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ON THE SOURCES OF COSMIC RAY ELECTRONS

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ABSTRACT

The spectrum of cosmic ray electrons is discussed in terms of the contributions from discrete sources distributed over the Galaxy. The analysis shows that one needs sources situated within a few hundred parsecs of the solar system, in order that the radiative losses of energy do not induce a premature cutoff in the energy spectrum. If the mean spatial density of the sources is roughly uniform over the galactic plane, then there should be at least 3×10^4 active sources accelerating cosmic rays in the Galaxy. Therefore it is very unlikely that supernovae are the only sources of cosmic ray electrons in the energy range 1–1000 GeV.

Subject headings: cosmic rays: general — galaxies: stellar content — stars: supernovae

I. INTRODUCTION

The astrophysical implications of the cosmic ray electron spectrum have been discussed extensively in the literature (Daniel and Stephens 1975). In these discussions it is generally assumed that the cosmic ray source function is continuous over the Galaxy. In view of the fact that supernovae and their remnants have been suggested as the cosmic ray sources, we wish to study here the effects of radiative losses of energy during the diffusive transport from a discrete set of sources. The importance of the discrete nature of the cosmic ray sources in controlling the distribution of high-energy electrons has been discussed earlier by Shen (1970) and Shen and Mao (1971). Our study here is an extension of their work.

II. TRANSPORT OF COSMIC RAYS FROM DISCRETE SOURCES

The geometrical model of the Galaxy which will be used in the mathematical formulation of the problem is shown in Figure 1. The so-called "storage-volume" for cosmic rays is bounded by two parallel planes at $Z = 0$ and $Z = h$, with the median plane being occupied by the galactic stellar disk. Now, the crux of the problem involves the solution of the diffusion equation in cylindrical coordinates:

$$\nabla\cdot(K\nabla G) - \frac{\partial G}{\partial t} = \delta(\boldsymbol{r}' - \boldsymbol{r})\cdot\delta(t)\,, \tag{1}$$

with the boundary condition that $G = 0$ at $Z = 0$ and $Z = h$ signifying escape into the intergalactic medium. Here K is the diffusion coefficient which we assume to be independent of energy, and $G(r, r', t)$ is the density of cosmic rays at the observation point (the solar system, say) at time t after a pulse of cosmic rays of unit strength is emitted at a source situated at r'. Following Carslaw and Jaeger (1959), the required solution which exhibits cylindrical symmetry is

$$G(\boldsymbol{r}, \boldsymbol{r}', t) = \frac{1}{8(\pi K t)^{3/2}} \exp(-r^2/4Kt) \sum_{n=-\infty}^{\infty} \left\{ \exp\left[-\frac{(2nh + Z - Z')^2}{4Kt}\right] - \exp\left[-\frac{(2nh + Z' + Z)^2}{4Kt}\right] \right\} \tag{2}$$

for $0 < Z < Z'$; and if $Z' < Z < h$, the result is the same with Z and Z' interchanged.

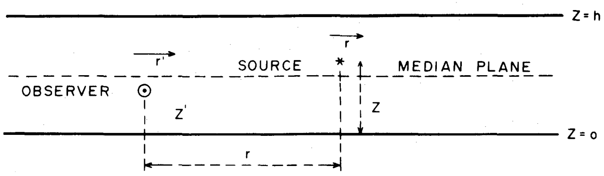

FIG. 1.—A geometrical model of the Galaxy used to describe propagation of cosmic rays from discrete sources

Using the Green's function given in equation (2), we can derive several important properties of the cosmic rays such as the anisotropies, the L/M ratios, etc., for any specified distribution of the sources in the Galaxy. Here we confine our discussions to the spectrum of electrons which lose energy through radiative processes like synchrotron emission and Compton scattering against starlight and the universal microwave background (Ginzburg and Syrovatskii 1964; Hayakawa 1969). The rate of loss of energy in such processes can adequately be approximated as

$$\frac{dE}{dt} = -bE^2 , \tag{3}$$

where the constant b depends upon the energy densities in the radiation and magnetic fields. Let us suppose that a source injects into the interstellar medium a spectrum of electrons at the rate of (S_j/E^β). Then the spectrum as seen by an observer at time t takes the form:

$$\begin{aligned} f(\boldsymbol{r}_j', \boldsymbol{r}, E, t) &= \frac{S_j}{E^\beta}(1 - bEt)^{\beta-2}G(\boldsymbol{r}, \boldsymbol{r}_j', t) \quad \text{for } E < \frac{1}{bt} \\ &= 0 \quad \text{for } E \geq \frac{1}{bt} \cdot \end{aligned} \tag{4}$$

