THE DISTRIBUTION OF RELATIVISTIC ELECTRONS IN THE GALAXY AND THE SPECTRUM OF SYNCHROTRON RADIO EMISSION

S. I. Syrovat-skii

P. N. Lebedev Physical Institute, Academy of Sciences, USSR

The problem of the diffusion of particles is solved, taking into account the regular changes of the particle energy during this process. The spatial distribution and the energy spectrum of electrons, whose energy changes because of radiation emission in the magnetic field, were found on the assumption that the sources occupy an ellipsoidal volume and inject into interstellar space relativistic electrons with an energy spectrum $QE^{-\gamma_0}$. The case when the distribution of the sources coincides with the flat subsystem of the galaxy and $\gamma_0 = 2$ is considered in detail. The energy spectra of electrons along the line of sight in different directions and the corresponding intensities of synchrotron radiation were calculated. It is shown that the energy spectrum of electrons along the line of sight can be represented in a limited energy region by the expression $KE^{-\gamma}$, where γ varies within the limits $2 < \gamma < 3$, depending on the choice of the diffusion coefficient. The choice of the diffusion coefficient of relativistic particles in interstellar space equal to $D = 10^{29}$ cm/sec and the intensity of the sources (the coefficient in the source spectrum) $Q = 10^{38}$ ergs/sec give agreement between the theoretical and observed spectra of nonthermal radio emission of the galaxy in the frequency region $\nu > 10$ Mc.

At the present time, it appears that the galactic nonthermal radio radiation is widely accepted to be the radiation from relativistic electrons moving in the galactic magnetic fields [1]. In this connection it is of interest to carry out a more detailed investigation of the spectrum and the intensity of this radiation, including such features as the distribution and source spectrum of the relativistic electrons, the diffusion of electrons towards the outer regions of the galaxy, and the energy losses incurred during diffusion. The estimates made in [2] show that these factors can be used to account for the observed dependence of the intensity and spectrum of the radio radiation on the galactic coordinates. In the following, this problem is solved in its general form and the electron-energy spectrum and the intensity of the radiation as a function of the direction of observation are also calculated on the assumption that the sources are situated near the galactic plane and produce electrons with the energy spectrum $Q(E) = QE^{-2}$. The results obtained show that with a reasonable choice of the parameters it is possible to obtain a satisfactory agreement between theory and the observational data and also to make an independent estimate of the diffusion coefficient for relativistic electrons moving in interstellar space.

1. The Distribution Function

We shall describe the spatial and energy distribution of the particles at a given time by the function $f(\mathbf{r}, E, t)$. If the distribution of particles is governed by the diffusion in space, the continuous loss or gain in energy during the diffusion process, the disappearance of the particles as the result of collisions, and the presence of the sources, then the distribution function $f(\mathbf{r}, E, t)$ satisfies the following equation:

$$\frac{\partial f}{\partial t} - D\Delta f + \frac{\partial}{\partial E}(Bf) + \frac{f}{T} = Q(\mathbf{r}, E, t). \tag{1}$$

In this equation Δ is the Laplace operator, $D = D(E)$ is the diffusion coefficient which, in general, is energy-dependent,

$$B(E) = \frac{dE}{dt} \tag{2}$$

is the rate at which the energy of the particles changes as the result of losses or the rate of gain of energy during the diffusion process, T is the average lifetime of the particles determined by the collisions which remove them from the process (nuclear interactions in the case of nucleons and heavy nuclei, radiation losses due to the emission of a high-energy quantum in the case of electrons, etc.), and, finally, $Q(\mathbf{r}, E, t)$ is the intensity of the sources or the rate at which the particles are produced (per unit volume, per unit energy interval). The possibility of fluctuations of the particle energy is not taken into account in Eq. (1), otherwise a term of the form $\partial^2 f/\partial E^2$ would appear in (1). In a number of cases and, in particular, in the problem of electron diffusion investigated below this term is not important.

It is not difficult to obtain the general solution of Eq. (1) for infinite space. With this aim, we make use of Green's function for Eq. (1),satisfying the following equation:

$$\frac{\partial f}{\partial t} - D\Delta f + \frac{\partial}{\partial E}(Bf) + \frac{f}{T} = \delta(\mathbf{r} - \mathbf{r}_0)\,\delta(E - E_0)\,\delta(t - t_0). \tag{3}$$

With the substitution (it is assumed that $B(E) \neq 0$)

$$f = \frac{e^{-t/T}}{B}\varphi \tag{4}$$

and the transformation to the new independent variables

$$t' = t - \tau, \text{where } (E, E_0) = \int_{E_0}^{E} \frac{dE}{B(E)} \tag{5}$$

and

$$\lambda = \int_{E_0}^{E} \frac{D(E)}{B(E)}\, dE \tag{6}$$

Equation (3) can be transformed into the heat-conduction equation

$$\frac{\partial \varphi}{\partial \lambda} - \Delta\varphi = e^{-t'/T}\frac{B}{|B|}\,\delta(\lambda)\,\delta(\mathbf{r} - \mathbf{r}_0)\,\delta(t' - t_0). \tag{7}$$

The solution of Eq. (7) for infinite space is given by

$$\varphi = e^{t'/T}\frac{B(E_0)}{|B(E_0)|}\, e^{-(\mathbf{r}-\mathbf{r}_0)^2/4\lambda}\,\frac{\delta(t' - t_0)}{(4\pi\lambda)^{3/2}}. \tag{8}$$

Going back to the function f, we obtain

$$f(\mathbf{r}, E, t;\ \mathbf{r}_0, E_0, t_0) = \frac{1}{|B(E)|}\, e^{-\tau/T}\, \frac{e^{-(\mathbf{r}-\mathbf{r}_0)^2/4\lambda}}{(4\pi\lambda)^{3/2}}\,\delta(t - t_0 - \tau), \tag{9}$$

where τ and λ are given by expressions (5) and (6). The appearance of the δ-function in Eq. (9) reflects the regular nature of the energy variation, as a result of which the energy at any given time is a single-valued function of the initial energy E_0.

