luminosity is connected thus with the accretion rate. UHECR are probably also connected with the accretion power of massive black holes through the production of Jets and generation of shock waves in the jets and radio lobes. According to recent detailed analysis [39, 40] the evolution of AGN seen in X-ray radiation can be described in terms of the factor  $(1 + z)^m$  up to  $z_c \approx 1.2$  and is saturated at larger z. In [39] the pure luminosity evolution and pure density evolution is allowed with m = 2.7 and m = 4.2, respectively and with  $z_c \approx 1.2$  for both cases. In [40] the pure luminosity evolution is considered as preferable with m = 3.2 and  $z_c = 1.2$ . These authors do not distinguish between different morphological types of AGN. It is possible that some AGN undergo weak cosmological evolution, or no evolution at all. For instance BL Lacs, which are suspected as sources of observed UHECR [32], show *negative* evolution (m < 0) [41], which should be most probably interpreted as weak or absent evolution.

The effect of the evolution of the sources of UHECR on the shape of the dip was discussed in [6] and in a recent review [42]. In the case of UHECR there is no need to distinguish between luminosity and density evolution, because the diffuse flux is determined by the comoving energy-density production rate (*emissivity*)  $\mathcal{L} = L_p n_s$ , where  $L_p$  is the cosmic ray luminosity and  $n_s$  is the space density of the sources.

In Fig. 6 we present the calculated spectrum for evolutionary models, inspired by the data cited above. For comparison we show also the case of absence of evolution m = 0. As our calculations show, in most cases the negative evolution (m < 0) results in the same shape of the dip as the no-evolution case m = 0.

The universal spectra, obtained for sources evolving up to  $z_c > 1$ , fit the observational data down to  $E \sim 3 \times 10^{17}$  eV and even at lower energies. However, for reasonable magnetic fields in the intergalactic medium, protons with these energies have small diffusion lengths and the universal spectrum fails at  $E < 1 \times 10^{18}$  eV, exhibiting a diffusion 'cutoff' that starts at energy  $E_{cr}$ .

We conclude that at present evolutionary models can fit the shape of the dip as well as models without evolution (m = 0).

# 4. Energetics

The universal spectrum, which fits the observed dip, requires an injection spectrum with a slope  $\gamma_g = 2.6 - 2.7$ . The normalization to the observed flux needs the emissivity (energy-density production rate) at  $t = t_0$   $\mathcal{L}_0 \propto E_{\min}^{-(\gamma_g - 2)}$ , where  $E_{\min}$  is the minimum acceleration energy. For low  $E_{\min} \sim 1$  GeV the required emissivity is too high. In order to prevent this energetic crisis, it was suggested phenomenologically in Refs. [6] and [43] that the generation rate per unit comoving volume  $Q(E_g)$  may have the form

$$Q_{\text{gen}}(E_g) = \begin{cases} \propto E_g^{-2} & \text{at } E_g \le E_c \\ \propto E_g^{-2.7} & \text{at } E_g \ge E_c, \end{cases}$$
(3)

where the spectrum  $\propto E^{-2}$  is due to non-relativistic shock acceleration. Recently, an interesting idea was put forward in Ref. [44], due to which the broken spectrum Eq. (3) can be realized. The authors of [44] observed that while the slope of the acceleration spectrum (e.g. 2.0 for non-relativistic shocks and 2.2 for relativistic shocks) is universal, the maximum acceleration energy  $E_{\text{max}}$  depends on individual characteristics of the sources, such as magnetic field and/or size. As a result the sources should be expected to have a distribution of maximum energies at the different acceleration sites,  $n_s(E_{\text{max}})$ . This distribution naturally results in a complex spectrum of the form given in Eq. (3), with  $E_c$  being a free parameter.

In section A 5 we propose another model for the complex spectrum (Eq. 3). It is based on a possible correlation of  $E_{\text{max}}$  with source luminosity. The distribution of sources over luminosities results in the complex spectrum in the form of Eq. (3) for the generation rate per comoving volume,  $Q_{\text{gen}}(E_q)$ .

With the complex spectrum Eq. (3) we obtain for the spectrum shown in the right panel of Fig. 3, the emissivity  $\mathcal{L}_0 = 3.7 \times 10^{46} \text{ erg Mpc}^{-3} \text{ yr}^{-1}$  for  $E_c = 1 \times 10^{18} \text{ eV}$ . Using the sources space density  $n_s = 2 \times 10^{-5} \text{ Mpc}^{-3}$ , as deduced from small scale anisotropy [27, 28], we arrive at the cosmic ray luminosity of a source  $L_p = 5.9 \times 10^{43}$ ,  $2.6 \times 10^{44}$  and  $1.2 \times 10^{45} \text{ erg/s}$  for  $E_c$  equals to  $1 \times 10^{18} \text{ eV}$ ,  $1 \times 10^{17} \text{ eV}$  and  $1 \times 10^{16} \text{ eV}$ , respectively. These luminosities fit well the energetics potential of AGN.

It is necessary to emphasize that low  $E_c$  in Eq. (3) does not change our conclusion about the transition from galactic to extragalactic cosmic rays, provided  $E_c$  is below or close to  $1 \times 10^{18}$  eV.

### 5. Chemical Composition

The presence of nuclei heavier than protons in the primary UHECR flux can substantially modify the proton dip [5, 45] and affect the agreement with observational data shown in Fig. 2. In Fig 7 the modification factors for helium and iron nuclei are presented in comparison with the proton modification factor. One can see that the presence of 15 - 20 % of nuclei in the primary flux affects the good agreement with observations.

