Observable traces of non-metricity: new constraints on metric-affine gravity

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Relaxing the Riemannian condition to incorporate geometric quantities such as *torsion* and *non-metricity* may allow to explore new physics associated with defects in a hypothetical space-time microstructure. Here we show that non-metricity produces observable effects in quantum fields in the form of 4-fermion contact interactions, thereby allowing us to constrain the scale of non-metricity to be greater than 1 TeV by using results on Bahbah scattering. Our analysis is carried out in the framework of a wide class of theories of gravity in the metric-affine approach. The bound obtained represents an improvement of several orders of magnitude to previous experimental constraints.

Introduction. The Einstein equivalence principle is the cornerstone of gravitational physics, supporting the idea that gravitation can be interpreted as a geometric phenomenon. Particles and radiation fields follow special paths determined by a curved geometry, whereas their local causal relations are determined by the metric. General Relativity (GR) and, more generally, metric theories of gravity are built under the assumption that the geometry is (pseudo-)Riemannian, i.e., that *the metric is the foundation of all*. However, the experimental limits of the Riemannian assumption are still not well established [1–8] and some steps should be taken to better understand whether geometric structures other than the metric could be needed to account for all space-time properties. In this regard, it is worth noticing that though much has been done to infer the potential existence of higher dimensions [9] or supersymmetry [10], the roles of torsion [12] and non-metricity [11] are much less known. The physical relevance of these magnitudes can be appreciated in condensed matter systems, where the underlying lattice structure gives rise to the emergence of a continuous geometry which cannot be described solely in terms of an effective metric [13–15]. Torsion and non-metricity become necessary to fully account for the physical characteristics of those systems, such as plasticity and viscoelasticity, which are intimately related with the presence of topological defects in the crystal lattice [13–17]. Given our limited understanding of gravitation in the high-energy regime, if the continuous space-time that we perceive had some kind of "microstructure", as expected in almost the totality of approaches to quantum gravity [18–41], it is legitimate to explore how the continuum may arise and the potential impact that geometrical objects other than the metric could have in a low-energy effective description. Though some observable effects of torsion have been experimentally tested [42–46], the observable consequences of non-metricity still represent a largely unknown territory.

The main purposes of this Letter are 1) to show that the observable effects of non-metricity may be more easily accessible than those of torsion and 2) set a lower limit to the energy scale Λ_{NM} at which non-metricity may become important. To this end, we consider the effects of non-metricity in quantum systems involving spinor fields in both relativistic and non-relativistic scenarios. To carry out this study, it is first necessary to work out the generalization of the covariant Dirac equation [47] for space-times with the most general form of non-metricity. The nontrivial new elements involved in this generalized equation of motion already suggest that non-metricity could have observable effects, but a precise quantitative analysis requires the definition of specific forms of non-metricity. For this purpose, we use as a guide the predictions of a wide class of metric-affine theories of gravity recently studied in the literature [48], which we call RBG (Ricci-Based Gravity) theories. These theories are defined by Lagrangians of the form $\mathcal{L}_{grav} = f(g_{\mu\nu}, R_{\mu\nu})$, where $R_{\mu\nu}$ denotes the (symmetrized) Ricci tensor, such that GR is recovered as a low-energy limit. Corrections to GR appear as terms proportional to inverse powers of Λ_{NM}, the scale which we here wish to constrain experimentally. Our results apply to the vast majority of metric-affine theories studied so far, e.g. $f(R)$, Born-Infeld (BI), and other extensions (see [49–53] for related reviews). Indeed, a common feature of all RBG models (with minimal matter couplings) is that non-metricity is sourced by the local densities of energy and momentum [53–68]. We explicitly show that this leads to geometry-induced interactions among the matter fields. As a result, to first order in perturbation theory, this coupling generates 4-particle fermion interactions whose amplitude can be orders of magnitude larger than the 4-fermion interactions typically associated with torsion. Within this approximation, the effects of torsion in RBGs are the same as in GR and, thus, can be consistently neglected. Given that, by using experimental data for Bhabha scattering from LEP [69, 70], we are able to constrain the energy scale at which non-metricity effects may arise, setting a lower bound for Λ_{NM} at the TeV scale.