With the help of Green's function (9), we can obtain the general solution of Eq. (1) for an arbitrary distribution of sources, namely,

$$f(\mathbf{r}, E, t) = \iiint_{-\infty}^{+\infty} d\mathbf{r}_0 \int_0^{\infty} dE_0 \int_{-\infty}^{t} dt_0 \, Q(\mathbf{r}_0, E_0, t_0) \, f(\mathbf{r}, E, t; \mathbf{r}_0, E_0, t_0), \tag{10}$$

where $d\mathbf{r}_0$ is an element of volume. In particular, problems in which the particle-source intensity is independent of the time, i. e., the process is stationary, are of interest. In the case of a stationary process, because of (9), (10) the source function has the form

$$f(\mathbf{r}, E; \mathbf{r}_0, E_0) = \int_{-\infty}^{t} dt_0 \, f(\mathbf{r}, E, t; \mathbf{r}_0, E_0, t_0) =$$

$$= \begin{cases} \dfrac{1}{(4\pi\lambda)^{3/2}} e^{-(\mathbf{r}-\mathbf{r}_0)^2/4\lambda} \dfrac{1}{|B(E)|} e^{-\tau/T}, & \text{for} \quad \tau > 0, \\ 0, & \text{for} \quad \tau < 0. \end{cases} \tag{11}$$

An analogous expression was derived in [3]. The above expressions are applicable to the diffusion of any type of particle.* We use these expressions below in the investigation of the electron diffusion.

2. The Distribution of Relativistic Electrons in the Galaxy

For the problem of diffusion in the galaxy and the determination of the electron spectrum we can restrict ourselves to the investigation of the stationary conditions since there are no reasons for considering that the number of relativistic electrons supplied by the sources is appreciably time-dependent. Therefore, we shall use the source function (11) for the stationary case.

To calculate the spatial distribution and the spectrum of the sources we have to know the source distribution $Q(\mathbf{r}_0, E_0)$. We shall assume that the sources supply electrons, on the average, with the same energy spectrum and, consequently,

$$Q(\mathbf{r}_0, E_0) = Q_1(\mathbf{r}_0) \, Q_2(E_0), \tag{12}$$

where $Q_1(\mathbf{r}_0)$ is the spatial distribution of the sources.

At the present time two main possibilities for the origin of relativistic cosmic electrons are being considered [1]: 1) their generation in gaseous nebulae – the envelopes of novae and supernovae, and 2) the production of relativistic electrons in nuclear interactions of heavy cosmic-ray particles. In the first case, the distribution of the sources coincides with the distribution of gaseous nebulae which form a flat subsystem in the galaxy. Roughly speaking, this subsystem forms an ellipsoid of revolution with a semimajor axis $a \simeq$ 10-15 kpc and a semiminor axis $b \simeq$ 100-150 pc. In the second case, the electrons are produced throughout the whole volume of the galaxy occupied by the cosmic rays. However, since the density of the cosmic-ray particles is apparently greater near the galactic plane than away from it and since approximately one-half of the interstellar matter is concentrated near the galactic plane in the flat subsystem, then the majority of the nuclear collisions will take place in the

* In the case of protons and heavy nuclei the influence of the galactic boundary has to be taken into account. In the case of electrons with an energy of $\sim 10^9$ ev this effect is negligible since they practically do not reach the boundary because of the energy losses to radiation in the magnetic fields.

same region as that taken above for the first case. Therefore, it is reasonable to assume that the spatial distribution of the sources is described by an ellipsoid of revolution with semiaxes a and b and can be described analytically by the Gaussian distribution function

$$Q_1(\mathbf{r}_0) = \frac{1}{\pi^{3/2} a^2 b} \exp\{-(x_0^2 + y_0^2)/a^2 - (z_0^2/b^2)\}. \tag{13}$$

With this notation the distribution function is normalized to unity, the normalization constant being incorporated into $Q_2(E_0)$. Expression (13) is sufficiently general since it includes various axially symmetric distributions from a spherical (a = b) to a flat one (b = 0). The spatial distribution of electrons from the sources described by (13) is found with the help of Eq. (11):

$$f(\mathbf{r}, E;\ E_0) = \iiint\limits_{-\infty}^{+\infty} Q_1(\mathbf{r}_0) f(\mathbf{r}, E;\ \mathbf{r}_0\, E_0)\, d\mathbf{r}_0\, . \tag{14}$$

After an elementary integration we obtain

$$f(\mathbf{r}, E;\ E_0) = \begin{cases} \dfrac{e^{-\tau/T}}{|B(E)|} \dfrac{\exp\left\{-\dfrac{x^2+y^2}{a^2+4\lambda} - \dfrac{z^2}{b^2+4\lambda}\right\}}{\pi^{3/2}(a^2+4\lambda)\sqrt{b^2+4\lambda}}, & \text{for } \tau > 0, \\ 0, & \text{for } \tau < 0. \end{cases} \tag{15}$$

In Eq. (15) the energy dependence of the diffusion coefficient, as indicated in expression (6), is taken into account. This energy dependence is important only for particle energies for which the radius of curvature R_H in the magnetic field is comparable to the dimensions of the nonuniformities of the field l, i. e., when

$$R_H = \frac{E}{300\,H} \not> l. \tag{16}$$

According to the estimates available, the intensity of the interstellar magnetic field is $3 \cdot 10^{-6}$ to 10^{-5} oersted. For definiteness, in the following we take $H = 7 \cdot 10^{-6}$ oersted as the most probable value. For $l \approx 10$ pc $= 3 \cdot 10^{19}$ cm, condition (16) is satisfied only for a particle energy $E \not> 6 \cdot 10^{16}$ ev. Since the major contribution to the observed radio radiation is given by electrons with a considerably lower energy ($< 10^{11}$ ev), in the following we assume that D = const, and, consequently, expressions (5) and (6) give

$$\lambda = D\tau. \tag{17}$$

In order to express τ as a function of E, it is necessary to adopt a specific expression for the rate of energy variation given by expression (2). During the diffusion process the electron energy decreases as the result of radiation emission in the magnetic field, as well as ionization and radiation losses.* Moreover, the electron energy can increase as the result of acceleration in interstellar magnetic fields. It appears that in the case of electrons moving in the interstellar medium the losses are considerably greater than any possible energy increase [1]. Therefore, in the following we shall not consider the possibility of acceleration. The relative importance of the different processes leading to the loss of energy by relativistic electrons has been investigated in detail in [1, 4].

For electrons, the radiation length in hydrogen is approximately 62 g/cm². If we take into account the presence of helium atoms in the interstellar matter (about 10%), this value will increase by approximately 6% since the bremsstrahlung losses are proportional to $Z(Z + 1)$, so that the losses in helium relative to those in hydrogen will be higher by a factor of three although the mass of a helium atom is four times as great as that of hydrogen.

* The losses due to the inverse Compton effect can be neglected by comparison with the processes considered, as has been shown in [1].

