Probing Nearby CR Accelerators with Milagro/IceCube Hot Spots

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Abstract: Both the acceleration of cosmic rays (CR) in supernova remnant shocks and their subsequent propagation through the random magnetic field of the Galaxy deem to result in an almost isotropic CR spectrum. Yet the MILAGRO TeV observatory and the IceCube discovered sharp ($\sim 10^{\circ}$) arrival anisotropies of CR nuclei. We suggest a mechanism for producing such a CR beam which operates en route to the observer. The key assumption is that CRs are scattered by a strongly anisotropic Alfven wave spectrum formed by the turbulent cascade across the local field direction. The strongest pitch-angle scattering occurs for particles moving almost precisely along the field line. Partly because this direction is also the direction of minimum of the large scale CR angular distribution, the enhanced scattering results in a weak but narrow particle excess. The width, the fractional excess and the maximum momentum of the beam are calculated from a systematic transport theory depending on a single scale l which can be associated with the longest Alfven wave, efficiently scattering the beam. The best match to all the three characteristics of the beam is achieved at $l \sim 1$pc. The distance to a possible source of the beam is estimated to be within a few 100pc. Possible approaches to determination of the scale l from the characteristics of the source are discussed. Alternative scenarios of drawing the beam from the galactic CR background are considered. The beam related large scale anisotropic CR component is found to be energy independent which is also consistent with the observations.

Keywords: acceleration of particles – cosmic rays – plasmas supernova remnants — turbulence – scattering – diffusion

Introduction The MILAGRO TeV observatory and the IceCube discovered collimated beams dominated by hadronic cosmic rays (CR) with a narrow ($\sim 10^{\circ}$) angular distribution in the 10 TeV energy range [2, 1]. This is surprising, since most of the CR acceleration and propagation models predict only a weak, large scale anisotropy.

In this paper we suggest a novel mechanism for producing a narrow CR beam. It is based on the strong anisotropy of the MHD turbulence in the ISM. Such anisotropy is expected when the turbulence is driven at a long (outer) scale, but unlike the isotropic Kolmogorov cascade, the incompressible MHD cascade is directed perpendicularly to the magnetic field in the wave vector space. This was shown by Goldreich & Sridhar [4] (GS). The cascade proceeds to $k_{\perp} r_g(p) \gg 1$ in the perpendicular wave number direction for the protons with the gyro-radii $r_g \sim 10^{16}$cm, typical for the particles of the beam energies $pc \sim 10$TeV and the ISM magnetic field of a few μG. Contrary to the $k_{\perp}$ direction the spectrum spreading in $k_{\parallel}$ is suppressed, so that $k_{\parallel} \sim k_{\perp}^{2/3} l^{-1/3} \ll k_{\perp}$, where l is the outer scale.

The CR scattering by the GS anisotropic spectrum was investigated in e.g., (Chandran 3). The pitch-angle scattering rate is peaked at $|\mu| = |\cos\vartheta| \approx 1$, i.e., for particles moving along the field line, since for these particles $k_{\perp} r_g(p_{\perp}) \lesssim 1$. Only particles with such small $p_{\perp}$, i.e., with pitch angles within $\sin^2\vartheta \lesssim \varepsilon \ll 1$ are scattered efficiently.

Angular distribution of particles For the purposes of this paper we need the *angular profile* the pitch-angle scattering coefficient near $|\mu| = 1$, which we evaluate below. Assuming the GS spectrum for the spectral wave density I,

$$I = \frac{1}{6\pi} k_{\perp}^{-10/3} l^{-1/3} g\left(\frac{k_{\parallel} l^{1/3}}{k_{\perp}^{2/3}}\right) e^{-\tau/\tau_k}, \qquad (1)$$

the pitch-angle scattering coefficient can be represented as follows (e.g., 3)

$$\begin{aligned} D_{\mu\mu} &= \frac{\pi}{3} l^{-1/3} \Omega^2 \left(1-\mu^2\right) \int_0^{\infty} k_{\perp}^{-7/3} dk_{\perp} \sum_{n=-\infty}^{\infty} \\ &\times \frac{n^2 J_n^2(\xi)}{\xi^2} \int_{-\infty}^{\infty} g\left(\frac{k_{\parallel} l^{1/3}}{k_{\perp}^{2/3}}\right) \delta\left(k_{\parallel} v_{\parallel} - n\Omega\right) dk_{\parallel} \end{aligned} \qquad (2)$$