Gravity-induced point-like interactions were already

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pointed out by Flanagan [71] (see also [72]) in the context of $1/R$ gravity using a scalar-tensor representation. The existence of a certain gauge freedom associated to the projective invariance of RBGs, however, does not allow to interpret those effects in $f(R)$ theories as due to the *non-metricity*, as a metric-compatible gauge choice (with torsion) of those theories is possible [48, 73, 74]. On the contrary, the broader class of gravity theories considered here possess genuine *non-metricity*, and we are able to ascribe the appearance of additional particle contact interactions directly to this term [see below Eq.(6) for more details].

Other kind of experimental constraints on the components of non-metric tensors have been recently considered in [75] in the context of Lorentz symmetry breaking. There the crucial assumption for breaking symmetries is that the non-metricity tensor has a constant non-zero expectation value. We believe that dynamically generated non-metricity should not introduce such violations, being the RBG models possible counter-examples, which opens different avenues to explore this sector of the gravitational interactions.

Fermions in non-Riemannian spaces. The standard covariant Dirac equation in a curved Riemannian spacetime can be easily obtained from the Lagrangian [47]

$$\mathcal{L} = \sqrt{-g}\left[\frac{1}{2}\left(\bar{\psi}\underline{\gamma}^{\mu}(\nabla_{\mu}\psi) - (\nabla_{\mu}\bar{\psi})\underline{\gamma}^{\mu}\psi\right) + \bar{\psi}m\psi\right] \quad (1)$$

and takes the form

$$\left(\underline{\gamma}^{\mu}\nabla_{\mu} + m\right)\psi = 0\ , \quad (2)$$

which is known as the Dirac-Weyl-Fock (DWF) equation (see e.g. [79]). Here $\nabla_{\mu}\psi \equiv (\partial_{\mu} - \Gamma_{\mu})\psi$, where $\Gamma_{\mu} \equiv \omega_{\mu}{}^{ab}\sigma_{ab}$, is the spinor connection, $\omega_{\mu}{}^{ab} \equiv \frac{1}{2}\left(\partial_{\mu}e^{b}{}_{\alpha} + e^{b}{}_{\beta}\Gamma_{\mu\alpha}{}^{\beta}\right)\eta^{ac}e_{c}{}^{\alpha}$, $\sigma_{ab} \equiv \frac{1}{4}[\gamma_b, \gamma_a]$[1]and $\underline{\gamma}^{\mu} = e_{a}{}^{\mu}\gamma^{a}$, being $e_{a}{}^{\mu}$ the vierbein defined by $e_{a}{}^{\mu}e_{b}{}^{\nu}g_{\mu\nu} = \eta_{ab}$ and γ^{a} the standard Dirac matrices. In non-Riemannian spacetimes, the above equation (2) is modified. Considering non-metricity ($Q_{\mu\nu\alpha} \equiv -\nabla_{\mu}g_{\nu\alpha}$) and torsion $S_{\mu\nu}{}^{\alpha} \equiv -2\Gamma_{[\mu\nu]}{}^{\alpha}$, the equation for spinor fields is (see Appendix A)

$$\left[\underline{\gamma}^{\mu}\nabla_{\mu} + \frac{1}{2}\left(S_{\mu\alpha}{}^{\alpha} + Q_{[\alpha\mu]}{}^{\alpha}\right)\underline{\gamma}^{\mu} + m\right]\psi = 0, \quad (3)$$

which recovers (2) in the Riemannian case.