Taking into account the composition of interstellar matter, we can take the average radiation loss to be*

$$\left(-\frac{dE}{dt}\right)_{\text{rad}} = 9.3\cdot 10^{-16}\, nE. \tag{18}$$

From this it follows that the time in which the energy decreases by a factor e is given by

$$T = 1/(9.3\cdot 10^{-16}\, n) = 1.1\cdot 10^{15}\,\frac{1}{n}\ \text{sec.} \tag{19}$$

If we assume that the average density of interstellar matter in the total volume of the galaxy (including the galactic corona) is $n = 0.01\ \text{cm}^{-3}$,** then

$$t = 1.1\cdot 10^{17}\ \text{sec} = 3.5\cdot 10^{9}\ \text{years} \tag{20}$$

Since there is a high probability that during the time T the electron emits a quantum whose energy is comparable with the initial energy of the electron, we can give approximately the same meaning to the time T as that given in Eq. (1).

The energy lost by a relativistic electron as the result of synchrotron emission is given by the expression

$$-\frac{dE}{dt} = 3.94\cdot 10^{-15}\, H_{\perp}^{2} E^{2}\ \text{ev/sec,} \tag{21}$$

where E is the electron energy in electron volts, and $H_{\perp}$ is the component of the magnetic field perpendicular to the instantaneous velocity of the particle. The value $H = 7\cdot 10^{-6}$ oersted is taken for the average magnetic field intensity. If the velocities are distributed isotropically,*** then the average value is

$$\overline{H_{\perp}^{2}} = \overline{(H\sin\vartheta)^{2}} = \frac{2}{3}H^{2} = 3.3\cdot 10^{-11}(\text{oersted})^{2}. \tag{22}$$

With this value of $H_{\perp}$, expression (21) becomes

$$-\frac{dE}{dt} = \beta E^{2},\quad \beta = 1.3\cdot 10^{-25}\quad \text{ev}^{-1}\cdot\text{sec}^{-1}. \tag{23}$$

Integrating Eq. (23), we find

$$t = -\int\limits_{E_0}^{E}\frac{dE}{\beta E^{2}} = \frac{1}{\beta E} - \frac{1}{\beta E_0}. \tag{24}$$

From this it follows that the time during which the energy is halved is

$$t_{1/2} = \frac{1}{\beta E_0}. \tag{25}$$

For $E_0 = 10^8$ ev, $t_{1/2} = 7.7\cdot 10^{16}$ sec $= 2.4\cdot 10^9$ years. Comparing this value with the lifetime T governed by the radiation losses (20), we see that for electrons of energy $E \geq 10^8$ ev radiation losses can be neglected by comparison with the losses due to the emission of radiation in the magnetic field. This also means that in the energy

* The value of the numerical factor in (18) has been increased by 15% as compared to the value given in [1] to take into account the composition of interstellar gas.

** This value is obtained if we assume that the total mass of gas in the galaxy is $10^9 M_{\odot} = 2\cdot 10^{42}$ g, and that the volume of the galaxy is approximately equal to 10^{68} cm^3.

*** The assumption made about the isotropic distribution of the velocities of the relativistic electrons is valid if the magnetic field varies little in the transition from the galactic disc to the corona, or if the adiabatic invariant for the particle is not conserved as the result of the accumulation of scattering deflections due to collisions with the small-scale nonuniformities of the magnetic field.

region $E > 10^8$ ev, we can put $e^{-\tau/T} \approx 1$ in Eq. (15) since for electrons the lifetime is mainly determined by the radiation losses (the inverse Compton effect is unimportant, while absorption by cosmic bodies is completely insignificant).

The ionization losses in atomic hydrogen are given by [1, 4]

$$-\left(\frac{dE}{dt}\right)_{\text{ion}} = 7.62\cdot 10^{-9} n \left(20.1 + 3 \ln \frac{E}{mc^2}\right) \text{ ev/sec.} \tag{26}$$

These losses are weakly dependent on the energy and therefore by comparison with the synchrotron losses they are important only at the lowest electron energies ($E \approx 10^8$ ev) that are of interest to us. For $E = 10^8$ ev, Eq. (26) gives $-(dE/dt)_{\text{ion}} = 2.7\cdot 10^{-7}$ n ev/sec; for n = 0.01, as adopted above for the average value for the galaxy, the ionization loss is $2.7\cdot 10^{-9}$ ev/sec. On the other hand, the synchrotron losses (23) at this energy are $1.3\cdot 10^{-9}$ ev/sec, i. e., approximately one-half of the ionization losses. However, already at an energy of $2\cdot 10^8$ ev the synchrotron losses will be twice as great as the ionization losses and will become dominant. A more accurate estimate of n, as well as the evaluation of the contribution from regions of ionized hydrogen in which at $E \approx 10^8$ ev the losses are more than twice as great as those in atomic hydrogen, can lead to a change in the value of the upper limit to the energy at which the ionization losses become important. However, it is clear that this limiting energy cannot be too different from 10^8 ev, since it varies with the density only as $n^{1/2}$.

In the following we neglect ionization losses, which is valid for electrons with energies higher than several times 10^8 ev, and we must keep in mind that the results obtained are not applicable to electrons with an energy $E \leq 10^8$ ev.

If only synchrotron losses are taken into account, which is possible in view of the estimates made above, then because of (15) and (17) the distribution function for electrons emitted from monoenergetic sources with an initial energy E_0 is given by

$$\begin{aligned} f(\mathbf{r}, E;\ E_0) &= \frac{1}{\beta E^2} \frac{\exp\left\{-\dfrac{x^2+y^2}{a^2+4D\tau} - \dfrac{z^2}{b^2+4D\tau}\right\}}{\pi^{3/2}(a^2+4D\tau)\sqrt{b^2+4D\tau}} && \text{for } \tau > 0, \\ f(\mathbf{r}, E;\ E_0) &= 0 && \text{for } \tau < 0, \end{aligned} \tag{27}$$

where

$$\tau = -\int\limits_{E_0}^{E} \frac{dE}{\beta E^2} = \frac{1}{\beta E} - \frac{1}{\beta E_0}. \tag{28}$$

The distribution function $f(\mathbf{r}, E)$ will be determined when the source energy spectrum $Q_2(E_0)$ is known. In general, for a given energy interval the source spectrum can be taken to be of the form

$$Q_2(E_0) = Q E_0^{-\gamma_0} \quad \text{for} \quad E_1 \leqslant E_0 \leqslant E_2, \tag{29}$$

where Q and γ_0 are constants. From Eq. (27) the electron distribution function for this case is

$$\begin{aligned} f(\mathbf{r}, E) &= \int f(\mathbf{r}, E;\ E_0)\, Q_2(E_0)\, dE_0 = \\ &= \frac{Q}{\pi^{3/2} \beta E^2} \int\limits_{E_{\min}}^{E_2} \frac{E_0^{-\gamma_0}\, dE_0}{(a^2+4D\tau)\sqrt{b^2+4D\tau}} \exp\left\{-\frac{x^2+y^2}{a^2+4D\tau} - \frac{z^2}{b^2+4D\tau}\right\}. \end{aligned} \tag{30}$$

Because of (27), the value of E_{min} in the above expression is the largest of the quantities E and E_1. The influence of the other regions of the source spectrum will be negligible if the condition $E_1 \leq E \leq E_2$ is satisfied. If this is so, then the lower limit of integration in (30) is E, while the upper limit with sufficient accuracy can be put equal to infinity. Electrons with energies $10^8 \leq E \leq 10^{10}$ ev contribute to the nonthermal component of the galactic radio radiation. Therefore, the simple expression (30) can be used if the energy spectrum at the source can be written as a power function (29) in the energy interval 10^8-10^{11} ev. We assume that this condition is satisfied.