Here, $g(x) = H(1-|x|)$, where H is the Heaviside function and $\tau_k = (l/V_A)(k_{\perp} l)^{-2/3}$ is the turbulence correlation time. For a small $\delta, \varepsilon \ll 1$ (where $\delta = V_A/v \approx V_A/c$,

$\varepsilon = v/l\Omega$, $\Omega = eB_0/p$ and $v \approx c$ is the particle velocity) one obtains

$$D_{\mu\mu} \simeq \frac{1}{6}\frac{v}{l}\delta\left[\ln\left(\frac{1}{\varepsilon}\right) - \frac{1}{2}\ln\left(1-\mu^2\right)\right]\left(1-\mu^2\right) \qquad (3)$$

Now we concentrate on the particular region, $1-\mu^2 \lesssim \varepsilon$, for which we obtain the following expression for the scattering coefficient [6]

$$D_{\mu\mu} = \frac{\pi}{2}\frac{v}{l}\left(1-\mu^2\right)\left[\frac{J_1^2(y)}{y^2} + ry^{4/3}\right] \qquad (4)$$

where $r \sim 10^{-2}$ and $y = \sqrt{(1-\mu^2)/\varepsilon}$. Clearly, we can neglect the small second term in the brackets altogether, and switch to the expression given by eq.(3) for $y \gtrsim j_1$, where $j_1 \approx 3.8$ being the first root of J_1. The most important part of the scattering coefficient $D_{\mu\mu}(y)$ is its sharp peak near $|\mu| = 1$ where it behaves as $D_{\mu\mu} \propto J_1^2(y)$. As y grows and approaches $y = j_1$, $D_{\mu\mu}/\left(1-\mu^2\right)$ falls down to $\sim \delta$ of its peak value at $|\mu| = 1$ and remains approximately constant, eq.(3). The other peak occurs at $\mu \approx 0$ but it is not important for our purposes.

Particle propagation Suppose that a source of CRs is within the same magnetic flux tube with the Earth. We calculate the CR propagation to the Earth below. Obviously, the degree of CR anisotropy near the source may be significantly higher than that observed at the Earth. The propagation problem may be considered being one dimensional and stationary with the only spatial coordinate z, directed along the flux tube from the source to the Earth.

The particle momentum is conserved and the transport problem is in only two variables, the coordinate z and the pitch angle ϑ (or $\mu \equiv \cos\vartheta$). The characteristic (ϑ-independent) pitch-angle scattering frequency ν_ϑ (typical for μ not too close to $\mu = 0, \pm 1$) can be written as:

$$\frac{D_{\mu\mu}}{1-\mu^2} \approx \nu_\vartheta \equiv \frac{v}{l}\left(\delta\ln\left(\frac{1}{\varepsilon}\right) + \varepsilon^{3/2}\right)/6 \qquad (5)$$

The equation for the CR distribution thus reads

$$(u+\mu)\frac{\partial f}{\partial z} = \frac{\partial}{\partial\mu}\left(1-\mu^2\right)D(\mu)\frac{\partial f}{\partial\mu} \qquad (6)$$

Here u is the bulk flow (scattering centers) velocity along z in units of the speed of light, $u \ll 1$, $\mu = \cos\vartheta$. The coordinate z is normalized to the pitch-angle scattering length $c/\nu_\vartheta \approx v/\nu_\vartheta$, so that $D(\mu) = \nu_\vartheta^{-1}D_{\mu\mu}/\left(1-\mu^2\right)$ being normalized to ν_ϑ, is close to unity except for the narrow peaks. Our purpose is to find a narrow feature (which may be a bump or a hole) on the otherwise almost isotropic angular spectrum $f(\mu)$. Clearly, this feature must be pinned to one of the peaks of $D(\mu)$.