If the source has been continuously active for a period much longer than h^2/K, the observed spectrum will be:

$$F(\boldsymbol{r}_j', \boldsymbol{r}, t) = \int_0^{1/6E} f(\boldsymbol{r}_j', \boldsymbol{r}, E, t)dt . \tag{5}$$

This spectrum can be evaluated in terms of error-functions for $\beta = 3$, but for other values of β we are compelled to use numerical methods.

A comparison of the observed spectrum of electrons with $F(\boldsymbol{r}, \boldsymbol{r}', E)$ would reveal many properties of the cosmic ray sources. But before we do this, let us estimate the total electron spectrum by summing over the contributions of all the sources. This summation can be performed only when the distribution of the sources in the Galaxy has been specified. As a specific example, let us say that all the cosmic ray sources are situated in the median plane (see Fig. 1) containing the stars and that their density is roughly uniform over the galactic plane. Let us further locate each of the sources at its mean distance from the solar system and write:

$$\langle r_j \rangle = \langle r_1 \rangle j^{1/2} . \tag{6}$$

Here the index j sequentially runs over from the nearest to the farthest source. We can now write for the spectrum due to all the sources:

$$\mathscr{F}(E) = \sum_{j=1}^{N} F(0, \langle r_j \rangle, E) . \tag{7}$$

In performing the sum in equation (7) we have approximated the small contributions for $j \geq 5$ by an integral.

III. COMPARISON WITH THE OBSERVATIONS AND A DISCUSSION OF THE IMPLICATIONS

We now need to specify h, the thickness of the galactic disk; K, the diffusion constant; and b, the parameter describing the rate of loss of energy by the electrons. Radio astronomical observations (Ilovaisky and Lequeux 1972) fix h to be $\sim 4.5 \times 10^{21}$ cm, and K is taken to be 10^{28} cm^2 s^{-1}, its canonical value. These values are quite consistent with other cosmic ray data. For example, the mean residence time of cosmic rays in the Galaxy is given by

$$\langle \tau \rangle = \frac{\sum_j \int G(\boldsymbol{r}, \boldsymbol{r}_j', t)tdt}{\sum_j \int G(\boldsymbol{r}, \boldsymbol{r}_j', t)dt} , \tag{8}$$

where the summations are carried over all the sources in the Galaxy. For the values of h and K chosen, $\langle \tau \rangle \approx 5 \times 10^6$ years consistent with cosmic ray data (Meyer 1975). If we choose the galactic magnetic field to be $\sim 3\mu$ gauss, the starlight density to be 0.4 eV cm^{-3}, and the universal background radiation to be at 2.7 K, b takes the value (Hayakawa 1969) $\sim 9 \times 10^{-20}$ (MeV$\cdot$s)$^{-1}$. Considerations that the value of the magnetic field strength and the starlight density may be somewhat smaller at heights beyond the scale height of gas and stars would reduce the value of b, but in no case to a value much smaller than $\sim 5 \times 10^{-20}$ (MeV$\cdot$s)$^{-1}$. Therefore we choose this value as a conservative average in the region of cosmic ray storage.

The cosmic ray electron spectrum has been measured by several authors (see Muller and Prince 1977) using a variety of techniques, the latest of these being with a transition radiation detector by Hartmann, Muller, and

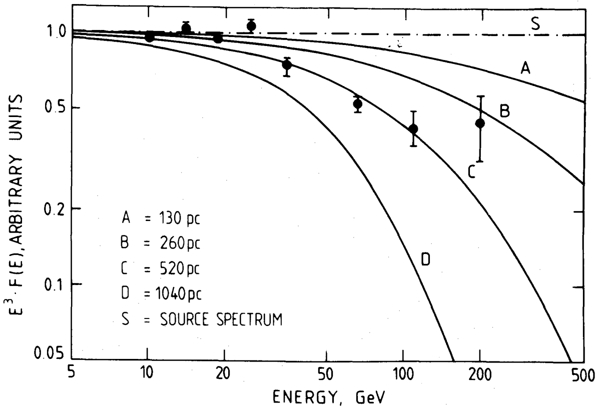

FIG. 2.—The calculated dependence of the spectrum of electrons on the distance of the source