Let us transform Eq. (30) and choose as the integration variable

$$\xi = \frac{4D\tau}{a^2} = \eta\left(1 - \frac{E}{E_0}\right), \tag{31}$$

where

$$\eta = \frac{4D}{a^2 \beta E}. \tag{32}$$

Further, let

$$p = \frac{b^2}{a^2}; \tag{33}$$

distances will be measured in units of a. Then

$$f(\mathbf{r}, E) = \frac{Q}{4\pi^{3/2} aD}\left(\frac{a^2\beta}{4D}\right)^{\gamma_0 - 2} E^{-2} \int_0^\eta \frac{(\eta - \xi)^{\gamma_0 - 2}}{(1+\xi)\sqrt{p+\xi}} \exp\left\{-\frac{x^2+y^2}{1+\xi} - \frac{z^2}{p+\xi}\right\} d\xi. \tag{34}$$

In the calculations that follow, the value of the source-spectrum exponent is taken to be

$$\gamma_0 = 2. \tag{35}$$

This choice is based on the assumption that the spectrum of relativistic electrons injected into interstellar space cannot be very much different from the energy spectrum of the other cosmic-ray particles for which γ = 1.9-2.2 in the energy range under consideration. This assumption appears to be valid both for the case when the electrons are accelerated by some type of primary source, as well as for the case when the electrons are produced in the nuclear interactions of the heavy cosmic-ray particles [1].

For $\gamma_0 = 2$, expression (34) leads to the following electron distribution:

$$f(\mathbf{r}, E) = \frac{Q}{4\pi^{3/2} aD} E^{-2} \int_0^\eta \frac{\exp\left\{-\frac{x^2+y^2}{1+\xi} - \frac{z^2}{p+\xi}\right\}}{(1+\xi)\sqrt{p+\xi}} d\xi. \tag{36}$$

In particular, at the galactic center ($\mathbf{r}$ = 0)

$$f(0, E) = \frac{Q}{4\pi^{3/2} aD} E^{-2} \frac{2}{\sqrt{1-p}} \operatorname{arc\,tg} \frac{\sqrt{1-p}\,(\sqrt{\eta+p} - \sqrt{p})}{1 - p + \sqrt{p}\sqrt{p+\eta}}. \tag{37}$$

At the center of the galaxy, the number of electrons per unit volume with energies $E \geq E_0$ is

$$N(E \geqslant E_0) = \int_{E_0}^{\infty} f(0, E)\, dE = \frac{Q a\beta}{8\pi^{3/2} D^2} \frac{1}{\sqrt{1-p}} \int_0^{\eta_0} \operatorname{arc\,tg} \frac{\sqrt{1-p}\,(\sqrt{\eta+p}-\sqrt{p})}{1-p+\sqrt{p}\sqrt{\eta+p}}\, d\eta, \tag{38}$$

where

$$\eta_0 = \frac{4D}{a^2\beta E_0}. \tag{39}$$

Let us introduce the integration variable

$$u = \sqrt{1-p}\,\frac{\sqrt{p+\eta}-\sqrt{p}}{1-p+\sqrt{p}\sqrt{p+\eta}} \tag{40}$$

and let

$$\delta = \frac{\sqrt{p}}{\sqrt{1-p}}. \tag{41}$$

Then

$$N(E \geqslant E_0) = \frac{Q a\beta}{4\pi^{3/2} D^2 \sqrt{1-p}} \int_0^{u_0} \frac{u+\delta}{(1-\delta u)^3} \operatorname{arc\,tg} u\, du =$$

$$= \frac{Q a\beta}{4\pi^{3/2} D^2 \sqrt{1-p}} \frac{(1+u_0^2)\operatorname{arc\,tg} u_0 - u_0(1-\delta u_0)}{2(1-\delta u_0)^2}, \tag{42}$$

where u_0 is the value of (40) when $\eta = \eta_0$. With the conditions

$$\delta \ll 1 \quad \text{and} \quad u_0\delta \ll 1, \tag{43}$$

it is easy to show that

$$N(E \geqslant E_0) = \frac{Q a\beta}{8\pi^{3/2} D^2} [(1+\eta_0)\operatorname{arc\,tg}\sqrt{\eta_0} - \sqrt{\eta_0}], \tag{44}$$

These expressions allow the relativistic-electron density in the galaxy to be calculated and they will be used in Section 5.

3. Energy Spectrum of the Electrons Along the Line of Sight

As will be shown in Section 4, in order to determine the spectral intensity of the synchrotron radiation incident on the earth from different regions of the sky, we have to know the energy spectrum of the electrons along the line of sight. The latter is obtained by the integration of expression (34) along a chosen direction. Let the point of observation be situated in the galactic plane (in the model adopted – in the plane of symmetry of the source distribution (13)), at a distance R from the center. We move the origin of the coordinates to this point, so that (R, 0, 0) become the coordinates of the galactic center. With this choice of the coordinate system, expression (36) will take the following form:

$$f(\mathbf{r}, E) = \frac{Q}{4\pi^{3/2} aD} E^{-2} \int_0^{\eta} \frac{\exp\left\{-\frac{(x-q)^2+y^2}{1+\xi} - \frac{z^2}{p+\xi}\right\}}{(1+\xi)\sqrt{p+\xi}}\, d\xi, \tag{45}$$

where

$$q = R/a \tag{46}$$

and $\mathbf{r} = \{x, y, z\}$ is the radius vector in the new coordinate system measured in units of a. Transforming to the spherical coordinates r, ϑ, φ whose axis is directed at the galactic pole and integrating along the direction $\bar{l}$ determined by the angles ϑ and φ, we have

$$f_1(E) = \int_0^\infty f(r, E)\, dl = \frac{QE^{-2}}{4\pi^{3/2} D} \int_0^\eta \frac{d\xi}{(1+\xi)\sqrt{p+\xi}} \int_0^\infty \exp \times$$
$$\times \left\{-\frac{r^2 \sin^2\vartheta - 2rq \sin\vartheta \cos\varphi + q^2}{1+\xi} - \frac{r^2 \cos^2\vartheta}{p+\xi}\right\} dr. \tag{47}$$