The modification factor for a mixed composition can be calculated by introducing a mixing parameter  $\lambda = Q_A^{\text{unm}}(E)/Q_p^{\text{unm}}(E)$  as

$$\eta(E) = \frac{\eta_p(E) + \lambda \eta_A(E)}{1 + \lambda},\tag{4}$$

where  $Q_A^{\text{unm}}(E)$  is the injection spectrum of nuclei with mass number A at the source. The mixing parameter  $\lambda$  is determined primarily by the ratio of the number densities  $n_A/n_H$  in the gas where acceleration occurs. The largest ratio  $n_A/n_H$  is given by helium which has basically a cosmological origin. The cosmological mass fraction of helium  $Y_p = 0.24$  results in  $n_{\text{He}}/n_H = 0.079$ . In Fig. 8 the modification factors for the mixed composition of protons and helium with  $\lambda = 0.1$  and  $\lambda = 0.2$  are shown. In the former case the agreement is sufficiently good, in the latter case the agreement is noticeably worse than for a pure proton composition.

Apart from the density ratio, the mixing parameter depends on the details of the acceleration mechanism. In principle, if acceleration takes place in a relatively cold medium, where helium is not ionized (the ionization potential for helium is very high: 24.4 eV and 54.4 eV for the first and second ionization potentials, respectively), helium nuclei may be not accelerated at all. This possibility can occur for the case of induced electric fields in the plasma. However, for shock acceleration the fraction of accelerated nuclei is determined by their injection into the shock acceleration region (see e.g. [46]), and even for low temperature of the upstream gas, the rate of injection of nuclei in the downstream region can be high. For non-relativistic shocks the ratio of temperatures downstream and upstream can be low if the Mach number of the shock is low, and injection of nuclei may be suppressed. This case is considered in more detail in section A 6.



FIG. 7: Modification factors for helium and iron nuclei in comparison with that for protons. Proton modification factors are given by curves 1 and 2. Modification factors for nuclei are shown as curve 3 (adiabatic and pair production energy losses) and curve 4 (with photodisintegration included).



FIG. 8: Modification factors for the mixed composition of protons and helium nuclei in comparison with the AGASA data. The left panel corresponds to mixing parameter  $\lambda = 0.1$ , and the right panel to  $\lambda = 0.2$ .

The more realistic possibility for suppression of nuclei is given by the case of ultra-relativistic shocks with large Lorentz factor (see section A 6 for a detailed discussion). The fraction of heavy nuclei  $\lambda$  can be suppressed exponentially in this case, see Eq. (A6).

Finally, UHE nuclei can be photo-dissociated in the photon field of a source [36, 47].

# **III. TRANSITION FROM GALACTIC TO EXTRAGALACTIC COSMIC RAYS**

From the analysis of the dip we obtained the indications that the transition from galactic to extragalactic cosmic rays takes place at energy  $E_{\rm cr} = 1 \times 10^{18}$  eV. The first indication is given by the measured modification factor which at this energy becomes larger than one (see Fig. 2) in contradiction with the definition of  $\eta$  that forces  $\eta(E) \leq 1$ . This signals the appearance of a new component at lower energies, which can be nothing but galactic cosmic rays.

The second indication is given by the low-energy 'diffusion cutoff' of the extragalactic spectrum (Fig. 5), which inevitably provides the dominance of the galactic component at energy  $E < E_{\rm cr}$ .

The energy at which the transition begins,  $E_{\rm cr}$ , is completely determined by the equality of the rate of pair production losses and the rate of adiabatic losses at the energy  $E_{\rm eq} = 2.3 \times 10^{18}$  eV [6]. As stressed in Section II A 1 the connection between  $E_{\rm cr}$  and  $E_{\rm eq}$  is given by  $E_{\rm cr} = E_{\rm eq}/(1 + z_{\rm eff})^2$ , and results in  $E_{\rm cr} \approx 1 \times 10^{18}$  eV. The transition must be observationally visible at some lower energy. It coincides thus with the second knee located at energy  $E_{\rm 2kn}$ , for which the different experiments give  $E_{\rm 2kn} \sim (0.4 - 0.8) \times 10^{18}$  eV. For the other models, where the transition also occurs at the second knee, see [17]-[19].

The explanation of the transition given above can be put in a simple and general way: from the plot of the modification factors in Fig. 2, one can see that the spectrum of extragalactic cosmic rays reproduces the generation spectrum when the modification factor tends to unity. The generation spectrum is always flatter than the spectrum of galactic cosmic rays  $\propto E^{-3.1}$ . This argument is further strengthened by the low-energy 'diffusion cutoff' in the extragalatic spectrum.

The dip-transition model works most naturally in the framework of the rigidity model for the origin of the first knee. There are two versions of this model: rigidity-confinement model and rigidity-acceleration model (e.g. [17]). In the first model, the position of the proton knee  $(E_p \approx 2.5 \times 10^{15} \text{ eV})$  is determined by magnetic confinement of protons in the galactic halo. In this case the knee in the spectrum of nuclei with charge Z is located at  $E_Z = ZE_p$ . The highest energy knee, due to iron, must be located at  $E_{\text{Fe}} = 6.5 \times 10^{16} \text{ eV}$ . This finding seems to be confirmed by Fig. 9, where the mean A is that corresponding to iron nuclei at roughly this energy. At energy  $E > E_{\text{Fe}}$  the total galactic flux, consisting mostly of iron, must be steeper. The acceleration rigidity model predicts the same behaviour of the spectra, but the knees appear due to different maximum acceleration energies  $E_{\text{max}}$  for different nuclei. The data of KASCADE confirm well the rigidity models when the SYBILL interaction model is adopted, less well with the QGSJET model [48]. In the framework of a rigidity dependent origin of the knees, the dip model describes the