RBGs and the non-metricity tensor. In order to explore the physics associated to the break-down of the metricity condition, $\nabla_{\alpha}g_{\mu\nu} = 0$, a non-metricity tensor must be specified. In this sense, RBG theories are a rather general but sufficiently simple family that nicely fits to our purposes [80–87]. These theories possess an Einstein frame representation [53, 80] in terms of an auxiliary metric $h_{\mu\nu}$ whose relation with $g_{\mu\nu}$ to lowest order can be parametrized as

$$g_{\mu\nu} = h_{\mu\nu} + \alpha T h_{\mu\nu} + \beta T_{\mu\nu}\ . \quad (4)$$

Here $T_{\mu\nu}$ is the stress-energy tensor of the matter fields, T its trace, and α and β are proportional to Λ_{NM}^{-4} up to some $\mathcal{O}(1)$ dimensionless coefficients. In RBG models [53], this energy scale takes the form[2] $\Lambda_{NM} = (8\pi G\lambda_{model}^2)^{-1/4} = (2\pi)^{1/4}(E_p\Lambda_{model})^{1/2}$ in units $h = c = 1$. Rather than finding these model-dependent coefficient we here aim to constrain the general energy scale Λ_{NM}. The Einstein frame representation shows that in the weak field limit $h_{\mu\nu} \approx \eta_{\mu\nu} + \delta h_{\mu\nu}$, with $\delta h_{\mu\nu}$ representing the usual Newtonian and post-Newtonian corrections, which can be neglected in non-gravitational experiments[3]. As a result, the metric and non-metricity tensor become

$$g_{\mu\nu} = \eta_{\mu\nu} + \alpha T\eta_{\mu\nu} + \beta T_{\mu\nu} \quad (5)$$

$$Q_{\alpha\mu\nu} = -\alpha(\nabla_{\alpha}T)\eta_{\mu\nu} - \beta\nabla_{\alpha}T_{\mu\nu}\ . \quad (6)$$

Though the α-dependent term in (6) can be gauged away, as in $f(R)$ theories [48], the β contribution is a genuine form of non-metricity. The effects of α, nonetheless, may still arise through the tetrads associated to (5). Thus, our scenario consists of a Minkowskian background corrected by terms that give non-metricity (plus torsional terms generated by the matter sector). Note that, as non-metricity is sourced locally by (derivatives of) the stress-energy tensor, then how energy and momenta are distributed locally plays a pivotal role in determining short-distance physics, since large departures from the flat Minkowski metric could arise at high enough densities. This opens the way to the search for departures from GR in the *high-density* regime rather than in the high-energy regime, as also suggested elsewhere relying on different arguments [88], with implications in stellar and nuclear matter models [53].

Particle interactions induced by non-metricity. From (5), we obtain up to $\mathcal{O}_1$ in α and β:

$$\begin{aligned}
e^{a}{}_{\mu} &= \delta^{a}{}_{\mu} + \frac{\alpha}{2}T\delta^{a}{}_{\mu} + \frac{\beta}{2}T^{a}{}_{\mu} + \mathcal{O}_2 \\
e_{a}{}^{\mu} &= \delta_{a}{}^{\mu} - \frac{\alpha}{2}T\delta_{a}{}^{\mu} - \frac{\beta}{2}T_{a}{}^{\mu} + \mathcal{O}_2 \\
\sqrt{-g} &= 1 + \frac{4\alpha+\beta}{2}T + \mathcal{O}_2 \\
\Gamma_{\mu} &= -\frac{1}{4}S_{\mu\alpha}{}^{\beta}\left[\sigma^{\alpha}{}_{\beta} + \frac{\beta}{2}\left(T^{b}{}_{\beta}\sigma^{\alpha}{}_{b} - T_{a}{}^{\alpha}\sigma^{a}{}_{\beta}\right)\right] + \mathcal{O}_2\ .
\end{aligned} \quad (7)$$

[1] σ_{ab} are the generators of the Lorentz group in its usual form within the spin representation.

[2] Here E_p is the Planck energy and $\Lambda_{model} = hc/\lambda_{model}$.