After an integration with respect to r we obtain

$$f_1(E) = \frac{QE^{-2}}{4\pi^{3/2} D} \int_0^\eta \frac{d\xi}{(1+\xi)\sqrt{p+\xi}} \exp \times$$
$$\times \left\{-\frac{p \sin^2\vartheta \sin^2\varphi + \cos^2\vartheta + \xi(1 - \sin^2\vartheta \cos^2\varphi)}{(1+\xi)(p \sin^2\vartheta + \cos^2\vartheta + \xi)}\, q^2\right\} \times$$
$$\times \left[\frac{(1+\xi)(p+\xi)}{p \sin^2\vartheta + \cos^2\vartheta + \xi}\right]^{1/2} \int\limits_{-\left[\frac{p+\xi}{(1+\xi)(p \sin^2\vartheta + \cos^2\vartheta + \xi)}\right]^{1/2} q \sin\vartheta \cos\varphi}^{\infty} e^{-y^2}\, dy. \tag{48}$$

From expression (48) it is possible to obtain the energy spectrum of the electrons along the line of sight for an arbitrary direction. In particular, for the three typical directions: 1) towards the center ($\vartheta = \pi/2$, $\varphi = 0$), 2) towards the anticenter ($\vartheta = \pi/2$, $\varphi = \pi$), and 3) towards the pole ($\vartheta = 0$ or $\varphi = \pi$), we have

$$f_{1,2}(E) = \frac{Q}{4\pi^{3/2} D} E^{-2} \int_0^\eta \frac{d\xi}{\sqrt{(1+\xi)(p+\xi)}} \int\limits_{\mp q/\sqrt{1+\xi}}^{\infty} e^{-y^2}\, dy \tag{49}$$

and

$$f_3(E) = \frac{Q}{8\pi D} E^{-2} \int\limits_{q^2/1+\eta}^{q^2} e^{-y} \frac{dy}{y}. \tag{50}$$

The function f_3 is expressed in terms of the known exponential integral function. The evaluation of the integral in expression (49) is carried out by means of a series expansion of the integral with respect to y. Since in our case $q = R/a < 1$, it is sufficient to retain a small number of terms. The result is

$$f_{1,2}(E) = \frac{QE^{-2}}{4\pi D}\left\{\ln \frac{\sqrt{\eta+1} + \sqrt{\eta+p}}{1+\sqrt{p}} \pm \frac{2}{\sqrt{\pi}}\left[\left(q - \frac{q^3}{6(1-p)} + \frac{3q^5}{80(1-p)^2}\right) \times\right.\right.$$
$$\times \frac{1}{\sqrt{1-p}} \operatorname{arc\,tg} \frac{\sqrt{1-p}\,(\sqrt{\eta+p} - \sqrt{p})}{1-p+\sqrt{p}\sqrt{\eta+p}} - \left(\frac{q^3}{6(1-p)} - \frac{3q^5}{80(1-p)^2}\right) \times$$
$$\left.\left.\times \frac{\sqrt{\eta+p} - (1+\eta)\sqrt{p}}{1+\eta} + \frac{q^5}{40} \frac{\sqrt{\eta+p} - (1+\eta)^2\sqrt{p}}{(1+\eta)^2} + \ldots\right]\right\}. \tag{51}$$

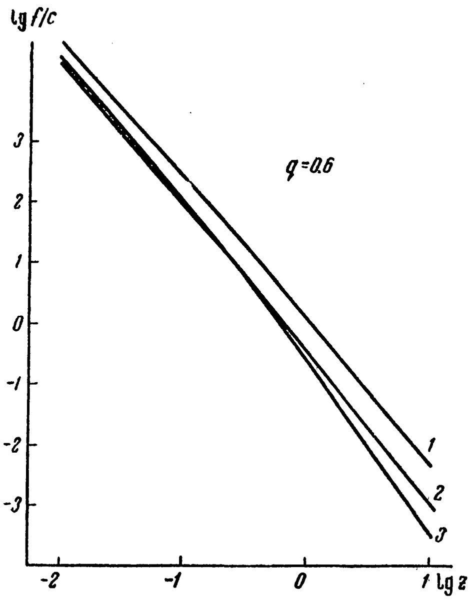

Fig. 1.

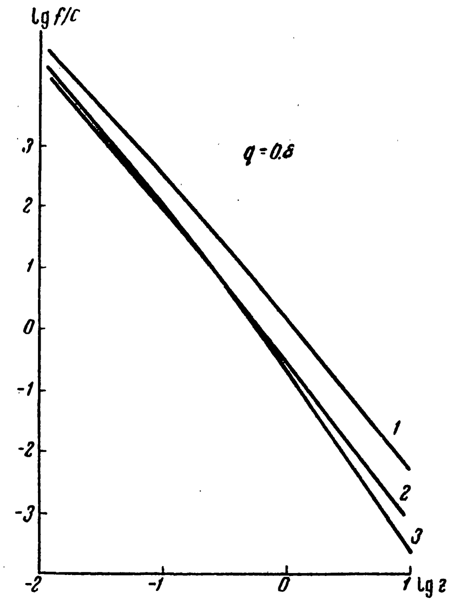

Fig. 2.

Figures 1 and 2 show, on a logarithmic scale, the curves for f_1, f_2, and f_3, as functions of the dimensionless parameter

$$z = \frac{1}{\eta} = \frac{a^2 \beta}{4D} E, \tag{52}$$

where z is the ratio of the radius of the source distribution to the diffusion pathlength in which the electron with energy E loses one-half of the energy as the result of synchrotron emission. The curves have been obtained for $P = 10^{-4}$ and two values of the parameter q: 0.6 and 0.8. The functions f are plotted along the vertical axis in units of

$$C = \frac{Q}{4\pi D}\left(\frac{a^2\beta}{4D}\right)^2. \tag{53}$$

The asymptotic expressions for high values of z, i. e., in the high-energy region, are

$$f_{1,2} \to \frac{Q}{8\pi D}\left(\frac{a^2\beta}{4D}\right)^2 \frac{2}{\sqrt{p\pi}} \int\limits_{\mp q}^{\infty} e^{-\xi^2} d\xi \, z^{-3}, \tag{54}$$

$$f_3 \to \frac{Q}{8\pi D}\left(\frac{a^2\beta}{4D}\right)^2 e^{-q^2} z^{-3}.$$

For small z

$$f_{1,2,3} \to -\frac{Q}{8\pi D}\left(\frac{a^2\beta}{4D}\right)^2 z^{-2} \ln z. \tag{55}$$

If the energy spectrum of the electrons along the line of sight is approximated by the expression $k_1 E^{-\gamma}$ over a narrow energy interval, then the value of the spectrum exponent γ varies from approximately $\gamma = 2$ in the region of small z to $\gamma = 3$ in the region of large z. It should be noted, however, that for small z the above expressions may turn out to be inapplicable since at low electron energies ionization losses become important.