FIG. 9: Mean logarithmic mass number of cosmic rays as a function of energy [49].

transition as the intersection of a steep galactic spectrum at  $E > E_{\rm Fe} = 6.5 \times 10^{16}$  eV with a flat extragalactic proton spectrum at  $E < E_{\rm cr} = 1 \times 10^{18}$  eV. Numerically this transition is shown in the left panel of Fig. 10 for the Bohm diffusion of extragalactic protons at energies below  $E_{\rm cr} = 1 \times 10^{18}$  eV. The beginning of the galactic steepening at  $E_{\rm Fe} = 6.5 \times 10^{16}$  eV and the extragalactic flattening below  $E_{\rm cr} =$ 

The beginning of the galactic steepening at  $E_{\rm Fe} = 6.5 \times 10^{16}$  eV and the extragalactic flattening below  $E_{\rm cr} = 1 \times 10^{18}$  eV are the results of separate pieces of Physics and nothing "unnatural" takes place in the proximity of these energies (they differ by one order of magnitude). The ratio of the fluxes  $J_{\rm gal}(E_{\rm Fe})/J_{\rm extr}(E_{\rm cr}) \sim 7 \times 10^3$  is large, and we do not see any fine-tuning in this model of the transition.

The galactic and extragalactic fluxes become equal at  $E_{\rm tr} = 5 \times 10^{17}$  eV. The transition is very prominent, if the iron and proton components are resolved, but in the total spectrum the transition appears as a faint feature known as the *second knee*. This property is the same as for the other knees (proton, helium, carbon, etc.) observed by KASCADE: while the transition between the knees is distinct in the spectra with fixed chemical composition, the resultant total spectrum has a smooth power-law shape.

For the fraction of iron and proton fluxes in the energy range  $1 \times 10^{17} - 1 \times 10^{18}$  eV see [16, 43].

The right panel of Fig. 10 shows the traditional transition from galactic to extragalactic cosmic rays at the ankle  $(E_a \approx 1 \times 10^{19} \text{ eV})$ . In this model the extragalactic component has a very flat generation spectrum  $\propto E^{-2}$  which naturally intersects the steep ( $\propto E^{-3.1}$ ) galactic component. The most attractive feature of this model is given by the



FIG. 10: Left panel: the second-knee transition. The extragalactic proton spectrum is shown for  $E^{-2.7}$  generation spectrum and for propagation in magnetic field with  $B_c = 1$  nG and  $l_c = 1$  Mpc, with the Bohm diffusion at  $E \leq E_c$ . The distance between sources is d = 50 Mpc.  $E_b = E_{cr} = 1 \times 10^{18}$  eV is the beginning of the transition,  $E_{Fe}$  is the position of the iron knee and  $E_{tr}$  is the energy where the galactic and extragalactic fluxes are equal. The dash-dot line shows the power-law extrapolation of the KASCADE spectrum to higher energies, which in fact has no physical meaning, because of the steepening of the galactic spectrum at  $E_{Fe}$ . Right Panel: the ankle transition, for the injection spectrum of extragalactic protons  $E^{-2}$ . In both cases the dashed line is obtained as a result of subtracting the extragalactic spectrum from the observed all-particle spectrum.

flatness of the extragalactic generation spectrum, which provides reasonable luminosities of the sources and a natural interpretation of the intersection of the galactic and extragalactic cosmic ray components. Being stimulated by the discovery of the ankle by the Haverah Park array in the '70s, this model has been considered recently in Refs. [12, 13]. Both models of the transition, at the second knee and at the ankle, have some advantages and problems, as summarized below:

- The second knee model is inspired by and based on the numerical confirmation of the existence of the dip as a spectral feature of extragalactic protons interacting with the CMB (see Fig. 2). The probability of an accidental agreement, estimated from the  $\chi^2$ , the number of free parameters and the number of energy bins in each of the four experiments, is very small. The ankle model explains the dip as a possible interplay between galactic and extragalactic spectra. It looks rather odd that such feature has exactly the same shape as that of the CMB-induced dip.
- The explanation of the transition is more straightforward in the ankle model: it is the simple intersection of the flat extragalactic spectrum with the steep galactic spectrum. This model naturally predicts a rather low luminosity of the sources and allows to incorporate an arbitrary fraction of heavy nuclei in the total flux at  $E > 1 \times 10^{19}$  eV, in case the future experiments will show that this is needed. The second knee transition is also based on the intersection of a steep galactic spectrum with a flat extragalactic spectrum. The flatness of the extragalactic spectrum (diffusion 'cutoff') appears quite naturally at energy close to  $E_{\rm cr} = 1 \times 10^{18}$  eV due to diffusion of protons with  $E < E_{\rm cr}$ . However, a low luminosity of the sources can be achieved only by postulating a distribution of maximum energies at the sources. The energy where the *effective* generation spectrum shows the steepening is a free parameter.
- The dip is modified by the presence of heavy nuclei in the primary radiation and it allows only small admixture of heavy nuclei at the dip and above it. This may in turn be interpreted as a possible signature of the model of transition at the dip.
- The model of the transition at the ankle requires that the galactic component of cosmic rays extends to energies

in excess of  $10^{19}$  eV. This requires a revision of the existing galactic models of propagation and acceleration, which predict the maximum acceleration energy to be less than  $1 \times 10^{18}$  eV for iron nuclei [51]. The model of transition at the dip appears to be in agreement with observations of the first knee combined with a rigidity dependent picture (either related to propagation or to acceleration) of the knees for nuclei more massive than hydrogen.