Prince (1977), who have also reviewed the earlier results. In view of the fact that the measured spectral shape is expected to be more accurately determined than the absolute fluxes, we have to use the spectra measured in individual experiments separately in making comparison with the theoretical predictions. As an example we choose the injection spectrum as E^{-3} and compare the results with the spectrum as given by Hartmann *et al.* because they quote the steepest spectrum. Presently we shall see the advantages of this specific choice of the experimental data.

In Figure 2 we compare the observed spectrum with those expected on the basis that only a single source situated at a specific distance contributes. We notice immediately that if this hypothetical source were any further than $\sim 1.6 \times 10^{21}$ cm ($\sim$500 pc), then the calculated spectrum would be too steep to explain the fluxes of the high-energy electrons. The energy losses due to radiative processes have become so severe that the electrons at high energies are severely depleted. Thus we may state that we definitely need a cosmic ray source closer than 500 pc from the solar system. We would have to place the nearest source much closer if we accepted the flatter spectra measured by the other authors (see Muller and Prince 1977 for a review of the observations).

On the other hand, it is reasonable to assume that there is a large number of cosmic ray sources sprinkled all over the galactic plane. Our theoretical predictions based on such an assumption are compared with the observed spectrum in Figure 3. The topmost curve, labeled 0, is for a continuous uniform distribution of sources over the galactic plane. The curves labeled 1–4 are for the distances to the nearest source r_1, being equal to 2×10^{20} cm, 4×10^{20} cm, 5.6×10^{20} cm, and 1.1×10^{21} cm, respectively. First of all, we notice a good agreement between the observed spectrum and the theoretical curves for $r_1 \leq 4 \times 10^{20}$ cm. For larger values of r_1, the spectrum steepens rapidly and fails to reproduce the intensities at high energies. The choice of an experimental spectrum given by any other experimentalist is in general flatter and will tend only to aggravate the discrepancy at high energies. Thus $r_1 \leq 4 \times 10^{20}$ is a reasonable upper limit for the typical spacing between the cosmic ray sources in the neighborhood of the solar system.

Now we may take recourse to the studies of the nonthermal emission of the galactic disk (Ilovaisky and Lequeux 1972; Webber 1968) and to the relative constancy of cosmic rays in the past (Schaeffer 1975) to assert that the solar system is not in any special juxtaposition with respect to the cosmic ray sources and that there are no substantial variations in the density of the sources over the galactic plane. Thus our result that the mean distance

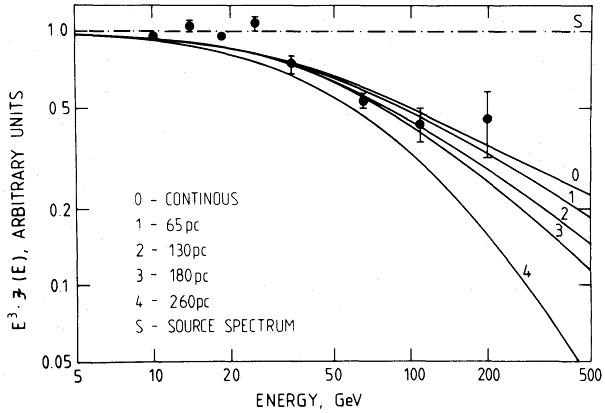

FIG. 3.—The spectrum of electrons summed over all the sources in the Galaxy; the parameters represent the typical spacing between the sources.

between sources is less than 4×10^{20} cm implies that there are more than $\pi(R_g/4 \times 10^{20})^2 \approx 3 \times 10^4$ sources required to explain the electron spectrum. Thus each of these need only produce $\lesssim 5 \times 10^{34}$ ergs s^{-1} in high-energy electrons. If one demands further that these sources accelerate cosmic ray nuclei as well, then their total emissivity should be 5×10^{36} ergs s^{-1} in cosmic rays. There limits are not changed substantially for any variations of the parameters within ranges generally allowed by our present knowledge of the Galaxy.