4. The Intensity of the Synchrotron Radiation

In the case of an isotropic magnetic field which, in general, is randomly orientated and an isotropic particle-velocity distribution, the intensity of the synchrotron radiation observed from the given point along some direction **l** is given by

$$I_l(\nu) = \frac{1}{8\pi}\int_0^\infty\int_0^\infty\int_0^\pi f(\mathbf{r}, E)\, p(\nu, E, H\sin\vartheta)\, e^{-k_\nu l}\, dl\, dE \sin\vartheta\, d\vartheta, \tag{56}$$

where k_ν is the absorption coefficient for the radio waves propagating in the interstellar medium and

$$p(\nu, E, H\sin\vartheta) = \sqrt{3}\,\frac{e^3 H\sin\vartheta}{mc^2}\,\frac{\nu}{\nu_c}\int_{\nu/\nu_c}^\infty K_{5/3}(x)\, dx \tag{57}$$

is the energy emitted per unit time by a relativistic particle of charge $\underline{e}$ at an angle ϑ to the magnetic field H [5]. In expression (57), $K_{5/3}(x)$ is the MacDonald function and

$$\nu_c = \frac{3eH\sin\vartheta}{4\pi mc}\left(\frac{E}{mc^2}\right)^2. \tag{58}$$

In (56) it is assumed that the magnetic field intensity is constant in absolute magnitude and the integration is carried out along the direction of observation.

With the exception of a thin layer close to the galactic plane which contains the clouds of ionized hydrogen, the absorption coefficient k_ν for the radio-frequency range of interest is everywhere small [6]. We therefore, neglect absorption. Taking into account that

$$\int_0^\infty f(\mathbf{r}, E)\, dl = f_l(E) \tag{59}$$

is the energy spectrum of the electrons along the line of sight in the given direction, we can rewrite expression (56) for $k_\nu = 0$ as

$$I_l(\nu) = \frac{1}{8\pi}\int_0^\pi \sin\vartheta\, d\vartheta \int_0^\infty f_l(E)\, p(\nu, E, H\sin\vartheta)\, dE. \tag{60}$$

Let us assume that in an energy interval $E_1 < E < E_2$ the electron spectrum along the line of sight can be represented as

$$f_l(E) = k_l E^{-\gamma}. \tag{61}$$

Substituting (57) and (61) into expression (60) and transforming to a new integration variable

$$y = \frac{\nu}{\nu_c} = \frac{4\pi mc}{3eH\sin\vartheta}\left(\frac{mc^2}{E}\right)^2\nu, \tag{62}$$

we obtain

$$I_1(\nu) = \frac{\sqrt{3}}{16\pi}\frac{e^3}{mc^2}\left[\frac{3e}{4\pi m^3c^5}\right]^{(\gamma-1)/2} k_l H^{(\gamma+1)/2}\nu^{-(\gamma-1)/2}\int_0^{\pi}(\sin\vartheta)^{(\gamma+3)/2}d\vartheta\int_{y_2}^{y_1} y^{(\gamma-1)/2}dy\int_y^{\infty}K_{5/3}(x)\,dx, \quad (63)$$

where y_1 and y_2 are the values of y given by (62) for $E = E_1$ and $E = E_2$, respectively.

The evaluation of the integrals on the right-hand side of expression (63) shows * that for $2 \le \gamma \le 3$, provided that

$$\bar{y}_1 \ll 0.03 \text{ and } \bar{y}_2 \gg 3, \quad (64)$$

where

$$\bar{y} = y\sin\vartheta = \frac{4\pi mc}{3eH}\left(\frac{mc^2}{E}\right)^2\nu, \quad (65)$$

the integration of expression (63) with respect to y can be extended to the region from 0 to ∞, the error incurred being less than 15%. This means that the major contribution (more than 85%) to the radiation with the frequency ν is given by electrons with energies

$$\left(\frac{1}{3}\frac{4\pi mc}{3e}\frac{\nu}{H}\right)^{1/2} \leqslant \frac{E}{mc^2} \leqslant \left(\frac{1}{0.03}\frac{4\pi mc}{3e}\frac{\nu}{H}\right)^{1/2}. \quad (66)$$

Hence, the departure of the spectrum outside this region from the spectrum (61) inside for the energy interval given by expression (66) will not appreciably affect the intensity of the radiation of frequency ν.

For a field of $H = 7\cdot10^{-6}$ oersted, condition (66) reduces to

$$5.4\cdot10^7\sqrt{\nu} \leqslant E \leqslant 5.4\cdot10^8\sqrt{\nu}, \quad (67)$$

where the frequency is expressed in megacycles and the energy in electron volts. Thus, if in the energy interval

$$5.4\cdot10^7\sqrt{\nu_1} \leqslant E \leqslant 5.4\cdot10^8\sqrt{\nu_2} \quad (68)$$

the energy spectrum of the electrons along the line of sight can be represented in the form given by expression (61) with a constant exponent γ, then the intensity of the synchrotron radiation in the frequency interval $\nu_1 < \nu < \nu_2$ is determined sufficiently accurately when expression (63) is integrated with respect to y from 0 to ∞.