• At energy  $E \ge 1 \times 10^{17}$  eV, i.e. much higher than the proton knee, at least 10% of the observed flux is in the form of protons. This fraction is most naturally explained in the context of the second knee model. It also represents a serious challenge for the ankle model.

As stressed above, both models make clear predictions, that can and should be used to prove or disprove them. The most critical observation is the measurement of the chemical composition in the energy region around  $E \sim 1 \times 10^{18}$  eV. While the dip model predicts a strong dominance of protons, in the ankle model a strong dominance of iron nuclei is expected. We think that the fluorescence measurements provide us with the most promising tool for discrimination of these two models. At present HiRes elongation rate data [20] show the transition to a proton-dominated mass composition at  $E \approx 1 \times 10^{18}$  eV as the dip model predicts, while HiRes predecessor, Fly's Eye, shows this transition at higher energies. Referring to this contradiction we want to emphasize the uncertainties, both experimental and theoretical, involved in present analysis, and which hopefully will be overcome in the future.

As we discussed above, the ankle model in its canonical form predicts a transition from galactic to extragalactic component at energy  $E_a \approx 1 \times 10^{19}$  eV, as observational data from Fig. 2 imply, and as most authors of the ankle models assume. However, there could be an intermediate possibility between the dip and the ankle models, when transition occurs at  $1 \times 10^{18} < E < 1 \times 10^{19}$  eV. The most elaborated model of this type is the mixed-composition model [50]. The transition is found to occur at energy  $E \sim 3 \times 10^{18}$  eV, and thus it softens the difficulties with the highest energy end of the galactic cosmic ray spectrum. In this model, the observed dip, as it is shown in Fig. 2, is reproduced exactly, provided that the galactic component of cosmic rays is fitted a *posteriori* by subtracting the calculated extragalactic component from the observed total spectrum (the model for the galactic spectrum is not discussed and the reason why the dip in this model coincides exactly with the dip calculated for extragalactic protons is left unanswered). Inspired by observations of galactic cosmic rays, the chemical composition of extragalactic cosmic rays is assumed to be mixed. The authors show that elongation rates, especially the ones from the data of Fly's Eye and Yakutsk, confirm better the mixed model than the pure proton model of the dip. We can add to this finding that Akeno data [24] confirm also the mixed composition, while data of HiRes [20], HiRes-MIA [21] and more reliable muon data of Yakutsk [22] support the proton-dominated composition at  $E > 1 \times 10^{18}$  eV. Based on these many contradictory sets of data we do not feel of sharing with the authors of [50] their trust in the accuracy of interaction-dependent analysis of the elongation rate at present, although we hope in its future progress.

# **IV. CONCLUSIONS**

The dip is a feature in the extragalactic cosmic ray spectrum, that originates from the Bethe-Heitler pair production of protons on the cosmic microwave background ([5],[6]). The dip appears at energy  $1 \times 10^{18} - 4 \times 10^{19}$  eV, with shape practically independent of the discreteness and inhomogeneity in the source distribution, independent of local overdensity and deficit of the sources and of the maximum acceleration energy, independent of the presence or absence of magnetic fields in the intergalactic space, and independent of fluctuations in  $p\gamma$  interactions. The cosmological evolution of UHECR sources, with the parameters taken from observations of AGN, do not affect the dip. The only phenomenon which modifies noticeably the shape of the dip is the presence of a large fraction of nuclei, heavier than hydrogen, in the generation spectrum. We consider the small fraction of nuclei, allowed by the dip shape, as an indication of possible mechanisms of acceleration or injection, operating in UHECR sources (Section A 6).

The predicted shape of the dip is in excellent agreement with the data of Akeno-AGASA, Fly's Eye, HiRes and Yakutsk detectors (the data of Auger are inconclusive because of the absence of data at  $E < 3 \times 10^{18}$  eV, essential for the dip). The energy calibration of these detectors by the position of the dip results in excellent agreement between the measured fluxes.

To our knowledge, the precision of the agreement between the predictions for the dip, which need only two free parameters, and observations (see Fig. 2) is the best that ever existed in cosmic ray physics. This, and the results of energy calibration of the detectors by the position of the dip (see Fig. 3), makes improbable that we are observing an accidental agreement between the predictions and the observations.

All the discussion presented so far in the paper was based on purely phenomenological and largely model independent grounds. Clearly, in order to establish a connection with existing theories of acceleration or propagation of cosmic rays, the dip needs to be related to models. In passing we notice however that the shape of the dip agrees well with data when AGN with their observed cosmological evolution are considered as sources of UHECR (Section II A 3). A

possible, though model dependent, correlation between the maximum acceleration energy and the luminosity of the sources might also solve the problem of the excessive energetics required by the dip scenario (Section A 5). A small fraction of heavy nuclei can also be accommodated in the model (Section II A 5).

In the dip scenario the transition from galactic to extragalactic cosmic rays takes place at  $E_{\rm cr} \approx 1 \times 10^{18}$  eV. The natural character of this transition is guaranteed by the fact that a flat extragalactic spectrum at  $E < E_{\rm cr}$  intersects a steep galactic spectrum at  $E > E_{\rm Fe} \sim 1 \times 10^{17}$  eV. Observationally the transition occurs at the second knee.

