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Deconvolution of energy spectra in the ATIC experiment

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The Advanced Thin Ionization Calorimeter (ATIC) balloon-borne experiment is designed to perform cosmic-ray elemental spectra measurements from below 100 GeV up to tens of TeV for nuclei from hydrogen to iron. The instrument is composed of a silicon matrix detector followed by a carbon target, interleaved with scintillator tracking layers, and a segmented BGO calorimeter composed of 320 individual crystals totalling 18 radiation lengths, used to determine the particle energy. The technique for deconvolution of the energy spectra measured in the thin calorimeter is based on detailed simulations of the response of the ATIC instrument to different cosmic ray nuclei over a wide energy range. The method of deconvolution is described and energy spectrum of carbon obtained by this technique is presented.

1. Deconvolution as ill-posed problem and the solution method with Tikhonov's regularization

The ATIC experiment measures the spectra of energy deposit $f(E_d)$ in a BGO-calorimeter for different primary particles. The problem is to calculate the initial spectra of primaries $\Phi(E_0)$ knowing the spectra of energy deposits. Deposit spectra are connected with primary energy spectra by the Fredholm equation

$$f(E_d) = \int A(E_d, E_0)\, \Phi(E_0)\, dE_0 \qquad (1)$$

where $A(E_d, E_0)$ is a response function of the instrument, which was determined in the present work by Monte Carlo simulation separately for different primaries. Determining $\Phi(E_0)$ for a given $f(E_d)$ is known as a deconvolution problem.

The actual experimental spectra is recorded as a number of counts in logarithmic bins dividing the total energy range $[E_{min}, E_{max}]$ into n intervals. If the primary energy spectra are found in the same form, then (1) can be rewritten as a system of equations

$$M_i = \sum_{j=1}^{n} a_{ij} N_j, \quad i = 1, 2, \dots n \qquad (2)$$

where M_i is the number of events in the energy deposit bin i, a_{ij} is an element of the response matrix, and N_j is the number of counts of the primary spectrum in bin j. In the present work $E_{min} = 10\,\text{GeV}$, $E_{max} = 10^3$ TeV, $n = 25$ (5 bins per one decade of energy).

The problem of finding the values of N_j from (2) refers to the class of ill-posed problems. The direct solution of the equation is possible, but the result is meaningless because there is no smooth connection between the initial data M_i and the result N_j. To obtain a meaningful solution of the deconvolution problem (2) we must suppress the oscillations of the solution N_j. A wide-spread practice is to use a square norm of second derivative of solution as a regularization term with weight τ, where τ is known as the *parameter of regularization*. To do this we can rewrite equation (2) as a minimization problem

$$S'(N_1,\dots,N_n) = \sum_{i=1}^{n}\left(\frac{\sum_{j=1}^{n} a_{ij}N_j - M_i}{\sigma_i}\right)^2 + \tau\cdot\sum_{j=2}^{n-1}\left(\frac{N_{j+1} - 2N_j + N_{j-1}}{\sigma_j}\right)^2 \to \min \tag{3}$$

where σ_i is a standard deviation of M_i.

But the function (3) is not very good for one reason. It is obvious that in the limit of very high τ the second item in (3) will dominate and the solution will tend to some straight line. But it is expected that the shape of the spectrum is generally like a power law with the index -2.75 for equidistant bins or -1.75 for logarithmic bins. So the idea is to minimize the second derivative of $N_j \times E_j^{1.75}$ instead of the second derivative of N_j. Then the solution will tend to a power law for high values of τ and distortion of the spectrum due to regularization is minimized.