Let us consider the particle scattering problem given by eq.(6) in a half space $z \geq 0$ and assume that at $z = 0$ (source) the distribution function is $f(0,\mu) = f_0(\mu)$. It is clear that if there are no particle sources at $z = \infty$, then $f(\infty,\mu) = f_\infty = const$, apart from the dependence of f on the particle momentum as a parameter. It is convenient to subtract f_∞ from f, $\Psi(z,\mu,p) = f(z,\mu,p) - f_\infty(p)$, so that the new function Ψ satisfies the same equation (6) as f and the following boundary conditions

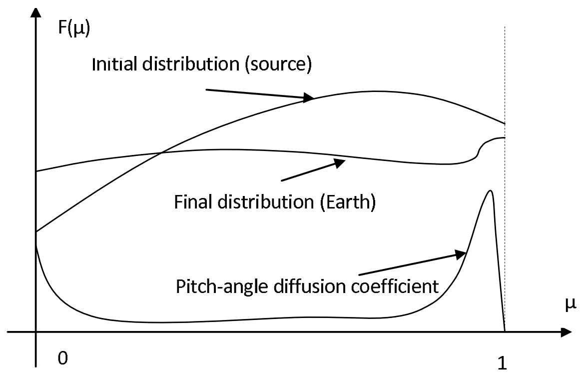

Figure 1: Schematic representation of initial and final pitch-angle distributions and that of the diffusion coefficient $D_{\mu\mu}(\mu)$.

$$\Psi = \begin{cases} \phi(\mu) = f_0(\mu) - f_\infty, & z = 0 \\ 0, & z = \infty \end{cases}$$

It is natural to expand the solution into the series of eigenfunctions Ψ_λ

$$\Psi = \sum_\lambda C_\lambda \Psi_\lambda(\mu)\, e^{-\lambda z} \qquad (7)$$

to be found from the following spectral problem

$$\frac{d}{d\mu}\left(1-\mu^2\right)D(\mu)\frac{d\Psi_\lambda}{d\mu} + \lambda(u+\mu)\Psi_\lambda = 0 \qquad (8)$$

As is well known (7, see also 5), there exists a complete set of the orthogonal eigenfunctions $\{\Psi_\lambda\}_{\lambda_i=-\infty}^{\lambda_i=\infty}$ with the discrete spectrum λ_i having no limiting points other than at $\pm\infty$. If we consider the formal solution given by eq.(7) at such a distance z where $(\lambda_2 - \lambda_1)z \gtrsim 1$, with $\lambda_{1,2}$ being the first (smallest) positive eigenvalues, the solution will be dominated by the first eigenfunction Ψ_{λ_1}. We know that the anisotropy at the Earth is very small ($\sim 10^{-3}$) and, assuming it being not so small at the source, we deduce that $\lambda_1 z \gg 1$ so that the inequality $(\lambda_2 - \lambda_1)z \gg 1$ should satisfy as well and we can limit our treatment of the spectral problem given by eq.(8) to the determination of only the first positive eigenvalue with the corresponding eigenfunction. Although the function $D(\mu)$ has a strong peak at $\mu \approx 1$, this peak is very narrow ($\sim \varepsilon$) and, as we mentioned, a perturbation theory applies.