An alternative possibility for the transition is given by the *ankle* model (see Section III).

The ankle model has many attractive features. It gives a simple and natural picture of the transition as the intersection of a steep galactic spectrum with a very flat extragalactic spectrum, resulting from a generation spectrum as predicted by the 'standard' acceleration models ( $E^{-2}$  for non-relativistic shock acceleration and  $E^{-2.2}$  for relativistic shocks). These generation spectra have no problem with energetics. The model can also easily accommodate a large fraction of heavy nuclei, if observations show that this is needed.

On the other hand the ankle model has weaknesses: the model requires that the galactic component extends to energies in excess of  $10^{19}$  eV, in apparent contradiction with the KASCADE data. We add also that it seems to contradict the most current models of cosmic ray acceleration in galactic sources [51]. The model also has difficulties in explaining the ~ 10% of protons observed in Akeno at  $E \sim 10^{17}$  eV (in the dip model these protons are extragalactic).

In conclusion, we want to emphasize the importance of having detectors working in the energy range  $3 \times 10^{17} - 1 \times 10^{19}$  eV, such as possible low-energy extensions of Auger, Telescope Array (TA) and low-energy extension of TA (TALE). These detectors can reliably solve the problem of measuring the mass composition in the transition region. It is important to observe that the two basic models of the transition, at the second knee and at the dip, give drastically different predictions for the chemical composition in the energy interval  $1 \times 10^{18} - 1 \times 10^{19}$  eV: proton-dominated in the dip model and iron-dominated in the ankle model. The updated fluorescent method with measurement of atmosphere transparency for each event can reliably distinguish these two models, in spite of existing experimental and theoretical (models of interaction) uncertainties.

One can add another signature common to both the dip and the ankle scenarios (see [50]) namely the appearance of a jump in  $x_{max}$  at energy  $E_{tr}$ , which equals  $\sim 5 \times 10^{17}$  eV for the dip model and  $\sim 10^{19}$  eV for the ankle model. This feature appears because of a sharp transition from a steep galactic (iron dominated) spectrum to a flat extragalactic (proton dominated) component (see Fig. 10). Such a jump is practically absent or much smoother in the mixed composition model. In contrast with the dip and ankle models, the mixed composition model needs a higher accuracy of the fluorescent method to distinguish the mixed extragalactic composition from the proton-dominated or iron-dominated compositions (there are no reliable predictions for the mass composition of extragalactic UHECR).

One should keep in mind the great predictive power of the *spectrum measurements* for the determination of the mass composition of extragalactic UHECR. The modification factors for protons and different nuclei (see Fig. 7 for comparison) differ very strongly in the energy region  $1 \times 10^{18} - 1 \times 10^{20}$  eV and even small admixtures of nuclei can be discovered as a distortion of the proton modification factor. Note that this possibility exists for all models: dip, ankle and intermediate transition.

In principle, the *anisotropy* discriminates the different models of transition. While the dip-transition model predicts an extragalactic anisotropy at  $1 \times 10^{18} - 1 \times 10^{19}$  eV, the ankle models predicts the galactic anisotropy at this energy range. However, in the case of an expected iron-dominated composition of galactic cosmic rays both models predict too small anisotropy.

We think that measurements of the spectrum and chemical composition in the energy region  $3 \times 10^{17} - 1 \times 10^{19}$  eV can resolve the existing problem of establishing where the transition from galactic to extragalactic cosmic rays takes place and much experimental effort should be put in aiming at this goal.

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# APPENDIX A: THEORETICAL ISSUES: THE DIP SCENARIO AND THE ACCELERATION OF EXTRAGALACTIC COSMIC RAYS

One of the arguments that is often put forward as being in favor of the ankle scenario for the transition from galactic to extragalactic cosmic rays is that the ankle scenario requires injection spectrum  $E^{-2} - E^{-2.3}$ , compatible with the prediction from the theory of particle acceleration at shocks. In this Section we discuss the acceleration aspects for

the dip scenario, demonstrating the existence of quite reasonable possibilities. Our discussion will concern mainly the shock acceleration mechanism.

# 1. The spectrum of particles accelerated at non relativistic shocks

The test particle theory of particle acceleration at collisionless shocks makes very clear predictions for the spectrum of accelerated particles. For a shock moving with Mach number M, the spectrum is a power law at all momenta larger than the injection momentum, and the slope is related to the Mach number as  $\gamma = 2(M^2 + 1)/(M^2 - 1)$ for adiabatic index 5/3. When the shock becomes transonic, the efficiency of particle acceleration vanishes and the spectrum becomes infinitely steep. If the shock moves at very high (but still non relativistic) speed, the spectrum tends asymptotically to  $E^{-2}$ , for  $M \to \infty$ . This is the case of strong shock which is commonly used in the literature. As long as the shock is non-relativistic, these results remain true irrespective of the choice of the diffusion properties in the shocked fluid, which instead are important for the determination of the maximum energy of the accelerated particles. It is worth stressing that a slope 2.7 would correspond in this context to a Mach number M = 2.6. An instance of a situation in which weak shocks like these develop are mergers of clusters of galaxies [52], though the maximum energy in this case is not expected to exceed  $10^{19}$  eV [53].

The predicted spectrum of particles accelerated at non relativistic shocks is substantially changed when the approximation of *test particles* is relaxed [54, 55, 56, 57]: the effect of the dynamical reaction of particles onto the shock is to steepen the low energy part of the spectrum and flatten the highest energy part compared with the *test particle* prediction. For strongly modified shocks the slope of the spectrum at the highest energies can become as flat as  $E^{-1.2}$  [58].