Let us define the solution N_j by $N_j \equiv x_j R_j^0$, where R_j^0 is some reasonable “initial approximation” for the expected result N_j. Then the function (3) can be rewritten as

$$S(x_1,\dots,x_n) = \sum_{i=1}^{n}\left(\frac{\sum_{j=1}^{n} a_{ij}R_j^0 x_j - M_i}{\sigma_i}\right)^2 + \tau\cdot\sum_{j=2}^{n-1}\left(\frac{\alpha R_{j+1}^0 x_{j+1} - 2R_j^0 x_j + \alpha^{-1}R_{j-1}^0 x_{j-1}}{\sigma_j}\right)^2 \tag{4}$$

where $\alpha \equiv (E_{j+1}/E_j)^{1.75}$. It is obvious that minimization of the first item of (4) is equivalent to solution of equation (2). Minimization of (4) means that there needs to be some compromise between the exact solution (the first sum in (4)) and the minimum of oscillations of N_j (the second sum in (4)). This procedure is called the method of Tikhonov’s regularization [1]. Different values of τ lead to different strength of regularization. The case $\tau = 0$ corresponds the absence of regularization. The problem is to choose a value of τ and to minimize function (4).

2. Problem of errors calculation and choice of regularization parameter τ

Let us suppose that we have chosen the parameter of regularization τ (from some consideration) and now we are able to minimize (4) and find the solution $(x_1^*,\dots,x_n^*)$. We know that the initial data M_i have errors σ_i and the values of matrix a_{ij} are also known with errors σa_{ij}. The question is: what is the statistical errors of our result x_j^*? It is rather difficult to trace analytically the influence of σ_i and σa_{ij} to the result x_j^*. The simplest way to answer this question is to utilize Monte Carlo simulations. Using back restored values of M_i:

$$\widetilde{M}_i = \sum_{j=1}^{n} a_{ij}N_j^*, \quad N_j^* = R_j^0 x_j^*, \tag{5}$$

along with initial values of matrix elements a_{ij}, standard deviations σ_i and σa_{ij} as standard errors for $\widetilde{M}_i$ and a_{ij} respectively, we can generate a set of L species of simulated experimental data $\{M_i^r, a_{ij}^r\}$ $(r = 1,\dots,L)$ and solve the deconvolution problem for each of them. This way we will obtain a set of results $\{(x_1^r,\dots,x_n^r)\}$.

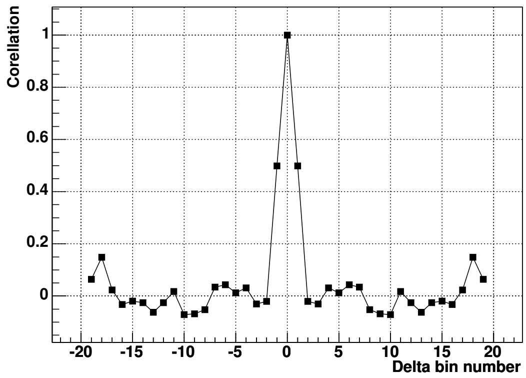

Figure 1. The mean correlation function of carbon.

From this set we can get the statistics of distribution of the deviations for the sought result $(x_1^*, \dots, x_n^*)$. This Monte Carlo method is very simple and was used by us to calculate the error of results in the deconvolution problem. We used a value of $L = 100$ because the relative error of error of N_j is reasonable: $1/\sqrt{L} = 10\,\%$. We need to obtain both standard deviations for the mean values of $(x_1^*, \dots, x_n^*)$ and the complete covariation matrix

$$\mathrm{Cov}_{lm} = \big\langle (x_l - x_l^*)(x_m - x_m^*) \big\rangle, \quad l, m = 1, 2, \dots, n \tag{6}$$

The reason is that the values of x_j^* for different j are not independent from each other unlike the experimental data M_i. The complete statistical properties of the final result $(x_1^*, \dots, x_n^*)$ are described by the complete covariation matrix, not by the deviations only. The standard deviations can be obtained by

$$\sigma x_j^* = \sqrt{\mathrm{Cov}_{jj}} \tag{7}$$

We calculate the complete covariation matrix in our method, but it is rather difficult to use this matrix directly to understand the statistics of the result. It is suitable to use some simplifications. The first step one can do is to say that the estimation of deviations (7) is a rough estimate of the errors for x_j^*. The errors for N_j^* will be $\sigma N_j^* = \sigma R_j^0 x_j^*$. The second step uses the value of "mean correlation function" of x_j^* with value of $x_{j+\Delta j}^*$, where $\Delta j = 0, \pm 1, \pm 2, \dots$ The correlation matrix is defined by equation