Outside of the peak region we assume $D = 1$ as an exact value for D. Therefore, for $\left(1-\mu^2\right) \gtrsim \varepsilon$, the zeroth order approximation of the outer expansion reads

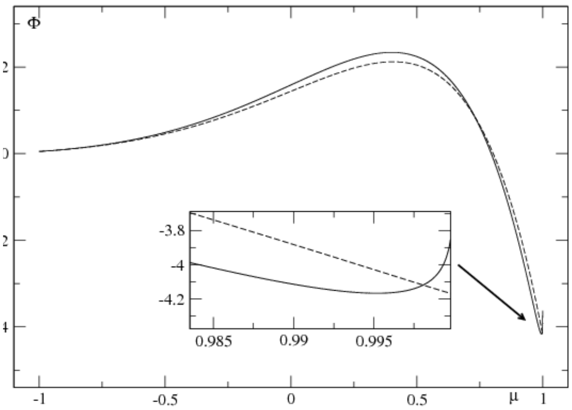

Figure 2: Unperturbed eigenfunction $\Phi(\mu) \equiv \Psi^{(0)}_{\lambda_1}$ (numerical solution of eq.[9], dashed line). Perturbed solution (solid line). The insert shows the solution behavior at the end point, including the logarithmic term of the outer solution.

$$\frac{d}{d\mu}\left(1-\mu^2\right)\frac{d\Psi^{(0)}_\lambda}{d\mu}+\lambda^{(0)}\mu\Psi^{(0)}_\lambda=0 \tag{9}$$

To find $\lambda^{(0)}$ we require the solution to be regular at the both singular points $\mu = \pm 1$. It easy to find the required single eigenvalue λ_1 and the corresponding eigenfunction by a direct numerical integration of the above equation. The result is shown in Fig.2 and $\lambda_1 \approx 14.54$.

Since $D \equiv 1$ in the outer region, the perturbation can be associated only with the perturbation of λ. Therefore, we expand λ and Ψ_λ as $\lambda = \lambda^{(0)} + \delta\lambda + \ldots$, $\Psi_\lambda = \Psi^{(0)}_\lambda + \delta\lambda\Psi^{(1)}_\lambda + \ldots$. Here λ can be an arbitrary point of the spectrum $\lambda = \lambda_i > 0$, but we are primarily interested in$\lambda = \lambda_1$. The equation for $\Psi^{(1)}_\lambda$ takes the following form

$$\frac{d}{d\mu}\left(1-\mu^2\right)\frac{d\Psi^{(1)}_\lambda}{d\mu}+\lambda^{(0)}\mu\Psi^{(1)}_\lambda=-\mu\Psi^{(0)}_\lambda \tag{10}$$

We can write the solution of the last equation as follows

$$\Psi^{(1)}_\lambda = -\Phi\int_{-1}^{\mu}\frac{U(\mu')\,d\mu'}{\Phi^2(\mu')\left(1-\mu'^2\right)} \tag{11}$$

where we have denoted $\Phi(\mu) \equiv \Psi^{(0)}_\lambda(\mu)$, and $U(\mu) \equiv \int_{-1}^{\mu}\mu'\Phi^2(\mu')\,d\mu'$.

Turning to the inner expansion of the solution of eq.(8), it is convenient to stretch the variable μ at $\mu = 1$ as follows $w = (1-\mu)/b$. Note that $b = \varepsilon j_1^2/2$ is chosen in such a way that $D(w=1) \approx 1$. Therefore, we represent D as $D(w) = a^{-1}F(w)+1$, $w \le 1$ and $D(w) = 1$, $w > 1$, where $F(w) = \left(\pi/2j_1^2 w\right)J_1^2\left(j_1\sqrt{w}\right)$. Here $a = \nu_\vartheta l/v \ll 1$ and Eq.(8) can be written as follows

$$\frac{d}{dw}\left[F(w)+a\right](2-bw)\,w\frac{d\Psi^i_\lambda}{dw}+ba\lambda\,(1-bw)\,\Psi^i_\lambda=0 \tag{12}$$

where the index i stands for the 'inner' solution. In contrast to the outer problem we must impose the regularity condition at $\mu = 1$ ($w = 0$).