Replacing the limits of integration in (63) y_1 and y_2 by 0 and ∞, respectively, and taking into account that (cf. also [7])

$$\int_0^{\pi}(\sin\vartheta)^{(\gamma+3)/2}d\vartheta = \sqrt{\pi}\,\frac{\Gamma\left(\frac{\gamma+5}{4}\right)}{\Gamma\left(\frac{\gamma+7}{4}\right)} \quad (69)$$

and

$$\int_0^{\infty} y^{(\gamma-1)/2}dy\int_y^{\infty}K_{5/3}(x)\,dx = \frac{2^{(\gamma+1)/2}}{\gamma+1}\Gamma\left(\frac{3\gamma-1}{12}\right)\Gamma\left(\frac{3\gamma+19}{12}\right), \quad (70)$$

* The evaluation of the integral is carried out by the expansion of the function $K_{5/3}(x)$ into a series or an asymptotic series for small and large x, respectively. Since the calculation is not complicated, although fairly lengthy, we do not give the details here. The evaluation for large y has been carried out in [7].

we find the final expression for the intensity of the synchrotron radiation:

$$I_1(\nu) = A(\gamma)\,\frac{e^3}{mc^2}\left(\frac{e}{m^3c^5}\right)^{(\gamma-1)/2} H^{(\gamma+1)/2} k_1 \nu^{-(\gamma-1)/2}, \tag{71}$$

where

$$A(\gamma) = \frac{\sqrt{2}}{8(\gamma+1)}\left(\frac{3}{2\pi}\right)^{\gamma/2} \Gamma\left(\frac{3\gamma-1}{12}\right)\Gamma\left(\frac{3\gamma+19}{12}\right)\frac{\Gamma\left(\frac{\gamma+5}{4}\right)}{\Gamma\left(\frac{\gamma+7}{4}\right)}. \tag{72}$$

In particular,

$$A(2,3) = 0.0352; \quad A(2,5) = 0.0284; \quad A(2{,}64) = 0.0246. \tag{73}$$

5. Comparison with Observational Data and Estimation of Parameters

From expressions (36), (48), and (71) it is possible to calculate the distribution and the spectrum of the relativistic electrons in the galaxy, the spectrum of the electrons along the line of sight in an arbitrary direction, and the spectral distribution of the synchrotron radiation observed in this direction. For this the following quantities have to be known: the diffusion coefficient of the interstellar medium D, the intensity of the sources of the relativistic electrons Q, the parameters a and b describing the distribution of the sources, and R, the distance between the point of observation and the galactic center. The last three parameters can be set sufficiently reliably on the assumption that the source distribution coincides with the flat subsystem of the galaxy. The thickness of the latter is 200-300 pc and the radius is equal to 10-15 kpc. Therefore, taking into account that the results are insensitive to changes in p for small p, we take

$$p = \frac{b}{a} = 10^{-4}. \tag{74}$$

The value of the parameter $q = R/a$ appears to lie within the limits 0.6 and 0.8 if we assume that the sun is at a distance of R = 7.2 kpc from the galactic center and that the semimajor axis of the source distribution is somewhat smaller than the radius of the galactic disc. Further evaluations will be carried out simultaneously for these two cases, namely,

$$q = 0.6; \quad a = 12\,\text{kpc} = 3.7\cdot 10^{22}\ \text{cm}; \tag{75}$$

$$q = 0.8; \quad a = 9\,\text{kpc} = 2.8\cdot 10^{22}\ \text{cm}. \tag{76}$$

At the present time there are no independent measurements available from which we could estimate reliably the values of the parameters D and Q. We shall therefore formulate the problem in a somewhat different way. Namely, we shall show that the results obtained from expressions (36), (48), and (71) can be fitted to the observational data for definite values of the parameters D and Q and in this way we shall evaluate these quantities.

Measurements of the intensity of the galactic radio radiation show* that in the direction of the galactic poles where the major part of the radio emission is of nonthermal origin, the effective temperature of the radiation in the frequency interval

$$10\ \text{Mc} < \nu < 400\ \text{Mc} \tag{77}$$

can be expressed in the following form:

$$T_{\text{eff}} = 1.8\cdot 10^{25}\nu^{-2.82}. \tag{78}$$

* See the review of radioastronomical data in [6], as well as [8].

The corresponding spectral intensity distribution is given by

$$I_3(\nu) = \frac{2k}{c^2}\nu^2 T_{\text{eff}} = 5.6\cdot 10^{-12}\nu^{-0.82}, \tag{79}$$

where the subscript 3 denotes the direction towards the galactic pole. Assuming that the radiation is the synchrotron radiation of electrons in a field of $H = 7\cdot 10^{-6}$ oersted, we can easily find from expression (61) the energy spectrum of relativistic electrons along the line of sight in the direction of the galactic pole. Comparing (71) and (79), we find

$$k_3 = 4.85\cdot 10^5, \quad \gamma_3 = 2.64 \tag{80}$$

or

$$f_3(E) = 4.85\cdot 10^5\ E^{-2.64}\ \text{cm}^{-2}\cdot\text{erg}^{-1}. \tag{81}$$

In view of the conditions (68) and (77), the relativistic electrons have the spectrum given by (81) in the energy interval

$$1.7\cdot 10^8 < E < 1.1\cdot 10^{10}\ \text{ev}; \tag{82}$$

these are the results that follow from the data provided by the radioastronomical observations.

On the other hand, the theoretical spectrum of the electrons along the line of sight in the direction of the galactic pole is given by expression (50) and, correspondingly, by the curves 3 in Figs. 1 and 2. As we have already seen, the exponent γ for this spectrum varies smoothly from a value close to 2 for small z to $\gamma = 3$ for large z. In order to obtain agreement with the observed spectrum (81), we have to determine the interval of log z in which the average exponent for the curves is equal to 2.64. From (82) and (52) we find that the length of this interval is

$$\lg z_2 - \lg z_1 = \lg 1.1\cdot 10^{10} - \lg 1.7\cdot 10^8 = 1.8. \tag{83}$$

This interval of log z for both the cases $q = 0.6$ and $q = 0.8$ is centered at the point

$$\lg z = -0.4. \tag{84}$$

Comparing this value of z and the value $E = 1.35\cdot 10^9$ ev obtained for the midpoint of the interval given by (83), we can establish the scale along the abscissas in Figs. 1 and 2 and estimate the value of the diffusion coefficient D from expression (52). This leads to the following values:

$$\begin{aligned} D &= 1.5\cdot 10^{29}\ \text{cm}^2/\text{sec for } q = 0.6; \\ D &= 8.5\cdot 10^{28}\ \text{cm}^2/\text{sec for } q = 0.8. \end{aligned} \tag{85}$$

The error in the spectrum exponent obtained from observational data and the approximate nature of the comparison made between the theoretical and observed spectra (the choice of the value of log z according to expression (84)) can result in a considerable error in the value of the diffusion coefficient. However, calculations have shown that the values given cannot be changed by more than a factor of two either way. Thus, one of the results of the above investigation is an independent method for the determination of the diffusion coefficient for relativistic particles moving in interstellar space.

Since $D = \frac{1}{3}vl$, where v is the average velocity of the particles along the magnetic-force line (for a relativistic particle $v \approx 10^{10}$ cm/sec) and l is the effective size of the uniform regions of the magnetic field, for $D = 10^{29}$ cm^2/sec we find that $l \approx 3\cdot 10^{19}$ cm = 10 pc, which does not contradict available estimates [9].