$$\mathrm{Corr}_{lm} = \frac{\mathrm{Cov}_{lm}}{\sqrt{\mathrm{Cov}_{ll}\mathrm{Cov}_{mm}}} \in [-1, 1], \quad \mathrm{Corr}_{ll} \equiv 1 \tag{8}$$

and can be easily calculated if one knows the covariation matrix (6). The value of $\mathrm{Corr}_{j,j\pm 1}$ shows what is the degree of correlation of the element in the j-th bin with neighbouring bins at the left or right side. The "mean correlation function" denotes an averaging of $\mathrm{Corr}_{j,j+\Delta j}$ over all j for fixed Δj

$$C(\Delta j) = \frac{1}{K(\Delta j)} \sum_j \mathrm{Corr}_{j,j+\Delta j}, \quad C(+\Delta j) \equiv C(-\Delta j), \quad \Delta j = 0, \pm 1, \pm 2, \dots \tag{9}$$

where $K(\Delta j) = n - |\Delta j|$ is a corresponding number of items in the sum. The value of $C(\Delta j)$ indicates the mean correlation between channels for given Δj and for the whole spectrum. Figure 1 shows an example of the mean correlation function.

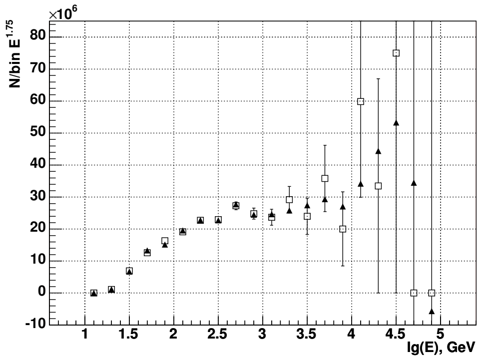

Figure 2. Spectrum of energy deposition of carbon. Open squares: experimental spectrum M_i, full triangles: back restored spectrum $\widetilde{M_i}$.

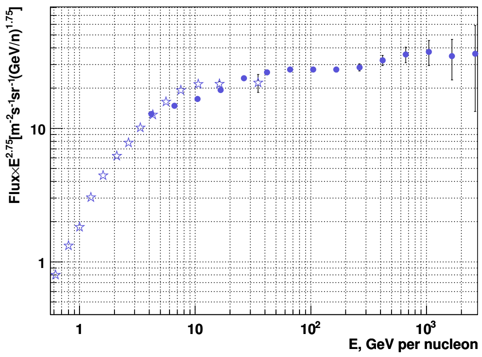

Figure 3. Spectrum of carbon. Full circles: present work, open stars: HEAO-3-C2 [2].

Consideration of the mean correlation function $C(\Delta j)$ gives us a way to choose the parameter of regularization τ. Strong oscillation of the solution without regularization means strong anticorrelation of neighbour bins, i.e. $C(1) \sim -1$. The idea is to choose τ such that strong anticorrelations would be changed to weak correlations in neighbour bins. This goal may be easily achieved because $C(1)$ monotonically depends on τ, decreasing from values close to $+1$ for high τ to the values close to -1 for low values of τ. The value of χ^2 per one degree of freedom for deviations of back-restored spectrum $\widetilde{M_i}$, and the experimental spectrum M_i, is expected to be about 1 because $\widetilde{M_i}$ should not trace statistical fluctuations of M_i, but should deviate on values of $\sqrt{\langle (M_i - \widetilde{M_i})^2 \rangle} \sim \sigma_i$. Such a value of $\chi^2 \approx 1$ corresponds to the value of $C(1) \approx 0.5$ which actually means weak correlation between neighbour channels, as expected. Our method is to choose values of τ such that $C(1) = 0.5$ exactly for the spectra of all species that we consider.

The result of analysis for carbon projectiles is shown on figures 2 and 3. The spectra for other nuclei will be discussed elsewhere.

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