[3] This simply reflects that curvature effects can be locally removed by a suitable choice of coordinates while the effects of non-metricity cannot.

Writing the Lagrangian (1) as $\mathcal{L} = \mathcal{L}^0 + \mathcal{L}^I$, where $\mathcal{L}^0$ is the usual spinor Lagrangian in Minkowski spacetime [47], we get[4]:

$$\mathcal{L}^I = \frac{\beta}{2}\left(T\left[\bar{\psi}\overleftrightarrow{\not\partial}\psi + \bar{\psi}m\psi\right] + T_a{}^{\mu}\left[\bar{\psi}\gamma^a\overleftrightarrow{\partial}_{\mu}\psi\right]\right) + \\ + \frac{3\alpha}{2}T\left[\bar{\psi}\overleftrightarrow{\not\partial}\psi\right] + \mathcal{O}_2\,, \tag{8}$$

where torsion has been neglected because, as shown in [92], torsion-induced interactions are beyond experimental reach unless a very-high density of spin (the source of torsion [91]) is considered. This behavior of torsion contrasts with that of non-metricity, since the latter is sourced by the energy-momentum density, which can be more easily controlled and magnified in particle colliders.

The Lagrangian $\mathcal{L}^I$ evidences that non-metricity in RBGs induces contact interactions between a fermion pair and any kind of field entering the stress-energy tensor (even self-interactions). Accordingly, we can constrain Λ_{NM} by requiring that the non-metric contribution to the cross-section of particle processes does not exceed the measurement error. This implies that theories with non-metricity of the form (6) should be regarded as effective theories because the lack of new dynamical degrees of freedom (as compared to GR) together with the existence of 4-fermion contact interactions (8) may lead to unitarity violations at the scale Λ_{NM} (unless some strong coupling mechanism beyond the linear approximation fixes this issue[5]).

Let us now focus on the process $e^+e^- \to e^+e^-$ in the ultra-relativistic regime ($m_e \approx 0$) for which up to $\mathcal{O}_1$

$$\mathcal{L}^I = -\beta\left[\bar{\psi}\left(\gamma_a\overleftrightarrow{\partial}^{\mu} + \gamma^{\mu}\overleftrightarrow{\partial}_a\right)\psi\right]\left[\bar{\psi}\gamma^a\overleftrightarrow{\partial}_{\mu}\psi\right] + \mathcal{O}_2, \tag{9}$$

Within the Standard Model, the contribution of (9) to the cross section of this process at tree level and order $\mathcal{O}_1$ in β is

$$\sigma_{NM} \simeq 0.040\beta \text{ pb}, \tag{10}$$

where $\beta = C_{model}\Lambda_{NM}^{-4}$, with C_{model} a model-dependent dimensionless constant typically of $\mathcal{O}(1)$. Current data on the process $e^+e^- \to e^+e^-$ can be found in [69, 70]. Measurements from LEP[6] at a center of mass energy of $\sqrt{s} = 207$ GeV show that the cross section for this process is $\sigma_{exp} = 256.9 \pm 1.4 \pm 1.3$ pb[7] [69, 70]. The requirement that any RBG model in the metric-affine approach has to be consistent with current data[8] sets a lower (upper) bound for Λ_{NM} (λ_{NM}) of about

$$\Lambda_{NM} \gtrsim 0.3\, C_{model}^{1/4}\,\text{TeV}, \tag{11}$$
$$\lambda_{NM} \lesssim 4\, C_{model}{}^{-1/4} \times 10^{-18}\text{ m}\,, \tag{12}$$

and, correspondingly, for Λ_{model} (λ_{model})

$$\Lambda_{model} \gtrsim 0.02\, C_{model}{}^{1/2}\,\text{meV}, \tag{13}$$
$$\lambda_{model} \lesssim 6\, C_{model}{}^{-1/2}\text{ cm}. \tag{14}$$