Working up to the second order in $b \ll 1$, and integrating eq.(12) by parts, we transform it into the following first order equation

$$\frac{d\Psi^i_\lambda}{dw}+\frac{\lambda b}{2}g'\left[1-\frac{\lambda b}{2}\left(\frac{h}{w}-g\right)\right]\Psi^i_\lambda=0$$

Comparing with the outer solution yields $\Psi^i_\lambda(0) \approx \Phi(1)/(1+\lambda b/2)$ and $\delta\lambda = b^2\lambda^2\Phi^2(1)/4U(1)$. Using the matching procedure we determined the initially unknown arbitrary constant of the inner solution $\Psi^i_\lambda(0)$ and the perturbation of the eigenvalue λ by matching the terms in both equations that are independent of w and proportional to $\ln w$, respectively. The linear and quadratic terms in w match automatically to the appropriate accuracy $\sim b^2$. This follows from the two further relations $\Phi'(1) = \lambda\Phi(1)/2$, $\Phi''(1) = \lambda\Phi'(1)/4$, which can be obtained from the Frobenius series of eq.(9) at the singular end point $\mu = 1$ with $\Psi^{(0)}_\lambda \equiv \Phi$.

Beam characteristics After we have determined the angular distribution of the beam, the question is whether it is consistent with at least the prominent MILAGRO hot spot A [2]. Two major beam parameters were calculated in terms of the small parameter of the theory, $\varepsilon = r_g(p)/l$, where r_g is the particle gyro-radius and l is maximum wave length beyond which particles interact with waves adiabatically. The first parameter of the beam is its angular width (in terms of $\mu = \cos\vartheta$) $b = j_1^2\varepsilon/2 \approx 7.3\varepsilon$ and the second is its strength, which can be conveniently expressed as the ratio of the beam excess to the amplitude of the first eigenfunction, $\delta\Phi(1)/\Phi(1) \approx \lambda_1 b/2 \approx 53.4\varepsilon$. Since $\varepsilon \propto p$, the spectrum of the beam should be one power harder than the CR large scale anisotropic component inside the flux tube. This is consistent with the Milagro beam spectrum, provided that Φ scales with momentum similarly to the galactic CR background.

According to the MILAGRO Region A observations, the beam width is about $\Delta\vartheta \sim 10°$, where $\Delta\vartheta \approx \cos^{-1}(1-b) \approx \sqrt{2b} = j_1\sqrt{\varepsilon}$ so that we obtain for ε the following constraint from the observed MILAGRO Spot A $\varepsilon \approx \Delta\vartheta^2/j_1 \approx 2.1\cdot 10^{-3}$. This estimate yields the strength of the beam at the level of ≈ 0.1 which is also consistent with the MILAGRO fractional excess of the beams A and B measured with respect to the large scale anisotropy.

Beam Sustainability Now that we have calculated the pitch-angle distribution of a narrow CR beam formed from a wide angleanisotropic CR flux by its interaction with the *background ISM turbulence*, we need to check whether the beam will survive the pitch-angle scattering by *self-generated waves*. Assuming a power-law momentum scaling for the background CRs, $F_C(p) \propto p^{-q_c}$ (with $q_c = 4.6-4.7$) we can obtain an expression for the beam instability threshold distribution $\mathscr{F}_{th}(p_\parallel) \equiv \delta \cdot F_C(p_\parallel)/(q_c-2)$, so that if $\mathscr{F}_B(p_\parallel) \le \mathscr{F}_{th}(p_\parallel)$, the beam can sustain its angular distribution. Otherwise, it will be spread in pitch angle to satisfy the last inequality. Using the expressions for the width of the beam and for its amplitude relative to $\Phi(\mu, p)$, we can represent $\mathscr{F}_B(p_\parallel)$ as follows

$$\mathscr{F}_B(p_\parallel) = \frac{\lambda_1 b^2}{2} F_0(p_\parallel) = \frac{1}{8}\lambda_1 j_1^4 \varepsilon^2 F_0(p_\parallel) \tag{13}$$

where we have denoted $F_0(p) \equiv \Phi(\mu = 1, p)$. Then, our constraint $\mathscr{F}_B(p_\parallel) \le \mathscr{F}_{th}(p_\parallel)$ can be represented in the following way