# 2. Acceleration and Reacceleration

The spectrum of cosmic rays accelerated in a region which is crossed by several shock waves of different strength (e.g. different Mach numbers) may be quite complex. The case of two shocks is illuminating in this respect. Let us consider the case of a shock wave with Mach number  $M_1$ , which accelerates particles to a power law spectrum with slope  $\gamma_1$  up to a maximum energy  $E_1$ . If a second shock front with Mach number  $M_2$  passes through the same region, the pre-existing population of cosmic rays is re-energized. It can be easily shown [59] that the final spectrum of cosmic rays at large momenta is a new power law with slope close to  $\gamma_1$  if  $M_2 < M_1$ , or approaching  $\gamma_2$  if  $M_2 > M_1$ . If however the second shock front is able to achieve a maximum energy  $E_2 > E_1$  and  $M_2 < M_1$ , then the final spectrum is  $E^{-\gamma_1}$  for  $E < E_1$  and  $E^{-\gamma_2}$  (steeper than at low energy) for  $E_1 < E < E_2$ . For instance if  $M_1 = 100$  and  $M_2 = 2.6$ , the final spectrum will be a broken power law with low energy slope  $\sim 2$  and high energy slope  $\sim 2.7$ . This exercise, of didactic interest, shows that the assumption of a single power law spectra, usually adopted for simplicity may be far from realistic situations.

# 3. The spectrum of particles accelerated at ultra-relativistic shocks

The spectrum of the particles accelerated at shocks moving with relativistic speed is not universal, in that it depends on the details of the scattering properties of the plasma in which the shock develops [60, 61, 64] and the equation of state of the downstream plasma [65]. Some sort of universality is achieved if the motion of the particles occurs in the regime of small pitch angle scattering and if the shock has Lorentz factor  $\Gamma_{sh} \gg 1$  (see [61] and references therein). In this special case, the spectrum of the accelerated particles is a power law with slope 2.32 [62] with a low energy cutoff at energy  $\sim \Gamma_{sh}^2 m_p c^2$ , if  $m_p$  is the mass of the accelerated particles.

Many physical phenomena affect this simple prediction, as discussed in detail in [61]. The spectrum has a slope  $\sim 2.7$  if the relativistic equation of state for the downstream gas is assumed and the shock speed in units of the speed of light is 0.9 [61]. Spectra flatter than 2.32 can easily be obtained when the scattering of the particles is assumed to be in the regime of large angle scattering. Recently the authors of [63] noticed that an isotropic Kolmogorov turbulence in the upstream plasma, compressed by the shock front in the direction parallel to the shock surface leads to a spectrum of accelerated particles with slope 2.7. The effect is due to the fact that the scattering in the downstream plasma becomes anisotropic. The general case of anisotropic scattering was recently studied in detail in [64]. All these cases show that the spectrum of the accelerated particles at a relativistic shock front is determined by many poorly known aspects of the microphysics of the scattering, and that universality can only be used as a sort of *rule of thumb*.

# 4. Other acceleration mechanisms

Many processes lead to particle acceleration, with spectra and efficiencies that depend on the specific situation. We briefly mention here two cases: particle acceleration driven by a relativistic wind, and particle acceleration through pinch instability.

The first acceleration mechanism was discussed in the context of the generation of UHECR in [66] and [67] and applied to the case of magnetized rapidly spinning neutron stars. Charged particles trapped on the open magnetic field lines on the star (namely outside the light cylinder) are pushed outwards with Lorentz factor close to that of the relativistic wind induced by the magnetic pressure. In this scenario, a spectrum of particles is generated due to the slowing down of the spinning neutron star. If the slowing down is dominated by magnetic dipole radiation, it was shown in [66] that the spectrum of the accelerated particles is  $\propto E^{-1}$ , therefore suitable only to explain the AGASA excess, if it is there.

Particle acceleration in jets due to the pinch mechanism was suggested first for tokamaks (where it was confirmed on the laboratory scale) and was then rescaled to astrophysical objects in [68]. The pinch mechanism of acceleration works due to the pinch-neck instability, which is accompanied by the generation of intense electric fields. The solution of the kinetic equation [68] predicts a power-law spectrum of accelerated particles  $Q(E) \propto E^{-\gamma}$  with  $\gamma = 1 + \sqrt{3}$ .

The maximum energy at the source may exceed  $10^{20}$  eV, if the results of laboratory experiments are extended to jets typical of AGNs.

### 5. A model for complex injection

In this section we discuss the possibility, first put forward in [44], that the sources of UHECR might be characterized by parameters of the spectrum of accelerated particles that differ from source to source. In particular, the maximum energy might depend on environmental parameters of the sources, and not only on the type of sources. The implication of this assumption is that the *effective* injection spectrum due to the sum over all the sources at the given redshift may appear as having different slopes in different energy ranges. A simple example may serve to clarify the basic



FIG. 11: Normalized injection spectrum  $Q(E)(E/E_0)^{\gamma}$  for  $\beta = 0, 0.5, 1$ .