In particular, for BI inspired models with Lagrangian $\left(|\det(\delta^{\mu}_{\nu} + \lambda^2_{BI}g^{\mu\alpha}R_{\alpha\nu})|^n - 1\right)/(8\pi G\lambda^2_{BI})$, one has $C_{BI} = \frac{1}{2n}$, with $n = 1/2$ corresponding to the so-called Eddington-inspired BI model. Picking out $n = 1/2$, the above bounds translate into $\Lambda_{BI} \gtrsim 0.02$ meV and $\lambda_{BI} \lesssim 6$ cm. It is worth mentioning that these bounds we here established are in the range recently highlighted in the naive estimations of [53]. Let us stress that this represents an improvement on the previous best limit on λ_{BI} (see e.g. [86, 87]) by more than 6 orders of magnitude. A worth feature of the above constraint is that it weakly depends on the details of the model considered. For astrophysical and cosmological bounds on the $n = 1/2$ model see [53].

Other tests of non-metricity. Let us now illustrate a preliminary proposal to perform tests also in the non-relativistic regime. As first shown by Parker [94–96], strong gravitational fields produce modifications in the atomic interaction Hamiltonian, which then induces specific shifts in their energy levels in regions of high curvature. One may wonder if similar effects could be sourced by high-density concentrations through non-metricity. Following Parker's approach, a non-metric interaction Hamiltonian can be defined from (3) by identifying $H = i\partial_t$ and stating $H_I \equiv H_D - H_D^M$, where H_D^M is the Dirac Hamiltonian in Minkowski spacetime. This leads to the following interaction Hamiltonian for fermions in a curved but non-metric space:

$$H_I = -i\left[\gamma^a\gamma^b\left(\frac{e_a{}^0 e_b{}^i}{g^{00}} + \delta^0_a\delta^i_b\right)\partial_i\frac{e_a{}^0 e_b{}^i}{g^{00}}(q_i - \Gamma_i) \right. \\ \left. + q_0 - \Gamma_0 + \left(\frac{e_a{}^0}{g^{00}} + \delta^0_a\right)m\gamma^a\right]\,, \tag{15}$$

where we defined $q_{\mu} \equiv 1/2\left(S_{\mu\alpha}{}^{\alpha} + Q_{[\alpha\mu]}{}^{\alpha}\right)$. Using (7), assuming conservation of $T_{\mu\nu}$, and neglecting again torsion for the aforementioned reasons, up to $\mathcal{O}_1$ we find

$$H_I = i\left[-\beta\left(T^{00}\delta_a{}^0\delta_b{}^i + \frac{1}{2}\left(\delta_a{}^0 T_b{}^i + T_a{}^0\delta_b{}^i\right)\right)\gamma^a\gamma^b\partial_i + \right. \\ \left. + \frac{3\alpha+\beta}{4}(\partial_a T)\gamma^0\gamma^a + \left(\frac{\alpha}{2}T\delta_a{}^0 - \frac{\beta}{2}T_a{}^0 - \beta T^{00}\delta_a{}^0\right)m\gamma^a\right] \tag{16}$$

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[4] $\bar{\psi}\overleftrightarrow{\partial}\psi \equiv \frac{1}{2}\left[\bar{\psi}(\partial_{\mu}\psi) - (\partial_{\mu}\bar{\psi})\psi\right]$

[5] In some RBGs black hole and cosmic singularities may be avoided in a non-perturbative way[89, 90].

[6] Let us mention that using LHC data for process of the type $q\bar{q} \to f\bar{f}$ would not improve the limit we here establish. See e.g. [93].

[7] We use the data with $\theta_{acol} < 10^o$ and $|\cos\theta_{e^{\pm}}| < 0.96$ [69, 70].

[8] This is done by requiring $\sigma_{SM} + \sigma_{NM}$ is compatible with the experimental value.