$$F_0(p) \le A \frac{V_A}{c} \frac{l^2}{r_g^2(p)} F_C(p) \tag{14}$$

where $r_g = pc/eB_0$ is the particle gyro-radius. We denoted by A the following numerical factor $A = 8/\lambda_1 j_1^4 (q_c-2) \approx 10^{-3}$. Due to the factor r_g^{-2} in the relation given by eq.(14), the function $F_0(p)$ is constrained at high momenta. Assuming that F_0 is not much steeper than the background distribution F_C, we infer from eq.(14) that there exists maximum momentum p_{Bmax}, beyond which the beam would spread in pitch-angle and dissolve in the CR background,

$$\frac{p_{Bmax}}{mc} \simeq \frac{1}{K}\sqrt{\frac{V_A}{c}\frac{A}{\alpha}} \tag{15}$$

where we have introduced the following parameter which is the major small parameter of the theory $K \equiv c/l\omega_c = \varepsilon mc/p$. Here ω_c is the proton cyclotron (non-relativistic) frequency and l is the maximum turbulence scale beyond which the particles response becomes adiabatic. Based on the two independent MILAGRO measurements of the width and the fractional excess of the Beam A, we inferred the parameter $\varepsilon \sim 10^{-3}$. Assuming that this value of ε relates to the 1TeV beam median energy, we obtain $K \sim 10^{-6}$. Taking $V_A/c \sim 10^{-4}$ and $\alpha \sim A \sim 10^{-3}$, we obtain $p_{Bmax} \sim 10$ TeV. This is encouragingly close to the MILAGRO estimates of the beam cut-off energy.

Summary The principal results of this paper are as follows. Assuming only a *large scale* anisotropic distribution of CRs (generated, for example by a nearby accelerator, such as a SNR) and a Goldreich & Sridhar [4] (GS) cascade of Alfvenic turbulence originating from some scale l, which is the longest scale relevant for the wave-particle interactions, we calculated the propagation of the CRs down their gradient along the interstellar magnetic field. It is found that the CR distribution develops a characteristic angular shape consisting of a large scale anisotropic part (first eigenfunction of the pitch-angle scattering operator) superposed by a beam, tightly focused in the momentum space in the local field direction. The large scale anisotropy carries the *momentum dependence* of the source, while both the beam angular width and its fractional excess (with respect to the large scale anisotropic component) grow with momentum (as $\sqrt{p}$ and p, respectively). Apart from the width and the fractional excess of the beam, the theory predicts its maximum momentum on the ground that beyond this momentum the beam destroys itself. All the three quantities are completely determined by the turbulence scale l. Even if l is considered unknown, it can be inferred from any of the three independent MILAGRO measurements. These are the width, the fractional excess and the maximum energy of the beam, and all the three consistently imply the same scale l ~1 pc. The calculated beam maximum momentum encouragingly agrees with that measured by MILAGRO (~10 TeV/c). The theoretical value for the angular width of the beam is found to be $\Delta\vartheta \simeq 4\sqrt{\varepsilon}$, where $\varepsilon = r_g(p)/l \ll 1$. The beam fractional excess related to the large scale anisotropic part of the CR distribution is $\simeq 50\varepsilon$. Both quantities also match the Milagro results for $E \sim 1-2$ TeV. So, the beam has a momentum scaling that is one power shallower than the CR carrier, it is drawn from.

The model suggested in this paper becomes devoid of free parameters, if the knee energy at $\sim$ 3PeV can be associated with the maximum CR energy of the source of the beam and thus the unknown parameter l can be associated with the gyroradius of a 3PeV particle. Even though such an association is not proven, our propagation model predicts the three beam characteristics: its width, fractional excess and maximum energy to be the functions of a single quantity, the longest wave-particle interaction scale l. They all give the correct MILAGRO values for $l \simeq 1$ pc, which is unlikely to be coincidental.

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