idea: let us assume that a class of sources of UHECR has a luminosity function given by

$$N(L) = N_0 \left(\frac{L}{L_{min}}\right)^{-\alpha},$$

and that the maximum energy of the sources correlates with the source luminosity as

$$E_{max}(L) = E_{max}^* \left(\frac{L}{L_{min}}\right)^{\beta},$$

where  $E_{max}^*$  is the maximum energy of the sources with luminosity  $L_{min}$  (in the general case the function N(L) might depend explicitly on redshift). Moreover, let us assume that the injection spectrum of each source has the form of a power law with an exponential cutoff:

$$Q_s(E) = \frac{(\gamma_g - 2)L}{E_0^2} \left(\frac{E}{E_0}\right)^{-\gamma_g} \exp\left[-\left(\frac{E}{E_{max}(L)}\right)\right].$$

The *effective* injection spectrum, i.e. the number of particles generated per unit comoving volume at the redshift z is therefore

$$Q(E) = \int_{L_{min}}^{L_{max}} dL \ N(L)Q_s(E) = Q_0 \left(\frac{E}{E_0}\right)^{-\gamma_g} \int_1^r dx \ x^{-\alpha+1} \exp\left[-\left(\frac{E}{E_{max}^* x^\beta}\right)\right],$$
(A1)

where  $r = L_{\text{max}}/L_{\text{min}}$  and  $Q_0 = (\gamma_g - 2)N_0(L_{\text{min}}/E_0)^2$ . For illustrative purposes we choose  $E_{max}^* = 10^{17}$  eV,  $r = 10^4$  and  $\alpha = 3$ . In Fig. 11 we plot  $Q(E)(E/E_0)^{\gamma}$  in arbitrary units for  $\beta = 0, 0.5, 1$ .

One can see that despite the fact that each source has a power law injection spectrum with given slope  $\gamma$ , the effective injection relevant for the calculations of the propagation of UHECR has the shape of a broken power law, with steeper slope at high energies. The attractiveness of this model is that it allows us to adopt a generation spectrum



FIG. 12: Spectrum of the extragalactic cosmic rays obtained with the complex injection spectrum having slope  $\gamma = 2.2$  for  $E < E_c$  and slope  $\gamma = 2.7$  for  $E > E_c$  ( $E_c = 10^{18}$  eV). The dashed line is obtained as a result of subtracting the extragalactic spectrum from the observed all-particle spectrum.

for each source that is a power law with slope 2 - 2.4 and obtain the *desirable* injection spectrum at high energy due to the superposition of many sources. The flat spectrum alleviates or solves the problem with energetics discussed in Sec. II (see subsection II A 4). On the other hand it becomes very difficult to make realistic predictions in this model, unless the sources of UHECR, the acceleration process and the evolution properties of the sources are very well known.

In Fig. 12 we plot the predicted spectrum of extragalactic cosmic rays in the case of an injection spectrum which is  $E^{-2.7}$  for  $E > E_c = 10^{18}$  eV and  $E^{-2.2}$  for  $E < E_c$ . The dashed line is the spectrum of galactic cosmic rays inferred by subtraction of the extragalactic flux from the all-particle observed spectrum. Again the transition takes place at energy  $E_{\rm tr} \approx 3.5 \times 10^{17} {\rm eV}.$ 

# 6. Injection to shock acceleration and nuclei-to-protons ratio in the accelerated flux

The agreement of the dip with observations for the best fit parameters implies the strong dominance of protons in the primary radiation. Meanwhile, the observed abundance of heavy nuclei in galactic cosmic rays is large enough to affect the good agreement of the proton dip with the data. Since the sources of UHECR are different from those for galactic cosmic rays, one may ask: Can acceleration mechanisms which operate in UHECR sources result in lower abundance of heavy nuclei?

Any source acceleration mechanism operates in the gas where heavy nuclei are present. The largest and guaranteed abundance is given by cosmological helium which results in the ratio of densities  $n_{\rm He}/n_{\rm H} = 0.079$ . We will focus on the He/p ratio in the acceleration flux, limiting our discussion to shock acceleration.

The He/p ratio in cosmic rays differ from the density ratio  $n_{\rm He}/n_{\rm H} = 0.079$  due to the injection process.

The injection of charged particles in collisionless shocks is a complex and still unsolved plasma physics problem. The shock is most likely formed in a collisionless plasma due to Weibel instability and its thickness is reasonably thought to be of the order of a few Larmor radii of the thermalized particles in the downstream plasma. This is however a quite heuristic argument based on the fact that Weibel instability generates wild magnetic fluctuations in the shock structure (even in the absence of an initial magnetic field). If these fluctuations are such that their amplitude exceeds the background field (this is certainly the case if there is no initial magnetic field), the thickness of the shock can be smaller than suggested by our heuristic argument. We are not aware of accurate predictions of the internal structure of a collisionless shock in the literature. This makes the investigation of the injection process very uncertain at the present time.

The energetics of the shock is expected to be dominated by protons, which are heated downstream to a temperature T which roughly corresponds to the isotropization of the momenta of upstream particles flowing in the shock rest frame as the cold fluid. The average momentum of the thermal protons is therefore  $p_{th}^{(p)} \approx m_p v_s \gamma_{sh}$ , where  $v_s$  and  $\gamma_{sh}$  are the velocity and Lorentz factor of the upstream fluid in the shock frame. For a non-relativistic shock this reduces to  $p_{th}^{(p)} \approx m_p v_s$ , which is approximate since it neglects the bulk motion of the downstream plasma in the shock frame. According with the argument given above, the thickness of the shock is  $R_{sh} \approx \xi p_{th}^{(p)} c/(eB)$ , with  $\xi = 2 - 4$  [69].