Before we can apply our calculations to investigate the possibility of supernovae and their remnants being the sources of the cosmic ray electrons, we should notice that these are transient sources whereas our calculations apply to sources continuous in time. However, it is easy to show that continuous sources always predict a flatter spectrum at any observation point than the transient sources, so one may use the spectra calculated here as an upper limit to the spectral intensities expected for the transient sources with identical spatial distributions. We now notice from a compilation of the supernova remnants down to 50 flux units (Woltjer 1972) that the nearest remnant, Vela, is 400 pc away. If Vela were the only source, one would get a spectral shape between curves B and C, in Figure 2, not in disagreement with the observed spectral shape. However, when one adds the contributions of all the other sources, including the more distant remnants, one would obtain a spectrum much steeper than curve 4 in Figure 3, far below the observed spectral intensities at high energies. We therefore feel that there must be other sources far more numerous ($\gtrsim 3 \times 10^4$) contributing dominantly to the observed spectrum. These conclusions are strengthened further if one considers the observations of the electron spectrum by the other groups which indicate reasonably high fluxes up to ~ 1000 GeV (Muller and Prince 1977). The only way these conclusions can be relaxed is by drastically increasing the value of K but enclosing the diffusion volume in side boundaries having very high reflectivity. This would allow one to sample a large number of sources and also maintain the residence time of $\sim 10^7$ years in conformity with the observations of the secondary cosmic ray nuclei.

In view of the fact that some of the ideas developed in this paper are also found in the work of Shen (1970) and Shen and Mao (1971), it is appropriate here to compare and contrast our approach with the earlier work. To the best of our knowledge ours is the first calculation of the net spectral shape obtained by a summation over the contributions of all the individual discrete sources. The total spectrum seen in Figure 3 exhibits a more rapid turnover than the spectrum of electrons from the nearest sources seen in Figure 2. This is due to the continued contributions at lower energies even from the more distant sources.

Another essential difference is that whereas we use a Green's function which takes into account the loss of particles from the sides of the galactic disk, Shen and Mao use a Green's function pertaining to diffusion in an infinite medium. Their Green's function allows old and also distant sources to contribute more importantly than in our formalism, and cosmic ray observations can be used to distinguish between the two possibilities. Notice, for example, that Shen's Green's function implies that the steady-state electron spectrum would be one half power steeper than the injected spectrum (Ramaty 1974) down to energies

$$E \geq \frac{K}{bR_{\max}{}^2} \approx 30\ \text{MeV}\,. \tag{9}$$

Here $R_{\max}$ is the distance to the farthest sources, $\sim 6 \times 10^{22}$ cm. We can now check this result directly from observation of the spectrum of cosmic ray positrons, as we do have an *a priori* knowledge of the injection spectrum of the positrons, these being secondary to the nucleonic component. Various estimates of the source spectra of positrons (Ramaty 1974; Daniel and Stephens 1975; Badhwar *et al.* 1975) are compared with the observations (Orth and Buffington 1976). It is seen that the observed spectrum has the same slope (~ -2.7) as the production spectrum of positrons, thus contradicting the predictions of the Shen's model. In contrast, when one allows particles to escape from the sides of the galactic disk, the injected spectrum remains unmodified at low energies (Fig. 3).

The basic difficulty in using an infinite diffusing medium is the long "effective residence time" that it implies. On the other hand, when escape is allowed for, the lifetime is limited and it is easy to show the consistency of our model with the lifetime estimates for the cosmic rays (eq. [8]). Accordingly, while calculations using an infinite diffusing medium suffice to indicate that the very high energy electrons are expected to come from only a few nearby sources, nevertheless considerations of particle losses at the boundaries are required to make a detailed comparison of theory and experiment.

IV. CONCLUSION

The spectrum of cosmic ray electrons, when discussed in terms of the contributions from discrete sources of cosmic rays situated in the Galaxy, indicates that the sources are very numerous, $\gtrsim 3 \times 10^4$, each emitting $\lesssim 3 \times 10^{36}$ ergs s^{-1} in cosmic rays. We will discuss the nature and dynamics of these sources elsewhere.

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