Keeping only the leading order terms in the non-relativistic limit as in [96][9], one finally has

$$H_I^{NR} = -i\frac{3\alpha+\beta}{4}\partial_0 T - \frac{m}{2}\left(\beta T^{00} - \alpha T\right). \tag{17}$$

In order to test non-metricity effects through energy shifts of atomic levels, one should be able to change the local distributions of energy and momentum around the atom minimizing the impact of undesired electromagnetic couplings. Clouds of dark matter particles and/ or intense neutrino fluxes, both having very weak or no couplings to the electromagnetic sector, could do the job. In the case of a Hydrogen atom traversed by a radiation flux modeled as an ideal null fluid[10], $T_{\mu\nu} = \rho l_\mu l_\nu$, (17) turns into

$$H_I^{NR} = -\frac{\beta}{2} m\rho \,. \tag{18}$$

Assuming an energy density profile that decays with the distance R to the center of the source as $\rho = \rho_s R_s^2/(R_s + R)^2$, being R_s the size of the source and ρ_s its initial density, the non-metricity correction to the energy levels is

$$\Delta^Q_{(n,l,m)} \simeq -\frac{\beta}{2} m\rho_s \left(1 + \frac{1}{R_s^2}\left\langle 4r^2\cos^2\theta - r^2\right\rangle_{nlm}\right), \tag{19}$$

where r measures the distance from the center of the atom, and terms of order $(r/R_s)^3$ and higher have been neglected. Then, for a state of the form $(n,0,0)$, one gets

$$\Delta^Q_{(n,0,0)} = -\frac{\beta}{2} m\rho_s \left(1 - \frac{1}{3}\left(\frac{na_0}{R_s}\right)^2 (5n^2+1)\right), \tag{20}$$

being a_0 the Bohr radius and m the electron mass. For the transition $(1000,0,0) \to (2,1,0)$, taking $\rho_s = 10^{31}$ J/m^3 (similar in magnitude[11] to the event GW150914 [8]) and requiring Δ^Q to be less than the error $\sim \alpha_{em}^2$ due to neglecting relativistic corrections, one finds $\lambda_{model} \lesssim 10^{17}$ m, which is orders of magnitude weaker than our relativistic estimates. Other avenues to explore non-metricity effects on atomic systems involve the study of rapid transients, which are sensitive to the coefficient α in (17). This requires a more detailed modeling of the sources and the use of time-dependent perturbation theory and will be explored elsewhere.

Outlook. We have shown that for RBG models in the metric-affine approach, non-metricity gives rise to potentially observable effects in microscopic systems, which can be used to impose tight constraints on the model parameters. We feel that recognizing this already provides some additional insight on our understanding of the space-time geometry and the potential impact of departures from GR on experiments at short scales. Indeed, using current data for $\mathbf{e}^+\mathbf{e}^-$ scattering, we have set a lower bound of the order of 1 TeV on the scale at which non-metricity could be present without being in conflict with experiments. We also found that a general consequence of non-metricity is the induction of contact 4-particle interactions among all particles (not just fermions). Consequently, Higgs physics at LHC or its impact on flavor physics could provide complementary bounds for non-metric effects[12]. Moreover, forthcoming accelerator experiments such as CLIC [97] will be used to perform high-precision measurements of $\mathbf{e}^+\mathbf{e}^-$ collisions with a center of mass energy around the TeV scale (almost one order of magnitude more than the LEP configuration we used). If no departures from the Standard Model are found in those experiments, our bounds might be improved by a few orders of magnitude. Also interesting would be understanding the role of non-metricity in the production of non-linearities of the cosmological perturbations that reflect into non-gaussianities in the Cosmic Microwave Background [98–100]. At the same time, we hope that non-metricity might open a new window to detect the presence of energy-momentum fluxes carried by weakly interacting sources. The non-relativistic interaction Hamiltonian (17) allowed us to explore the impact of such fluxes using atomic energy levels, which unfortunately turned out to be extremely tiny. Nonetheless, we hope that transient processes and nuclear physics scenarios are likely to provide more insightful experimental constraints. The rapid progress experienced (and expected) in atomic interferometry could also help explore this sector of gravitational physics through a variety of new experiments.