We consider now nuclei with mass number A and charge Z in the downstream region. In order to cross the shock front and proceed towards upstream, their Larmor radius  $R_L = pc/ZeB$  must exceed  $R_{\rm sh}$ , and hence only nuclei with momenta higher than  $p_{\rm inj}^A = ZeBR_{\rm sh}/c$  can be accelerated. In this way we arrive at the rigidity-dependent injection mechanism, when injection momenta for nuclei and protons are related by

$$p_{\rm inj}^A = Z p_{\rm inj}^p. \tag{A2}$$

This simple argument suggests that the injection of nuclei is easier than the injection of protons, in agreement with what is usually assumed in the current literature [70]. It is however difficult to quantify this difference on theoretical grounds. Based on the observations of cosmic rays in the Galaxy, nuclei are systematically more abundant in cosmic rays than they are in the interstellar medium in the solar neighborhood [46, 70].

If we assume, as suggested by Eq. (A2) that  $p_{inj}^A = Z p_{inj}^p$ , and we also assume that the injection spectrum is a power law in momentum up to a maximum momentum, then the number of accelerated nuclei with momentum p is  $Q_A(p) = K_A(p/p_{inj}^A)^{-\gamma_g}$ , where A = 1 corresponds to protons,  $K_A = \epsilon_A n_A (\gamma_g - 1)/p_{inj}^A$  with  $\epsilon_A$  being the fraction of thermal nuclei involved in acceleration. For large enough momenta p one obtains

$$\frac{Q_A(p)}{Q_p(p)} = \frac{\epsilon_A}{\epsilon_p} \frac{n_A}{n_H} Z^{\gamma_g - 1},\tag{A3}$$

where  $\epsilon = \epsilon_A/\epsilon_p \ge 1$  depends on the distribution of thermal nuclei at momenta  $\ge p$ .

From Eq. (A3) it follows that in most cases the presence of He nuclei (Z=2) with cosmological number density  $n_H$  results in a too strong distortion of the proton modification factor. An exception may be given by the case of acceleration in a cold gas. We remind the reader that the first and second potentials for helium are very high, 24.6 eV and 54.4 eV, respectively. In the extreme case of neutral helium the mixed modification factor will be  $\eta(E) \approx \eta_p(E)$ . If the temperature of the gas is low enough to prevent a second ionization, the condition for injection is given by  $Z_{\text{eff}} = 1$  and the dangerous term  $Z^{\gamma_g-1}$  in Eq. (A3) disappears. Note, that at larger energies, the He nuclei become fully ionized, but the total number of accelerated nuclei is determined by the injection condition with  $Z_{\text{eff}} = 1$ .

Let us illustrate this possibility by using the case of a non-relativistic shock with Mach number M propagating in a warm upstream gas with temperature  $T_u$ . Injection occurs in the downstream region, where the temperature  $T_d$  of the gas found from the Rankine-Hugoniot relations is given by

$$\frac{T_d}{T_u} = \frac{1}{16} (5M^2 - 1)(1 + \frac{3}{M^2}),\tag{A4}$$

valid for a gas with adiabatic index 5/3. In the limit of large Mach number  $M \gg 1$   $T_d/T_u = (5/16)M^2$  and the spectral index of accelerated particles tends to  $\gamma_g = 2$ . In the case of a small Mach number  $\gamma_g = 2(M^2 + 1)/(M^2 - 1)$  and the increase of  $T_d$  is modest. For example, in the case of M = 2.59,  $\gamma_g = 2.7$ , as needed to fit the dip, and the ratio of temperatures  $T_d/T_u = 2.9$ . In the case  $T_u \sim 10^4 - 10^5$  K the downstream temperature is below the second ionization potential and injection of He-nuclei is suppressed. For the case of M = 4 when compression  $\sigma$  is close to 4:  $\sigma = 3.4$ ,  $\gamma_g = 2.3$  and  $T_d/T_u = 5.9$ .

Another example when the fraction of accelerated nuclei might be suppressed is given by relativistic shock acceleration with large  $\Gamma_{\rm sh}$ . In this case the cold upstream particles have a downstream energy  $E_d \sim \Gamma_{\rm sh} Am_p$  and the temperature of the downstream gas T can be very high. We shall assume that the temperature of nuclei and protons is the same  $T_A \approx T_p$ . This assumption is based on poorly known physics of thermalization of the gas downstream. The thermalization occurs most probably due to collective processes and it is not known how fast these processes are. If they are slow, the equilibrium condition  $T_A \approx T_p$  can be reached on average too far from the shock front for efficient injection to take place. Our assumption implies a fast thermalization. Thus we assume that particles downstream have a thermal distribution

$$n_A^{\rm th}(p) = \frac{n_A}{2T^3} p^2 \exp(-p/T),$$
 (A5)

where  $n_A$  is the momentum integrated density and A=1 corresponds to protons. Particles are injected in the regime of acceleration if  $p \ge p_{\text{inj}}^A$ . The density of accelerated particles is  $n_A^{\text{acc}}(p) = K p^{-\gamma_g}$  at  $p \ge p_{\text{inj}}^A$  and the condition of particle number conservation reads  $n_A^{\text{th}}(\ge p_{\text{inj}}^A) = n_A^{\text{acc}}(\ge p_{\text{inj}}^A)$ . Solving these equations one obtains

$$\frac{n_A^{\text{acc}}(p)}{n_p^{\text{acc}}(p)} = \frac{n_A}{n_H} Z^{\gamma_g+1} \exp\left[-\frac{p_{\text{inj}}^p}{T}(Z-1)\right]$$
(A6)

For  $p_p^{\text{inj}} > 2T$  the fraction of nuclei with  $Z \ge 2$  is exponentially suppressed.

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