In order to determine the paramter Q, we have to establish the vertical scale (the value of the constant C) in Figs. 1 and 2 by using the observed spectrum (81). Because of the smooth behavior of the curves in Figs. 1 and 2, we can replace them in the interval given by (83) and (84) by straight-line segments, such that at the point $\log z = -0.4$, $\log \frac{f_3}{c} = 0.45$ for $q = 0.6$ and $\log \frac{f_3}{c} = 0.38$ for $q = 0.8$. On the other hand, from (81) we find that $f_3 = 5.3 \cdot 10^{12}$ at an energy $E = 1.35 \cdot 10^9$ ev which corresponds, as has been taken above, to the value $\log z = -0.4$. Consequently, $C = 1.9 \cdot 10^{12}$ for $q = 0.6$ and $C = 2.2 \cdot 10^{12}$ for $q = 0.8$. From (53) and (85) with these values of C, we find

$$Q = 10^{38} \text{ for } q = 0.6; \qquad Q = 7 \cdot 10^{37} \text{ for } q = 0.8. \tag{86}$$

Knowing Q, we can estimate the energy production of the relativistic-electron sources:

$$U = \int_{E_1}^{E_2} Q(E)\, E dE = Q \ln \frac{E_2}{E_1} = (5-7) \cdot 10^{38} \text{ ergs/sec.} \tag{87}$$

Expressions (29) and (35) have been used to obtain this equation and U is the energy supplied to electrons with energies in the interval $10^8 \leq E \leq 10^{11}$ ev. As has been shown in [1], this power output is available in the envelopes of novae and supernovae for the generation of relativistic electrons.

Let us estimate the density of relativistic electrons in the galactic plane. For simplicity we use expression (42) for the density of electrons at the galactic center. Since with the above choice of the parameters, $p = 10^{-4}$ and $\eta_0 = 3.4$ for $E_0 = 10^9$ ev [cf. (39)]. the conditions given by expression (43) are satisfied and we can make use of expression (44). This gives

$$N(E > 10^9 \text{ ev}) = 8.5 \cdot 10^{-13} \text{cm}^{-3}, \text{ for } q = 0.6; \qquad N(E > 10^9 \text{ev}) = 1.4 \cdot 10^{-12} \text{cm}^{-3}, \text{ for } q = 0.8. \tag{88}$$

If we take into account that in the vicinity of the sun the electron density must be smaller by approximately a factor e^{q^2}, as shown by equation (36), then the values (88) are not in contradiction with the available data on the density of the relativistic electrons at the earth [1, 4].

Thus, the evaluation of the parameters leads to values which lie within acceptable limits and which are almost the same as those commonly adopted [1].

Up to now we have investigated the spectrum (50) in the direction of the galactic pole and by comparing it with the observed spectrum we were able to estimate the unknown parameters. In addition to this spectrum, the above analysis also gives us the spectra in the direction of the galactic center and anticenter (49). As can be seen from Figs. 1 and 2 (curve 2), the spectrum in the direction of the anticenter for the interval of log z under consideration [(83), (84)] is very close to the spectrum in the direction of the galactic pole, although it is somewhat less steep than the latter (the average exponent is $\gamma_2 = 2.5$ in the interval considered). It appears that this is in agreement with observations [6].

Let us determine the spectrum in the direction of the galactic center. In the interval of log z given by (83) and (84), the average exponent of the electron spectrum is equal to (curve 1, Figs. 1 and 2)

$$\gamma_1 = 2.30. \tag{89}$$

With the values of D and Q given by (85) and (86), it is not difficult to determine the value of the coefficient K in the spectrum of the electrons along the line of sight (61) and the corresponding spectrum of the synchrotron radiation (71) with the help of expression (49) or the curves 1 in Figs. 1 and 2. This leads to the following expressions for the intensity distribution of the radiation:

$$I_1 = 9.6 \cdot 10^{-13} \nu^{-0.65}, \quad \text{for} \quad q = 0.6;$$
$$I_1 = 1.3 \cdot 10^{-12} \nu^{-0.65}, \quad \text{for} \quad q = 0.8. \tag{90}$$

From this it follows that the effective temperature of the radiation incident along the direction from the galactic center is given by

$$T_1 = 3.1 \cdot 10^{24} \nu^{-2.65}, \quad \text{for} \quad q = 0.6;$$
$$T_1 = 4.2 \cdot 10^{24} \nu^{-2.65}, \quad \text{for} \quad q = 0.8. \tag{91}$$

In particular, for a wavelength of $\lambda = 3.5$ m this leads to a temperature of 3000°K for q = 0.6 and 4000°K for q = = 0.8. The spectrum given by (91) does not include the radiation from the galactic nucleus, which according to available data is fairly appreciable, as well as the thermal radiation from clouds of ionized gas. Since both of these effects are operative only in the immediate vicinity of the galactic plane, it is possible to eliminate them from the observations through the use of receivers with a high resolving power and the extrapolation of the observed intensity from higher latitudes to the region of the galactic center. The preliminary data obtained in this way* [10] appear to be in agreement with (91). It should also be noted that the difference between the spectra in the direction of the pole (78) and in the direction of the galactic center (91) is in agreement with the observed concentration of the radio emission towards the galactic center and its frequency dependence [6].

Thus, within the framework of the above investigation of the diffusion of relativistic electrons supplied by sources situated in the galactic plane, the energy losses due to the radiation in the magnetic fields being taken into account, it is possible to explain the main observations made in the frequency interval 10-400 Mc. It is not possible to extend the results obtained to the frequency region below 10 Mc, since for electrons with energies $E < 10^8$ ev (cf. (82)) the ionization losses neglected above become important. In this connection it should be noted that according to the latest data [8], the spectrum of radio radiation changes appreciably for $\nu < 10$ Mc; in the range from 2-10 Mc the intensity is practically independent of the frequency. If this change is governed by a change in the electron spectrum as the result of ionization losses, then the estimate of the average gas density in the total volume of the galaxy, $n \approx 0.01$ cm^{-3}, made above receives an independent confirmation.

More accurate data on the radio radiation observed in different directions will make it possible to increase the accuracy of the determinations of the various parameters and also to determine the value of q which lies within the limits 0.6-0.8. The numerical estimates made must be regarded as preliminary, particularly since it is possible that the source-spectrum exponent is slightly different from that adopted by us (35).

In conclusion, we note that the results obtained are valid both when the relativistic electrons are secondaries from the nuclear interaction of heavy cosmic-ray particles, as well as when they are directly produced in sources. The problem of which of these processes plays the dominant role requires a special investigation.

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