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[9] $\gamma^i = -i\tilde{\beta}\tilde{\alpha^i} \sim \mathcal{O}(\alpha_{em})$, $\gamma^0 = -i\tilde{\beta} \sim \mathcal{O}(1)$, $\partial_i \sim p_i \sim \mathcal{O}(\alpha_{em})$

[10] Here ρ is the energy density of the fluid and l_μ is a null vector.

[11] Gravitational waves do not appear in the matter $T_{\mu\nu}$ and, therefore, strictly speaking cannot be regarded as a source of non-metricity. We take this example for illustrative purposes only, as it represents one of the most energetic events known in Nature.

[12] The interaction Lagrangian for a real Klein-Gordon field is: $\mathcal{L}^I_{KG} = -\frac{1}{2}\left[\alpha T\eta^{\mu\nu} + \beta T^{\mu\nu}\right]\partial_\mu\phi\partial_\nu\phi + \mathcal{O}_2$

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Appendix A

We here provide the reader with more details about the derivation of Eq. (3) in the main text. In spaces with non-metricity and torsion, the Dirac equation (2) is modified due to two reasons: 1) that the relation between the divergence operator and the covariant derivative is not the usual one, and 2) that the curved Dirac matrices are no longer covariantly constant. Consider a smooth vector field A in a general smooth n-dimensional manifold $\mathcal{M}$ with a volume form dV, its divergence is defined by: $Div(A)dV \equiv d(i_A dV)$ [101]. If in some chart $dV = f dx^{\mu_1} \wedge ... \wedge dx^{\mu_n}$ then $Div(A) = 1/f \partial_\mu (f A^\mu)$. If $\mathcal{M}$ admits an affine structure Γ, $(\mathcal{M}\ ,\Gamma)$ has an associated covariant derivative which satisfies [102]:

$$\nabla_\mu A^\mu = \partial_\mu A^\mu + {\Gamma_{\mu\alpha}}^\mu A^\alpha \tag{A1}$$
$$\nabla_\mu f = (\partial_\mu f - {\Gamma_{\mu\alpha}}^\alpha f)\,. \tag{A2}$$

Therefore $Div(A)$ can also also be written:

$$Div(A) = \frac{1}{f}\nabla_\mu (f A^\mu) + {S_{\mu\alpha}}^\alpha\,, \tag{A3}$$

where $S^\alpha_{\mu\nu} \equiv -2{\Gamma_{[\mu\nu]}}^\alpha$ is the torsion of the affine structure. Now if $f = \sqrt{|Det(g)|}$, being g the metric, then one can also write

$$Div(A) = \nabla_\mu A^\mu - \left(\frac{1}{2}{Q_{\mu\alpha}}^\alpha + {S_{\alpha\mu}}^\alpha\right) A^\mu\,, \tag{A4}$$

where we used $Q_{\mu\nu\alpha} \equiv -\nabla_\mu g_{\nu\alpha}$. This is the divergence operator appearing in Stoke's theorem [101] which allow us to identify some integrals in the bulk with vanishing boundary terms. Finally, taking into account that

$$\nabla_\mu \underline{\gamma}^\alpha = \frac{1}{2}{Q_{\mu\nu}}^\alpha \underline{\gamma}^\nu\,, \tag{A5}$$

we have for the Dirac equation in spaces with general non-metricity and torsion

$$\left[\underline{\gamma}^\mu \nabla_\mu + \frac{1}{2}\left({S_{\mu\alpha}}^\alpha + {Q_{[\alpha\mu]}}^\alpha\right)\underline{\gamma}^\mu + m\right]\psi = 0\,, \tag{A6}$$

i.e. Eq. (3). Note that for Riemannian spaces (2) is recovered as expected.